Wavefront Analysis for Annular Ellipse Aperture

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Abstract

The orthonormal annular ellipse Zernike polynomials are important for wavefront analysis of annular ellipse aperture (elliptical aperture with an elliptical obscuration) for their property of orthgonalization over such aperture and representing balanced aberration. In this paper, the relationship between the Zernike annular ellipse polynomials and third order Siedel aberrations were studied. Then the standard deviations of balanced and **unbalanced** primary aberrations have been calculated for this aperture.

Keywords: Zernike polynomials, annular ellipse aperture, aberration, standard deviation

1. Introduction

There are huge references describe the Zernike polynomials for circular aperture. Deferent researchers were focused in transforming these Polynomials to be appropriate for other apertures. V.N. Mahajan and G. M. Dai derived the Zernike polynomials for other apertures like annular circle aperture, elliptical aperture, square aperture, rectangular aperture, hexagonal aperture [1-3].

These different groups of Zernike polynomials were used in many researches to find different properties. In 1982 Virendra N. Mahajan considered imaging systems with circular and annular pupils aberrated by primary aberrations, and he discussed both classical and balanced (Zernike) aberrations[4].

In 1992, James C. Wyant found that the first-order wavefront properties and third-order wavefront aberration coefficients for circular aperture can be obtained from the Zernike polynomials coefficients[5].

In this work, the third order wavefront aberration coefficients for the elliptical aperture with elliptical obscuration (annular ellipse aperture) were found in terms of Zernike annular ellipse polynomials which have been found in a previous paper[6]. Also the balanced and unbalanced primary aberrations with annular ellipse aperture were considered.

2. Rotationally Symmetric Systems

The wavefront aberration function may be analyzed into several components by expanding the function as a power series. For the relatively simple case of a rotationally symmetric system, one expansion of the wavefront aberration is as follows [7]:

$$W(r,\rho,\cos\phi) = {}_0W_{00} + {}_1W_{11} r\rho\cos\phi + {}_0W_{20}\rho^2 + {}_0W_{40}\rho^4 + {}_1W_{31} r\rho^3\cos\phi + {}_2W_{20} r^2\rho^2 + {}_2W_{22} r^2\rho^2\cos^2\phi + {}_3W_{11} r^3\rho\cos\phi + \ldots \quad (1)$$

This takes into account the distance of the image point from the optic axis(r) and expresses the pupil sphere coordinates (x,y) in polar terms, where ρ is the distance from the optic axis, and angle ϕ is measured from the vertical meridian. The coefficient W before each polynomial is subscripted with three numbers that indicate, from left to right, the power of the r, ρ and $\cos\phi$ terms, respectively. The first three terms describe first order wavefront errors (piston, tilt, and defocus respectively). The next five terms correspond to the third order Seidel aberrations, (Spherical, comma, fied curvature, astigmatism, and distorsion respectively). Addition of fifth order or higher terms will improve the polynomial approximation to the wavefront aberration, though the higher terms normally account for a much smaller portion of the aberration.

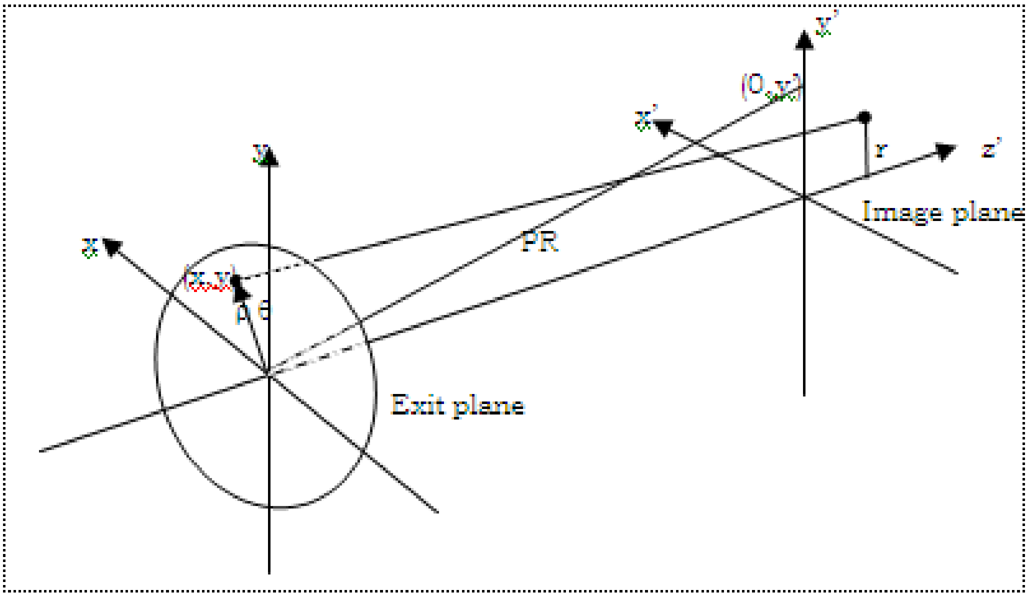

Figure.1 Coordinates for defining transverse ray aberrations [7].

If there is no field dependence in these terms, they are not true Seidel aberrations. Wavefront measurement using an interferometer provides data at a single field point only. This causes field curvature to look like focus, and distortion to look like tilt[8,9].

$$W(r, \rho, \cos\phi) = {}_0W_{40}\,\rho^{4} + {}_1W_{31} r \rho^{3} \cos\phi + {}_2W_{20} r^{2} \rho^{2} + {}_2W_{22} r^{2} \rho^{2} \cos^{2}\phi + {}_3W_{11} r^{3} \rho \cos\phi \qquad (2)$$

3. Zernike polynomials

The Zernike polynomials were recommended for describing wave aberration functions over circular pupils with unit radius. Individual terms, or modes, of a Zernike polynomial are mutually orthogonal over the unit circle and are easily normalized to form an orthonormal basis. These polynomials were lost their orthogonolity for pupils other than circular.

In a previous paper the annular Zernike polynomials for annular ellipse were found, (see fig. 1),from that of circular Zernike polynomials [6], and table (1) represents the first nine orthogonal polynomials for annular aperture of aspect ratio(ratio of the primary to the secondary axes)=b and obscuration ratio (the ratio of radii of outer and inner ellipses)=k. and table 2, represents the first nine orthonormal polynomials for annular ellipse aperture of aspect ratio=b and obscuration ratio=0.5.

