

Face Recognition Using Fuzzy Moments Discriminant Analysis

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Abstract

In this work, an enhanced feature extraction method for holistic face recognition approach of gray intensity still image, namely Fuzzy Moment Discriminant Analysis is used. Which is first, based on Pseudo-Zernike Moments to extract dominant and significant features for each image of enrolled person, then the dimensionality of the moments features vectors is further reduced into discriminant moment features vectors using Linear Discriminant Analysis method, for these vectors the membership degrees in each class have been computed using Fuzzy K-Nearest Neighbor, after that, the membership degrees have been incorporated into the redefinition of the between-classes and within-classes scatter matrices to obtain final features vectors of known persons. The test image is then compared with the faces enrollment images so that the face which has the minimum Euclidean distance with the test image is labeled with the identity of that image.

Keyword: Zernike Moments, LDA, Fuzzy K-Nearest Neighbor.

1. Introduction

As life in our planet is growing so rapidly, its complexity is increasing, and businesses between individuals become more sophisticated, Discrimination between people is now crucial in almost all life fields.

Development of person identity grew from the traditional knowledge-based (what person knows) and token-based (what person has) approaches to the revolutionary Biometric scheme (what person is). Focus now is moving away from traditional methods -towards Biometrics- due to defaults in their usage, For example, password can be guessed, forgotten or shared; card key can also be lost or snooped easily. While Biometric characteristics cannot be borrowed, stolen or forgotten [1].

Biometric is the science of recognizing the identity of a person based on the physical or behavioral characteristics possessed by him (based on the fact that each individual has unique features whether organic - related to his physique e.g. fingerprint, face, iris, etc.- or functional -related to his behavior e.g. gait, signature, hand writing, etc.-) . Among all Biometric characteristics, face based recognition has become one of the most intensively researched topics in biometrics, due to three reasons: First, faces play a crucial role in the regulation of our social behaviors and interactions. Second, it is easy to recognize and categorize conspecifics' faces, and lastly unlike other biometrics, it requires no cooperation of the individual. Thus it successfully had overcome the interruption or discomfort that may possible happen in other biometrics [2].

Unlike human (who can recognize faces easily), automated face recognition remains a great challenge in computer-based automated recognition research due to variability of human faces under different conditions such as: facial expression, illumination change, pose, occlusions, eyeglasses, etc, which result in a high complex distribution and deteriorate the recognition performance.

Face recognition methods can be classified into two broad categories according to feature extraction schemes for face representation: feature-based approach and appearance-based (holistic) approach. The first approach extracts local features such as the locations and local statistics of the eye, nose, mouth, etc., manually, this type of methods are rely heavily on the accurate detection of facial features, And the feature extraction techniques needed for this type of approach are still not accurate enough. While the second approach called appearance-based (holistic) approach generally operates directly on raw image data and processes them as 2D holistic patterns to avoid difficulties associated with 3D modeling, and shape or landmark detection. Consequently, this class of methods tends to be easier to implement, more practical and reliable as compared to the geometric feature-based methods [3] [4].

Holistic face recognition system includes three stages. The first stage requires extraction of a suitable representation of the face region. The second stage classifies the facial image based on representation obtained in the previous stage. Finally, a comparison between the facial image and database (gallery) is made and reports a match.

2. Literature Review

In 1991, M.Turk,and A.Pentland presented an approach to the identification of human faces namely Principle Component Analysis (PCA). This approach transforms face images into a small set of characteristic

features images, called “eigenfaces”. The individual face is characterized by a weighted sum of the eigenface features which will be compared to the weights of known individuals to perform the recognition [5].

In 1997, P.Belhumeur, J.Hespanha, and D.Kriegman developed a face recognition algorithm which is insensitive to large variation in lighting direction and facial expression. They apply first PCA for dimension reduction and then LDA which maximizes the ratio of between-class scatter to that of within-class scatter. This method, which they call FisherFaces produce well separated classes in a low dimensional subspace, even under severe variation in lighting and facial expressions [6].

In 1999, A. A. Abdual-Rahman, R. A. Mehdi, and R. S. N.aoum proposed new human face construction and recognition system. The implementation procedure of this system includes constructing the facial image of a specific person, extracting meaningful features from it, and finally being able to recognize or reject these features from a pre-stored database of known faces. The construction process employs some of the image manipulation techniques and graphical capabilities to enable a non-artist user to compose a facial image from a given database then they extract features vector from this image, the features vector is a set of geometrical distances between the feature points of the face which will enter to recognition module, this module employs an artificial Neural Net classifier to map the constructed face to any of the pre-defined faces [7].

