Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol.20, 2013

Calculation of Energy Levels for Nuclei ^{34}S,^{34}Ar,^{34}Cl by using Surface Delta Interaction

A.K.Hasan[1] , F.M.Hussain[2]
Department of Physics, College of Education for Girls, University of Kufa [1]
Department of Physics, College of Education for Pure Sciences, University of Babylon [2]

Abstract
The energy levels have been calculated by using the nuclear shell model and adopting the surface delta interaction for the nuclei ^{34}S, ^{34}Ar and ^{34}Cl with two additional nucleons which are located outside close core ^{32}S. A comparison had been made between our theoretical predictions and the recent available experimental data . A reasonable agreement were obtained from these comparisons .
Key words*:* shell model, energy levels, Surface delta interaction.

1. Introduction

The nuclear shell model is one of the important and useful models that became mainstream theoretical model in the nuclear physics structure [1, 2]. In the model ,we fill the shells with in order of increasing energy, consistent with the requirement of the Pauli principle, when we do so, we obtain an inert core of filled shells and some number of valence nucleons, the model then assumes that nuclear properties are determined primarily by the valence nucleons [3].

In this paper, Surface delta interaction has been used as interaction residual in two nucleons by applying nuclear shell model to calculate the energy levels for three nuclei equivalent in mass numbers and different in nucleons type, these nuclei are: $^{34}S_{18}$, $^{34}Ar_{16}$ and $^{34}Cl_{17}$. The core taken at ^{32}S with two nucleons distributed over $(0d_{3/2}0f_{7/2})$.

2. Theory

The shell model uses two principle assumptions are there exists an inert core, made of close shells, which acts with central force on valence nucleons and there exists a residual interaction caused by two body forces action between the valence nucleons [4]. Residual interaction is defined as the force that produces when nucleons collide with each other and this interaction is happen the perturbation in Hamiltonian operator that equal summing two particles potential and represent Hamiltonian operator to perturbation state from equation [5]:

$$H = H_\circ + \sum_{i\langle j} V_{ij} \qquad (1)$$

V_{ij} is the residual two-body interaction , H_0 is represent Hamilton operator without perturbation.

To calculate the spectrum of this nucleus we assume that the residual nucleon-nucleon interaction V_{ij} is the surface-delta potential (SDI) ,is given by [5]:-

$$V = -4\pi V_\circ \delta(\underline{r}_i - \underline{r}_j) \qquad (2)$$

Where V_0 is interaction strength and $\underline{r}_i$, $\underline{r}_j$ are the position vectors.

This form was introduced by Green and Moszkowski has served as a remarkably good effective interaction for shell model calculations in many regions of the periodic table [6]. Tow -body matrix elements are given by [5]:

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$$E_J(j_1 j_2; j_3 j_4) = \left\langle \Psi_{JM}(j_1 j_2) \middle| V_{ij} \middle| \Psi_{JM}(j_3 j_4) \right\rangle \qquad (3)$$

And matrix element of SDI for two body outside core is given by following form [5]:

$$E_{JT}(j_1 j_2; j_3 j_4) = -(-1)^{j_1+j_2+j_3+j_4}\frac{V_\circ \bar{R}}{4(2J+1)}\times\left\{1+(-1)^{l_1+l_2+l_3+l_4}\right\}$$

$$\times\left[\frac{(2j_1+1)(2j_2+1)(2j_3+1)(2j_4+1)}{(1+\delta_{j_1 j_2}\delta_{l_1 l_2})(1+\delta_{j_3 j_4}\delta_{l_3 l_4})}\right]^{\frac{1}{2}}$$

$$\times\left[\{(1+(-1)^T\}\left\langle j_1 j_2 \tfrac{1}{2}\tfrac{1}{2}\,|\,J1\right\rangle\times\left\langle j_3 j_4 \tfrac{1}{2}\tfrac{1}{2}\,|\,J1\right\rangle+(-1)^{l_2+l_4+j_2-j_4}\cdot\right. \tag{4}$$

$$\left.\times\left\{1-(-1)^{J+T+l_3+l_4}\right\}\left\langle j_1 j_2 \tfrac{1}{2}-\tfrac{1}{2}\,|\,J0\right\rangle\left\langle j_3 j_4 \tfrac{1}{2}-\tfrac{1}{2}\,|\,J0\right\rangle\right]$$

Together with the restriction that the magnitude of the radial integrals involved be set equal to a constant

$$\bar{R} = \int R_{j_1}(r)R_{j_2}(r)R_{j_3}(r)R_{j_4}(r)r^2 dr$$

The matrix element as explicitly evaluated with wave functions that are positive at $r \to 0$.

$$\bar{R} = (-1)^{n_1+n_2+n_3+n_4}R_0$$

Where n_i is the number of radial nodes for the state j_i, R_0 is appositive number and $\langle\ |\ \rangle$ is the Clebsch- Gordon coefficients.