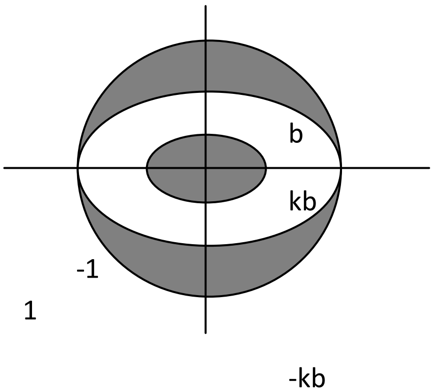

Fig.2. annular ellipse aperture inside a unit circle.

Table 1. the first nine orthogonal polynomials for annular aperture of aspect ratio=b and obscuration ratio=k[6].

Orthogonal Zernike polynomials		
A1=1	A2= x	A3= y
A4= $\sigma+2\rho^2$	A5= $\alpha_1 x^2+\alpha_2\rho^2+\alpha_3$	A6=2xy
A7=$(3\rho^2-\gamma)x$	A8=$(3\rho^2-\gamma 1)y$	A9= $6\rho^4+\delta\rho^2+\delta_1 x^2+\delta 2$

$\sigma= -1/2(1+b^2)(k^2+1)$ $\alpha=k^4(3b^4-2b^2+3)-b^2(8k^2-3b^2-2)+3$

$\alpha_1=[2b^2(k^2(-2k^2-8+3b^2k^2)-2+3b^2)+6k^4+6]/\alpha$

$\alpha_2=[2b^2(k^2(k^2+4)+1)-6k^4-6]/\alpha$ $\alpha_3=[b^2(k^2(k^4-b^2k^4+2k^2-2b^2k^2-2b^2+2)-b^2+1)]/\alpha$

$\gamma=[(3+b^2)(k^4+k^2+1)/(2k^2+2)$ $\gamma 1=[(1+3b^2)(k^4+k^2+1)/(2k^2+2)$

$\delta=[(18b^2-6)k^6+(17b^2-12)k^4-(17b^2+12)k^2-18b^2+6]/[4(k^4+k^2+1)]$

$\delta 1=[(12b^2-12)k^6+(-15b^2+39)k^4+(15b^2+39)k^2+12b^2-24]/[4(k^4+k^2+1)]$

$\delta 2=[(2b^2+3)k^8+(15b^4+10b^2+15)k^6+(18b^4+12b^2+18)k^4-(15b^4+10b^2+15)k^2+(3b^4+2b^2+3](2b^2)/[4(k^4+k^2+1)]$

Table (2).First nine orthonormal Zernike polynomials for unit elliptical aperture obscured by elliptical obscuration, with aspect ratio=b and obscuration ratio=k=0.5[6].

Orthonormal annula Zernike polynomials			
A1= 1	A2=$\frac{4}{\sqrt{5}}$ x	A3= $\frac{4}{b\ \sqrt{5}}y$	A4= $\frac{16\rho^2-5(1+b^2)}{\sqrt{17b^4+17-22b^2}}$

A5= $[\sqrt{\frac{2}{21(17b^4+17-22b^2)}}\,[(-68+44b^2)\rho^2+(68b^4-88b^2+68)x^2-35b^2(1-b^2)]/b^2$

A6=$\left[\frac{8\ \sqrt{2}}{b\ \sqrt{7}}\right]$ xy A7= $\left[\frac{8}{\sqrt{5}}\right]$ $[40\rho^2-21-7b^2]$ $x/(229b^4-326b^2+361)^{1/2}$

A8=$\left[\frac{8}{b\ \sqrt{5}}\right][40\rho^2-7-21b^2]y/(361b^4-326b^2+229)^{1/2}$

A9= $\left[\frac{1}{7b^2}\sqrt{\frac{35}{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}}\right][1792\rho^4-(1520b^4-560b^2-160)\rho^2+160(-b^6+b^4-6b^2-1)x^2+b^2(231b^4+154b^2+231)]$

4. Standard deviation

The Variance of the classical aberration is defined by the relation[10]

$$\sigma^2=\overline{\Delta W^2}-(\overline{\Delta W})^2 \tag{3}$$

Where $\overline{\Delta W}$ and $\overline{\Delta W^2}$ represent the mean and the mean square value of the aberration function and, for annular ellipse aperture, they were given by[6]

$$\overline{W}(x,y)=\frac{\int_{-1}^{1}\int_{-b\sqrt{1-x^2}}^{b\sqrt{1-x^2}}W(x,y)dydx-\int_{-k}^{k}\int_{-b\sqrt{k^2-x^2}}^{b\sqrt{k^2-x^2}}W(x,y)dydx}{\int_{-1}^{1}\int_{-b\sqrt{1-x^2}}^{b\sqrt{1-x^2}}dydx-\int_{-k}^{k}\int_{-b\sqrt{k^2-x^2}}^{b\sqrt{k^2-x^2}}dydx} \tag{4}$$

And

$$\overline{W^2}(x,y)=\frac{\int_{-1}^{1}\int_{-b\sqrt{1-x^2}}^{b\sqrt{1-x^2}}W^2(x,y)dydx-\int_{-k}^{k}\int_{-b\sqrt{k^2-x^2}}^{b\sqrt{k^2-x^2}}W^2(x,y)dydx}{\int_{-1}^{1}\int_{-b\sqrt{1-x^2}}^{b\sqrt{1-x^2}}dydx-\int_{-k}^{k}\int_{-b\sqrt{k^2-x^2}}^{b\sqrt{k^2-x^2}}dydx} \quad (5)$$

the standard deviation σ is the square root of the Variance, σ^2, and it can be found by making benefit of the important property of Zernike polynomials, which is orthogonality.

By having a glance to eq.s (4) and (5), it can be concluded that the variance of the aberration is given simply by the sum of the squares of the Zernike coefficient excluding the piston coefficient, since Zernike annular ellipse polynomials form an orthogonal set in the limits of the unit annular ellipse aperture.