In 2005, K. Kwak and W. Pedrycz tried to enhance the performance of the Linear Discriminant Analysis through present a novel method called Fuzzy Fisher classifier, this method used PCA as feature extraction and then it used the fuzzy K-nearest neighbor class assignment that produces the corresponding degrees of class membership. After that it incorporated the membership degree into the definition of the within and between class scatter matrices. The Fuzzy Fisherface show improved classification rates and reduced sensitivity to variations between face images caused by changes in illumination and viewing directions [8].

In 2007, M. Hassan, I. Osman, and M. Yahia proposed a new facial features extraction approach known as Walsh-Hadamard Transform (WHT). The basic functions of this transform are based on square or rectangular waves with peaks of ± 1 . This approach depends on the correlation between local pixels of the face image. Its primary advantage is the simplicity of its computation because all they need to do, after project an image onto the basis functions, is multiply each pixel by ± 1 [9].

In 2009, S. M. Lajevardi, and Z. M. Hussain, presented a facial expression recognition system using an orthogonal invariant moment namely Zernike moment (ZM) as a feature extraction and Linear Discriminant Analysis as a classifier method. They show that the recognition rate is improved with higher order Zernike moments in noise and rotation images while the recognition rate is retrograde with bank of Gabor filters as feature extractor in noise and rotation images[10].

In 2010, L. chan, S. Hussain, C. Ting presented a comparative study between two subspace methods PCA and LDA in two biometric modes Face Identification and Face Verification. This study had proved in two modes, LDA is able to provide better discriminant ability in feature extraction for face biometrics, when given sufficient training samples [11].

In this research, Pseudo Zernike moments have been used to extract features vectors from facial images. The LDA method have been used to reduce the dimension of the moments features vectors, then the Fuzzy LDA is used to obtain the optimal features vectors and the Euclidean distance metric have been used in matching.

3. Face Recognition

The **Face Recognition** is a process of allocating one image of a person as a member of one of the different classess. The entire task of Face Recognition takes place into three phases: data acquisition, data preprocessing and feature extraction and decision classification, as shown in Fig. (1), In data acquisition phase, data are collected from the database and converted into digital format $I(r,c)$. The digital data are then input into the second phase and grouped be classified. In Fig. (1) The set of data at A, B and C are in the person space, feature space, and classification space, respectively [12][13][14].

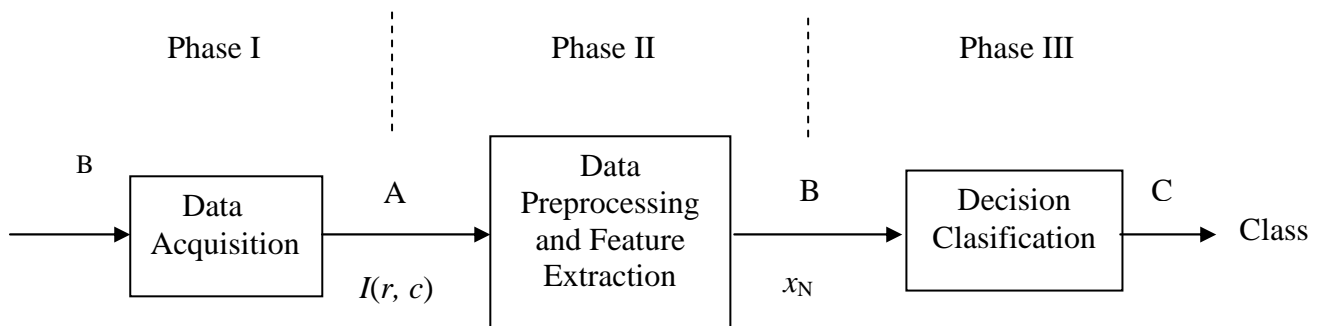


Fig. (1) Face Recognition Phases' Diagrammatic Representation.

