

A Computer Programme to Determine the Bending and Pitting Stresses of Gears and the Effect of Varying the AGMA Stress Equation Parameters on the Stress Values

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Abstract

Gears are one of the most critical components in mechanical power transmission systems. The bending and surface strength of the gear tooth are considered to be the main contributors for the failure of the gears in a gear set. Thus, analysis of these stresses has become imperative in the area of research on gears to reduce errors and omissions in order to prevent the failures, and therefore, optimize the design of gears. The objective of this project is to write a computer programme developed from Microsoft Excel and Matlab softwares, the application of which is aimed at reducing the errors and omissions when calculating the bending and pitting stresses using the AGMA methodology effectively, efficiently and quickly in the design and analysis of spur and helical gears. The procedures employed include extraction of all the figures from the various graphs, obtaining an equation of the graphs extracted using the curve fitting tool function in Matlab and using the graph equations together with other equations of AGMA. The computer programme has been tested successfully and it has been established that its application is capable of determining the bending and pitting stresses of any spur and helical gears and their corresponding safety factors using the AGMA methodology.

Keywords: Gears; Transmission system; Gear set; Bending and surface strength.

1. Introduction

The crucial requirement of effective power transmission in various machines, automobiles, elevators, generators, ships, aircrafts, etc, has created an increasing demand for more accurate analysis of the characteristics of gear systems. This rapid development of these heavy machines requires advanced application of gear technology (Alemu, 2007).

From the design viewpoint, failure by bending of the teeth as well as pitting failure of tooth surfaces are the most important criteria when designing a gears et because each gear tooth may experience billions of load cycles (Stoker et al., 2010).

The manual method of analyzing and designing of spur and helical gears such as using a pen and a paper come with a lot of inefficiencies apart from it being time consuming when the AGMA methodology is employed. This increase in errors and spending so much time on calculating these stresses has therefore made it essential to have a computer software that will be used to calculate the stresses effectively, efficiently and quickly.

2. American Gears Manufacturers Association (AGMA) Standpoint and Contribution to Gear Manufacturing

2.1 Information about AGMA

AGMA is a voluntary association of companies, consultants and academicians with a direct interest in the design, manufacture, and application of gears and flexible couplings (Anon., 2012). AGMA was founded in 1916 by nine companies in response to the market demand for standardized gear products; it remains a member- and market—driven organization to this day. AGMA provides a wide variety of services to the gear industry and its customers and conducts numerous programs which support these services.

Some of these services and programs are

Standards: AGMA develops all U.S. gear related standards through an open process under the authorization of the American National Standards Institute (ANSI) (Anon., 2012). **ISO Participation:** AGMA is Secretariat to TC60, the committee responsible for developing all international gear standards. TC60 is an ISO (International Organization of Standardization) committee.

Market Reports and Statistics: AGMA's Operating Ratio Report, Wage & Benefit Survey, and Monthly Market Trend Reports help you stay competitive by giving you up-to-date information on the gear industry (Anon., 2012).

The Marketing and Statistical Councils enhance your competitiveness by sharing information and by developing creative solutions to common industry problems.

Gear Expo: This is the only trade show dedicated solely to the gear industry.

The AGMA Training School for Gear Manufacturing uses current technology to offer.

News Digest, AGMA's quarterly newsletter, offers you timely, useful information you can use immediately (Anon., 2012).

2.2 Gears under Consideration: Spur and Helical Gears

2.2.1 General Spur Gears

Spur gears are the most commonly used gear type. These gears have teeth parallel to the axis of the wheel as shown in Fig.2.1 (Khurmi and Gupta, 2008). Spur gears are by far the most commonly available, and are generally the least expensive. The basic descriptive geometry for a spur gear is as shown below.

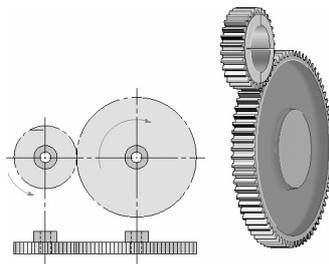


Fig. 1 Spur Gear
(Source: Suman, 2003)

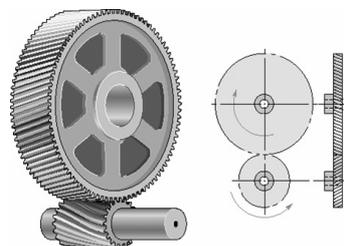


Fig. 2 Helical Gear
(Source: Suman, 2003)

2.2.1.1 Limitations

Spur gears generally cannot be used when a direction change between the two shafts is required.

