Performance Evaluation of Two-Node Tandem Communication Network with DBA having Compound Poisson Binomial Bulk Arrivals

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ABSTRACT

Communication Network is an important consideration for optimal utilization of resources. Several communication networks are studied through descriptive modelling dealing with characterizing the performance of the network. However, for optimal utilization of resources such as bandwidth, transmitters, routers, etc., the perspective modelling and analysis of communication networks is needed. In this paper, the optimal operating policies of a two-transmitter communication network are developed and analysed. Here, the arrivals of the network are characterized by compound binomial Poisson process and transmission of both the transmitters is characterized by Poisson process. The dynamic bandwidth allocation policy for transmission is considered. With suitable cost considerations, the expected total revenue function is derived and maximized with respect to the mean arrival rate, mean transmission rates. The sensitivity of the optimal policies with respect to the cost and input parameters is also studied through numerical illustrations. It is observed that the optimal policies are highly influenced by the input parameters and costs. This model is useful for scheduling the Internet, ATM, LAN, and WAN in several places.

Keywords: Optimal policy of a network, Dynamic Bandwidth Allocation, Compound Poisson process, two-node tandem communication network.

1. INTRODUCTION

Congestion occurs in communication systems due to unpredicted demand on transmission lines. The traditional best-effort Internet is developing into a flexible network that can offer various multimedia real time services in addition to the conventional data services and can improve the quality of service that guarantee to different users. The key concern in communication systems is to transmit the data/voice with high quality of service (QoS). Packet switching gives enhanced utilization over circuit or message switching and yields comparatively short delay and improve the QoS. The delays in packet switching can be reduced by using the statistical multiplexing in communication networks. Several communication networks which support teleprocessing applications are mixed with statistical multiplexing and dynamic engineering skills (Gaujal and Hyon, 2002).

In store and forward communication systems, the transmitters are joined in tandem having more than one transmitter. The voice quality over transmission is much effected by the transmission strategies, when the transmitters are coupled through buffers. The statistical multiplexing with load dependent strategy has been evolved through bit-dropping and flow control techniques to decrease congestion in buffers (Sriram et al, (1994), Srinivasa Rao et al (2001), Kim, (2002)). The variation on transmission rates based on content of the buffer is to be considered for efficient transmission with high quality. This kind of adjusting the transmission rates based on the content of the buffer is known as dynamic bandwidth allocation (DBA) (Srinivasa Rao et. al (2008)), Marco Mezavilla (2011), Patras (2009).

Few works have been reported in literature concerning communication networks with DBA strategy (P.S.Varma et al, (2007), and Padmavathi.G,et.al (2009)). They considered communication network models with the assumption that the transmission rate of packet is adjusted instantaneously depending on the content of the buffer. However, they assumed that the packets arrive to the buffers connected to the transmitters are in single and follows a Poisson process. But, in practice the messages arrived at the source are converted into a random number of packets depending on the size of the message. As a result of it, the arrival of packets to the buffers is in bulk and the arrival process can be characterized with compound Poisson process. The compound Poisson process characterises the statistical nature of the bulk arrival of packets to the buffers and analyzes the communication systems more close to the reality. Poisson process is a particular case of compound Poisson process.

Realizing the importance of compound Poisson process, Kuda Nageswara Rao et al (2010, 2011) and Suhasini et al (2012) have developed and analyzed communication network models with dynamic bandwidth allocation having bulk arrivals. They assumed that the number of packets in any arriving module is random and follows a uniform distribution. The assumption of batch size distribution being uniform work well only when the message length/the content of the message is uniform. But, in many practical situations, the message length may not be uniform and the number of packets that the message may be converted is skewed/symmetric depending on the length of the message. A close look into the arrival stream of messages in a communication network reveals that the number of packets that can be converted from a message follows a binomial distribution.

In general, in communication networks the models are analysed under steady state behaviour, due to its simplicity. But, in many communication networks, the steady state measures of system performance simply do not make sense when the practitioner needs to know how the system operates up to some specified time (P.R.Parthasarathy et al, (2001)). The behaviour of the system could be understood more effectively with the help of time dependent analysis. The laboratory experimentation is time consuming and expensive, hence it is desirable to develop communication network models and their analysis under transient conditions.

In addition to this, in communication networks the utilization of the resources is one of the major considerations. In designing the communication networks two aspects are to be considered. They are congestion control and packet scheduling. Earlier these two aspects are dealt separately. But, the integration of these two is needed in order to utilize resources more effectively and efficiently. Little work has been reported in literature regarding optimization of communication networks. Matthew Andrews (2006) considered the joint optimization of scheduling and congestion control in communication networks. He considered a constrained optimization problem under non-parametric methods of characterizing the communication network. In general the non-parametric methods are less efficient than parametric methods of modelling. Hence, in this paper we develop and analyse a scheduling and routing algorithm for the two-transmitter tandem communication network with dynamic bandwidth allocation having binomial bulk arrivals. This communication network model is much useful for improving the quality of service (QoS) avoiding wastage in internet, intranet, LAN, WAN and MAN.

Using the difference differential equations, the transient behaviour of the communication network is analyzed. Section 2 deals with the development and analysis of two transmitter tandem communication network with dynamic bandwidth allocation having compound Poisson binomial bulk arrivals. Section 3 deals with performance evaluation of the proposed communication network. Section 4 deals with optimal operating policies of the communication network. The solution procedure is demonstrated in section 5 through numerical illustration. The sensitivity of the optimal policies with respect to the changes in model parameters and costs is discussed in section 6. In section 7, the conclusions along with scope for further work is discussed.

2. QUEUING MODEL

Consider a two-transmitter tandem communication network in which the messages arrive to the network are converted into a random number of packets. The arrival process of the messages is random and a number of packets (X) that a message can be converted follows a binomial distribution with parameters m and p i.e., the arrival modules follows a compound Poisson binomial process with composite arrival rate α. E(X). The probability mass function of the number of packets that a message can be converted is

$$C_k = \frac{{}^{m}C_k p^k (1-p)^{m-k}}{1-(1-p)^m}; \quad K=1,2,\ldots.m.\ 0<p<1,$$

The transmission process at each transmitter follows a Poisson process. The transmission rate at each transmitter depends on the content of the buffer connected to it. Let β_1 and β_2 are the transmission rates of transmitter 1 and transmitter 2 respectively. The schematic diagram representing the communication network is shown in Figure1.

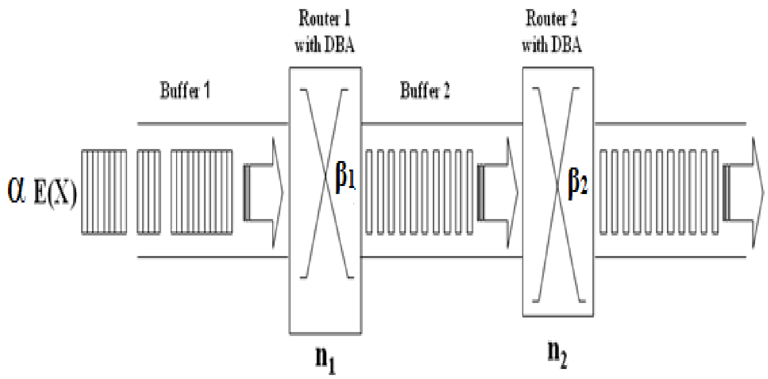

Figure 1: Communication network with dynamic bandwidth allocation and bulk arrivals

Let $P_{n_1,n_2}(t)$ be the probability that there are n_1 packets in the first buffer and n_2 packets in the second buffer at time t. With this structure, the difference - differential equations of the Communication network are:

$$\frac{\partial P_{n_1,n_2}(t)}{\partial t} = -(\alpha + n_1\beta_1 + n_2\beta_2) P_{n_1,n_2}(t) + (n_1+1)\beta_1 P_{n_1+1,n_2-1}(t) + (n_2+1)\beta_2 P_{n_1,n_2+1}(t) + \alpha\left[\sum_{k=1}^{n_1} P_{n_1-k,n_2}(t)\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\right] \quad (1)$$

$$\frac{\partial P_{n_1,0}(t)}{\partial t} = -(\alpha + n_1\beta_1) P_{n_1,0}(t) + \beta_2 P_{n_1,1}(t) + \alpha \sum_{k=1}^{n_1} P_{n_1-k,0}(t)\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m} \quad (2)$$

$$\frac{\partial P_{0,n_2}(t)}{\partial t} = -(\alpha + n_2\beta_2) P_{0,n_2}(t) + \beta_1 P_{1,n_2-1}(t) + (n_2+1)\beta_2 P_{0,n_2+1}(t)$$
(3)

$$\frac{\partial P_{0,0}(t)}{\partial t} = -\alpha P_{0,0}(t) + \beta_2 P_{0,1}(t) \quad (4)$$

$$\frac{\partial P_{1,0}(t)}{\partial t} = -(\alpha + \beta_1) P_{1,0}(t) + \beta_2 P_{1,1}(t) + \alpha P_{0,0}(t) C_1 \quad (5)$$

$$\frac{\partial P_{0,1}(t)}{\partial t} = -(\alpha + \beta_2) P_{0,1}(t) + \beta_1 P_{1,0}(t) + 2\beta_2 P_{0,2}(t) \quad (6)$$

with initial conditions

$P_{00}(0) = 1; P_{n_1,n_2}(0) = 0$ for n_1, $n_2>0$

Let $P(Z_1,Z_2;t)$ be the joint probability generating function of $P_{n_1,n_2}(t)$ and C_Z is the probability generating function of { C_k }. Then

$$P(z_1, z_2; t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} Z_1^{n_1} Z_2^{n_2} P_{n_1,n_2}(t) \text{ and } C_z = \sum_{k=1}^{m} \frac{{}^{m}C_k p^k (1-p)^{m-k}}{1-(1-p)^m} Z^k \quad (7)$$

