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Vibration Characteristics of Non-Homogeneous Visco-Elastic Square Plate

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Abstract

A mathematical model is presented for the use of engineers, technocrats and research workers in space technology, mechanical Sciences have to operate under elevated temperatures. Two dimensional thermal effects on frequency of free vibrations of a visco-elastic square plate is considered. In this paper, the thickness varies parabolic in X- direction and thermal effect is vary linearly in one direction and parabolic in another direction. Rayleigh Ritz method is used to evaluate the fundamental frequencies. Both the modes of the frequency are calculated by the latest computational technique, MATLAB, for the various values of taper parameters and temperature gradient.

Keywords: visco-elastic, Square plate, vibration, Thermal gradient, Taper constant, non-homogeneous.

1 Introduction

In the engineering we cannot move without considering the effect of vibration because almost machines and engineering structures experiences vibrations. Structures of plates have wide applications in ships, bridges, etc. In the aeronautical field, analysis of thermally induced vibrations in non-homogeneous plates of variable thickness has a great interest due to their utility in aircraft wings.

As technology develops new discoveries have intensified the need for solution of various problems of vibrations of plates with elastic or visco-elastic medium. Since new materials and alloys are in great use in the construction of technically designed structures therefore the application of visco-elasticity is the need of the hour. Tapered plates are generally used to model the structures. Plates with thickness variability are of great importance in a wide variety of engineering applications.

The aim of present investigation is to study two dimensional thermal effect on the vibration of visco-elastic square plate whose thickness varies parabolic in X-direction and temperature varies linearly in one direction and parabolic in another direction. It is assumed that the plate is clamped on all the four edges and its temperature varies linearly in both the directions. Assume that non homogeneity occurs in Modulus of Elasticity. For various numerical values of thermal gradient and taper constants; frequency for the first two modes of vibration are calculated with the help of latest software. All results are shown in Graphs.

2 Equation Of Motion

Differential equation of motion for visco-elastic square plate of variable thickness in Cartesian coordinate [1]:

$$[D_1\left(W,_{xxxx}+2W,_{xxyy}+W,_{yyyy}\right)+2D_1,_x\left(W,_{xxx}+W,_{xyy}\right)+2D_{1,y}\left(W,_{yyy}+W,_{yxx}\right)+$$

$$D_{1,xx}(W,_{xx}+\nu W,_{yy})+D_{1,yy}(W,_{yy}+\nu W,_{xx})+2(1-\nu)D_{1,xy}W,_{xy}]-\rho hp^2W=$$

(1)

which is a differential equation of transverse motion for non-homogeneous plate of variable thickness. Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = Eh^3/12(1-\nu^2) \qquad (2)$$

and corresponding two-term deflection function is taken as [5]

$$W=[(x/a)(y/a)(1-x/a)(1-y/a)]^2[A_1+A_2(x/a)(y/a)(1-x/a)(1-y/a)] \qquad (3)$$

Assuming that the square plate of engineering material has a steady two dimensional, one is linear and another is parabolic temperature distribution i.e.

$$\tau = \tau_0 (1 - x/a)(1 - y^2/a^2) \qquad (4)$$

where, τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "*a*" is the length of a side of square plate. The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this

$$E = E_0(1 - \gamma\tau) \qquad (5)$$

where , E_0 is the value of the Young's modulus at reference temperature i.e. $\tau = 0$ and γ is the slope of the variation of E with τ . The modulus variation (2.5) become

$$E = E_0[1 - \alpha(1 - x/a)(1 - y^2/a^2)] \qquad (6)$$

where, $\alpha = \gamma\tau_0 \, (0 \le \alpha < 1)$ thermal gradient.

It is assumed that thickness also varies parabolically in one direction as shown below:

$$h = h_0(1 + \beta_1 x^2/a^2) \qquad (7)$$

where, β_1 & β_2 are taper parameters in x- & y- directions respectively and $h=h_0$ at x=y=0.

Put the value of E & h from equation (6) & (7) in the equation (2), one obtain

$$D_1 = [E_0[1 - \alpha(1 - x/a)(1 - y^2/a^2)]h_0^3(1 + \beta_1 x^2/a^2)^3]/12(1 - v^2) \qquad (8)$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(V^* - T^*) = 0 \qquad (9)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$\left. \begin{aligned} W = W_{,x} = 0 &, \quad x = 0, a \\ W = W_{,y} = 0 &, \quad y = 0, a \end{aligned} \right\} \qquad (10)$$