5. Present work

5.1 (Relationship Between Zernike Polynomials of Annular Ellipse Aperture and Third Order Aberration)

First-order wavefront properties and third-order wavefront aberration coefficients (eq. 2) for annular ellipse aperture can be obtained from the first nine Zernike annular ellipse polynomials (table.1), as follows

$$W(x,y)= Z_0+ Z_1x+ Z_2y+Z_3(2x^2+2y^2+\mu) + Z_4(\alpha_1x^2-\alpha_2y^2+\alpha_3) + Z_5(2xy)+ Z_6(3x^3+3xy^2-\gamma x)+ Z_7(3x^2y+3y^3-\gamma_1 y)+ Z_8(6x^4+ 12x^2y^2+6y^4+\delta x^2+\delta y^2+\delta_1x^2+\delta_2) \quad (6)$$

Re-arranging these terms gives

$W(x,y)=Z_0-\mu Z_3+\alpha_3Z_4+\delta_2Z_8$	Piston
$+(Z_1-\gamma Z_6)x+(Z_2-\gamma_1Z_7)y$	Tilt
$+(2Z_3+\alpha_2Z_4+\delta Z_8)x^2 + (2Z_3+\alpha_2Z_4+\delta Z_8)y^2+2Z_5xy+Z_4\,\alpha_1x^2$	focus + astigmatism
$+(3Z_6x+3Z_7y)(x^2+y^2)$	Coma
$+6\rho^4Z_8$	Spherical

(7)

Simplifying the last equation using the following identity,

$$A\cos\alpha+B\sin\alpha=(A^2+B^2)^{1/2}\cos\{\alpha-tan^{-1}(A/B)\}$$

where $\alpha = \tan^{-1}\left(\frac{y}{x}\right)$, $\cos(\alpha) = \frac{x}{(x^2+y^2)^{1/2}}$ $\sin(\alpha) = \frac{y}{(x^2+y^2)^{1/2}}$, $x^2 + y^2 = \rho^2$

So, the tilt term is simplified to

$$(Z_1 - \gamma Z_6)x + (Z_2 - \gamma_1 Z_7)y = \rho\sqrt{(Z_1 - \gamma Z_6)^2 + (Z_2 - \gamma_1 Z_7)^2}\cos\left(\theta - tan^{-1}\left(\frac{Z_2 - \gamma_1 Z_7}{Z_1 - \gamma Z_6}\right)\right) \quad (8)$$

And for the focus and astigmatism term is simplified to:

$$(2Z_3 + \alpha_2 Z_4 + \delta Z_8)x^2 + (2Z_3 + \alpha_2 Z_4 + \delta Z_8)y^2 + 2Z_5 xy + \alpha_1 Z_4 x^2$$

$$= (2Z_3 + \alpha_2 Z_4 + \delta Z_8)\rho^2 + \alpha_1 Z_4 x^2 + 2Z_5 xy$$

$$= (2Z_3 + \alpha_2 Z_4 + \delta Z_8)\rho^2 + \alpha_1 Z_4 \rho^2\left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right) + Z_5\rho^2\sin(2\theta)$$

$$= (2Z_3 + (\alpha_2 + \alpha_1/2)Z_4 + \delta Z_8)\rho^2 + \frac{1}{2}\alpha_1 Z_4 \rho^2\cos(2\theta) + Z_5\rho^2\sin(2\theta)$$

$$= (2Z_3 + (\alpha_2 + \alpha_1/2)Z_4 + \delta Z_8)\rho^2 + \rho^2\sqrt{\left(\frac{1}{2}\alpha_1 Z_4\right)^2 + Z_5^2}\,cos\left(2\theta - tan^{-1}\left(\frac{2Z_5}{\alpha_1 Z_4}\right)\right)$$

$$= \left(2Z_3 + (\alpha_2 + \alpha_1/2)Z_4 + \delta Z_8 - \sqrt{\left(\frac{1}{2}\alpha_1 Z_4\right)^2 + Z_5^2}\right)\rho^2 + 2\rho^2\sqrt{\left(\frac{1}{2}\alpha_1 Z_4\right)^2 + Z_5^2}\,cos^2\left(\theta - \frac{1}{2}tan^{-1}\left(\frac{2Z_5}{\alpha_1 Z_4}\right)\right) \quad (9)$$

While for coma

$$(3Z_6 x + 3Z_7 y)\rho^2 = 3\rho^3\sqrt{Z_6^2 + Z_7^2}\cos\left(\theta - tan^{-1}\left(\frac{Z_7}{Z_6}\right)\right) \quad (10)$$

then substituting (8-10) in (7) gives the wavefront aberration terms as

$$W(x,y) = Z_0 - \mu Z_3 + \alpha_3 Z_4 + \delta_2 Z_8 \qquad \text{Piston}$$

$$\rho\sqrt{(Z_1 - \gamma Z_6)^2 + (Z_2 - \gamma_1 Z_7)^2}\cos\left(\theta - tan^{-1}\left(\frac{Z_2 - \gamma_1 Z_7}{Z_1 - \gamma Z_6}\right)\right) \qquad \text{Tilt}$$

$$\left(2Z_3 + (\alpha_2 + \alpha_1)Z_4 + \delta Z_8 \mp \sqrt{\left(\frac{1}{2}\alpha_1 Z_4\right)^2 + Z_5^2}\right)\rho^2 \qquad \text{Focus}$$

$$\pm 2\rho^2\sqrt{\left(\frac{1}{2}\alpha_1 Z_4\right)^2 + Z_5^2}\,cos^2\left(\theta - \frac{1}{2}tan^{-1}\left(\frac{2Z_5}{\alpha_1 Z_4}\right)\right) \qquad \text{Astigmatism}$$

$$+3\rho^3[(Z_6)^2 + (Z_7)^2]^{1/2}\cos\{\alpha - \tan^{-1}\frac{Z_7}{Z_6}\} \qquad \text{Coma}$$

$$+6\rho^4 z_8 \qquad \text{Spherical}$$

(11)

The magnitude , sign , and angle of the field independent aberration function coefficients are listed in table (3) .Note that focus has the sign chosen to minimize the magnitude of the coefficient, and astigmatism uses the sign opposite to that chosen for focus .