Multiplying the equations (1) to (7) with corresponding $Z_1^{n_1}, Z_2^{n_2}$ and summing overall n_1=0, 1, 2, 3, … and n_2=0, 1, 2, 3, … one can get

$$\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\frac{\partial}{\partial t}P_{n_1,n_2}(t).Z_1^{n_1}Z_2^{n_2}=-\left[\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}(\alpha+n_1\beta_1+n_2\beta_2)P_{n_1,n_2}(t).Z_1^{n_1}Z_2^{n_2}\right]$$

$$+\sum_{n_1=1}^{\infty}\sum_{n_2}^{\infty}(n_1+1)\beta_1P_{n_1+1,n_2-1}(t).Z_1^{n_1}Z_2^{n_2}$$

$$+\sum_{n_1=1}^{\infty}\sum_{n_2}^{\infty}(n_2+1)\beta_2P_{n_1,n_2+1}(t).Z_1^{n_1}Z_2^{n_2}$$

$$+\sum_{n_1=1}^{\infty}\sum_{n_2}^{\infty}\left[\alpha\sum_{k=1}^{m}P_{n_1-k,n_2}(t).\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}\right]Z_1^{n_1}Z_2^{n_2} \tag{8}$$

After simplification, we have

$$\frac{\partial P(Z_1,Z_2;t)}{\partial t}=\left[\alpha(c(Z_1)-1)\right]P(Z_1,Z_2;t)+\left[\beta_1(Z_2-Z_1)\right].\frac{\partial P(Z_1,Z_2;t)}{\partial Z_1}+\left[\beta_2(1-z_2)\right].\frac{\partial P(Z_1,Z_2;t)}{\partial Z_2}$$

Rearranging the terms we get

$$\frac{\partial P(Z_1,Z_2;t)}{\partial t}-\left[\beta_1(Z_2-Z_1)\right].\frac{\partial P(Z_1,Z_2;t)}{\partial Z_1}-\left[\beta_2(1-z_2)\right].\frac{\partial P(Z_1,Z_2;t)}{\partial Z_2}=\left[\alpha(c(Z_1)-1)\right]P(Z_1,Z_2;t) \tag{9}$$

Using the Lagrangian's method, the auxiliary equations of the equation (9) are

$$\frac{\partial t}{1}=\frac{-\partial Z_1}{\beta_1(Z_2-Z_1)}=\frac{-\partial Z_2}{\beta_2(1-Z_2)}=\frac{\partial P(Z_1Z_2;t)}{\left[\alpha(c(Z_1)-1)\right]P(Z_1,Z_2;t)} \tag{10}$$

$$u=(Z_2-1)e^{-\mu_2t} \tag{11}$$

$$v=\left[(Z_1-1)+\frac{\beta_1}{\beta_2-\beta_1}(Z_2-1)\right]e^{-\beta_1t} \tag{12}$$

and $$w=P(Z_1,Z_2,t).\exp\left[-\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}({}^kC_r)({}^rC_J)\left(\frac{u\beta_1}{\beta_2-\beta_1}\right)^J v^{(r-J)}\frac{e^{[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right] \tag{13}$$

where, **u** and **v** are as given in (10) and (11) respectively. Therefore

$$P(Z_1,Z_2,t)=w.\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}({}^kC_r)({}^rC_J)\left(\frac{u\beta_1}{\beta_2-\beta_1}\right)^J v^{(r-J)}\frac{e^{[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right]$$

Substituting the value of '**w**' one can get

$$P(Z_1,Z_2,t)=\left(\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}({}^kC_r)({}^rC_J)\left(\frac{u\beta_1}{\beta_2-\beta_1}\right)^J v^{(r-J)}\frac{1}{J\beta_2+(r-J)\beta_1}\right]\right)$$

$$\left(\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}({}^kC_r)({}^rC_J)\left(\frac{u\beta_1}{\beta_2-\beta_1}\right)^J v^{(r-J)}\frac{e^{[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right]\right)$$

This implies

$$P(Z_1, Z_2, t) = \exp\left[\alpha \sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{u\beta_1}{\beta_2-\beta_1}\right)^J v^{(r-J)}\left(\frac{e^{[J\beta_2+(r-J)\beta_1]t}-1}{J\beta_2+(r-J)\beta_1}\right)\right]$$

Substituting the values of **u** and **v** in the above equation and simplifying, one can get the joint probability generating function of the number of packets in the first transmitter and second transmitter as

$$P(Z_1, Z_2, t) = \exp\left[\alpha \sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1(Z_2-1)}{\beta_2-\beta_1}\right)^J \left((Z_1-1)+\frac{\beta_1(Z_2-1)}{\beta_2-\beta_1}\right)^{r-J}\frac{\left(1-e^{[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right] \quad (14)$$

3. PERFORMANCE MEASURES OF THE NETWORK

In this section, we derive and analyze the performance measures of the communication network under transient condition. From the equation (14), the joint probability generating function of the number of packets in both the buffers is

$$P(Z_1, Z_2, t) = \exp\left[\alpha \sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1(Z_2-1)}{\beta_2-\beta_1}\right)^J \left((Z_1-1)+\frac{\beta_1(Z_2-1)}{\beta_2-\beta_1}\right)^{r-J}\frac{\left(1-e^{[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right] \quad (15)$$

Taking $Z_2=1$, we get the probability generating function of the first buffer size distribution as

$$P(Z_1, t) = \exp\left[\alpha \sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r\,(Z_1-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right] \quad (16)$$

Expanding the equation $P(Z_1,t)$ and collecting the constant terms, we get the probability that the first buffer is empty as

$$p_{0.}(t) = \exp\left[\alpha \sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r\,(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right] \quad (17)$$

The mean number of packets in the first buffer is

$$L_1 = \frac{\alpha}{\beta_1}\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}.k\left(1-e^{-\beta_1 t}\right)\right] \quad (18)$$

The utilization of the first transmitter is

$$U_1 = 1 - p_{0.}(t)$$

$$U_1 = 1-\exp\left[\alpha \sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r\,(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right] \quad (19)$$

The throughput of the first transmitter is

$$Thp_1 = \beta_1.U_1 = \beta_1(1-\exp)(\alpha \sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r\,(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}) \quad (20)$$

The average delay in the first buffer is

$$W\left(N_1\right)=\frac{L_1}{Thp_1}=\frac{\left(\frac{\alpha}{\beta_1}\right)\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}.k\left(1-e^{-\beta_1 t}\right)\right]}{\beta_1\left(1-\exp\right)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]} \tag{21}$$

The variance of the number of packets in the first buffer is

$$Var\left(N_1\right)=E\left[N_1^2-N_1\right]+E\left[N_1\right]-\left(E\left[N_1\right]\right)^2$$

$$=\alpha\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}k(k-1)\left(\frac{1-e^{-2\beta_1 t}}{2\beta_1}\right)+\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}k\left(\frac{1-e^{-\beta_1 t}}{\beta_1}\right)\right] \tag{22}$$

The coefficient of variation of the number of packets in the first buffer is

$$cv\left(N_1\right)=\frac{\sqrt{Var\left(N_1\right)}}{L_1} \tag{23}$$

Similarly, taking Z_1 =1 in (14), we get the probability generating function of the second buffer size distribution as

$$P(Z_2,t)=\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1(Z_2-1)}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right] \tag{24}$$

Expanding the equation $P(Z_{2,}t)$ and collecting the constant terms, we get the probability that the second buffer is empty as

$$p_{.0}(t)=\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right] \tag{25}$$

The mean number of packets in the second buffer is

$$L_2=\frac{\alpha}{\beta_2}\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}.k\left[\left(1-e^{-\beta_2 t}\right)+\frac{\beta_2}{\beta_2-\beta_1}\left(e^{-\beta_2 t}-e^{-\beta_1 t}\right)\right]\right] \tag{26}$$

The utilization of the second transmitter is

$$U_2=1-p_{.0}(t)$$

$$=1-\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right] \tag{27}$$

The throughput of the second transmitter is

$$Thp_2=\beta_2.U_2$$

$$=\beta_2\left[1-\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right]\right] \tag{28}$$

The average delay in the second buffer is

$$W(N_2)=\frac{L_2}{Thp_2}=\frac{\frac{\alpha}{\beta_2}\left[\sum_{k=1}^{m}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}.k\left(1-e^{-\beta_2t}\right)+\frac{\beta_2}{\beta_2-\beta_1}\left(e^{-\beta_2t}-e^{-\beta_1t}\right)\right]}{\beta_2(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}({}^kC_r)({}^rC_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right]} \quad (29)$$

The variance of number of packets in the second buffer is

$$Var(N_2)=E\left[N_2^2-N_2\right]+E\left[N_2\right]-\left(E\left[N_2\right]\right)^2$$

$$=\left\{\left(\sum_{k=1}^{m}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}.k(k-1)\right)\left(\frac{\beta_1}{\beta_1-\beta_2}\right)^2\left[\left(\frac{1-e^{-2\beta_1t}}{2\beta_1}\right)-2\left(\frac{1-e^{-(\beta_1+\beta_2)t}}{\beta_1+\beta_2}\right)+\left(\frac{1-e^{-2\beta_2t}}{2\beta_2}\right)\right]\right\}$$

$$+\left\{\alpha\sum_{k=1}^{m}k.\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}\left[\left(\frac{1-e^{-\beta_2t}}{\beta_2}\right)-\left(\frac{e^{-\beta_2t}-e^{-\beta_1t}}{\beta_1-\beta_2}\right)\right]\right\} \quad (30)$$

The coefficient of variation of the number of packets in the second buffer is

$$cv(N_2)=\frac{\sqrt{Var(N_2)}}{L_2} \quad (31)$$

Expanding the equation (14) and collecting the constant terms, we get the probability that the network is empty as

$$P_{00}(t)=\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}({}^kC_r)({}^rC_J)(\beta_1)^J\frac{(-\beta_2)^{r-J}}{(\beta_2-\beta_1)^r}\frac{\left(1-e^{[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right] \quad (32)$$

The mean number of packets in the network is

$$L_N=L_1+L_2 \quad (33)$$

where,
L_1 is the mean number of packets in the first transmitter
L_2 is the mean number of packets in the second transmitter

4. Performance Evaluation of the Network

In this section, the performance of the proposed network is discussed through numerical illustration. Different values of the parameters are considered for bandwidth allocation and arrival of packets. After interacting with the technical staff at the Internet providing station, it is considered that the message arrival rate (α) varies from $1x10^4$ messages/sec to $4x10^4$ massages/sec. The number of packets that can be converted into a message varies from 1 to 30 depending on the length of the message. Hence, the number of arrivals of packets to the buffer are in batches of random size. The batch size is assumed to follow Binomial distribution with parameters (m, p). After transmitter 1, the packets are forwarded to the second buffer connected to the second transmitter, with forward transmission rate (β_1**)** varies from $1x10^4$ packets/sec to $5x10^4$ packets/sec. the packets leave the second transmitter with a transmission rate (β_2) which varies from $1x10^4$ packets/sec to $5x10^4$ packets/sec. In both the nodes, dynamic bandwidth allocation is considered i.e. the transmission rate of each packet depends on the number of packets in the buffer connected to it at the instant.

