Quantum Natures of Single-Mode Displaced Squeezed Vacuum State

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Abstract
The displaced squeezed vacuum state is produced by application of displaced operator on squeezed vacuum state. With help of density operator we find Q function, with the Q function mean, variance and quadrature variance would be calculated. From this we can determine the system has superpoissonian statics, the squeezed parameter is direct proportion with both mean and variance of photon number, but inversely proportion with quadrature variance. The squeezing occurs in plus quadrature with the maximum squeezing of 99.7% for r=3.
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1 Introduction

Squeezed states of light have been observed in a variety of quantum optical systems, which are used to enhance the measurement sensitivity in optomechanics [1], and even in biology [2]. In squeezed states of light, the noise of the electric field at certain phases falls below that of the vacuum state. This means that, when we turn on the squeezed light, we see less noise than no light at all. This apparently paradoxical feature is a direct consequence of quantum nature of light and cannot be explained within the classical framework [3].

They are described interims of single-mode, two-mode and as the mixtures with the other quantum states of light. The single-mode squeezed light is produced by a degenerate parametric amplifier, consisting of a nonlinear crystal pumped by coherent light. And the two-mode squeezed light is generated by a nondegenerate subharmonic system consisting of nonlinear crystal pumped by coherent light. Two-mode squeezed vacuum state is defined by applying the two-mode squeezed operator to the two-mode vacuum state. We can use the squeezed states of light with mixing them with other quantum states of light. The displaced number state is obtained by application of squeezing operator on displaced and number states [4-9].

In this paper we seek to determine the quantum nature of displaced squeezed vacuum state. We obtained it by application of displaced states on the squeezed vacuum, and by calculating its density operator we determine its quantum nature as described below.

2 Displaced Squeezed vacuum state

2.1 Single-mode squeezed vacuum state

Single-mode squeezed vacuum state is the prototype of a degenerate parametric amplifier consists of nonlinear crystals pumped by coherent light [10]. The Hamiltonian for degenerate parametric amplifier is given by,

$$H = \frac{i\epsilon}{2}(\hat{a}^2 - \hat{a}^{+2}). \tag{1}$$

The state vector of light for single-mode light initially in coherent state $|\gamma>$ can be expressed as

$$|\varphi(r)> = \hat{S}(r)|\gamma> , \tag{2}$$

Where the squeezed operator takes the form

$$\hat{S}(r) = e^{\frac{r}{2}(\hat{a}^2 - \hat{a}^{+2})} , \tag{3}$$

To this end, we can express the operator $a(r)$ by;

$$a(r) = \hat{S}^{+}(r) a \hat{S}(r), \tag{4}$$

Differentiating the above equation wrt r and using $I = \hat{S}^{+}(r)\hat{S}(r)$, Eq. (4) takes the form

$$a(r) = \hat{a}\cosh r - \hat{a}^{+}\sinh r, \tag{5}$$

On account of Eqs. (5), (4) and Eq. (3) we can re write the state vector for single-mode squeezed vacuum light as;

$$|\varphi(r)> = \hat{S}(r)|0> . \tag{6}$$

To this end, we can write the displaced squeezed vacuum state in the form of [11]

$$|\alpha, r> = \hat{D}(\alpha)\hat{S}(r)|0>. \tag{7}$$

Where

$$\hat{D}(\alpha) = \exp[\alpha\hat{a}^{+} - \alpha^{*}\hat{a}] , \tag{8}$$

From this we can easily determine the density operator for displaced squeezed vacuum state as

$$\hat{\rho}_{dsv} = \hat{D}(\alpha)\hat{S}(r)\hat{\rho}_0\hat{D}^{+}(\alpha)\,\hat{S}^{+}(r). \tag{9}$$

Where $\hat{\rho}_0$=|0> <0| is density operator for vacuum state

3 The Q Function

The Q function is defined in [12] as

$$Q(\alpha^*,\alpha,r)=\frac{1}{\pi}\int d^2\gamma\varphi_a(\gamma^*,\gamma,r)\mathrm{e}^{(\gamma^*\alpha-\gamma\alpha^*)}\ . \tag{10}$$

Where

$$\varphi_a(\gamma^*,\gamma,r)=Tr(\hat{\rho}_{dsv}e^{\alpha\hat{a}^+}e^{-\alpha^*\hat{a}}e^{\gamma a(r)}e^{-\gamma\hat{a}^+}), \tag{11}$$

Applying Backer Huasdrof identity

$$e^Ae^B=e^{A+B+\frac{1}{2}[A,B]}. \tag{12}$$

Eq. (11) takes the form

$$\varphi_a(\gamma^*,\gamma,r)=\frac{1}{\pi}e^{-y\gamma^*\gamma+\frac{x}{2}(\gamma^2+\gamma^{*2})}\ . \tag{13}$$