If e_j is the energy of the single –particle relative to the close core and $E_J(jj;jj)$ is the interaction energy when the two particles couple to angular momentum J the energy of this state relative to the closed shell is [5]:

$$\left\langle \Psi_{JM}(jj)\middle|H\middle|\Psi_{JM}(jj)\right\rangle = 2e_j + E_J(jj;jj) \tag{5}$$

For simplicity assumes there are two state denoted by $\Psi_{JM}(j_1 j_2)$ and $\Psi_{JM}(j_3 j_4)$, then the energies with respect to the core are given by [5]:

$$\left.\begin{aligned}\langle H\rangle_{11} &= \varepsilon_{j_1}+\varepsilon_{j_2}+E_J(j_1 j_2; j_1 j_2)\\ \langle H\rangle_{22} &= \varepsilon_{j_3}+\varepsilon_{j_4}+E_J(j_3 j_4; j_3 j_4)\end{aligned}\right\} \tag{6a}$$

$$\langle H\rangle_{12} = \langle H\rangle_{21} = E_J(j_1 j_2; j_3 j_4) \tag{6b}$$

Equation (6a) calculates the energies for pure configurations and to calculate the energies in mixing configurations (6b) should be added to (6a).

The allowable angular momentum states for two particles calculate from two theorems [5]:
First theorem, if two identical particles in the same single particle orbit j (j half integer) can only couple their spins to even values of J:

$$J = 0,2,4,.....(2J+1) \tag{7a}$$

But if two different particles the configuration $(j)^2$ have odd values of J (J=1, 3...) too.
Second theorem, if two particles in the states j1 and j2 (j1≠j2) the allowable angular Momentum values are

$$J = j_1+j_2, j_1+j_2-1,.....\left|j_1-j_2\right| \tag{7b}$$

3. Calculation, Results and Discussion

In this work, the energy levels have been calculated for ^{34}S, ^{34}Ar and ^{34}Cl in the model space ($0d_{3/2}\,0f_{7/2}$) with allowed angular momentum J.

3.1. The even-even ^{34}S, ^{34}Ar nuclei.

In our calculations ^{32}S is considered as the close core with 2 neutrons and 2 protons distributed over ($0d_{3/2}0f_{7/2}$) outside the core for ^{34}S and ^{34}Ar, respectively. The single particle energies for $0d_{3/2}$, $0f_{7/2}$ are taken to be -8.643MeV, -5.709MeV for neutrons and -2.277MeV, -0.408MeV for protons [7, 8, 9]. From eq. (7a) and (7b) the allowed angular momentum states have been assigned for these nuclei are:

J=0, 2, 3,4,5,6

A computer program written by Mathematica to calculate the matrix element of (SDI) residual interaction eq. (4), and hence the energy levels are calculated by using eq. (6a) and (6b).

The calculated levels spectra for ^{34}S are compared with the recent available experimental data [9, 10, 11], as shown in Fig.1 with their values tabulated in Table 1. Good agreement was obtained for this nucleus in both, positive and negative states. If we focus our attention to the first 2^+ from ground band, our work predicts this level at 2.385MeV which is very close to the value experiment, and we can notice the level 5^- in our work at 4.683MeV is deviated to some extent from the experiment value 5.689MeV, the difference between the two values is 1MeV.

Fig.2 and table 2, in this work, each level from calculated energy levels for ^{34}Ar in comparison with experimental data [9, 10, 11]. The comparison was obtained in good agreement, our calculations predict the first level 2^+ from the ground band at 2.048MeV is in good agreement with experimental value is 2.09MeV. From this compare we can see that this model space in reasonable agreement with experiment up to $J \leq 6^+$.

Table 1

Comparison between calculated excitation energy levels with experimental data for ^{34}S.