In the present study, the phase II preprocessing step means the process of images normalization to eliminate them from artifacts, poor uniformity and noise in the original images. The normalization is achieved by using the following equation [15]:

$$NI_{nsty} = \begin{cases} M_0 + \sqrt{\frac{std_0 \times (f(i, j) - M)^2}{std}}, & \text{if } f(i, j) > M \\ M_0 - \sqrt{\frac{std_0 \times (f(i, j) - M)^2}{std}}, & \text{otherwise} \end{cases} \dots\dots\dots (1)$$

Where $f(i, j)$ denotes the gray value at pixel (i, j) , M and std are the estimated mean and standard deviation of image, and M_0, std_0 are the desired mean and standard deviation.

3.1 Features Extraction methods

Features extraction methods that have been beneficial in the present study can be described in the following sections:

3.1.1 Pseudo-Zernike Moments (PZM)

Pseudo-Zernike denoted a set of complex polynomials which form a perfectly orthogonal set over the polar coordinate space inside a unit circle (i.e. $x^2 + y^2 = 1$), The proposed polynomials, termed as Pseudo-Zernike polynomials, which proposed by Bhatia and Wolf [16], are defined by $V_{pq}(r, \theta)$ in polar coordinates[17][15]:

$$PV_{pq}(r, \theta) = PR_{pq}(r)e^{-jq\theta} = PR_{pq}(r)(\cos q\theta - j \sin q\theta) \dots\dots\dots(2)$$

Where p positive integer or zero

q positive and negative integers subject to constraint of $|q| \leq p$

r length of vector from the origin of unit circle to (x, y) pixel

θ angle between vector r and x axis in counterclockwise direction

$$j = \sqrt{-1}$$

$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$e^{-j\theta} = \cos \theta - j \sin \theta$$

$PR_{pq}(r)$ is a set of real-valued radial polynomials given by

$$PR_{pq}(r) = \sum_{s=0}^{p-|q|} (-1)^s \frac{(2p+1-s)!}{s!(p-|q|-s)!(p+|q|+1-s)!} r^{p-s} \dots\dots\dots (3)$$

Note that $PR_{p,-q}(r) = PR_{p,q}(r)$ and $PR_{pq}(r)$ also subject to constraints: $|q| \leq p$.

The two-dimensional Pseudo-Zernike moments of order p with repetition q of an image intensity function $f(r, \theta)$ are denoted as:

$$PZ_{pq} = \frac{p+1}{\pi} \int_0^1 \int_0^{2\pi} PV(r, \theta) f(r, \theta) r dr d\theta \dots\dots\dots(4)$$

For digital image, the integral in equation (4) are replaced by summation to get [35]:

$$PZ_{pq} = \frac{p+1}{\pi} \sum_x \sum_y f(x, y) PV_{pq}(r, \theta), x^2 + y^2 \leq 1 \dots(5)$$

To compute Pseudo-Zernike moments for a given image, the image is normally mapped to unit circle using polar coordinate, where the center of the image is the origin of the unit circle. Pixels locating outside the unit circle are not taken into consideration, but unfortunately, this might cause important information to be lost. To avoid that, a square-to-circular image transformation has been used, which maps all pixels to the same circle.

The equations of this transform are [18]:

$$c_1 = \frac{\sqrt{2}}{N-1}, \quad c_2 = -\frac{1}{\sqrt{2}} \dots\dots\dots(6)$$

$$x_i = c_1 i + c_2, \quad y_j = c_1 j + c_2 \dots\dots\dots(7)$$

$$r_{ij} = \sqrt{x_i^2 + y_j^2} \dots\dots\dots(8) \quad \theta_{ij} = \tan^{-1} \left(\frac{y_j}{x_i} \right) \dots\dots\dots(9)$$

Where N is the number of pixels along each axis of the image. Using the above square-to-circular image transformation, then the discrete version Pseudo-Zernike moments are defined as:

$$PZ_{pq} = \delta(p, N) \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} PR_{pq}(r_{ij}) e^{-jq\theta} f(i, j) \dots\dots\dots(10)$$