Where

$$y=cosh^2r. \tag{14}$$

And

$$x=-\cosh r\sinh r. \tag{15}$$

Substituting Eq. (13) into Eq. (10) we see that

$$Q(\alpha^*,\alpha,r)=\frac{1}{\pi}\int\frac{d^2\gamma}{\pi}e^{(-y\gamma^*\gamma+\gamma^*\alpha-\gamma\alpha^*+\frac{x}{2}(\gamma^{*2}+\gamma^2))}. \tag{16}$$

Carrying out integration with help of [Beyene], the Q function for displaced squeezed vacuum state takes the form

$$Q(\alpha^*,\alpha,r)=\frac{(u^2-v^2)^{\frac{1}{2}}}{\pi}\exp\left[-u\alpha^*\alpha+v(\alpha^2+\alpha^{*2})\right]. \tag{17}$$

Where

$$u=\frac{y}{y^2-x^2},\text{ and } v=\frac{x}{y^2-x^2}. \tag{18}$$

4 Photon Statistics

Here we seek to calculate the mean and variance of photon number

4.1 mean photon number

Mean photon number is described in terms of Q function as

$$\begin{aligned}\bar{n}&=\int d^2\alpha\ Q(\alpha^*,\alpha,r)[(1-u)^2\alpha^*\alpha+v\alpha^2]\\&=(1-u)R_1+vR_2,\end{aligned} \tag{19}$$

Where

$$R_1=\int d^2\alpha\ Q(\alpha^*,\alpha,r)\alpha^*\alpha, \tag{20}$$

And

$$R_2=\int d^2\alpha\ Q(\alpha^*,\alpha,r)\alpha^2. \tag{21}$$

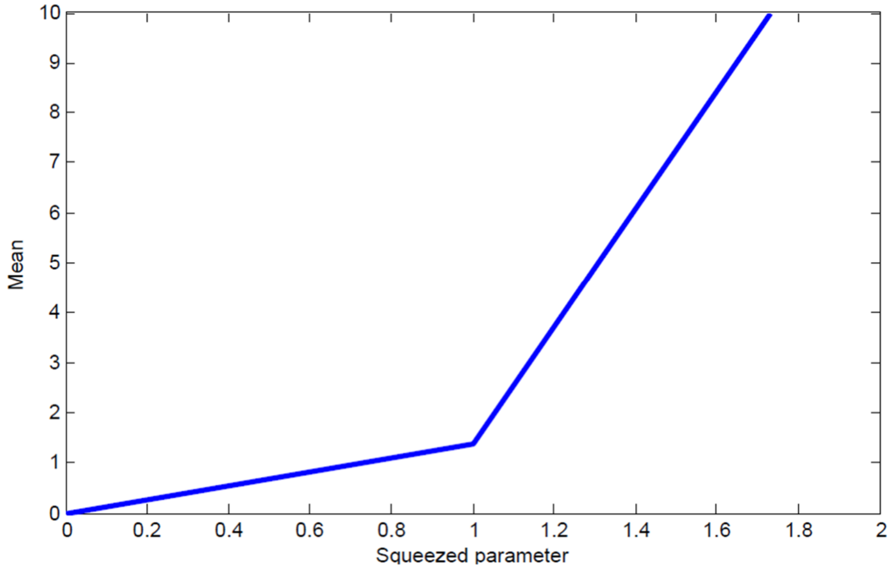
Using Eq. (17) into Eq. (20) and (21) and carrying out integration wrt α we get

$$R_1=y=cosh^2r. \tag{22}$$

$$R_2=x=-\cosh r sinh\ r. \tag{23}$$

On account of Eq. (22) and (23) into Eq. (19) and using trigonometric identity we set the mean photon number for displaced squeezed vacuum state as;

$$\overline{n}=sinh^2r. \tag{24}$$



Figuare 1: *Plots of mean photon number Eq.* (24) *versus squeeze parameter(r)*

Fig. (1) shows the relation between mean of photon number with squeezed parameter, and we can understand that both mean photon number and squeezed parameter have the directly proportional.