Pre. Res.		*Exp .Res. [9,10,11]*	
J^π	$E[MeV]$	J^π	$E[MeV]$
0_1^+	0.0	0^+	0.0
2_1^+	2.385	2^+	2.127
5^-	4.683	5^-	5.689
3^-	5.384	3^-	5.679
2_2^-	5.706	2^-	5.68
4_1^-	5.706	4^-	6.251
0_2^+	6.346	(0)	6.685
2_3^+	7.886	2^+	7.805
4_2^+	8.245	(4^+)	8.264
6^+	8.443	6^+	8.503

Table 2

Comparison between calculated excitation energy levels with experimental data for ^{34}Ar.

Pre. Res.		*Exp. Res. [9,10,11]*	
J^π	$E[MeV]$	J^π	$E[MeV]$
0_1^+	0.0	0^+	0.0
2_1^+	2.048	2^+	2.091
5^-	4.175	-	4.127
3^-	4.789	3^- -	4.513 4.865
2_2	5.07	-	5.225
4_1^-	5.07	-	5.225
0_2^+	5.710	0^+	5.909
2_3^+	7.093	2^+	7.322
4_2^+	7.409	(2^+)	7.499
6^+	*7.583*	-	*7.925*

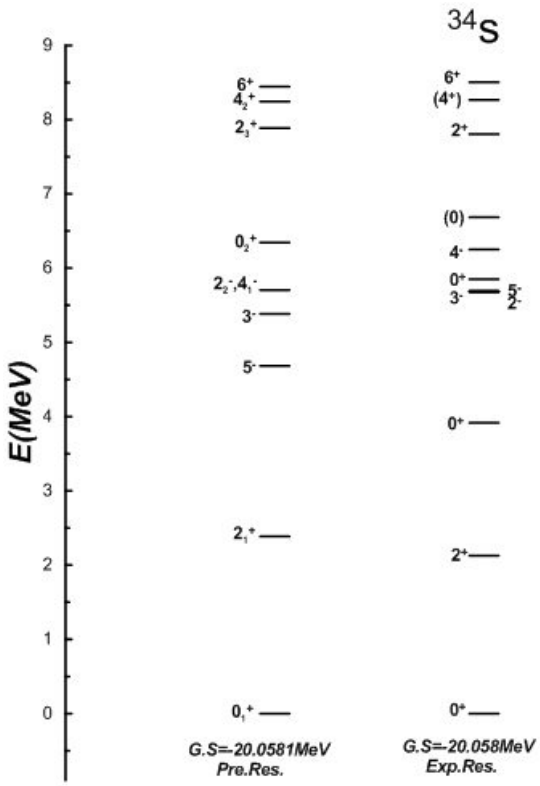

Fig .2- A comparison between theoretical energy levels and the experimental values taken from [9,10,11] for ^{34}S

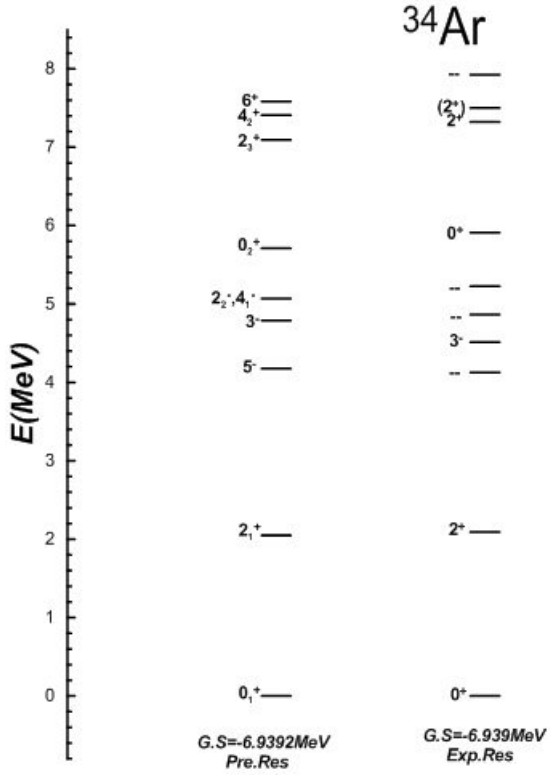

Fig .2- A comparison between theoretical energy levels and the experimental values taken from [9,10,11] for ^{34}Ar.