Since performance characteristics of the communication network are highly sensitive with respect to time, the transient behaviour of the model is studied through computing the performance measures with the following set of values for the model parameters:

t= 0.1, 0.2, 0.3, 0.4 seconds
m= 5, 6, 7, 8
p= 0.1, 0.2, 0.3, 0.4xm
α= 1, 2, 3, 4 (with multiplication of 10^4 packets/sec)

β_1= 1, 2, 3, 5 (with multiplication of 10^4 packets/sec)
β_2= 1, 2, 3, 5 (with multiplication of 10^4 packets/sec)

From equations (20) to (30), the transmission rate of first transmitter and the transmission rate of second transmitter are computed for different values of t, m, p, α, β_1, β_2 and given in Table 1.The relationship between the parameters and the probability of emptiness are shown in Figure 2.

Table 1: **Values of Network and Buffers Emptiness Probabilities of The Communication Network With Dynamic Bandwidth Allocation And Binomial Bulk Arrivals**

t*	**m**	**p**	**α#**	**$\beta_1^{\$}$**	**$\beta_2^{\$}$**	**$P_{00}(t)$**	**$P_{0.}(t)$**	**$P_{.0}(t)$**
0.1	3	0.2	2	4	8	0.761	0.843	0.968
0.2	3	0.2	2	4	8	0.707	0.747	0.915
0.3	3	0.2	2	4	8	0.360	0.686	0.866
0.4	3	0.2	2	4	8	0.025	0.595	0.768
0.2	**5**	0.2	2	4	8	0.605	0.735	0.902
0.2	**6**	0.2	2	4	8	0.547	0.730	0.895
0.2	**7**	0.2	2	4	8	0.485	0.725	0.889
0.2	**8**	0.2	2	4	8	0.421	0.720	0.882
0.2	3	**0.1**	2	4	8	0.751	0.753	0.921
0.2	3	**0.2**	2	4	8	0.707	0.747	0.915
0.2	3	**0.3**	2	4	8	0.657	0.740	0.907
0.2	3	**0.4**	2	4	8	0.602	0.732	0.899
0.2	3	0.2	**1**	4	8	0.841	0.804	0.956
0.2	3	0.2	**2**	4	8	0.707	0.747	0.915
0.2	3	0.2	**3**	4	8	0.594	0.645	0.875
0.2	3	0.2	**4**	4	8	0.500	0.558	0.837
0.2	3	0.2	2	**1**	8	0.585	0.691	0.972
0.2	3	0.2	2	**2**	8	0.622	0.711	0.949
0.2	3	0.2	2	**3**	8	0.663	0.729	0.930
0.2	3	0.2	2	**5**	8	0.707	0.747	0.915
0.2	3	0.2	2	4	**1**	0.564	0.747	0.876
0.2	3	0.2	2	4	**2**	0.579	0.747	0.882
0.2	3	0.2	2	4	**3**	0.596	0.747	0.889
0.2	3	0.2	2	4	**5**	0.634	0.747	0.900

*=Seconds, #=Multiples of 10, 000 messages/seconds, $=Multiples of 10,000 packets/second

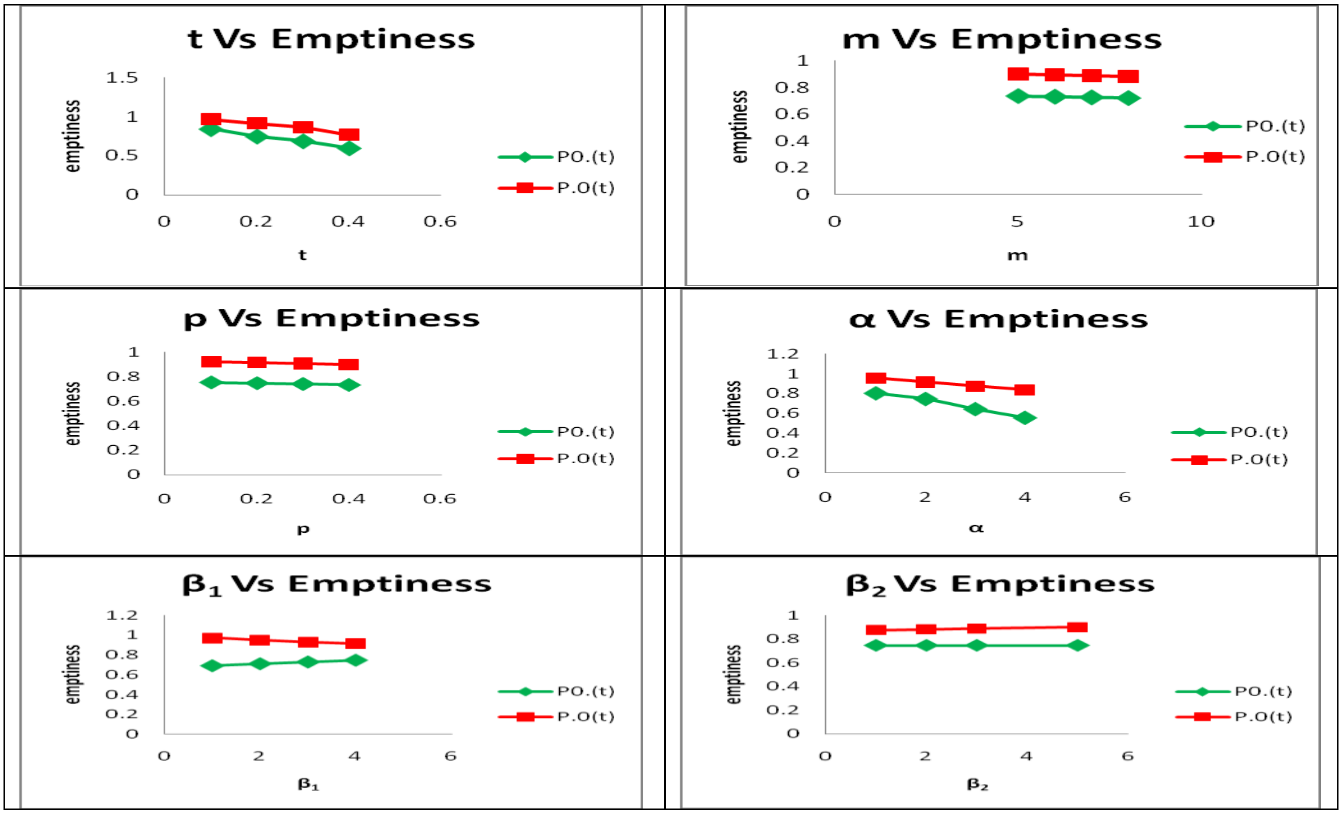

Figure 2: The relationship between Emptiness probability and various parameters

It is observed that the probability of emptiness of the communication network and the two buffers are highly sensitive with respect to changes in time. As time (t) varies from 0.1 to 0.4 second, the probability of emptiness in the network reduces from 0.761 to 0.025 when other parameters are fixed at (3, 0.2, 2, 4, 8) for (m, p, α, $\beta_{1,}$ β_2). Similarly, the probabilities of emptiness of the two buffers reduce from 0.843 to 0.595 and 0.968 to 0.768 for node 1 and node 2 respectively. The decrease in node 1 is more rapid when compared to node 2.

When the batch size distribution of number of packets a message can be converted (m) varies from 5 to 8, the probability of emptiness of the network decreases from 0.605 to 0.421 when other parameters are fixed at (0.2, 0.2, 2, 4, 8) for (t, p, α, $\beta_{1,}$ β_2). The same phenomenon is observed with respect to the first and second nodes. The probabilities of emptiness of the first and second buffers decrease from 0.735 to 0.720 and 0.902 to 0.882 respectively.

When the batch size distribution parameter (p) varies from 0.1 to 0.4, the probability of emptiness of the network decreases from 0.751 to 0.602 when other parameters are fixed at (0.2, 3, 2, 4, 8) for (t, m, α, $\beta_{1,}$ β_2). The same phenomenon is observed with respect to the first and second nodes. The probabilities of emptiness of the first and second buffers decrease from 0.753 to 0.732 and 0.921 to 0.899 respectively.

The influence of arrival of messages on system emptiness is also studied. As the arrival rate (α) varies from $1x10^4$ messages/sec to $4x10^4$ message/sec, the probability of emptiness of the network decreases from 0.841 to 0.500 when other parameters are fixed at (0.2, 3, 0.2, 4, 8) for (t, m, p, $\beta_{1,}$ β_2). The same phenomenon is observed with respect to the first and second nodes.

This decline is more in first node and moderate in the second node.

When the transmission rate of first transmitter (β_1) varies from $1x10^4$ packet/sec to $5x10^4$ packet/sec, the probability of emptiness of the network and the first buffer increases from 0.585 to 0.707 and 0.691 to 0.747 respectively and the probability of emptiness of the second buffer decreases from 0.972 to 0.915 when other parameters remain fixed at (0.2, 3, 0.2, 2, 8) for (t, m, p, α, β_2).