4.2 The Variance of Photon Number

The variance of photon number for single-mode state is settled in anti-normal order form as;

$$(\Delta n)^2 = < \hat{a}^{+2}\hat{a}^2 > +2\,\overline{n} - \overline{n}^2. \quad (25)$$

Where

$$< \hat{a}^{+2}\hat{a}^2 > = \int d^2\alpha\; Q(\alpha^*,\alpha,r)[(1-u)\alpha^{*2}\alpha^2 + v\alpha^2 + 2v(1-u)\alpha^*\alpha^3 + v^2\alpha^2],$$

$$= vI_1 + (1-u)^2 I_2 + 2v(1-u)I_3 + v^2 I_{4.} \quad (26)$$

Where

$$I_1 = \int d^2\alpha\; Q(\alpha^*,\alpha,r)\alpha^2, \quad (27a)$$

$$I_2 = \int d^2\alpha\; Q(\alpha^*,\alpha,r)\alpha^{*2}\alpha^2, \quad (27b)$$

$$I_3 = \int d^2\alpha\; Q(\alpha^*,\alpha,r)\alpha^*\alpha^3\,, \quad (27c)$$

And

$$I_4 = \int d^2\alpha\; Q(\alpha^*,\alpha,r)\alpha^4. \quad (27d)$$

Substituting Eq. (17) into all of Eqs. (27) And carrying out integration wrt . After substituting values of $I_1,\, I_{2,},\, I_3\; and\;\, I_4$ Eq. (26) takes the form;

$$< \hat{a}^{+2}\hat{a}^2 > = 2y^2 + x^2. \quad (28)$$

Using Eqs. (14) And (15) into Eq. (28) and with the help of Eq. (24) we can put the variance of photon number as;

$$(\Delta n)^2 = 3sinh^2 r + 2sinh^4 r - coshrsinhr. \quad (29)$$

Fig. (2) Shows the variance of photon number is greater than mean photon number, so the radiation of displaced squeezed vacuum state is super-posonian photon statics.

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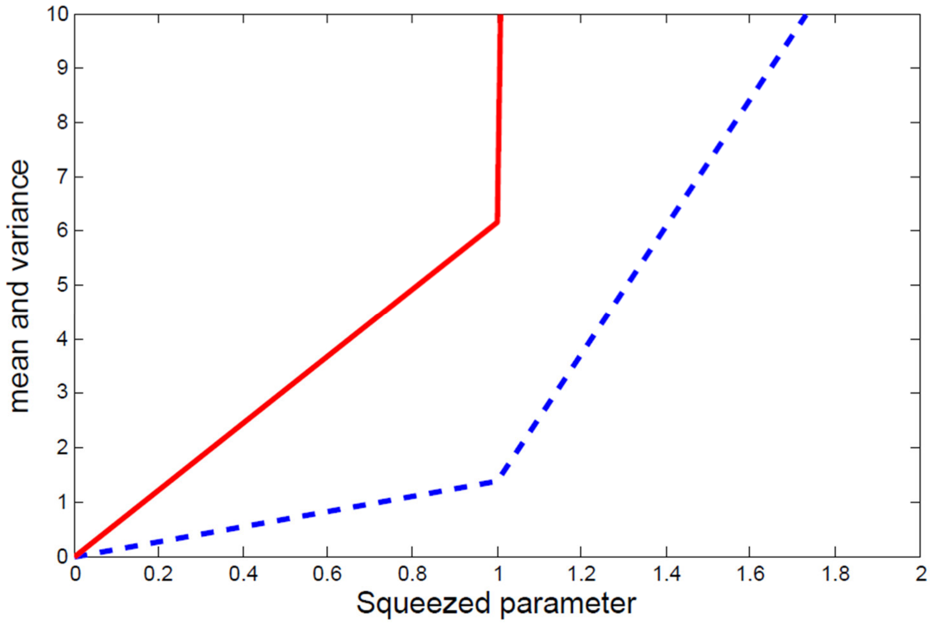

Figure 2: plots of mean (Eq. (24)) and variance of photon number (Eq. (29))

4.3 Quadrature squeezing

Quadrature squeezing is used to determine the squeezing properties of the given light. The single-mode light is said to be squeezed light if $\Delta a_+ < 1$ or $\Delta a_- < 1$, and it must satisfies $\Delta a_+ \Delta a_- \geq 1$. Where plus and minus quadrature variance can be related by

$$\widehat{a_\pm} = \sqrt{\pm 1}(\hat{a}^+ \pm \hat{a}) \tag{30}$$

Which must satisfies the commutation relation of

$$[a_+, a_-]=2i \tag{31}$$

Then one can describe the quadrature variance depend on the above equation as;

$$\Delta\, a_\pm{}^2 = <\widehat{a_\pm},\, \widehat{a_\pm}> \tag{31}$$

On account of Eq. (30) the above equation takes the form of;

$$< \Delta\, \widehat{a_\pm} >^2 = 1 \pm < \hat{a}^2 > \pm < \hat{a}^{+2} > +2 < \hat{a}^+ \hat{a} > \mp < \hat{a}^+ >^2 - 2 < \hat{a} >< \hat{a}^+ >. \tag{32}$$