2.2.1.2 Advantages

Spur gears are easy to find, inexpensive, and efficient.

2.2.2 General Helical Gears

Helical gears are similar to the spur gear except that the teeth are at an angle to the shaft, rather than parallel to it as in a spur gear. The resulting teeth are longer than the teeth on a spur gear of equivalent pitch diameter. The longer teeth cause helical gears to have the following differences from spur gears of the same size:

- Tooth strength is greater because the teeth are longer,
- Greater surface contact on the teeth allows a helical gear to carry more load than a spur gear
- The longer surface of contact reduces the efficiency of a helical gear relative to a spur gear.

Helical gears may be used to mesh two shafts that are not parallel, although they are still primarily use in parallel shaft applications. Helical gears run quieter and have a greater strength and capacity than spurs (Gretchen, 2001). The basic descriptive geometry for a helical gear is essentially the same as that of the spur gear, except that the helix angle must be added as a parameter (Suman, 2003).

Limitations: Helical gears have the major disadvantage that they are expensive and much more difficult to find. Helical gears are also slightly less efficient than a spur gear of the same size (see below).

Advantages of helical gears: Helical gears can be used on non-parallel and even perpendicular shafts, and can carry higher loads than can spur gears (Suman, 2003).

2.3 Modes of Gear Failure

Gear failure can occur in various modes. If care is taken during the design stage itself to prevent each of these failure a sound gear design can be evolved. The gear failure is explained by means of flow diagram in Fig.3.

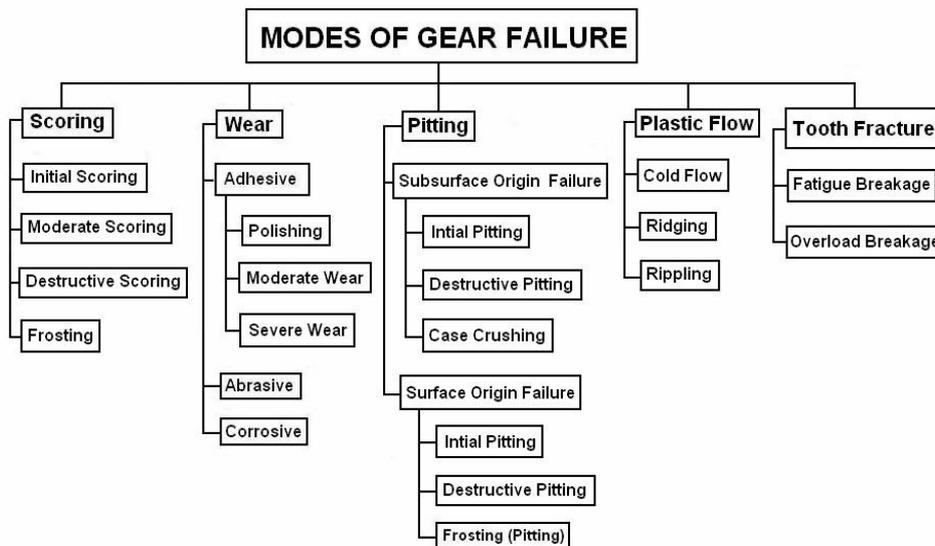


Fig. 3 Different Modes of Failure
 (Source: Gopinath and Mayuram, 2010)

2.3.1 Scoring

Scoring is due to combination of two distinct activities: First, lubrication failure in the contact region and second, establishment of metal to metal contact. Later on, welding and tearing action resulting from metallic contact removes the metal rapidly and continuously so far as the load, speed and oil temperature remain at the same level (Gopinath and Mayuram, 2010).

2.3.2 Wear

As per gear engineer's point of view, the wear is a kind of tooth damage where in layers of metal are removed more or less uniformly from the surface. It is nothing but progressive removal of metal from the surface. Consequently, tooth thins down and gets weakened. Three most common causes of gear tooth wear are metal-to-metal contact due to lack of oil film, ingress of abrasive particles in the oil and chemical wear due to the composition of oil and its additives (Gopinath and Mayuram, 2010).