When the transmission rate of second transmitter (β_2) varies from $1x10^4$ packet/sec to $5x10^4$ packet/sec, the probability of emptiness of the network and the second buffer increases from 0.564 to 0.634 and 0.876 to 0.900 respectively when other parameters remain fixed at (0.2, 3, 0.2, 2, 4) for (t, m, p, α, β_1).

From the equations (19 to 26 and 27), the mean number of packets and the utilization of the network are computed for different values of t, m, p, α, $\beta_{1,}$ β_2 and are given in Table 2. The relationship between mean number of packets in the buffers, utilization of the nodes with the parameters t, m, p, α, $\beta_{1,}$ β_2 is shown in Figure 3.

Table 2: Values of mean number of packets and utilization of the communication network with dynamic bandwidth allocation and bulk arrivals

t*	**m**	**p**	**α#**	**$\beta_1^\$$**	**$\beta_2^\$$**	**L_1**	**L_2**	**U_1**	**U_2**	**Ln**
0.1	3	0.2	2	4	8	0.165	0.033	0.157	0.032	0.198
0.2	3	0.2	2	4	8	0.275	0.093	0.253	0.085	0.369
0.3	3	0.2	2	4	8	0.349	0.150	0.314	0.134	0.500
0.4	3	0.2	2	4	8	0.399	0.306	0.405	0.232	0.705
0.2	**5**	0.2	2	4	8	0.275	0.113	0.265	0.098	0.388
0.2	**6**	0.2	2	4	8	0.275	0.123	0.270	0.105	0.399
0.2	**7**	0.2	2	4	8	0.275	0.134	0.275	0.111	0.410
0.2	**8**	0.2	2	4	8	0.275	0.146	0.280	0.118	0.421
0.2	3	**0.1**	2	4	8	0.275	0.084	0.247	0.079	0.359
0.2	3	**0.2**	2	4	8	0.275	0.093	0.253	0.085	0.369
0.2	3	**0.3**	2	4	8	0.275	0.104	0.260	0.093	0.379
0.2	3	**0.4**	2	4	8	0.275	0.116	0.268	0.101	0.391
0.2	3	0.2	**1**	4	8	0.138	0.470	0.136	0.044	0.184
0.2	3	0.2	**2**	4	8	0.275	0.630	0.253	0.085	0.369
0.2	3	0.2	**3**	4	8	0.413	0.840	0.355	0.125	0.553
0.2	3	0.2	**4**	4	8	0.551	0.986	0.442	0.613	0.737
0.2	3	0.2	2	**1**	8	0.363	0.290	0.309	0.028	0.391
0.2	3	0.2	2	**2**	8	0.330	0.530	0.289	0.051	0.383
0.2	3	0.2	2	**3**	8	0.301	0.750	0.271	0.070	0.375
0.2	3	0.2	2	**4**	8	0.275	0.930	0.253	0.085	0.369
0.2	3	0.2	2	4	**1**	0.275	0.143	0.253	0.124	0.418
0.2	3	0.2	2	4	**2**	0.275	0.134	0.253	0.118	0.409
0.2	3	0.2	2	4	**3**	0.275	0.125	0.253	0.111	0.401
0.2	3	0.2	2	4	**5**	0.275	0.111	0.253	0.100	0.386

*=Seconds, #=Multiples of 10, 000 messages/seconds, $=Multiples of 10,000 packets/second

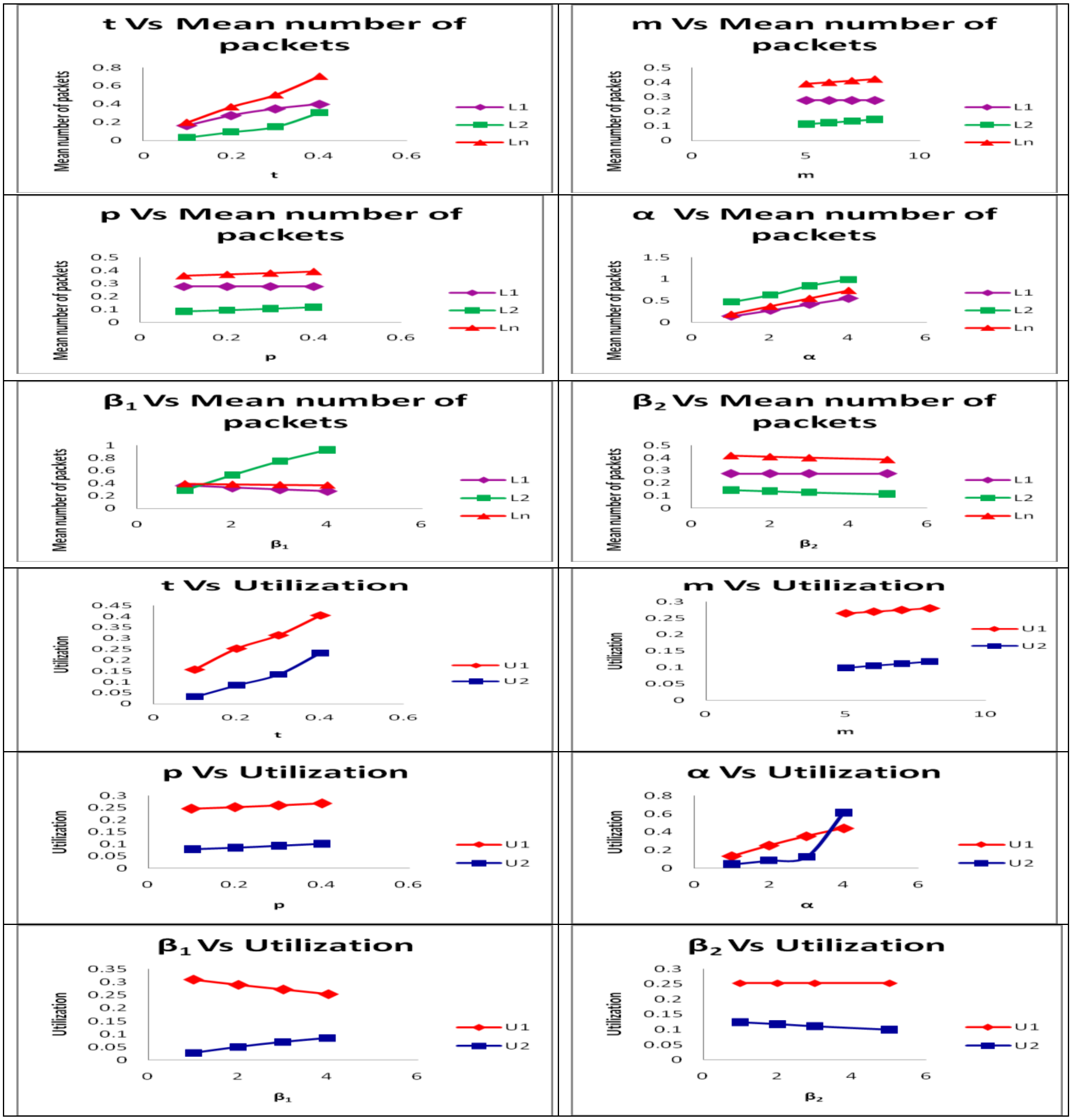

Fig 3: The relation between Mean number of packets and utilization for various parameters

It is observed that after 0.1 seconds, the first buffer is having on an average of 1650 packets, after 0.4 seconds it rapidly raised to an average of 3990 packets for fixed values of other parameters (3, 0.2, 2, 4, 8) for (m, p, α, $\beta_{1,}$ β_2). It is also observed that as time (t) varies from 0.1 to 0.4 seconds, average content of the second buffer and the network increases from 0330 packets to 3060 packets and from 1980 to 7050 packets respectively.

As the batch size distribution parameter (m) varies from 5 to 8, the first buffer values remains unchanged as 2750 packets, the second buffer and the network average content increase from 1130 packets to 1460 packets and 3880 packets to 4210 packets. As the batch size distribution parameter (p) varies from 0.1 to 0.4, the first buffer value remains unchanged and the second buffer, network average content increases from 1130 packets to 1460 packets 3880 packets to 4210 packets respectively when other parameters remain fixed.

As the arrival rate of messages (α) varies from $1x10^4$ messages/sec to $4x10^4$ messages/sec the, the first buffer, second buffer and the network average content increase from 1380 packets to 5510 packets, 4700 packets to 9860 packets and 1840 packets to 7370 packets respectively when other parameters remain fixed at (0.2, 3, 0.2, 4, 8) for (t, m, p, β_1, β_2). As the transmission rate of first transmitter (β_1) varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec, the first buffer and the network average content decrease from 3630 packets to 2750 packets and from 3910 packets to 3690 packets, the second buffer average increases from 2900 packets to 9300 packets respectively when other parameters remain fixed at (0.2, 3, 0.2, 2, 8) for (t, m, p, α, β_2).

As the transmission rate of second transmitter (β_2) varies from $1x10^4$ packets/sec to $5x10^4$ packets/sec, the second buffer and the network average content decreases from 1430 packets to 1110 packets and from 4180 packets to 3860 packets respectively when other parameters remain fixed at (0.2, 3, 0.2, 2, 4) for (t, m, p, α, β_1). It is revealed that the utilization characteristics are similar to mean number of packet characteristics. Here, as the time (t) and the arrival rate of messages (α) increase, the utilization of both the nodes increase for fixed values of the other parameters. As the batch size distribution parameters (m) and (p) increase, the utilization of both the nodes increase when the other parameters are fixed at (0.2, 2, 4, 8) for (t, α, β_1, β_2).

It is also noticed that as the transmission rate of first transmitter (β_1) increases, the utilization of the second node increases while the utilization of the first node decreases when other parameters remain fixed.

From the equations (3.4.6 to 3.4.14 and 3.4.15), the throughput and average delay of the network are computed for different values of t, m, p, α, β_1, β_2 and are given in Table 3. The relationship between throughput, average delay and parameters is shown in Figure 4.