Now assuming the non-dimensional variables as

$$X = x/a, Y = y/a, \overline{W} = W/a, \overline{h} = h/a \qquad (11)$$

The kinetic energy T* and strain energy V* are [2]

$$T^* = (1/2)\rho p^2 \overline{h_0} a^5 \int_0^1 \int_0^1 [(1 + \beta_1 X^2)\overline{W^2}] dY dX \qquad (12)$$

and

$$\begin{aligned} V^* = Q\int_0^1 \int_0^1 [1 - \alpha(1 - X)(1 - Y^2)](1 + \beta_1 X^2)^3 \{(\overline{W}_{,XX})^2 + (\overline{W}_{,YY})^2 \\ + 2v\overline{W}_{,XX}\overline{W}_{,YY} + 2(1 - v)(\overline{W}_{,XY})^2\} dY dX \end{aligned} \qquad (13)$$

Using equations (12) & (13) in equation (9), one get

$$(V^{**} - \lambda^2 T^{**}) = 0 \qquad (14)$$

where,

$$\begin{aligned} V^{**} = \int_0^1 \int_0^1 [1 - \alpha(1 - X)(1 - Y^2)](1 + \beta_1 X^2)^3 \{(\overline{W}_{,XX})^2 + (\overline{W}_{,YY})^2 \\ + 2v\overline{W}_{,XX}\overline{W}_{,YY} + 2(1 - v)(\overline{W}_{,XY})^2\} dY dX \end{aligned} \qquad (15)$$

and

$$T^{**} = \int_0^1 \int_0^1 [(1 + \beta_1 X)\overline{W^2}] dY dX \qquad (16)$$

Here, $\lambda^2 = 12\rho(1 - v^2)a^2 / E_0 h_0^2$ is a frequency parameter.

Equation (16) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of W. These two constants are to be determined as follows

$$\partial(V^{**} - \lambda^2 T^{**}) / \partial A_n \quad , n = 1, 2 \tag{17}$$

On simplifying (17), one gets

$$bn_1 A_1 + bn_2 A_2 = 0 \quad , n = 1, 2 \tag{18}$$

where, bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (18) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \tag{19}$$

With the help of equation (19), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1(Mode1) & λ_2(Mode2) for different values of taper constant and thermal gradient for a clamped plate.

3 Results and Discussion

All calculations are carried out with the help of latest Matrix Laboratory computer software. Computation has been done for frequency of visco-elastic square plate for different values of taper constants β_1 and β_2, thermal gradient α, at different points for first two modes of vibrations have been calculated numerically.

In Fig I: - It is clearly seen that value of frequency decreases as value of thermal gradient increases from 0.0 to 1.0 for $\beta_1 = \beta_2 = 0.0$ for both modes of vibrations.

In Fig II: - Also it is obvious to understand the decrement in frequency for $\beta_1 = \beta_2 = 0.6$. But it is also noticed that value of frequency is increased with the increment in β_1 and β_2

In Fig III: - It is evident that frequency decreases continuously as thermal gradient increases, $\beta_1 = 0.2$, $\beta_2 = 0.4$ respectively with the two modes of vibration.

In Fig IV :- Increasing value of frequency for both of the modes of vibration is shown for increasing value of taper constant β_2 from 0.0 to 1.0 and $\beta_1 = 0.2$, $\alpha = 0.4$ respectively. Note that value of frequency increased.