Where $f(i, j)$ is the image intensity function [19]. And

$$\delta(p, N) = \frac{(p+1)}{\pi} \dots\dots\dots(11)$$

Since the Pseudo-Zernike moments are sets of complex polynomials, after their application to extract feature vectors, the resultant feature vectors contain imaginary parts. We find, by taking the absolute value of a complex number, we can eliminate the imaginary part of complex number, as follows [20]:

$$|x + iy| = \sqrt{x^2 + y^2} \dots\dots\dots(12)$$

3.1.2 Linear Discriminant Analysis (LDA)

Linear Discriminant Analysis (LDA), also called Fisher Linear Discriminant (FLD) Analysis or Fisherface tries to find a line that best isolates the different points. In terms of face recognition this means grouping images of the same class and isolate images of different classes. So the images are projected from a N -dimensional space, where N is the number of pixels in the image, to a $M-1$ dimensional space, where M is the number of classes of images [21][22][23]. i.e. the LDA method maximizes the ratio of the determinant of the between-class scatter matrix of the projected samples to the determinant of the within-class scatter matrix of the projected samples. In other words, maximizing the between-class scatter matrix S_b , while minimizing the within-class scatter matrix S_w in the projective subspace[23][6], the S_w and S_b are defined as follows:

$$S_w = \frac{1}{N} \sum_{i=1}^C \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)(x_{ij} - \bar{x}_i)^T \dots\dots\dots(13)$$

$$S_b = \frac{1}{N} \sum_{i=1}^C n_i (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})^T \dots\dots\dots(14)$$

Where x_{ij} is the j th training sample of the i th class, n_i is the number of samples of the i th class, C is the number

of classes, $\bar{x} = \sum_{i=1}^C \sum_{j=1}^{n_i} x_{ij} / N$ is the global mean vector, N is the total number of training samples,

$\bar{x}_i = \sum_{j=1}^{n_i} x_{ij} / n_i$ is the mean vector of class i.

The optimal projection matrix W_{FLD} is chosen in such a manner so that it forms a matrix with orthonormal columns that maximizes the ratio of the determinant of the between-class scatter matrix of the projected samples and the determinant of the within-class scatter matrix of the projected patterns [24][25][13], i.e.,

$$W_{FLD} = \arg \max_w \frac{|W^T S_b W|}{|W^T S_w W|} = [w_1, w_2, \dots, w_m] \dots \dots \dots (15)$$

Where $\{w_i | i = 1, 2, \dots, m\}$ is the set of generalized eigenvectors of S_b and S_w corresponding to the $c-1$ largest generalized eigenvalues $\{\lambda_i | i = 1, 2, \dots, m\}$ that is $S_b w_i = \lambda_i S_w w_i, i = 1, 2, \dots, m$ (16)

So the feature vectors $V = \{v_1, v_2, \dots, v_N\}$ for any train images x_i can be calculated as follows [26][23]:

$$v_i = W_{FLD}^T x_i = W_{FLD}^T E^T (x_i - \bar{x}) \dots \dots \dots (17)$$

3.1. 3 Fuzzy Linear Discriminant Analysis (FLDA)

The classical LDA method dwells on the concept of a binary (yes-no) class assignment meaning that the faces come fully assigned to the given classes, So K.C. Kwak in 2005 attempt to enhance this approach by coalescing a gradual level of assignment to class being regarded as a membership grade with anticipation that such discrimination helps improve classification results. More specifically, when he gets a set of feature vectors transformed by PCA, $X = \{x_1, x_2, \dots, x_N\}$, he complete a Fuzzy K-nearest neighbor class assignment that produces the corresponding degree of membership of each vector to the classes (Where the membership matrix $[\mu_{ij}] (i = 1, 2, \dots, c, j = 1, 2, \dots, N)$ can be got from equation (22)), Then he used the results of the Fuzzy K-NN of class assignment in the computations of the statistical properties of the patterns such as the mean value and scatter covariance matrices [8][24](the computations being corner stone of the fisherface method).

Taking into account the membership grades, the mean vector of each class \bar{m}_i is calculated as follows:

$$\bar{m}_i = \frac{\sum_{j=1}^N \mu_{ij}^p x_j}{\sum_{j=1}^N \mu_{ij}^p} \dots \dots \dots (18)$$

And the between-class fuzzy scatter matrix SF_b and within-class fuzzy scatter matrix SF_w incorporate the membership values in their calculations:

$$SF_b = \sum_{i=1}^c \mu_{ij}^p (\bar{m}_i - \bar{m})(\bar{m}_i - \bar{m})^T \dots \dots \dots (19)$$

$$SF_w = \sum_{i=1}^c \sum_{j=1}^N \mu_{ij}^p (x_k - \bar{m}_i)(x_k - \bar{m}_i)^T \dots \dots \dots (20)$$