Table.3 The magnitude, sign, and angle of field independent wavefront aberration

Term	Description	Magnitude	Angle
W_{11}	Tilt	$\sqrt{(Z_1-\gamma Z_6)^2+(Z_2-\gamma_1 Z_7)^2}$	$tan^{-1}(\frac{Z_2-\gamma_1 Z_7}{Z_1-\gamma Z_6})$
W_{20}	Focus	$\left(2Z_3+(\alpha_2+\alpha_1)Z_4+\delta Z_8 \mp \sqrt{(\frac{1}{2}\alpha_1 Z_4)^2+Z_5^2}\right)$ sign chosen to minimize	
W_{22}	Astigmatism	$\pm 2\sqrt{(\frac{1}{2}\alpha_1 Z_4)^2+Z_5^2}$ sign opposite to that chosen in focus term	$\frac{1}{2}tan^{-1}(\frac{2Z_5}{\alpha_1 Z_4})$
W_{21}	Coma	$3[(Z_6)^2+(Z_7)^2]^{1/2}$	$\tan^{-1}\frac{Z_7}{Z_6}$
W_{40}	Spherical	$6Z_8$	

5.2 Standard deviation of balanced and unbalanced primary aberrations

As the Zernike annular ellipse polynomials were found, they were representing balanced polynomials. Now, it is appropriate to find the standard deviation of the balanced and unbalanced Siedel aberrations as follows:

The aberrations tilt in x and y (distortion) and focus (curvature of field) are having the same form in annular ellipse as that in circular aperture.

the fifth orthonormal polynomial for annular ellipse with obscuration ratio (k=0.5) represents balanced astigmatism and it can be written as (from table (2))

$A_5=[\sqrt{\frac{2}{21(17b^4+17-22b^2)}}\,[(-68+44b^2)\rho^2+4(17b^4-22b^2+17)\,x^2-35b^2(1-b^2)]/b^2$

or

$A_5=[\sqrt{\frac{32(17b^4+17-22b^2)}{21}}\,[\rho^2\cos^2\theta+\frac{68+44b^2}{4(17b^4+17-22b^2)}\,\rho^2]/b^2$ +cons.

It's clear here that astigmatism is balanced by defocus, and

$[\rho^2\cos^2\theta+\frac{-68+44b^2}{4(17b^4+17-22b^2)}\,\rho^2]=b^2\sqrt{\frac{21}{32(17b^4+17-22b^2)}}A_5$

and so the balanced astigmatism standard deviation is

$\sigma_{ba}=b^2\sqrt{\frac{21}{32(17b^4+17-22b^2)}}$

To determine σ of Seidel astigmatism, the aberration should be written in terms of the annular ellipse polynomials:

$\rho^2\cos^2\theta=b^2\sqrt{\frac{21}{32(17b^4+17-22b^2)}}A_5-\frac{(-68+44b^2)(17b^4+17-22b^2)^{1/2}}{4*16(17b^4+17-22b^2)}A_4+cA_1$

where *c* is a constant independent of the pupil coordinate.

As stated before, σ is the square root of the variance which is in turn can be found by summing the squares of the coefficients of the polynomial except that of A_1, then

$$\sigma_a=\left(\frac{21b^4}{32(17b^4+17-22b^2)}+\frac{(-68+44b^2)^2}{16*256(17b^4+17-22b^2)}\right)^{1/2}=0.066$$

The value of σ is independent of the aspect ratio b, this is because Seidel astigmatism x^2 varies only along the x axis for which the unit ellipse has the same length as a unit circle, and hence it is equal the value of σ for annular ellipse pupil of obscuration ratio k=0.5.
For comma, the 7th polynomial is

$$A_7=\left[\frac{8}{\sqrt{5}}\right]\,[40\rho^3\cos\theta(21+7b^2)\rho\cos\theta]\ /(229b^4-326b^2+361)^{1/2}$$

or

$$A_7=\left[\frac{320}{\sqrt{5}}\right]\,[\rho^3\cos\theta-(21+7b^2)\rho\cos\theta/40]\ /(229b^4-326b^2+361)^{1/2}+\text{cons.}$$

i.e. the comma aberration is balanced by tilt, and

$$[\rho^3\cos\theta-(21+7b^2)\rho\cos\theta/40]=\left[\frac{\sqrt{5(229b^4-326b^2+361)}}{320}\right]A_7+\text{cons.}$$

and so the balanced comma standard deviation is

$$\sigma_{bc}=\frac{\sqrt{5(229b^4-326b^2+361)}}{320}$$

To determine σ of Seidel comma, the aberration should be written in terms of the annular ellipse polynomials:

$$\rho^3\cos\theta=\left[\frac{\sqrt{5(229b^4-326b^2+361)}}{320}\right]A_7+(21+7b^2)\frac{\sqrt{5}}{160}A_2+cA_1$$

$$\sigma_c=\left(\frac{5(229b^4-326b^2+361)}{102400}+\frac{(21+7b^2)^2*5}{25600}\right)^{1/2}=\left(\frac{(425b^4+950b^2+1403)}{20480}\right)^{1/2}$$

For spherical aberration, the 9th polynomial is

$$A_9=\left[\frac{1}{7b^2}\sqrt{\frac{35}{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}}\right][1792\rho^4+(1520b^4-560b^2-160)\rho^2+160(-b^6+b^4-6b^2-1)\rho^2\cos^2\theta]+\text{cons.}$$

$$=\left[\frac{1792}{7b^2}\sqrt{\frac{35}{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}}\right]\left[\rho^4+\frac{(1520b^4-560b^2-160)\rho^2}{1792}+\frac{160(-b^6+b^4-6b^2-1)x^2}{1792}\right]$$

It's obvious that the spherical aberration ρ^4 is not balanced by only the defocus ρ^2 as in the circular aperture but also balanced with a small relative value of astigmatism $\rho^2 cos^2\theta$. This is because of unequal axes, so

$$\left[\rho^4+\frac{(1520b^4-560b^2-160)\rho^2}{1792}+\frac{160(-b^6+b^4-6b^2-1)x^2}{1792}\right]$$
$$=\left[\frac{7b^2}{1792}\sqrt{\frac{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}{35}}\right]A_9+\text{cons.}$$

then balanced spherical aberration standard deviation is

$$\sigma_{bs}=\frac{7b^2}{1792}\sqrt{\frac{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}{35}}$$

To determine σ of Seidel spherical aberration,

$$\rho^4=\left[\frac{7b^2}{1792}\sqrt{\frac{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}{35}}\right]A_9-\frac{(1520b^4-560b^2-160)\sqrt{17b^4+17-22b^2}}{1792*16}A_4$$

$$-\frac{160(-b^6+b^4-6b^2-1)}{1792}\left[b^2\sqrt{\frac{21}{32(17b^4+17-22b^2)}}A_5 - \frac{(-68+44b^2)}{4*16(17b^4+17-22b^2)^{1/2}}A_4\right]+cA_1$$

$$\rho^4=\left[\frac{7b^2}{1792}\sqrt{\frac{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}{35}}\right]A_9$$

$$-\frac{160(-b^6+b^4-6b^2-1)b^2}{1792}\sqrt{\frac{21}{32(17b^4+17-22b^2)}}A_5 + \frac{5(-1408b^8+3584b^6-1092b^4+11760b^2-2208)}{1792*16\sqrt{17b^4+17-22b^2}}A4+cA_1$$

By summing squares and then taking square root

$$\sigma = \left[\left(\frac{7b^2}{1792}\right)^2\left(\frac{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}{35}\right) + \left(\frac{160(-b^6+b^4-6b^2-1)b^2}{1792}\right)^2\left(\frac{21}{32(17b^4+17-22b^2)}\right) + \frac{25(345b^8-591b^6+607b^4-245b^2-66)^2}{(1792*16)^2(17b^4+17-22b^2)}\right]^{1/2}$$

Table (7) summarize the values of standard deviation of a primary and a balanced primary aberration for annular ellipse aperture with obscuration ratio =0.5.