Table 3: Values of Throughput and mean delay of the communication network with dynamic bandwidth allocation and bulk arrivals

t*	m	p	α#	$\beta_1^\$$	$\beta_2^\$$	Thp1	Thp2	W(N1)	W(N2)
0.1	3	0.2	2	4	8	0.628	0.254	0.263	0.131
0.2	3	0.2	2	4	8	1.013	0.683	0.272	0.136
0.3	3	0.2	2	4	8	1.255	1.070	0.279	0.140
0.4	3	0.2	2	4	8	1.619	1.857	0.297	0.165
0.2	**5**	0.2	2	4	8	1.059	0.785	0.260	0.143
0.2	**6**	0.2	2	4	8	1.080	0.839	0.255	0.147
0.2	**7**	0.2	2	4	8	1.100	0.892	0.250	0.151
0.2	**8**	0.2	2	4	8	1.119	0.945	0.246	0.154
0.2	3	**0.1**	2	4	8	0.987	0.631	0.250	0.133
0.2	3	**0.2**	2	4	8	1.013	0.683	0.257	0.136
0.2	3	**0.3**	2	4	8	1.041	0.742	0.265	0.140
0.2	3	**0.4**	2	4	8	1.071	0.807	0.272	0.144
0.2	3	0.2	**1**	4	8	0.543	0.349	0.253	0.133
0.2	3	0.2	**2**	4	8	1.013	0.683	0.272	0.136
0.2	3	0.2	**3**	4	8	1.418	1.002	0.291	0.139
0.2	3	0.2	**4**	4	8	1.769	1.308	0.311	0.143
0.2	3	0.2	2	**1**	8	0.309	0.223	1.173	0.129
0.2	3	0.2	2	**2**	8	0.579	0.405	0.570	0.132
0.2	3	0.2	2	**3**	8	0.812	0.557	0.370	0.134
0.2	3	0.2	2	**5**	8	1.013	0.683	0.272	0.136
0.2	3	0.2	2	4	**1**	1.013	0.124	0.272	1.148
0.2	3	0.2	2	4	**2**	1.013	0.235	0.272	0.569
0.2	3	0.2	2	4	**3**	1.013	0.333	0.272	0.376
0.2	3	0.2	2	4	**5**	1.013	0.498	0.272	0.222

*=Seconds, #=Multiples of 10, 000 messages/seconds, $=Multiples of 10,000 packets/second

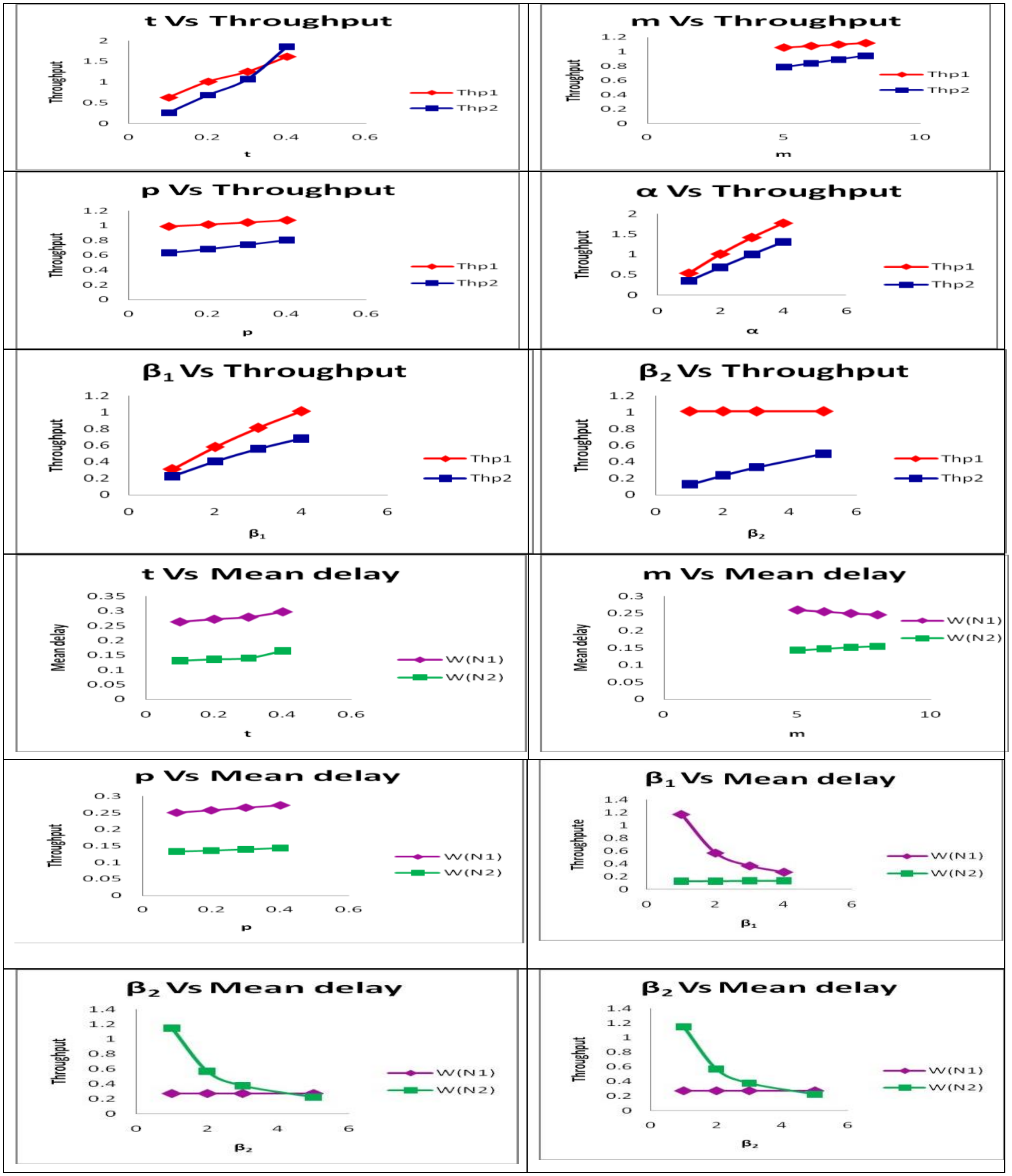

Fig 4: The relationship between Throughput, Mean Delay and various parameters

It is observed that as the time (t) increases from 0.1 seconds to 0.4 seconds, the throughput of the first and second nodes increase from 6280 packets to 16190 packets and from 2540 packets to 18570 packets respectively and there after stabilized when other parameters remain fixed at (3, 0.2, 2, 4, 8) for (m, p, α, β_1, β_2). As the batch size distribution parameter (m) varies from 5 to 8, the throughput of the first and second node increases from 10590 packets to 11190 packets and 7850 packets to 9450 packets respectively when other parameters remain fixed at (0.2, 0.2, 2, 4, 8) for (t, p, α, β_1, β_2).

As the batch size distribution parameter (p) varies from 0.1 to 0.4, the throughput of the first and second nodes increase from 9870 packets to 10710 packets and 6310 packets to 8070 packets respectively when other parameters remain fixed at (0.2, 3, 2, 4, 8) for (t, m, α, β_1, β_2). As the arrival rate (α) varies from $1x10^4$ messages/sec to $4x10^4$ messages/sec, it is observed that the throughput of the first and second nodes increase 5430 packets to 17690 packets and from 3490 packets to 13080 packets respectively, when other parameters remain fixed at (0.2, 3, 0.2, 4, 8) for (t, m, p, β_1, β_2).

As the transmission rate of first transmitter (β_1) varies from $1x10^4$ packets/sec to $5x10^4$ packets/sec, the throughput of the first and second nodes increase from 3090 packets 10130 packets and from 2230 packets to 6830 packets respectively, when other parameters remain fixed at (0.2, 3, 0.2, 2, 8) for (t, m, p, α, β_2). As the transmission rate of second node (β_2) varies from $1x10^4$ packets/sec to $5x10^4$ packets/sec, the throughput of second node increases from 1240 packets to 4980 packets, when other parameters remain fixed at (0.2, 3, 0.2, 2, 4) for (t, m, p, α, β_1).

From Table 3, it is also observed that as time (t) varies from 0.1 to 0.4 seconds, the mean delay of the first and second buffers increase from 26.300 µs to 29.700 µs and from 13.100 µs to 16.500 µs respectively, when other parameters remain fixed (3, 0.2, 2, 4, 8) for (m, p, α, β_1, β_2). As the batch size distribution parameter (m) varies from $5x10^4$ packets/sec to $5x10^4$ packets/sec, the mean delay of the first and second buffers increase from 26.00 µs to 28.600 µs and 14.300 µs to 15.400 µs respectively when other parameters remain fixed at (0.2, 0.2, 2, 4, 8) for (t, p, α, β_1, β_2). As the batch size distribution parameter (p) varies from $0.1x10^4$ packets/sec to $0.4x10^4$ packets/sec, the mean delay of the first and second buffers increase from 25.000 µs to 27.200 µs and 13.300 µs to 14.400 µs respectively when other parameters remain fixed at (0.2, 3, 2, 4, 8) for (t, m, α, β_1, β_2).

When the arrival rate (α) varies from $1x10^4$ messages/sec to $4x10^4$ messages/sec, the mean delay of the first and second buffers increase from 25.300 µs to 31.100 µs and 13.300 µs to 14.300 µs respectively, when other parameters remain fixed at (0.2, 3, 0.2, 4, 8) for (t, m, p, β_1, β_2). As the transmission rate of first transmitter (β_1) varies from $1x10^4$ packets/sec to $5x10^4$ packets/sec, the mean delay of the first buffer decreases from 117.30 µs to 27.200 µs and the mean delay of the second buffer increases from 12.900 µs to 13.600 µs, when other parameters remain fixed at (0.2, 3, 0.2, 2, 8) for (t, m, p, α, β_2). As the transmission rate of second transmitter (β_2) varies from $1x10^4$ packets/sec to $5x10^4$ packets/sec, the mean delay of the second buffer decreases from 114.800 µs to 22.200 µs, when other parameters remain fixed at (0.2, 3, 0.2, 2, 4) for (t, m, p, α, β_1).

