

Complete Solution for Particles of Nonzero Rest Mass in Gravitational Fields

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Abstract

In a paper "The Golden Dynamical Equation of Motion for Particles of Nonzero Rest in Gravitational Fields" (Howusu 2004, Physics Essays 17(3)), the planetary orbital equation of motion for particles of nonzero rest mass in gravitational fields was derived. In this paper we used the series method to calculate the angle of deflection for a photon that grazes the edge of the sun in the sun's Gravitational field. Our results were found to fall within experimental, measurements.

Keywords: Passive mass, Inertial mass, Gravitational field, Gravitational Potential, Deflection angle.

1. Introduction

It is known that the Newton's dynamical law of motion for a particle of nonzero rest mass in a gravitational field may be stated as

$$ {}^{d}/_{dt}\{M_i u\} = -M_p \nabla \Phi_g \tag{1}$$

In all inertial reference frames, where M_i and M_p are the inertial and passive masses, respectively; Φ_g is the external gravitational field; and u is the instantaneous velocity.

According to Newtonian physics, for particle of nonzero rest mass m_0

$$M_p = M_i = m_0 \tag{2}$$

Newton's equation of motion in a gravitational field was given explicitly by

$$ {}^{d}/_{dt}\, u = -\nabla \Phi_g \tag{3}$$

But it is now well established experimentally (Rindler, 1977) that the instantaneous passive and inertial masses for a particle of nonzero rest mass are given by

$$M_p = M_i = \left(1 - {}^{u^2}/_{c^2}\right)^{-1/2} m_0 \tag{4}$$

Where c is the speed of light in vacuum. It therefore follows that according to the experimental facts available today, a most natural generalization of Newton's dynamical equation for a particle of nonzero mass in a gravitational field is given by equation (1) as (Howusu, 1991),

$$ {}^{d}/_{dt}\left\{\left(1 - {}^{u^2}/_{c^2}\right)^{-1/2} u\right\} = -\left(1 - {}^{u^2}/_{c^2}\right)^{-1/2} \nabla \Phi_g \tag{5}$$

The equation (5) clearly completes the corresponding pure Newtonian equation (3) with correction terms of all orders of c^{-2}. This equation also opened up the doors for theoretical investigation and experimental applications.

2. Theoretical Framework

Now the instaneous total kinetic energy T for a particle of nonzero rest mass m_0 moving with an instantaneous speed u is given by (Weinberg, 1972)

$$T = \left(1 - {u^2}/{c^2}\right)^{-1/2} m_0 c^2 \quad (6)$$

In all inertial reference frames also it follows from (4) that the instantaneous gravitational potential energy V_g for a particle of nonzero rest mass in a gravitational field is given by (Logunor and Mestuirishvilli, 1989)

$$V_g = \left(1 - {u^2}/{c^2}\right)^{-1/2} m_0 \Phi_g \quad (7)$$

in all inertial reference frames. In a paper (Howusu, 2004), it was shown that the generalized law of motion (5) can be written as

$$d/dt\left[\left(1 + {1}/{c^2}\,\Phi_g\right)^{-1} u\right] = -\left(1 + {1}/{c^2}\,\Phi_g\right)^{-1} \nabla\Phi_g \quad (8)$$

If we consider a particle of nonzero rest mass m_0 moving in the gravitational field of a homogeneous spherical sun of radius R and rest mass m_0, then in spherical polar coordinates $(r,\theta,\phi)$with the origin at the centre of the sun the gravitational scalar potential Φ_g is given by (Moller, 1982)

$$\Phi_g(r) = -\frac{K}{r}; \quad (9)$$

Where

$$K = Gm_0 \quad (10)$$

And G is the universal gravitational constant. If the particle moves in the equatorial plane of the body, then its polar angular coordinates is given by

$$\theta = {\pi}/{2} \quad (11)$$

Hence , according to equation (8), the equations of motion for the particle are given by

$$\ddot{r} - r\dot{\phi}^2 - \frac{K\dot{r}^2}{c^2 r^2}\left(1 - \frac{K}{c^2 r}\right)^{-1} = -\frac{K}{r^2} \quad (12)$$

And

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} - \frac{K\dot{r}\dot{\phi}}{c^2 r}\left(1 - \frac{K}{c^2 r}\right)^{-1} = 0 \quad (13)$$

Now to integrate the azimuthal equation (13), we divide by $r\dot{\phi}$ to obtain

$$\frac{\ddot{\phi}}{\dot{\phi}} + \frac{2\dot{r}}{r} - \frac{K\dot{r}}{c^2 r^2}\left(1 - \frac{K}{c^2 r}\right)^{-1} = 0 \quad \text{or}$$

$$\dot{\phi}(r) = \frac{l}{r^2}\left(1 - \frac{K}{c^2 r}\right) \quad (14)$$

Where l is a constant of the motion. This is the instantaneous angular speed of the particle in terms of the radical coordinate according to equation (8).