Table (4) values of standard deviation of a primary and a balanced primary aberration for annular ellipse aperture with obscuration ratio =0.5.

Type	σ
Tilt (σ_t)	$\frac{\sqrt{5}}{4}$
defocus (σ_f)	$\frac{\sqrt{17b^4+17-22b^2}}{16}$
Balanced astigmatism (σ_{ba})	$b^2\sqrt{\frac{21}{32(17b^4+17-22b^2)}}$
astigmatism (σ_a)	0.066
Balanced comma (σ_{bc})	$\frac{\sqrt{5(229b^4-326b^2+361)}}{320}$
comma (σ_c)	$\left(\frac{(425b^4-326b^2+361)}{21200}\right)^{\frac{1}{2}}$
Balanced spherical aberration (σ_{bs})	$\frac{7b^2}{1792}\sqrt{\frac{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}{35}}$
spherical aberration (σ_s)	$\left[\left(\frac{7b^2}{1792}\right)^2\left(\frac{9475(1+b^8)-18340b^2(1+b^4)+21762b^4}{35}\right) + \left(\frac{160(-b^6+b^4-6b^2-1)b^2}{1792}\right)^2\left(\frac{21}{32(17b^4+17-22b^2)}\right) + \frac{25(345b^8-591b^6+607b^4-245b^2-66)^2}{(1792*16)^2(17b^4+17-22b^2)}\right]^{1/2}$

6. Discussion and Conclusions

As stated, before, if there is no field dependence, the terms of aberration function are not true Seidel wavefront aberration, i.e. field curvature looks like focus and distorsion looks like tilt.
Field independent third order aberration function coefficients for annular ellipse aperture were found as a function of Zernike annular coefficients as shown in table 3. This table shows that the tilt aberration magnitude and direction is look like those of circular aperture in that they relate to the coefficients Z_1, Z_2, Z_6 , and Z_7, but here Z_6 and Z_7 were multiplied by γ and γ_1 instead of 2 in circular (see reference[5]).
The focus error is different from that for circular in including the term $(\alpha_2+\alpha_1)Z_4$, or $(3(b^4-1)(k^4+1))Z_4$. This happened because of the unequal axes of the aperture which caused asymmetrical coefficients for x^2 and y^2 in the fifth term of Zernike annular polynomials, and it is obvious that this term is equal to zero when b=1.
The same reason causes Z_4 to accompany by $\alpha 1/2$ in astigmatism. While the terms of coma and spherical aberrations are save there their forms of magnitude and direction as like as that of circular aperture.
Table 4 shows the values of standard deviation, primary and balanced aberrations for annular ellipse aperture with obscuration ratio=0.5.
Fig. 3 shows values of standard deviations as a function of b (aspect ratio). It is clear that the tilt and astigmatism are independent on b, i. e. they were constants. This behavior is similar to that for elliptical and circular apertures (return to reference [3]). The behavior of focus standard deviation is down till the value of b reachs 0.8 then it rice.
This curve can be compared with the curve drown from values of Mahagan and Die for ellipse[3], as illustrated in fig. 5. It's clear that σ of focus error for annular ellipse with k=0.5 is more than that for ellipse till b reaches the value of 0.3 then it becomes lower. Here, as in the rest figures, the straight line represents the value of s for circular aperture and it is drawn just for comparison.
Returning to coma aberration, it is seen that the standard deviation is increased with b, and fig 6 shows that σ is lower than that for ellipse till b reaches to the value of 0.7 then it becomes higher.
Standard deviation for spherical aberration is so small for annular ellipse with k=0.5 as compared to that of ellipse (see fig. 9).
The curves of balanced standard deviation versus b are also plot as illustrated in fig .4, and they were compared with those of ellipse and circular as illustrated in fig. 7 for balanced comma, and in fig. 8 for balanced astigmatism and fig. 10 for balanced spherical aberration.

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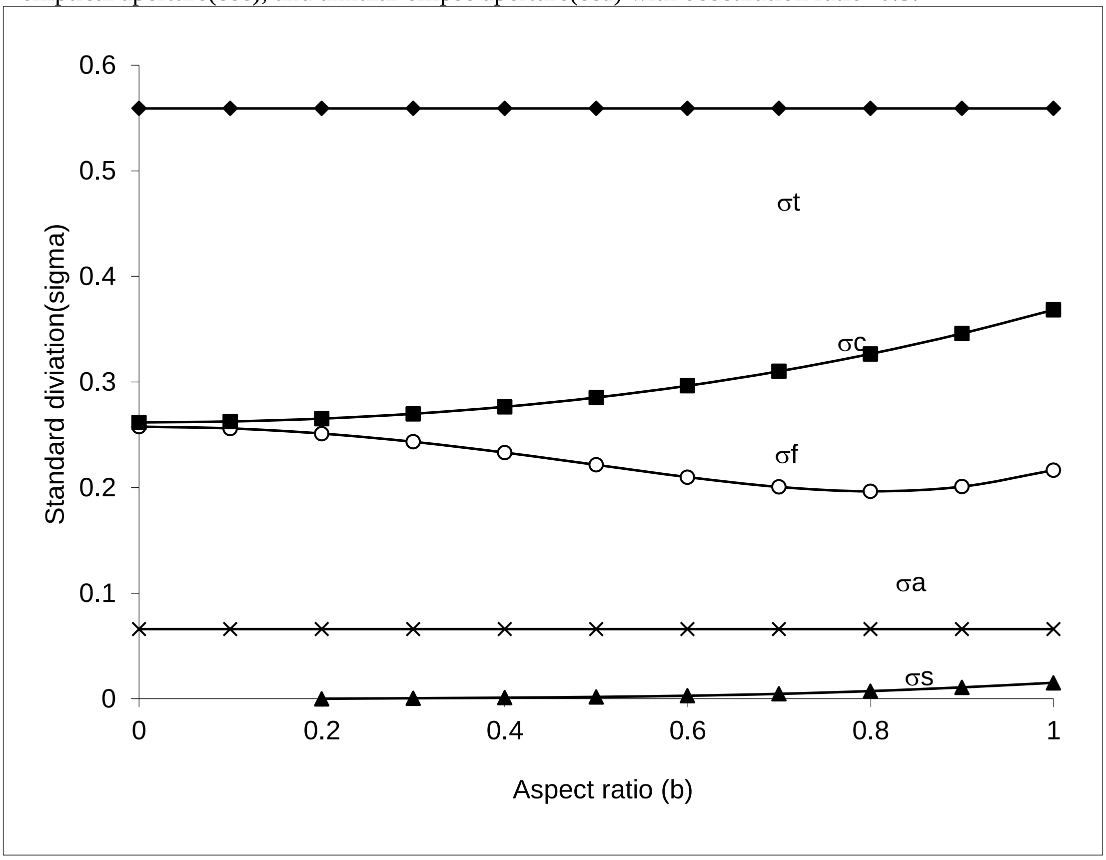