2.3.3 Pitting of Gears

Pitting is a surface fatigue failure of the gear tooth. It occurs due to repeated loading of tooth surface and the contact stress exceeding the surface fatigue strength of the material. Material in the fatigue region gets removed and a pit is formed. The pit itself will cause stress concentration and soon the pitting spreads to adjacent region till the whole surface is covered. Subsequently, higher impact load resulting from pitting spreads may cause fracture of already weakened tooth. However, the failure process takes place over millions of cycles of running. There are two types of pitting, initial and progressive (Gopinath and Mayuram, 2010).

2.3.4 Initial / Incipient Pitting

In the helical gear shown in Fig.4 pitting started as a local overload due to slight misalignment and progressed across the tooth in the dedendum portion to mid face. Here, the pitting stopped and the pitted surfaces began to polish up and burnish over. This phenomenon is common with medium hard gears. On gears of materials that run in well, pitting may cease after running in, and it has practically no effect on the performance of the drive since the pits that are formed gradually become smoothed over from the rolling action. The initial pitting is non-progressive.



Fig. 4 Initial Pitting
(Source: Gopinath and Mayuram, 2010)

2.3.5 Progressive or Destructive Pitting

During initial pitting, if the loads are high and the corrective action of initial pitting is unable to suppress the pitting progress, then destructive pitting sets in. Pitting spreads all over the tooth length. Pitting leads to higher pressure on the unpitted surface, squeezing the lubricant into the pits and finally to seizing of surfaces.

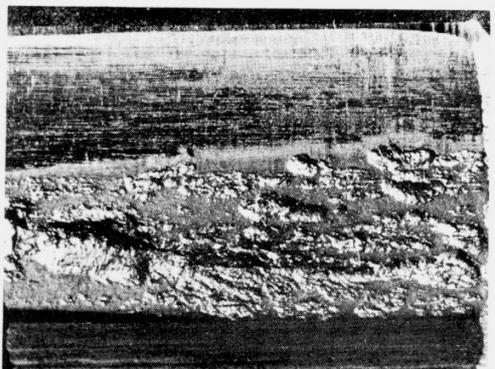


Fig.5 Tooth Surface Destroyed by Extensive Pitting
(Source: Gopinath and Mayuram, 2010)

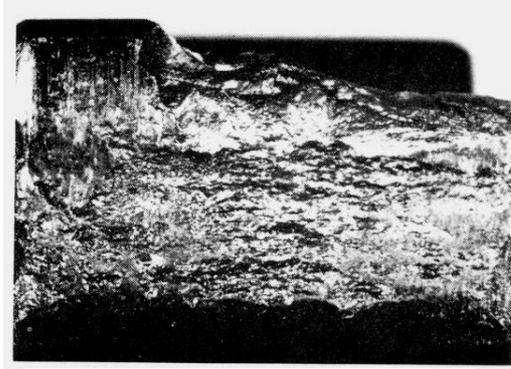
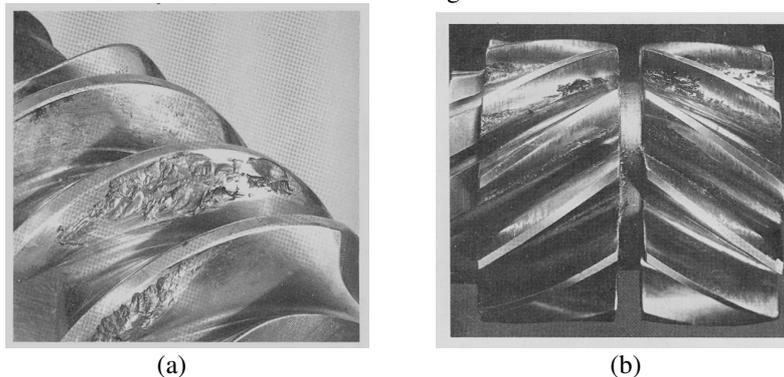


Fig. 6 Whole Tooth is Destroyed by Extensive Pitting
(Source: Gopinath and Mayuram, 2010)

Tooth faces are subjected to pitting only in rare cases. Fig. 2.5 shows how in destructive pitting, pitting has spread over the whole tooth and weakened tooth has fractured at the tip leading to total failure as in Fig.6.