After evaluating all expectation values in above equation the quadretuare variance of displaced squeezed vacuum takes the form;

$$< \Delta\, \widehat{a_\pm} >^2 = 2sinh^2 r + 1 \mp 2\cosh r sinh\, r. \tag{33}$$

Expressing the trigonometric function interims of exponential notation Eq. (33) takes the form

$$< \Delta\, \widehat{a_\pm} >^2 = e^{\mp 2r}. \tag{34}$$

Eq. (34) is the quadrature variance for displaced squeezed vacuum state. From this equation we can easily states that the squeezing occurs in plus quadrature.

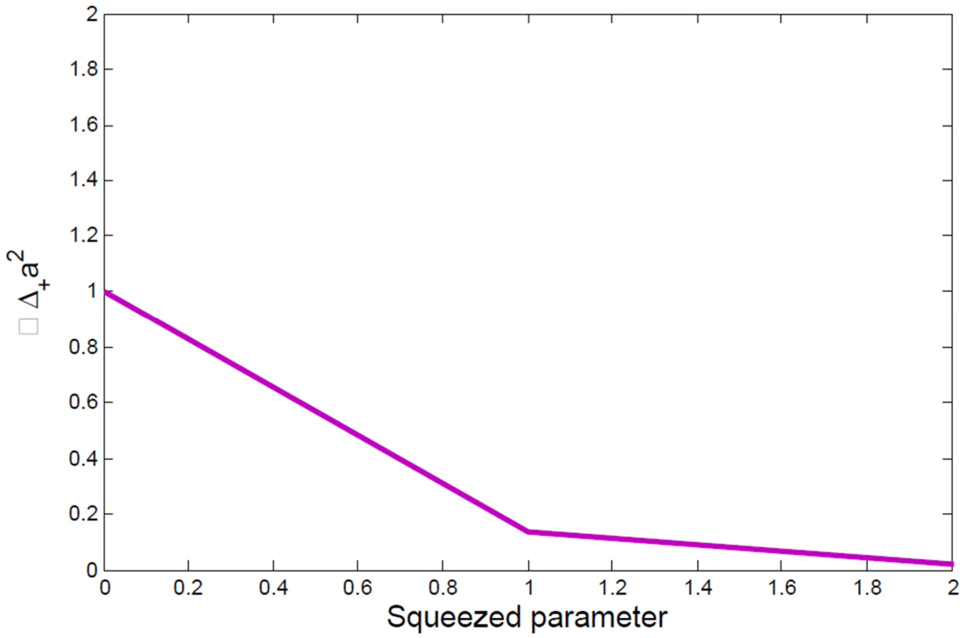

Figure 3: shows the relation between quadrature variance and squeezed parameter

Fig (3) shows the quadrature variance and squeezed parameters are inversely proportional. The other thing is that the graph of quadrature variance does not extend above 1. This indicates the calculation full fill the squeezing properties.

To this end, we can drive quadrature squeezing relative to vacuum state as;

$$s = \frac{(\Delta\, a_{\pm}{}^{2})_{v} - \Delta\, a_{+}{}^{2}}{(\Delta\, a_{\pm}{}^{2})_{v}}. \tag{35}$$

We can calculate $(\Delta\, a_{\pm}{}^{2})_{v}$ from Eq. (34) by taking the squeezing parameter zero (r=0) then Eq. (35) takes the form;

$$s = 1 - \Delta\, a_{+}{}^{2}. \tag{36}$$

With the help of Eq. (34) we can set quadrature squeezing as;

$$s = 1 - e^{-2r}. \tag{37}$$

5. Conclusions

In this paper we determine the quantum nature of DSVS, with help of Q function we calculate mean, variance of photon number, quadrature variance and quadrature squeezing. And we find the mean is greater than variance which shows the supperposonity of the system and squeezing occurs in plus quadrature. The other are described in the bellow table.

Squeezed parameter(r)	$\Delta\, a_{+}{}^{2}$	Quadrature squeezing(S)	Degree of squeezing
1.5	0.049	0.95	95%
2	0.0183	0.981	98.1%
2.5	0.0067	0.993	99.3%
3	0.0024	0.997	99.7%

Table 1: Shows relation between Squeezed parameter(r), $\Delta\, a_{+}{}^{2}$, Quadrature squeezing(S) and Degree of squeezing.

From table 1 we can investigate as squeezing parameter increase quadrature variance decrease but quadrature squeezing(S) increase. And we gate the maximum degree of squeezing to be 99.7% for r=3.

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