Fig.3 standard deviation (σ) versus aspect ratio(b) of the aberrations, spherical (s), astigmatism (a), coma(c), focus(f), and tilt (t) for annular ellipse aperture with obscuration ratio=0.5.

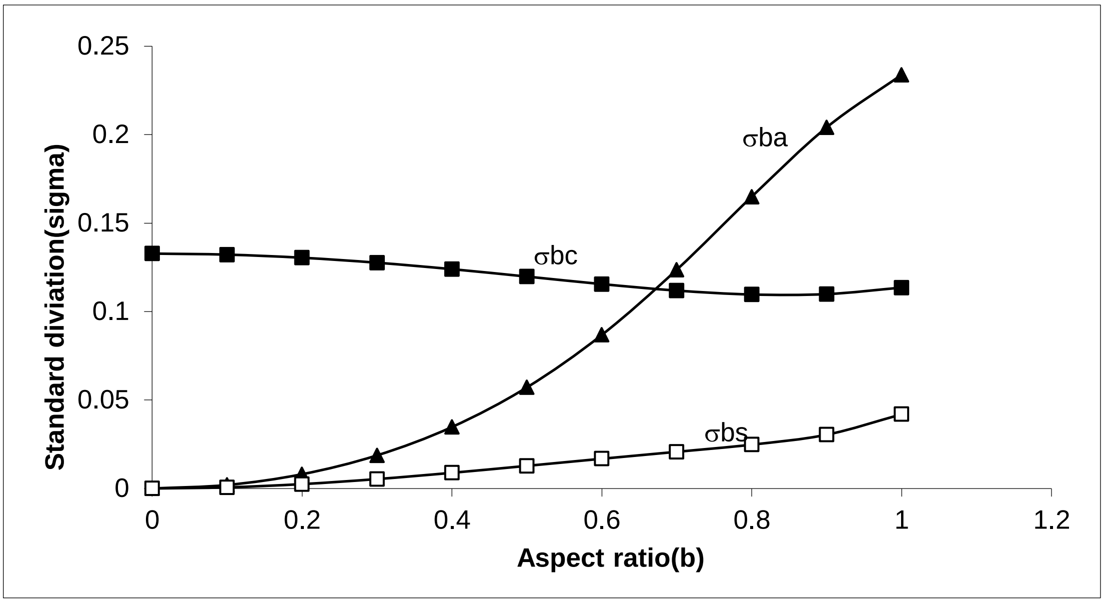

Fig.4 standard deviation (σ) versus aspect ratio(b) of the balanced aberrations, spherical (bs), astigmatism (ba), coma(bc) for annular ellipse aperture with obscuration ratio=0.5.

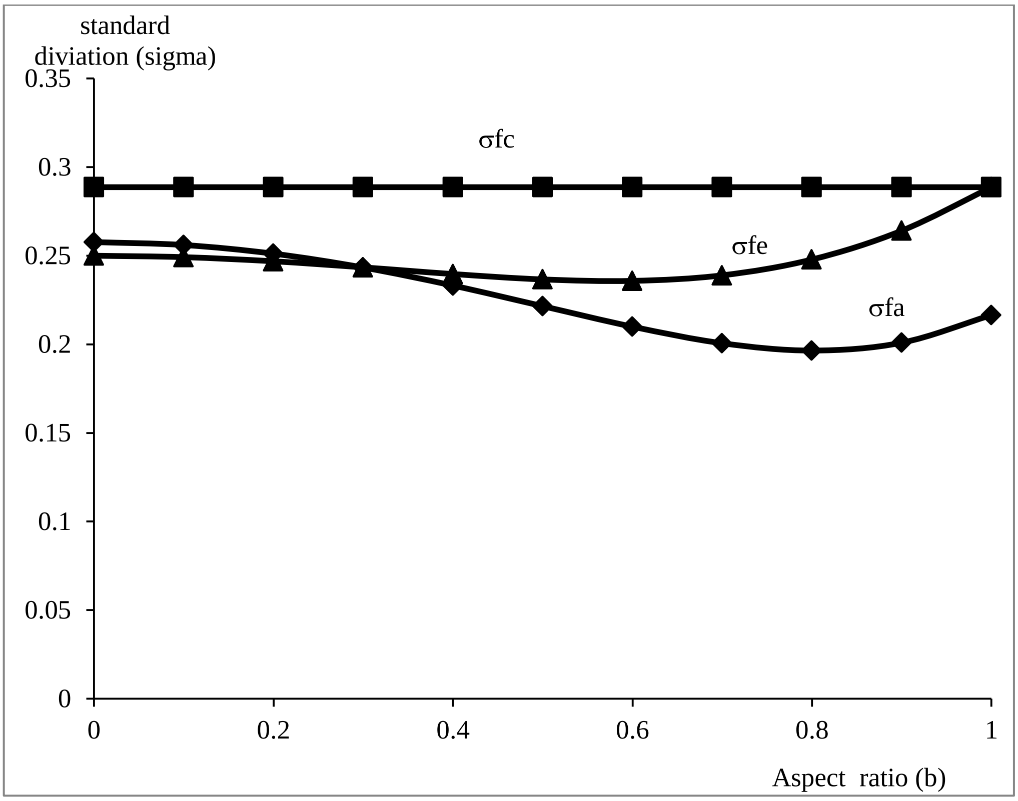

Fig.5 standard deviation (σ) versus aspect ratio(b) with focus aberration for circular aperture(fc), elliptical aperture(fe), and annular ellipse aperture(fa) with obscuration ratio=0.5