2.3.6 Flaking / Spalling

In surface-hardened gears, the variable stresses in the underlying layer may lead to surface fatigue and result in flaking (spalling) of material from the surface as shown in Fig.2.7



(a) (b)
Fig.7 Flaking / Spalling
(Source: Gopinath and Mayuram, 2010)

2.3.7 Pitting - Subsurface Origin Failure

Fig.8 shows the subsurface origin failure.

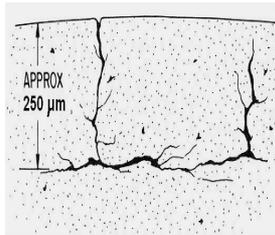


Fig.8 Subsurface Origin Failure
(Source: Gopinath and Mayuram, 2010)

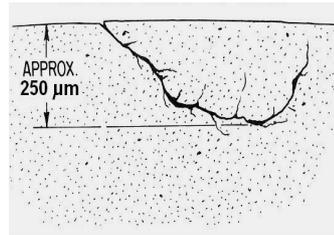


Fig.9 Surface Origin Failure
(Source: Gopinath and Mayuram, 2010)

2.3.8 Pitting - Surface Origin Failure

Failure modes in gear namely the surface origin failure is shown in Fig.9.

2.3.9 Progressive Pitting

The progressive pitting is shown in Fig.10.

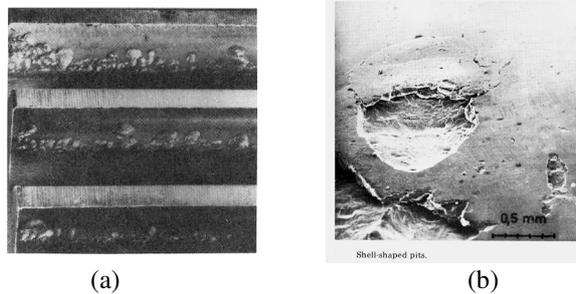


Fig. 10 Progressive pitting
(Source: Gopinath and Mayuram, 2010)

2.3.10 Pitting - Frosting

Frosting usually occurs in dedendum portion of the driving gear first and later on the addendum as shown in Fig.11. The wear pattern doesn't have normal metal polish but has etched-like finish.



Fig.11 Frosting
(Source: Gopinath and Mayuram, 2010)

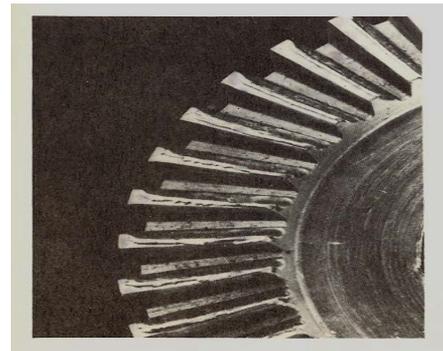


Fig.12 Plastic Flow - Cold Flow
(Source: Gopinath and Mayuram, 2010)

Under magnification, surface reveals very fine micro-pits of 2.5µm deep. These patterns follow the higher ridges caused by cutter marks. Frosting results from very thin oil film and some asperity.

2.3.11 Pitting Failure

Surface endurance strength determines the selection of dimensions and material for almost all gearing operating under conditions of the best possible lubrication.

2.3.12 Plastic Flow – Cold Flow

Plastic flow of tooth surface results when it is subjected to high contact stress under rolling cum sliding action. Surface deformation takes place due to yielding of surface or subsurface material. Normally, it occurs in softer gear materials. But it can occur even in heavily loaded case hardened gears. Cold flow material over the tooth tip can be seen clearly in the bevel gear shown in the Fig. 12.

2.3.13 Tooth Fracture

Tooth fracture is the most dangerous kind of gear failure and leads to disablement of the drive and frequently to damage of other components (shafts, bearings, etc.) by pieces of the broken teeth. Tooth breakage may be the result of high overloads of either impact or static in nature, repeated overloads causing low-cycle fatigue, or multiple repeated loads leading to high cycle fatigue of the material.