So the optimal fuzzy projection W of Fuzzy Fisherface follows the expression:

$$W_{FLDA} = \arg_w \max \frac{|W^T S_{FB} W|}{|W^T S_{FW} W|} \dots\dots\dots (21)$$

3.2 Classification Methods

A number of pattern classification techniques have been utilized for the recognition of faces, one of these methods is:

3.2.1 The Fuzzy K-NN Classifier

The **Fuzzy K-NN (FK-NN) classifier** differs from the crisp version in the form of its results because it assigns class membership to a sample vector rather than assigning the vector to a particular class. The advantage is that no randomly assignments are made by the algorithm of Fuzzy K-NN. In addition, the vector's membership values should provide a level of assurance to accompany the resultant classification. For example, if a vector is assigned 0.9 membership in one class and 0.05 membership in two other classes we can reasonably sure the class of 0.9 membership is the class to which the vector belongs.

The fuzzy algorithm is similar to the crisp version in the sense that it must also search the labeled sample set for the K-nearest neighbors. Beyond obtaining these K samples, the procedures differ considerably. Because the basis of the algorithm is to assign membership as a function of the vector's distance from its K-nearest neighbors and those neighbors' memberships in possible classes.

The **FK-NN** computes Euclidean distance matrix between pairs of feature vectors in the training, then it sort the distance matrix(treat each of its column separately) in ascending order and collect the class labels of the patterns under consideration (as we are concerned with "k" neighbors, this returns a list of "k" integers). Then it compute membership grade to class "i" for the jth patterns using equation (22) which is proposed in the literature [27].

$$\mu_{ij} = \begin{cases} 0.51 + 0.49(n_{ij} / k), & \text{if } i = \text{the same as the label of } j\text{th pattern} . \\ 0.49(n_{ij} / k), & \text{if } i \neq \text{the same as the label of } j\text{th pattern} . \end{cases} \dots\dots\dots(22)$$

Where n_{ij} stands for the number of the neighbors of the jth data (pattern) that belong to the ith class.

4. The Stages of Proposed System

The designed face recognition system consists of the following stages:

1. **Image Acquisition Stage:** Obtain the face images from database of face images. Then the images will be digitalized and converted into BMP format by using any format converting program.
2. **Normalization Stage:** Each image should be normalized by subtracting their mean and dividing their standard deviation, in order to reduce the sensibility to lighting conditions.
3. **Enrollment Stage:** Persons are requested to provide their images (called training images) for registration in the system, then the system will learn who the person in the image is, by performing the following steps on the training images:
 - a. Using Pseudo-Zernike Moments as Feature Extractor and dimension reducer: since the Pseudo-Zernike Moments posses advantages of geometrical invariance, robustness to noise, optimal feature representation and nearly zero information redundancy, so this method was used to extract the dominant and significant image features and to inhibit the redundant information of the image.
 - b. Applying the Linear Discriminant Analysis: the LDA has been applied for further reduction to the dimensions of the features vectors obtained from the Pseudo-Zernike Moments.
 - c. Performing Fuzzy K-Nearest Neighbor (FK-NN) class assignment: this method produces the corresponding degrees of class membership for each vector of training images.
 - d. Extracting the Optimal Feature Vectors Using Fuzzy Fisherface Method: by applying this method, the problem of the LDA which assumes the same level of typicality of each face to the corresponding class can be overcome, furthermore the fuzzy LDA considers the discriminative information in the null space of fuzzy

within-class scatter matrix that was ignored in LDA by taking the eigenvectors of matrix $FS_b FS_w^{-1}$ rather than maximize the ratio of their criterion.

After these four steps the training images projected into subspace of these steps.

4. **Testing Stage:** Plays the face recognition function of the system, whereas the test image is input into PZM analysis to be transformed into feature vector consisting of the expressive features. This vector is then subtracted from its mean and projected into the same subspace of the training images that projected into it. The test image is then compared with the face training images so that the face which has the minimum distance with the test image is labeled with identity of that image. The minimum distance can be computed using Euclidean distance metric. Fig. (2) show these stages of the proposed system.