3.2. The nucleus odd-odd represented by Chorine nucleus ^{34}Cl.

Nucleus ^{34}Cl contains one proton and one neutron outside close core ^{32}S distributed over the model space ($0d_{3/2}0f_{7/2}$). The allowed angular momenta can be calculated by using eqs. (7a, 7b) are: J=0, 1,2,3,4,5,6,7

The calculation have been carried out using the effective interaction (SDI) eq.(4), and we depended the single particles energies for neutron and proton.

In fig.3 and table 3, present the comparison of results obtained in this work for ^{34}Cl with experimental data [9,10,11]. From Fig.3 and Table 3, we can notice that this model is in good agreement with experiment up to 9.998MeV. Mixed arrangement adopted in our calculations, we can see the increasing energy levels that conform well with the experimental data. As show the calculated first level 1^+ at 1.567MeV, in this range no approach experimental value. The higher value was obtained for this nucleus according to the recent available experimental is 8.793MeV, comparing with our calculations was obtained the level 3^+ by 9.998MeV that exceed experimental data.

Table 3

Comparison between calculated excitation energy levels with experimental data for ^{34}Cl.

Pre. Res.		*Exp. Res. [9,10,11]*	
J^π	$E[MeV]$	J^π	$E[MeV]$
0_1^+	0.0	0^+	0.0
1_1^+	1.567	-	-
3_1^+	1.811	3^+	2.181
2_1^+	2.469	2^+	2.158
5_1^-	3.181	5^-	3.631
3_2^-	4.397	$(3)^-$	4.461
2_2^-	5.55	(2^-)	5.576
4_1^-	5.55	$(4)^-$	5.54
5_2^-	5.680	$(3)^-$	5.785
4_2^-	5.799	$4(-)$	6.207
2_3^-	5.801	2^-	5.805
0_2^+	6.219	(0)	6.479
1_2^+	6.964	1^+	6.991
7^+	7.089	-	7.061
3_3^-	7.668	3^-	7.677
5_3^+	7.681	-	7.666
2_4^+	7.723	$(2)^+$	7.761
4_3^+	8.084	$(4)^+$	8.012
6^+	8.284	-	8.280
3_4^+	9.998		

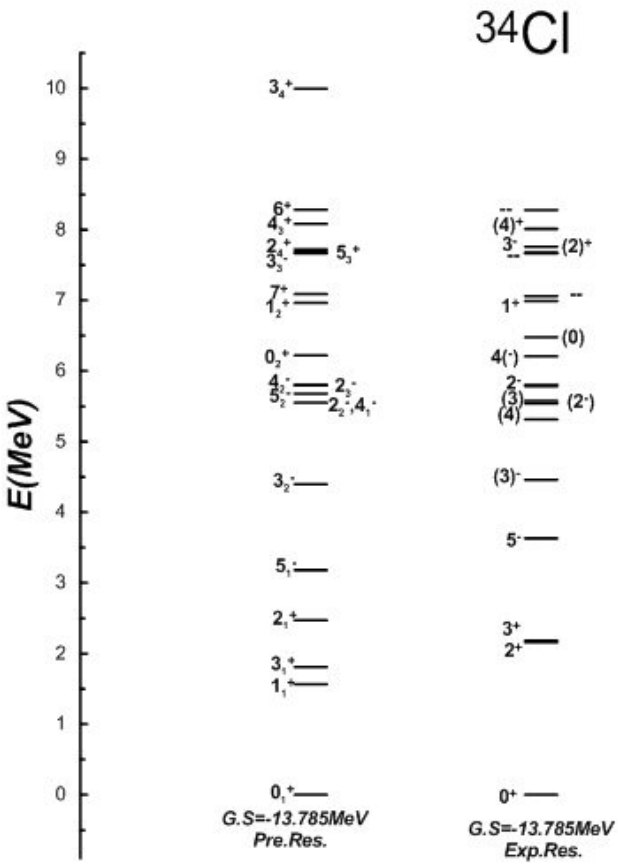

Fig.3- A comparison between theoretical energy levels and the experimental values taken from [9,10,11] for ^{34}Cl.

4. Conclusions

The nuclear shell model calculations are in good agreement with the experimental data by adopting the surface delta interaction (SDI) as residual interaction with suitable single particle energies. Configuration mixing are taken into account and enhances the calculations of the excitation energies. These calculations can be repeated by adopting the modified surface delta interaction (MSDI) with suitable parameters adjusted for this mass region and might improve the calculations.

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