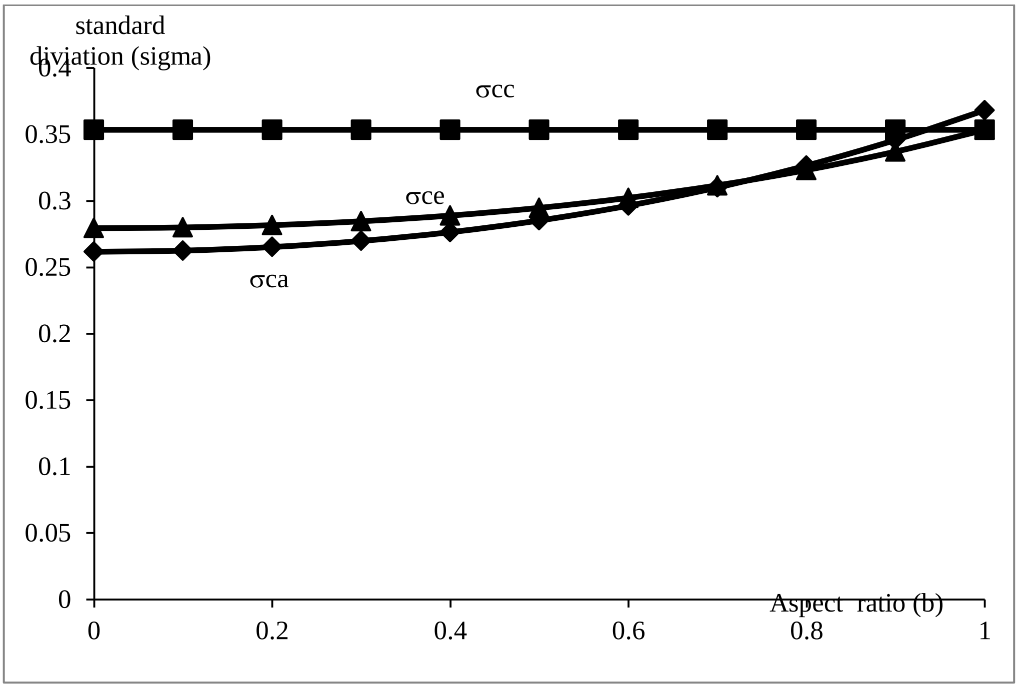

Fig.6 standard deviation (σ) versus aspect ratio(b) with coma aberration for circular aperture(cc), elliptical aperture(ce), and annular ellipse aperture(ca) with obscuration ratio=0.5.

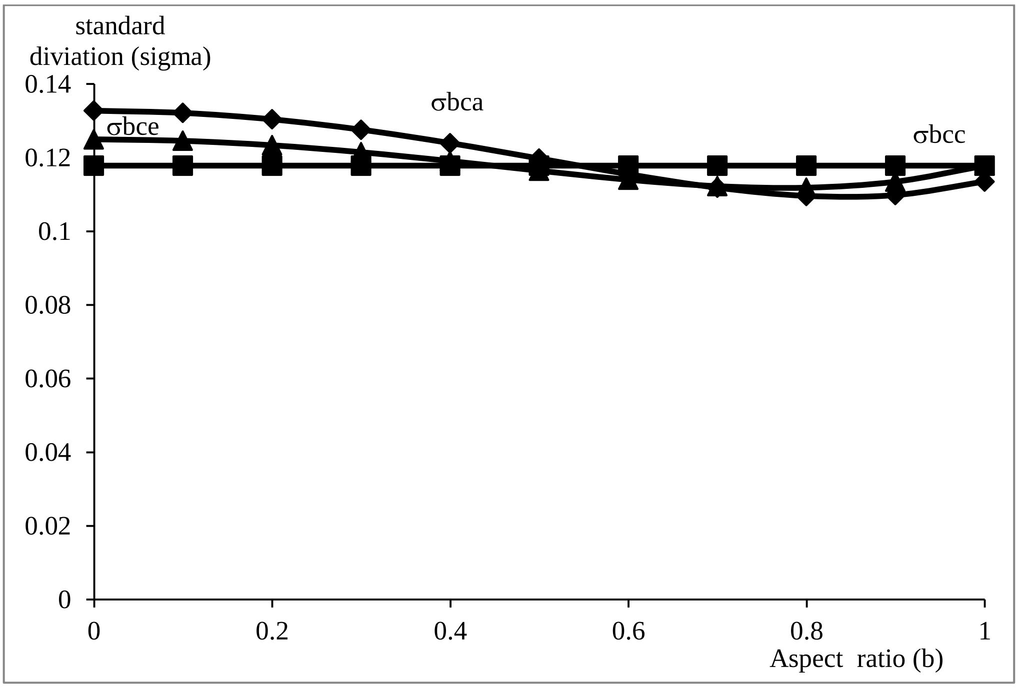

Fig.7 standard deviation (σ) versus aspect ratio(b) with balanced coma aberration for circular aperture(bcc), elliptical aperture(bce), and annular ellipse aperture(bca) with obscuration ratio=0.5.