The variance of the number of packets in each buffer, the coefficient of variation of the number of packets in first and second buffers are computed and given in Table 4.

Table 4: Values of Variance and Coefficient of Variation of the number of packets the communication network with dynamic bandwidth allocation and bulk arrivals

t*	m	p	α#	$\beta_1^{\$}$	$\beta_2^{\$}$	Var(N1)	Var(N2)	cv(N1)	cv(N2)
0.1	3	0.2	2	4	8	0.270	0.035	3.154	5.561
0.2	3	0.2	2	4	8	0.437	0.097	2.400	3.345
0.3	3	0.2	2	4	8	0.541	0.157	2.106	2.638
0.4	3	0.2	2	4	8	1.413	0.352	2.078	1.938
0.2	**5**	0.2	2	4	8	0.647	0.122	4.038	3.104
0.2	**6**	0.2	2	4	8	0.772	0.137	3.751	2.997
0.2	**7**	0.2	2	4	8	0.912	0.152	3.468	2.900
0.2	**8**	0.2	2	4	8	1.066	0.168	3.192	2.810
0.2	3	**0.1**	2	4	8	0.349	0.086	2.146	3.489
0.2	3	**0.2**	2	4	8	0.437	0.097	2.400	3.345
0.2	3	**0.3**	2	4	8	0.541	0.111	2.672	3.202
0.2	3	**0.4**	2	4	8	0.666	0.126	2.963	3.060
0.2	3	0.2	**1**	4	8	0.218	0.051	3.394	4.825
0.2	3	0.2	**2**	4	8	0.437	0.970	2.400	3.345
0.2	3	0.2	**3**	4	8	0.655	0.144	1.960	2.713
0.2	3	0.2	**4**	4	8	0.873	0.190	1.697	2.341
0.2	3	0.2	2	**1**	8	0.608	0.029	2.151	3.100
0.2	3	0.2	2	**2**	8	0.541	0.055	2.231	3.345
0.2	3	0.2	2	**3**	8	0.484	0.077	2.314	3.722
0.2	3	0.2	2	**4**	8	0.437	0.097	2.400	4.384
0.2	3	0.2	2	4	**1**	0.437	0.153	2.400	2.738
0.2	3	0.2	2	4	**2**	0.437	0.142	2.400	2.825
0.2	3	0.2	2	4	**3**	0.437	0.133	2.400	2.911
0.2	3	0.2	2	4	**5**	0.437	0.116	2.400	3.085

*=Seconds, #=Multiples of 10, 000 messages/seconds, $=Multiples of 10,000 packets/second

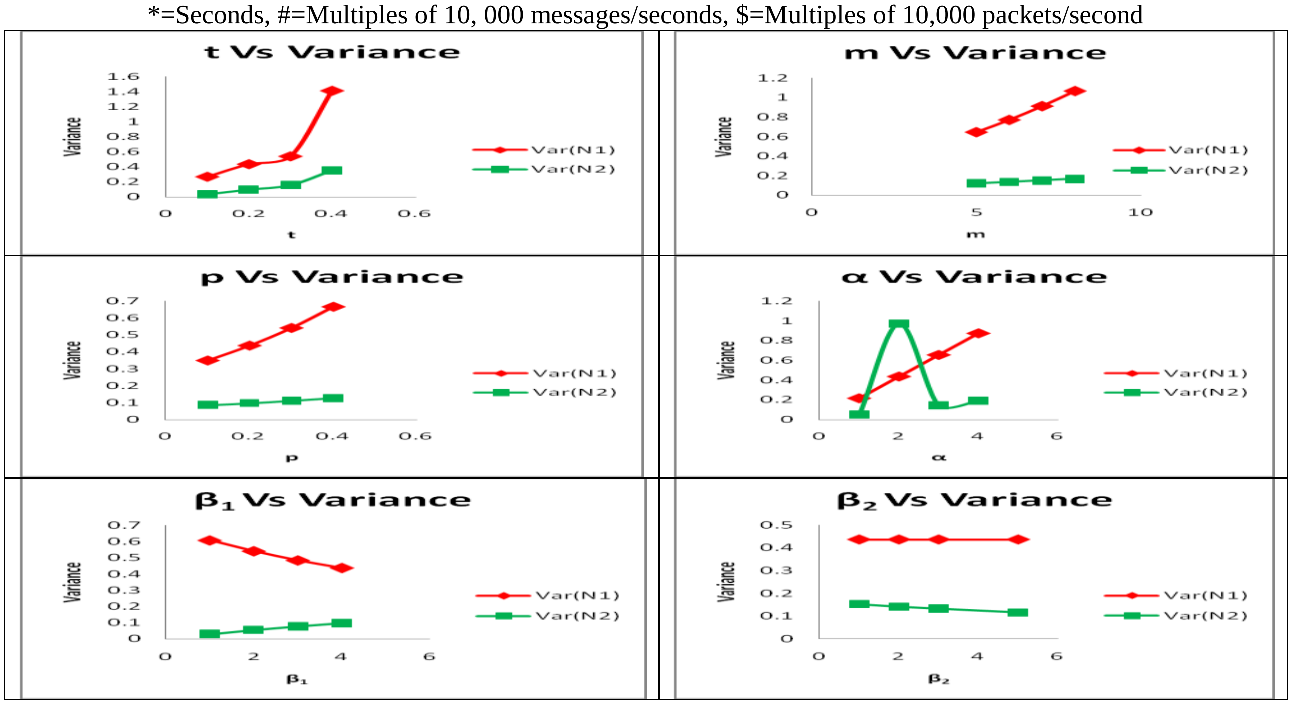

Fig 5: The relationship between Variance and various parameters

If the variance increases then the burstness of the buffers will be high. Hence, the parameters are to be adjusted such that the variance of the buffer content in each buffer must be small. The coefficient of variation of the number of packets in each buffer helps us to understand the consistency of the traffic flow through buffers. If coefficient of variation is large then the flow is inconsistent. It also helps us to compare the smooth flow of packets in two or more nodes.

It is observed that, as the time (t) and the batch size distribution parameter (m) increase, the variance of first and second buffers increased and the coefficient of variation of the number of packet in the first and second buffers decreased. As the batch size distribution parameter (p) increases, the variance of first and second buffers are increasing and the coefficient of variation of the number of packets in the first buffer is increasing and for the second buffer is decreasing.

From this analysis it is observed that the dynamic bandwidth allocation strategy has a significant influence on all performance measures of the network. It is further observed that the performance measures are highly sensitive towards smaller values of time. Hence, it is optimal to consider bulk arrivals with dynamic bandwidth allocation and evaluate the performance under transient condition. It is also observed that the congestion in buffers and delays in transmission can be reduced to a minimum level by adopting dynamic bandwidth allocation. This phenomenon has a vital bearing on quality of transmission (service).

5. Optimal policies of the model:

In this section, we derive the optimal operating policies of the communication networks under study. Here, it is assumed that the service provider of the communication network is interested in maximization of the profit function at a given time t. Let the service provider gets an amount of R_i units per every unit of time of the system busy at i^{th} transmitter (i=1,2). In other words, he gets revenue of R_i units per every unit of throughput of the ith transmitter. Therefore, the total revenue of the communication network at time t is,

$R(t)=R_1$(Throughput of first transmitter) + R_2(Throughput of second transmitter)

(34)

$$R(t)=R_1(\beta_1)(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r\,(-1)^{3r}\,\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]$$

$$+R_2(\beta_2)(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right] \quad (35)$$

Let A is the set up cost for operating the communication network. C_1 is the penalty cost due to waiting of a customer in the first transmitter. C_2 is the penalty cost due to waiting of a customer in the second transmitter. Therefore, the total cost for operating the communication network at time t is,

$C(t)$= A+ C_1(Average waiting time of a customer in first transmitter) + C_2(Average waiting time of a customer in second transmitter)

(36)

$$C(t)=A+C_1\frac{\left(\frac{\alpha}{\beta_1}\right)\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}k.\left(1-e^{-\beta_1 t}\right)\right]}{\beta_1(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r\,(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]}$$

$$+C_2\frac{\left(\frac{\alpha}{\beta_2}\right)\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}k\left(1-e^{-\beta_2 t}\right)\right]}{\beta_2(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r\,(-1)^{3r}\frac{\left(1-e^{-r\beta_2 t}\right)}{r\beta_2}\right]} \quad (37)$$

Substituting the values of R(t) and C(t) from equation (36) and (37) respectively we get total cost function as,

$$P(t)=R_1\beta_1(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]$$
$$+R_2\beta_2(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right]$$
$$-C_1\frac{\left(\frac{\alpha}{\beta_1}\right)\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}k\left(1-e^{-\beta_1 t}\right)\right]}{\beta_1(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]}$$
$$-C_2\frac{\left(\frac{\alpha}{\beta_2}\right)\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}k\left(1-e^{-\beta_2 t}\right)+\left(\frac{\beta_2}{\beta_2-\beta_1}\right)\left(e^{-\beta_2 t}-e^{-\beta_1 t}\right)\right]}{\beta_2(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{j=0}^{r}(-1)^{3r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right]}$$

(38)

To obtain the optimal values of β_1 and β_2, maximizing P(t), with respect to β_1 and β_2 and verify the hessian matrix $\frac{\partial P(t)}{\partial\beta_1}=0$ implies

$$\frac{\partial P(t)}{\partial\beta_1}=R_1\beta_1(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]$$
$$+R_2\beta_2(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right]$$
$$-C_1\frac{\frac{\alpha}{\beta_1}\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}.k\left(1-e^{-\beta_1 t}\right)\right]}{\beta_1(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]}$$
$$-C_2\frac{\frac{\alpha}{\beta_2}\left[\sum_{k=1}^{m}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}.k\left(1-e^{-\beta_2 t}\right)+\frac{\beta_2}{\beta_2-\beta_1}\left(e^{-\beta_2 t}-e^{-\beta_1 t}\right)\right]}{\beta_2(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{j=0}^{r}(-1)^{3r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right]}=0$$