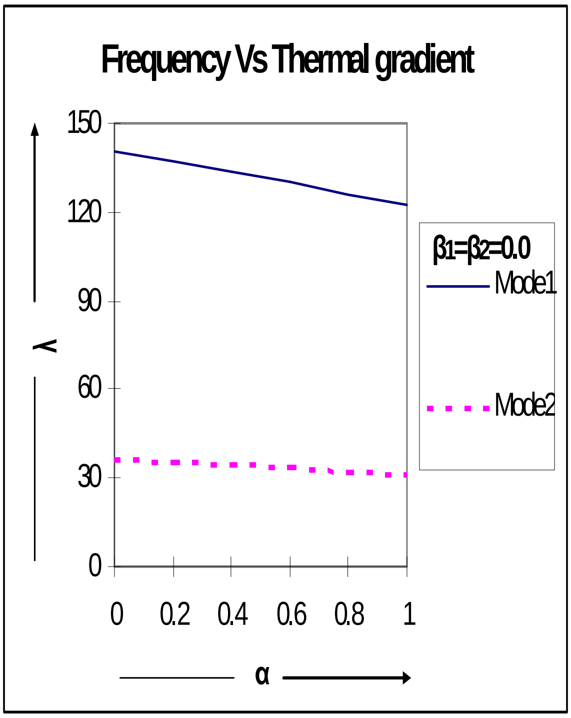

Fig I:- Frequency vs. thermal gradient at $\beta_1 = \beta_2 = 0.0$

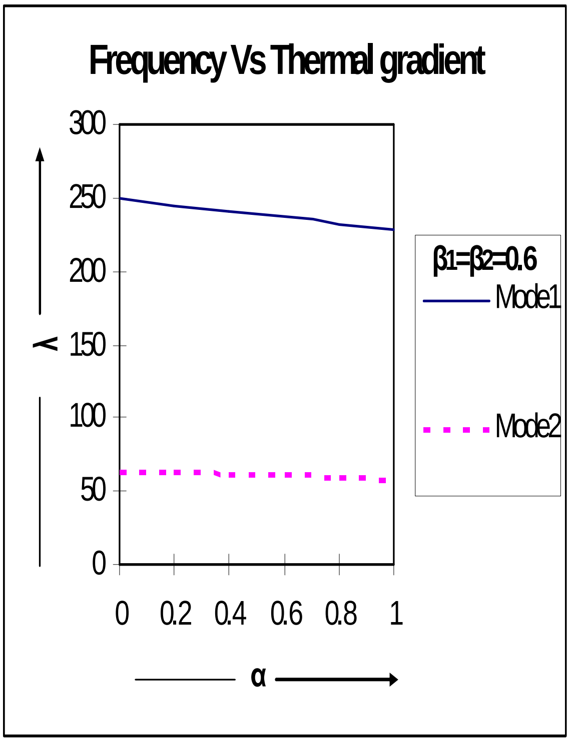

Fig II:- Frequency vs. thermal gradient at $\beta_1 = \beta_2 = 0.6$

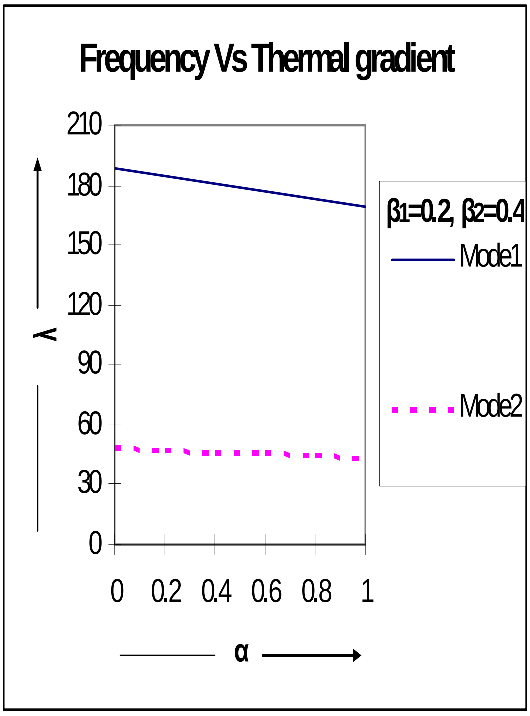

Fig III:- Frequency vs. thermal gradient at β_1=0.2, β_2=0.4

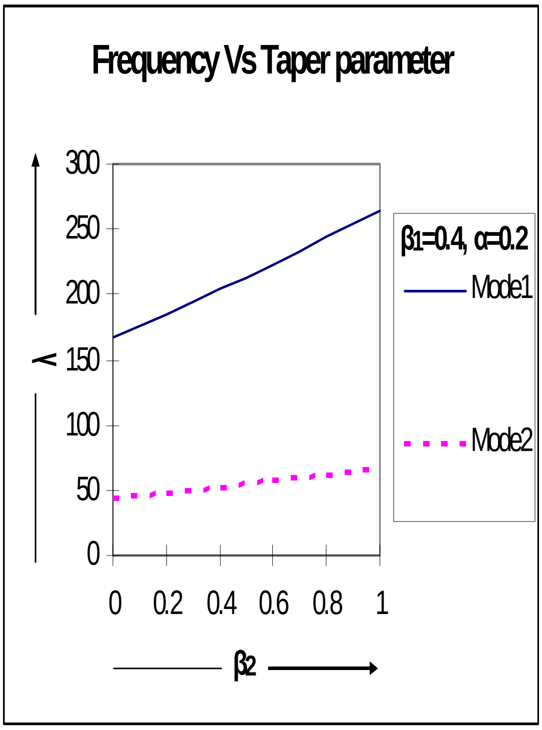

Fig IV:- Frequency vs. Taper constant at β_1=0.2, α=0.2

4. Conclusion

Results of the present paper are compared with [5] and it is found close agreement between the values of frequency for the corresponding values of parameters. Main aim for this research is to develop a theoretical mathematical model for scientists and design engineers so that they can make a use of it with a practical approach for the welfare of entire planet.

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