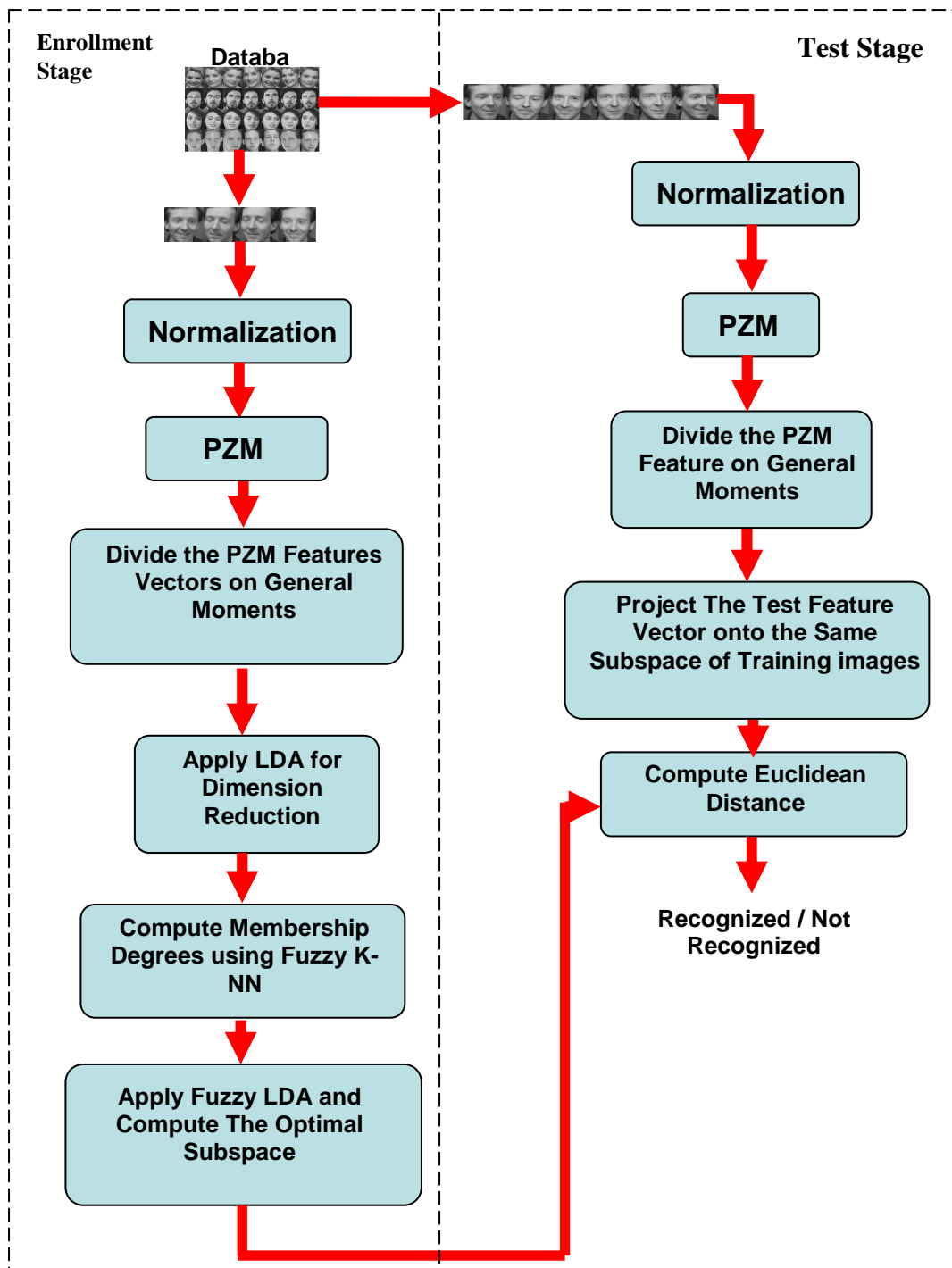


Fig. 2 Block Diagram of the Proposed Face Recognition System

5. Experimental Results

Experimental results of the proposed Fuzzy Moment Discriminant Analysis approach and its comparison with Fuzzy LDA are shown in this section. these two methods applied on two face databases: ORL Database, and BioID Database using visual basic 6.0 language.

5.1 First Experiment

This experiment was performed using ORL face database, from which randomly we choose only 22 persons, and from the ten images of each person the first six images were used for training, and the remaining four images for testing. All these images are manually cropped (after cropping each image its bit depth has been changed from 8 to 24), resized to 40×40 pixels, and normalized.