(39)

$\frac{\partial P(t)}{\partial\beta_2}=0$ Implies

$$\frac{\partial P(t)}{\partial\beta_2}=R_1\beta_1(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}\,{}^{k}C_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]$$
$$+R_2\beta_2(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{j=0}^{r}(-1)^{3r-J}\frac{{}^{m}C_k p^k(1-p)^{m-k}}{1-(1-p)^m}({}^{k}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right]$$

$$-C_1\frac{\frac{\alpha}{\beta_1}\left[\sum_{k=1}^{m}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}k\left(1-e^{-\beta_1 t}\right)\right]}{\beta_1(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}{}^kC_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]}$$

$$-C_2\frac{\frac{\alpha}{\beta_2}\left[\sum_{k=1}^{m}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}k\left(1-e^{-\beta_2 t}\right)+\frac{\beta_2}{\beta_2-\beta_1}\left(e^{-\beta_2 t}-e^{-\beta_1 t}\right)\right]}{\beta_2(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}({}^kC_r)({}^rC_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\frac{\left(1-e^{-[J\beta_2+(r-J)\beta_1]t}\right)}{J\beta_2+(r-J)\beta_1}\right]}=0 \quad (40)$$

The determinant of the Hessian matrix is, $|D|=\begin{vmatrix}\frac{\partial^2P(t)}{\partial\beta_1^2} & \frac{\partial^2P(t)}{\partial\beta_1\partial\beta_2}\\ \frac{\partial^2P(t)}{\partial\beta_1\partial\beta_2} & \frac{\partial^2P(t)}{\partial\beta_2^2}\end{vmatrix}<0$

(41)

Solving the equations (39) and (40) with respect to β_1 and β_2 and verifying the condition (41), for the given parameters of α, m, p and t we get the optimal values of transmission rates at transmitter 1 and transmitter 2 as β_1^* and β_2^* respectively.

Substituting the values of β_1^* and β_2^* in equation (40), we get the optimal value of the profit at given time t as

$$P^*(t)=R_1\beta_1^*(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}{}^kC_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1^* t}\right)}{r\beta_1^*}\right]$$

$$+R_2\beta_2^*(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}({}^kC_r)({}^rC_J)\left(\frac{\beta_1^*}{\beta_2^*-\beta_1^*}\right)^r\frac{\left(1-e^{-[J\beta_2^*+(r-J)\beta_1^*]t}\right)}{J\beta_2^*+(r-J)\beta_1^*}\right]$$

$$-C_1\frac{\frac{\alpha}{\beta_1^*}\left[\sum_{k=1}^{m}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}.k\left(1-e^{-\beta_1^* t}\right)\right]}{\beta_1^*(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}{}^kC_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1^* t}\right)}{r\beta_1^*}\right]}$$

$$-C_2\frac{\frac{\alpha}{\beta_2^*}\left[\sum_{k=1}^{m}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}.k\left(1-e^{-\beta_2^* t}\right)+\frac{\beta_2^*}{\beta_2^*-\beta_1^*}\left(e^{-\beta_2^* t}-e^{-\beta_1^* t}\right)\right]}{\beta_2^*(1-\exp)\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^mC_kp^k(1-p)^{m-k}}{1-(1-p)^m}({}^kC_r)({}^rC_J)\left(\frac{\beta_1^*}{\beta_2^*-\beta_1^*}\right)^r\frac{\left(1-e^{-[J\beta_2^*+(r-J)\beta_1^*]t}\right)}{J\beta_2^*+(r-J)\beta_1^*}\right]} \quad (42)$$

5. Numerical Illustration:

In this section, we demonstrate the solution procedure through a numerical illustration. Consider the service provider to a communication network is operating a two-transmitter tandem communication network. Let the estimated cost of revenue per unit throughput of transmitter 1 is R_1 varies from 0.3 to 0.6 and the revenue per unit of throughput of transmitter 2 R_2 varies from 0.3 to 0.6. It is estimated the composite arrival rate of messages per unit time is α varies from 1.5 to 1.9. The batch size distribution parameters of the number of packets that are generated from a message are considered to varies from 4 to 8 for m and 0.1 to 0.9 for p. Let the penalty cost per a packet waiting time at transmitter 1 for transmission per unit time is C_1 varies from 0.1 to 0.10. The penalty cost per a packet waiting time at transmitter 2 for transmission per unit time is C_2 varies from 0.1 to 0.10. With these costs and parameters, the optimal values of transmission rates β_1 and β_2 are obtained using Newton Rampsons Method and Mathcad. The optimal value of transmission rate of first transmitter and transmission rate of second transmitter, optimal profit parameters are computed and shown in Table 6.

t	m	p	α	R_1	R_2	C_1	C_2	β_1*	β_2*	P*(t)	D_1^*
5.1	4	0.1	1.5	0.5	0.5	0.1	0.1	3.354	3.351	3.131	-0.256
5.2	4	0.1	1.5	0.5	0.5	0.1	0.1	3.331	3.342	3.087	-0.242
5.4	4	0.1	1.5	0.5	0.5	0.1	0.1	3.102	3.324	2.917	-0.219
5	**5**	0.1	1.5	0.5	0.5	0.1	0.1	3.447	3.660	3.272	-0.198
5	**6**	0.1	1.5	0.5	0.5	0.1	0.1	3.461	3.960	3.397	-0.186
5	**8**	0.1	1.5	0.5	0.5	0.1	0.1	3.729	4.526	3.780	-0.152
5	4	**0.3**	1.5	0.5	0.5	0.1	0.1	2.714	3.013	2.526	-0.215
5	4	**0.5**	1.5	0.5	0.5	0.1	0.1	2.735	3.351	2.663	-0.202
5	4	**0.9**	1.5	0.5	0.5	0.1	0.1	2.795	4.090	2.969	-0.165
5	4	0.1	**1.6**	0.5	0.5	0.1	0.1	3.447	3.513	3.214	-0.389
5	4	0.1	**1.7**	0.5	0.5	0.1	0.1	3.468	3.750	3.320	-0.371
5	4	0.1	**1.9**	0.5	0.5	0.1	0.1	3.507	4.201	3.518	-0.331
5	4	0.1	1.5	**0.3**	0.5	0.1	0.1	3.117	3.013	2.808	-0.452
5	4	0.1	1.5	**0.4**	0.5	0.1	0.1	3.117	3.051	2.822	-0.441
5	4	0.1	1.5	**0.6**	0.5	0.1	0.1	3.117	3.092	2.838	-0.404
5	4	0.1	1.5	0.5	**0.3**	0.1	0.1	3.117	3.051	2.822	-0.574
5	4	0.1	1.5	0.5	**0.4**	0.1	0.1	3.117	3.051	2.841	-0.551
5	4	0.1	1.5	0.5	**0.6**	0.1	0.1	3.117	3.051	2.991	-0.513
5	4	0.1	1.5	0.5	0.5	**0.07**	0.1	3.161	2.288	2.551	-0.291
5	4	0.1	1.5	0.5	0.5	**0.08**	0.1	3.175	2.394	2.602	-0.272
5	4	0.1	1.5	0.5	0.5	**0.10**	0.1	3.201	2.581	2.692	-0.235
5	4	0.1	1.5	0.5	0.5	0.1	**0.07**	3.114	3.321	2.924	-0.783
5	4	0.1	1.5	0.5	0.5	0.1	**0.08**	3.114	3.328	2.927	-0.764
5	4	0.1	1.5	0.5	0.5	0.1	**0.10**	3.114	3.345	2.933	-0.721

Table 6: Numerical representation of Optimal Values of $\beta_1$and β_2

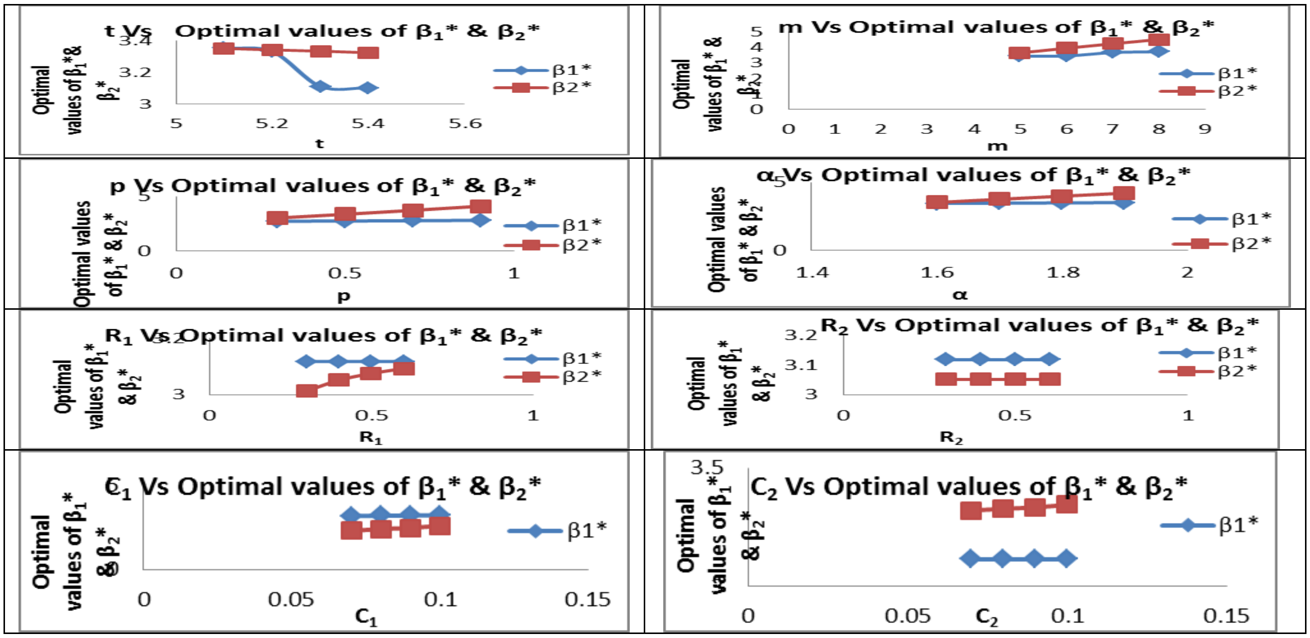

Figure 6: Numerical representation of Optimal Values of $\beta_1$and β_2

It is observed that the transmission rate of first transmitter (β_1) and the transmission rate of second transmitter (β_2) are highly sensitive with respect to changes in time. As time (t) varies from 5.1 to 5.4 second, the transmission rate of first transmitter (β_1) reduces from 3.354 to 3.102, the transmission rate of second transmitter (β_2) reduce from 3.351 to 3.324 respectively and the total optimal cost function decreases from 3.131 to 2.917 when other parameters are fixed at (4, 0.1, 1.5, 0.5, 0.5, 0.1, 0.1) for (m, p, α, R_1, R_2, C_1, C_2).