2.3.13.1 Tooth Breakage – Bending Fatigue

Bending fatigue failure occurs over a long period of time. The initiation of crack takes place at the weakest point, normally at the root of the tooth or at the fillet where high stress concentration exists together with highest tensile stress from bending or from the surface defects as shown in Fig.13. The crack slowly propagates over 80 to 90% of the life.

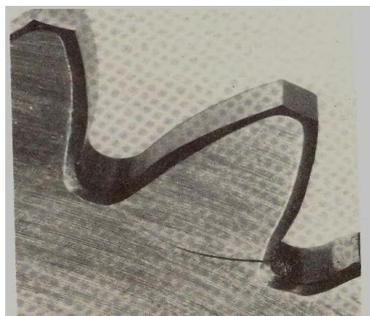


Fig. 13 Root Crack
(Source: Gopinath and Mayuram, 2010)

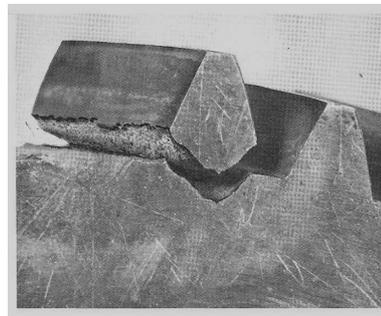


Fig. 14 Tooth Breakage
(Source: Gopinath and Mayuram, 2010)

Then crack propagates fast and suddenly results in fracture of the tooth as shown in Fig. 2.14. The fractured surface will exhibit beach marks in the slow crack propagation region and brittle fracture behavior in sudden fracture region. Since time taken for the failure is very long, it is known as high cycle fatigue.

2.3.13.2 Tooth Breakage – High Cycle Fatigue

The tooth breakage in case of high cycle fatigue is shown in Fig..15.



Fig. 15 High Cycle Fatigue
(Source: Gopinath and Mayuram, 2010)

2.3.13.3 Tooth Breakage – Low Cycle Fatigue (Over Load)

Overload breakage or short (low) cycle fatigue causes stringy fibrous appearance in broken ductile material. In harder materials this break has a more silky or crystalline appearance as shown in Fig.16.

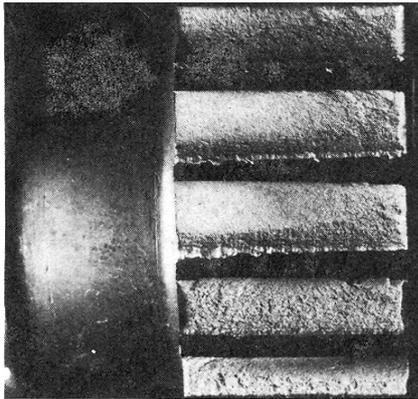


Fig.16 Low Cycle Fatigue (Over Load)
(Source: Gopinath and Mayuram, 2010)

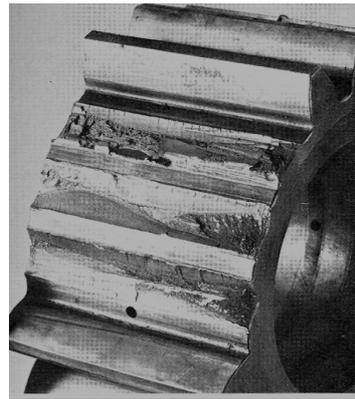


Fig. 17 Bending Fatigue
(Source: Gopinath and Mayuram, 2010)

2.3.13.4 Tooth Breakage – Bending Fatigue

Fig.17 shows tooth fatigue by bending fatigue.

2.3.13.5 Tooth Breakage

Breakage is often due to load concentration along the tooth length as a result of errors in machining and assembly or of large elastic deformation of the shafts; tooth wear leading to weakening of the teeth results in increased dynamic loads. Shifting of sliding gears into mesh takes place without stopping the rotation of the shafts. Cracks are usually formed at the root of the teeth on the side of the stretched fibers where the highest tensile stresses occur together with local stresses due to the shape of the teeth. Fracture occurs mainly at a cross section through the root of the teeth.