5.1.1 Enrollment Phase

In which, the six samples images for each of the 22 persons were enrolled by the system, the features vectors for these images have been extracted using PZM with order $p=10$ and repetition $q=8$, then by using LDA the dimension for the extracted features vectors have been reduced to 11 elements for each vector, after that the Fuzzy K-NN computes the membership degrees for these features vectors, and lastly the Fuzzy LDA uses the membership degrees to extract the final features vectors which represent the train images.

5.1.2 Test Phase

To evaluate the test phase performance, both the percentage of correct recognition (reported as the correct recognition rate (CRR)) and the percentage of false recognition (reported as the false recognition rate (FRR)) have been used. These rates defined as follows:

$$\text{CRR} = \frac{\text{No. of successfully recognized images}}{\text{Total number of images used in test phase}} \times 100\% \quad (4.1)$$

$$\text{FRR} = \frac{\text{No. of failed to be recognized images}}{\text{Total number of images used in test phase}} \times 100\% \quad (4.2)$$

persons enrolled into the system in train phase. In this phase we entered the no. of image which we want to test it, and we extract the features vector from it as shown in fig. (2), after that we perform the recognition process by matching the features vector of the test image to classes (persons) of facial images already enrolled in a system, using Euclidean distance metric. we can see that the proposed method CRR= 81 % while it's FRR=19 % (see Fig. (3) which show the ability of proposed method to recognize face). When we selected a value of order p other than 10 (more or less) or a repetition value other than 8 (more or less), the recognition performance of proposed method was markedly influenced and might decrease to 55 %. And when we apply the Fuzzy LDA on the same images we obtain CRR=70%.

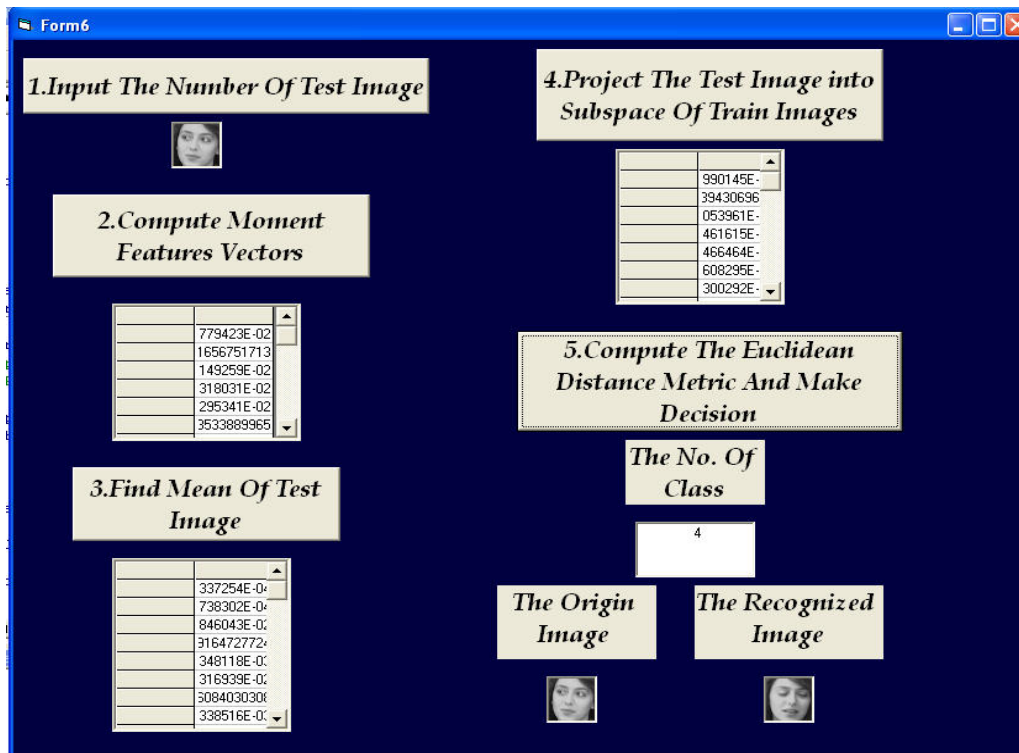


Fig. (3) Test Phase of Proposed Method on ORL database

5.2 Second Experiment

This experiment was performed using BioID face database, from which randomly we choose only 13 persons, and from the 66 images of each person we choose only 12 images which have strong variation in expression, face size, and hair style, where the first six images were used for training, and the remaining six images for testing. All these images are manually resized to 40×40 pixels, and normalized. In this experiment we didn't crop the image because we want to see the ability of the proposed method to recognize the images which have variations in hair style, and face sizes.