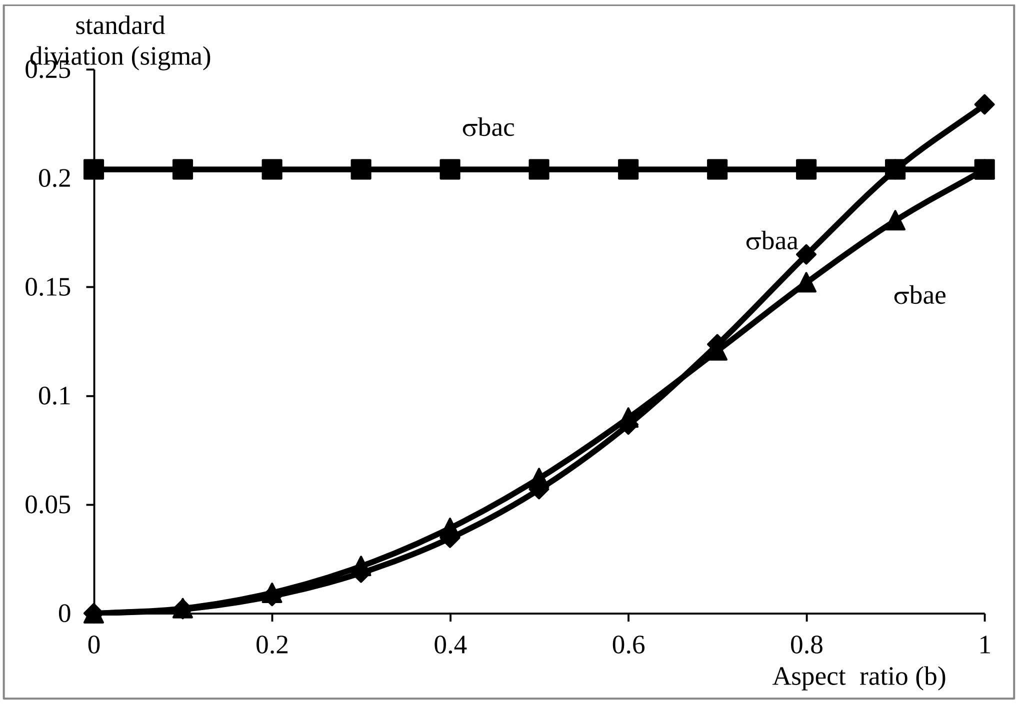

Fig.8 standard deviation (σ) versus aspect ratio(b) with balanced astigmatism aberration for circular aperture(bac), elliptical aperture(bae), and annular ellipse aperture(baa) with obscuration ratio=0.5.

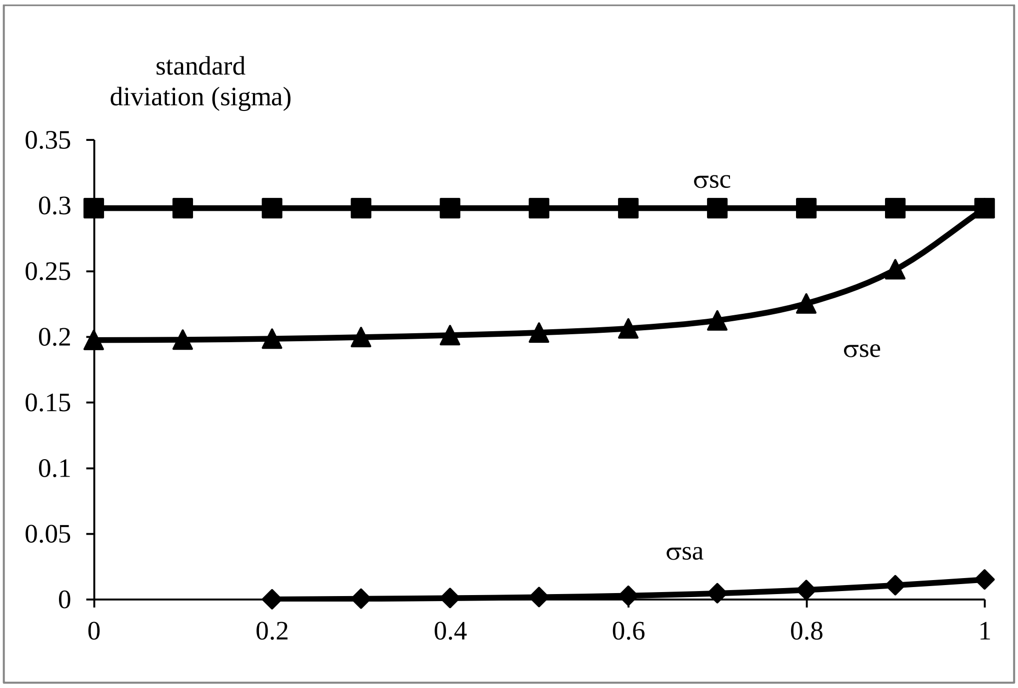

Fig.9 standard deviation (σ) versus aspect ratio(b) with spherical aberration for circular aperture(sc), elliptical aperture(se), and annular ellipse aperture(sa) with obscuration ratio=0.5.

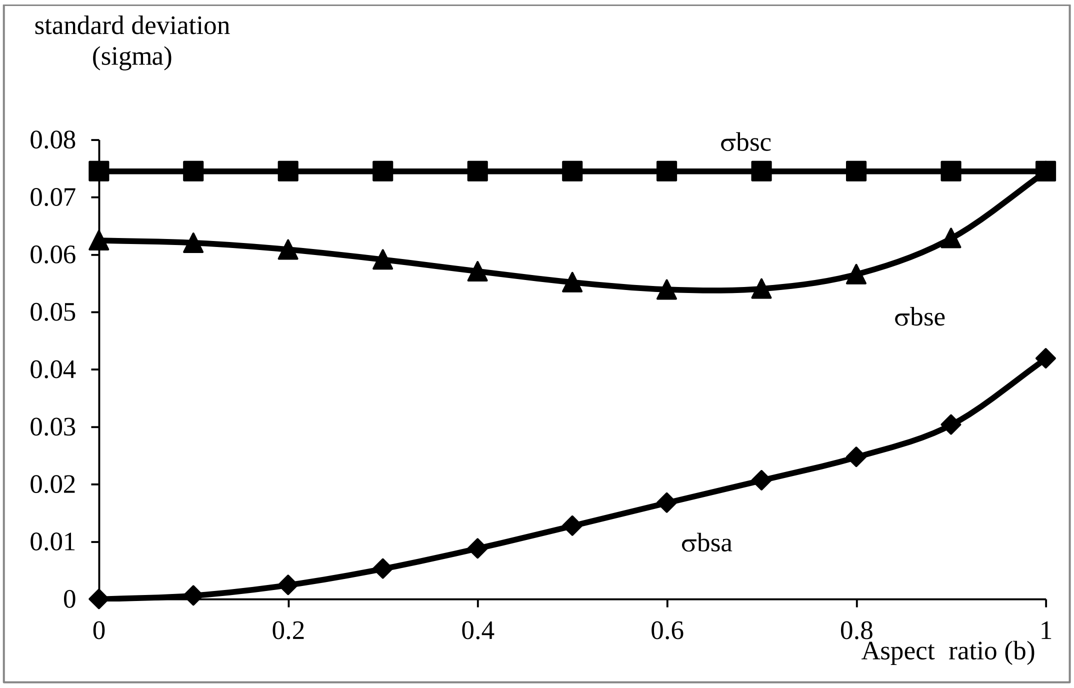

Fig.10 standard deviation (σ) versus aspect ratio(b) with balanced spherical aberration for circular aperture(bsc), elliptical aperture(bse), and annular ellipse aperture(bsa) with obscuration ratio=0.5.

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