When the batch size distribution of number of packets a message can be converted (m) varies from $5x10^4$ packets/sec to $8x10^4$ packets/sec, the transmission rate of first transmitter (β_1) increases from 3.447 to 3.729 and the transmission rate of second transmitter (β_2) increases from 3.660 to 4.526 and the total optimal cost function increases from 3.272 to 3.780 when other parameters are fixed at (5, 0.1, 1.5, 0.5, 0.5, 0.1, 0.1) for (t, p, α, R_1, R_2, C_1, C_2).

When the batch size distribution parameter (p) varies from $0.3x10^4$ packets/sec to $0.9x10^4$ packets/sec, the transmission rate of first transmitter (β_1) increases from 2.714 to 2.795, the transmission rate of second transmitter (β_2) increases from 3.013 to 4.090 and the total optimal cost function increases from 2.526 to 2.969 when other parameters are fixed at (5, 4, 1.5, 0.5, 0.5, 0.1, 0.1) for (t, m, α, R_1, R_2, C_1, C_2).

As the arrival rate (α) varies from $1.6x10^4$ messages/sec to $1.9x10^4$ messages/sec, the transmission rate of first transmitter (β_1) increases from 3.447 to 3.507, the transmission rate of second transmitter (β_2) increases from 3.513 to 4.020 and the total optimal cost function increases from 3.214 to 3.518 when other parameters are fixed at (5, 4, 0.1, 0.5, 0.5, 0.1, 0.1) for (t, m, p, R_1, R_2, C_1, C_2).

When the revenue parameter (R_1) varies from 0.3 to 0.6, the transmission rate of first transmitter (β_1) remain unchanged as 3.117, the transmission rate of second transmitter (β_2) increases from 3.013 to 3.092 and the total optimal cost function increases from 2.808 to 2.838 when other parameters are fixed at (5, 4, 0.1, 1.5, 0.5, 0.1, 0.1) for (t, m, p, α, R_2, C_1, C_2).

When the revenue parameter (R_2) varies from 0.3 to 0.6, the transmission rate of first transmitter (β_1) remain unchanged as 3.117, the transmission rate of second transmitter (β_2) remain unchanged as 3.051 and the total optimal cost function increases from 2.822 to 2.991 when other parameters are fixed at (5, 4, 0.1, 1.5, 0.5, 0.1, 0.1) for (t, m, p, α, R_1, C_1, C_2).

When the cost parameter (C_1) varies from 0.07 to 0.10, the transmission rate of first transmitter (β_1) increases from 3.161 to 3.201, the transmission rate of second transmitter (β_2) increases from 2.288 to 2.581 and the total optimal cost function increases from 2.551 to 2.692 when other parameters are fixed at (5, 4, 0.1, 1.5, 0.5, 0.5, 0.1) for (t, m, p, α, R_1, R_2, C_2).

When the cost parameter (C_2) varies from 0.07 to 0.10, the transmission rate of first transmitter (β_1) remains unchanged as 3.114, the transmission rate of second transmitter (β_2) increases from 3.321 to 3.345 and the total optimal cost function increases from 2.924 to 2.933 when other parameters are fixed at (5, 4, 0.1, 1.5, 0.5, 0.5, 0.1) for (t, m, p, α, R_1, R_2, C_1).

6. Sensitivity Analysis:

The sensitivity analysis of the Transmission rate parameters β_1* and β_2*, and the total cost function p*(t) are studied with respect to the parameters t, m, p, α, R_1, R_2, C_1 and C_2.

Sensitivity analysis of the model is carried out with respect to the parameters t, m, p, α, R_1, R_2, C_1, and C_2 on the transmission rate of first transmitter (β_1) and the transmission rate of second transmitter (β_2)

The following data has been considered for the sensitivity analysis.

t = 5 sec, m=4 x10^4 packets/sec, p=0.1, α = 2x10^4 packets/sec, R_1 = 0.7, R2 = 0.2, C_1 =0.4 and C_2 =0.3

The performance measure of the model is computed by variation of -15%, -10%, -5%, 0%, +5%, +10% and +15% on the input parameters t, p, α, R_1, R_2, C_1 and C_2 and -75%, -50%, -25%, 0%, +25%, +50% and +75% on the batch size distribution parameter m to retain them as integers. The computed values of the performance measures are given in Table 7.

Parameter	**Performance Measure**	**% change in parameters**						
		-15%	**-10%**	**-5%**	**0**	**+5%**	**+10%**	**+15%**
t=5	**β_1***	3.547	3.578	3.592	3.610	3.632	3.671	3.693
	β_2*	3.679	3.716	3.754	3.793	3.832	3.872	3.912
	P*(t)	3.139	3.163	3.181	3.201	3.222	3.251	3.272
P=0.1	**β_1***	2.979	2.999	3.029	3.061	3.092	3.124	3.153
	β_2*	2.897	2.929	2.960	2.993	3.025	3.058	3.091
	P*(t)	2.617	2.638	2.664	2.691	2.717	2.743	2.769
α =2	**β_1***	2.452	2.772	2.976	3.241	3.342	3.501	3.746
	β_2*	2.463	2.702	2.934	3.158	3.377	3.588	3.793
	P*(t)	2.178	2.446	2.629	2.832	2.952	3.090	3.256
R_1=0.7	**β_1***	3.107	3.156	3.189	3.217	3.241	3.276	3.299
	β_2*	3.061	3.096	3.110	3.175	3.189	3.201	3.231
	P*(t)	2.742	2.772	2.792	2.827	2.843	2.863	2.883
R_2=0.2	**β_1***	3.056	3.089	3.106	3.125	3.141	3.168	3.192
	β_2*	3.053	3.071	3.098	3.114	3.128	3.142	3.175
	P*(t)	2.709	2.731	2.749	2.763	2.776	2.793	2.816
C_1=0.4	**β_1***	3.151	3.167	3.189	3.207	3.226	3.241	3.263
	β_2*	3.132	3.151	3.176	3.189	3.202	3.235	3.267
	P*(t)	2.782	2.796	2.814	2.827	2.840	2.859	2.879
C_2=0.3	**β_1***	3.188	3.204	3.223	3.245	3.263	3.289	3.307
	β_2*	3.151	3.172	3.189	3.201	3.225	3.241	3.278
	P*(t)	2.805	2.820	2.835	2.849	2.865	2.882	2.907
		-75%	**-50%**	**-25%**	**0**	**+25%**	**+50%**	**+75%**
m=4	**β_1***	3.461	3.485	3.512	3.541	3.568	3.592	3.610
	β_2*	3.422	3.514	3.607	3.702	3.797	3.893	3.983
	P*(t)	3.018	3.059	3.101	3.144	3.185	3.225	3.260

Table 7: Sensitivity Analysis of case 1

The performance measures are highly affected by varying the time (t) and the batch size distribution parameters of arrivals, a time (t) increases to 15% the average number of packets transmitting through the two buffers increases along with the two transmitters. Similarly, as the arrival rate of messages (α) increases by 15%, the average number of packets transmitted through two transmitter's increases. Over all analysis of the parameters reflects that, dynamic bandwidth allocation strategy for congestion control tremendously reduces the mean delay in communication and improve voice quality by reducing burstness in buffers.

The sensitivity analysis of the Transmission rate parameters β_1* and β_2*, Arrival rate α and the total cost function p*(t) are studied with respect to the parameters t, m, p, R_1, R_2, C_1 and C_2. Sensitivity analysis of the model is performed with respect to the parameters t, m, p, R_1, R_2, C_1, and C_2 on the arrival rate (α), transmission rate of first transmitter (β_1) and the transmission rate of second transmitter (β_2)

The performance measures are highly affected with the variation in time (t) and the batch size distribution parameters of arrivals. As time (t) increases by 15% the average number of packets transmitting through the two buffers increases along with the two transmitters and the arrival rate of the packets increases. As the batch size distribution parameter p increases to 15%, the average number of packets transmitting through the two buffers increases along with the two transmitters and the arrival rate of the packets increases. Over all analysis of the parameters reflects that dynamic bandwidth allocation strategy for congestion control tremendously reduces the mean delay in communication and improve voice quality by reducing burstness in buffers.

7. CONCLUSION

In this paper, the optimal operating policies of a two-transmitter tandem communication network with binomial bulk arrivals are derived. Here, the arrival process is characterized by compound Poisson binomial process. The transmission process at both transmitters are characterized by Poisson processes. To control the congestion at buffer, the Dynamic Bandwidth Allocation (DBA) strategy is adopted. With suitable cost considerations, the total profit rate function is derived. The optimal operating policies of the network for scheduling the transmission rates are developed by maximizing the profit function. It is observed that the bulk size distribution parameters have significant influence of the optimal operating policies of the network. This network is useful for scheduling Intranet, Internet, LAN, WAN and MAN. This communication network model can be extended for the case of non-marchovian transmission times.

8. REFERENCES

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