5.2.1 Enrollment Phase

In which, the six samples images for each of the 13 persons were enrolled by the system, as in first experiment we select order $p=10$ and repetition $q=8$. we can see that the proposed method $CRR=82\%$ while it's $FRR=18\%$ (see Fig. (4) which show the ability of proposed method to recognize face). Also when we selected a value of order p other than 10 (more or less) or a repetition value other than 8 (more or less), the recognition performance of proposed method was markedly influenced and might decrease to 55%. The average training time of the proposed face recognition system during the enrollment stage was computed (in seconds), which is equals to 1080s due to complex calculation of PZM.

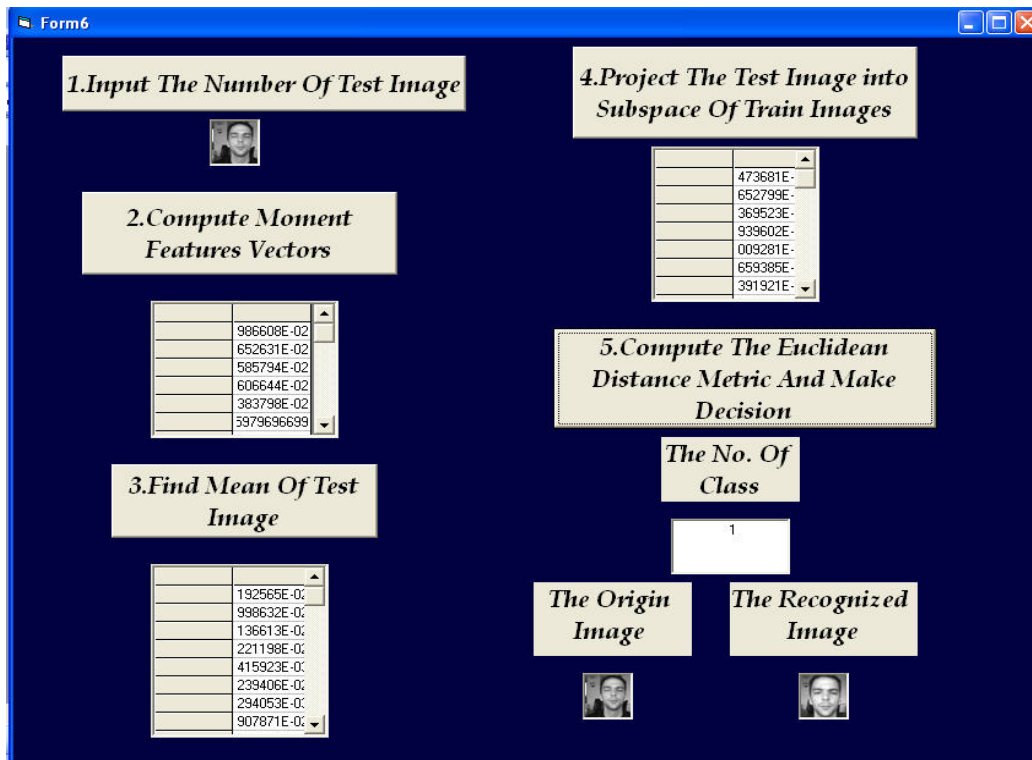


Fig. (4) Test Phase of Proposed Method on BioID database

6. Conclusions

In this work a new method of features extraction and recognition, namely, the fuzzy moment discriminant analysis (FMDA) is used, which is based on PZM, LDA, and fuzzy set, led to robust face recognition algorithm under extreme facial expressions variability (happy, sad, sleepy, normal, surprised, and wink), where this method achieved 82% accuracy under extreme facial expressions variability and variance in face size, while it achieved 81% under variance in pose, illumination, and expression. Performing PZM before applying LDA helps to achieve a more stable numerical computation and better solution in small-sample-size problem. The values of order and its repetition for PZM influence the recognition. And Normalize the values of PZM improve the recognition rate in an average of 2 % (since the PZM has large values).

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