In the case of fatigue failure, the fracture is of concave form in the body of the gear; it is of convex form when the failure is from overload. The teeth of herringbone or wide-face helical gears usually break off along a slanting cross section. To prevent tooth breakage, the beam strength of the gear teeth is checked by calculations. Fatigue pitting of the surface layers of the gear teeth is the most serious and widespread kind of tooth damage that may occur in gears even when they are enclosed, well lubricated and protected against dirt (Gopinath and Mayuram, 2010).

2.4 Analytical Methods of Gear Design

2.4.1 The Lewis Bending Equation

The design of gear strength is based on two models: bending stress and contact stress models. The former is related to the stress at the gear base, while the latter is related to the wear at the contact surface. The bending equation was introduced by Wilfred Lewis in 1892. Since then, this equation has remained the standard for gear design (Budynas and Nisbett, 2011; Dudley, 1984). Lewis calculated stress in the gear base using a cantilevered beam under an applied bending moment. By using this simple model, an accurate bending stress could be determined (Budynas and Nisbett, 2011). This calculation assumes that the load is applied at the location of the pitch radius (Colbourne, 1987). In practice, however, contact between the gear and pinion occurs at various locations during the rotation.

To derive the basic Lewis equation, refer to Fig.2.18, which shows a cantilever of cross-sectional dimensions F and t , having a length l and a load Wt , uniformly distributed across the face width F . The section modulus I/c is $Ft^2/6$, and therefore the bending stress is

$$\sigma = \frac{M}{I/c} = \frac{6Wtl}{Ft^2} \quad (1.a)$$

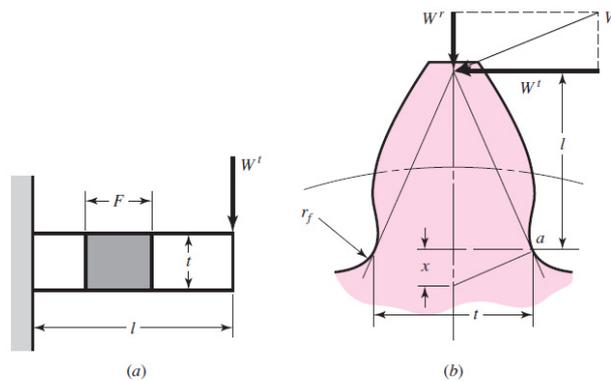


Fig. 2.18 Gear Tooth as Cantilever Beam
 (Source: Budynas and Nisbett, 2011)

Referring to Fig. 18, we assume that the maximum stress in a gear tooth occurs at point *a*. By similar triangles, we can write

$$\frac{t/2}{x} = \frac{l}{t/2} \text{ OR } x = \frac{t^2}{4l} \quad (1.b)$$

By rearranging Eq. (2.a)

$$\sigma = \frac{6Wtl}{Ft^2} = \frac{Wt}{F} \frac{1}{t^2/6l} = \frac{Wt}{F} \frac{1}{t^2/4l} \frac{1}{\frac{4}{6}} \quad (1.c)$$

If we now substitute the value of *x* from Eq. (2.b) in Eq. (2.c) and multiply the numerator and denominator by the circular pitch *p*, we find

$$\sigma = \frac{Wtp}{F\left(\frac{2}{3}\right)xp} \quad (1.d)$$

Letting $y = 2x/3p$, we have

$$\sigma = \frac{Wt}{Fpy} \quad (2)$$

This completes the development of the original Lewis equation. The factor *y* is called the *Lewis form factor*, and it may be obtained by a graphical layout of the gear tooth or by digital computation (Budynas and Nisbett, 2011).

2.11.2 AGMA Stress Equations

Two fundamental stress equations are used in the AGMA methodology, one for bending stress and another for pitting resistance (contact stress). In AGMA terminology, these are called *stress numbers*, as contrasted with actual applied stresses, and are designated by a lowercase letter *s* instead of the Greek lower case σ . (Budynas and Nisbett, 2011).