In the paper (Howusu, 2004) the planetary orbital equation according to equation (8) was approximated as

$$\frac{d^2u}{d\emptyset^2} - \frac{K}{c^2}\left(\frac{du}{d\theta}\right)^2 + u = 3\frac{K}{c^2}u^2 \tag{15}$$

In this paper we used the series method to solve the planetary equation (15).

3. Mathematical Deductions

In solving equation (15), we seek a series solution of the form

$$u(\phi) = \sum_{n=0}^{\infty} A_n e^{in\omega_0\phi} \tag{16}$$

Where A_n, ω_0 are constants to be determined. Differentiating equation (16) and substituting in equation (15) and comparing coefficients we obtain the following results

For n=0

$$A_0=0; \text{ or } A_0 = \frac{c^2}{2K} \tag{17}$$

For n=1, we have four solutions;

If

$$A_0=0, \tag{18}$$

$$\omega_0 = \pm 1 \tag{19}$$

and if

$$A_0 = \frac{c^2}{2K}, \tag{20}$$

$$\omega_0 = \pm i \tag{21}$$

For n=2, we have the following results;

If

$$A_0=0, \tag{22}$$

$$\omega_0 = \pm 1 \tag{23}$$

and

$$A_2 = -\frac{K}{c^2}{A_1}^2, \tag{24}$$

where A_1 is an arbitrary constant

If

$$A_0 = \frac{c^2}{2K}, \tag{25}$$

$$\omega_0 = \pm i \tag{26}$$

and

$$A_2 = \frac{K}{c^2}{A_1}^2, \tag{27}$$

Now we investigate the solutions by substituting in the constants we have determined.

$$A_0 = \frac{c^2}{2K},\ \omega_0 = -i \text{ and } A_2 = -\frac{K}{c^2}{A_1}^2$$

Then the equation (16) becomes

$$u(\phi) = A_0 + A_1 e^{\phi} + A_2 e^{2\phi} + \cdots$$

$$= \frac{c^2}{2K} + A_1\left[1 + \phi + \frac{(\phi)^2}{2!} + \cdots\right]$$

$$+A_2\left[1 + 2\phi + \frac{(2\phi)^2}{2!} + \cdots\right]$$

$$+A_3\left[1+3\phi+\frac{(3\phi)^2}{2!}+\cdots\right] \quad (28)$$

Collecting the first terms of the series and applying grazing condition, i.e.

$$u(\phi)\to\frac{1}{R},\ \text{as}\ \phi\to\frac{\pi}{2}$$

We have

$$A_1=\left(\frac{1}{R}-\frac{c^2}{2K}\right)\left(1-\frac{\pi}{2}\right)^{-1}, \quad (29)$$

or

$$A_1=\frac{c^2}{K}\left[\frac{\pi}{2}-1\right](1+\pi)^{-1}-\left(\frac{1}{R}-\frac{c^2}{2K}\right)\left(1+\frac{\pi}{2}\right)^{-1} \quad (30)$$

Similarly if

$$A_0=\frac{c^2}{K},\ \omega_0=-i\ \text{and}\ A_2=\frac{-K}{c^2}A_1,\ \text{and}$$

applying grazing condition and solving using quadratic formula, we obtain two values for A'_1 as

$$A'_1=\left(\frac{1}{R}-\frac{c^2}{2K}\right)\left(1-\frac{\pi}{2}\right)^{-1} \quad (31)$$

or

$$A'_1=\frac{c^2}{K}\left[\frac{\pi}{2}-1\right](1-\pi)^{-1}-\left(\frac{1}{R}-\frac{c^2}{2K}\right)\left(1-\frac{\pi}{2}\right)^{-1} \quad (32)$$

These values of A_1 and A'_1 gave us two deflection angles ϕ and ϕ' respectively as the photon grazes the edge of the sun in the gravitational field of the sun.

$$\phi_\infty=2.06\ \text{x (General Relativity Prediction)} \quad (33)$$

$$\phi'_\infty=1.07\ \text{x (General Relativity Prediction)} \quad (34)$$

4. Conclusion

In this paper, we have taken $\pi/2$ as the angle at the point the photon just touches the edge of the sun. It can also be seen that all the possible values obtained using the dynamical equation fall within experimental measurements. It should be noted that the results for which the coefficients

$$A_0=\frac{c^2}{2K},\ \omega_0^2=1\ \text{and}\ A_2=\frac{-K}{c^2}A_1^2,\ \text{e.t.c}$$

have solutions that are physically unattainable.

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