The fundamental equations are

$$\sigma = WtK_oK_vK_s \frac{1}{bmt} \frac{K_H K_B}{Y_j} \quad (\text{SI unit}) \quad (3)$$

where (for SI units),

W' is the tangential transmitted load, (N); *K_o* is the overload factor; *K_v* is the dynamic factor

K_s is the size factor; *F* (*b*) is the face width of the narrower member, (mm)

K_H is the load-distribution factor; *K_B* is the rim-thickness factor; (*m_i*) is the transverse metric module. *Y_j* is the geometry factor for bending strength (which includes root fillet stress-concentration factor *K_f*)

The fundamental equation for pitting resistance (contact stress) is

$$\sigma = ZE \sqrt{WtK_oK_vK_s \frac{K_H}{d_w1b} \frac{Z_R}{Z_I}} \quad (\text{S.I}) \quad (4)$$

where *W_t*, *K_o*, *K_v*, *K_s*, *K_m*, *F*, and *b* are the same terms as defined for Eq. (2.2).

For SI units, the additional terms are

Z_E is an elastic coefficient, ($\sqrt{\text{N/mm}^2}$); *Z_R* is the surface condition factor

d_{w1} is the pitch diameter of the *pinion*, (mm); *Z_I* is the geometry factor for pitting resistance

2.12 Matlab Programming Language

Matlab is a high-level language and interactive environment for numerical computation, visualization, and programming. Using MATLAB, you can analyze data, develop algorithms, and create models and applications. The language, tools, and built-in math functions enable you to explore multiple approaches and reach a solution faster than with spreadsheets or traditional programming languages, such as C/C++ or Java (Anon., 2013). Fig 2.19 shows the interface of the Matlab programming language.

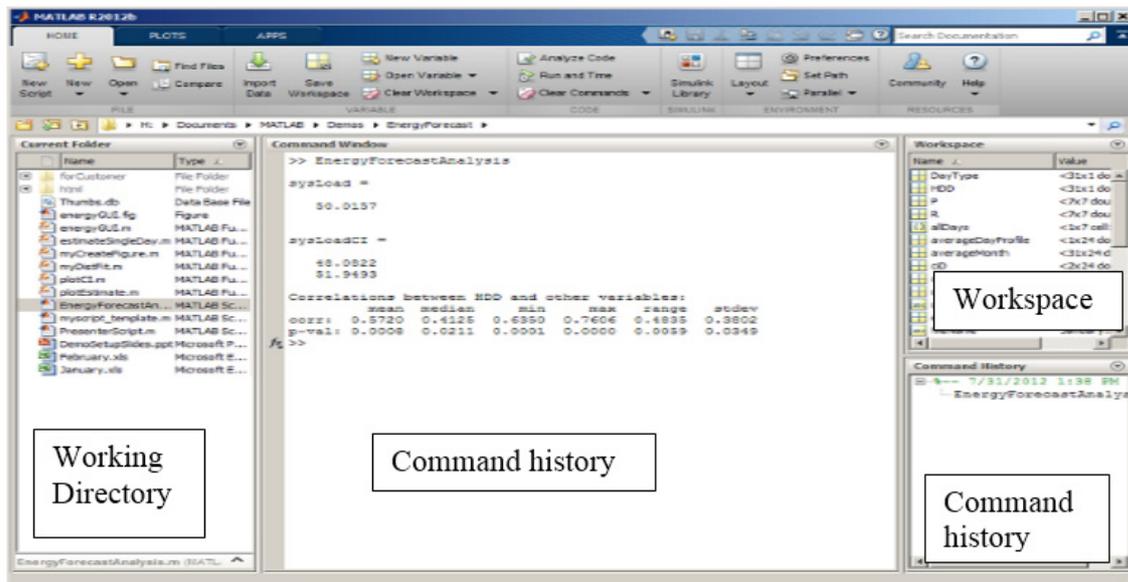


Fig.19 Interface of the Matlab Programming Language, IDE
(Source: Anon., 2013)

3. Application Architecture, its Development and Installation

3.1 AGMA Stress Calculator Tool (ASCT)

The AGMA Stress Calculator Tool (ASCT) is a software application that determines the bending and pitting stresses of spur and helical gears and their corresponding factors of safety. This was designed using Matlab R2012a, Microsoft excel and Microsoft word.

3.2 Application Architecture

The application is structured to operate with a little user interactions by taking off most of the manual processes and automating them behind the scene, thereby reducing errors caused by human interactions. The flow chart below shows the system architecture of the application. It shows a quick way for understanding the development process for the system.

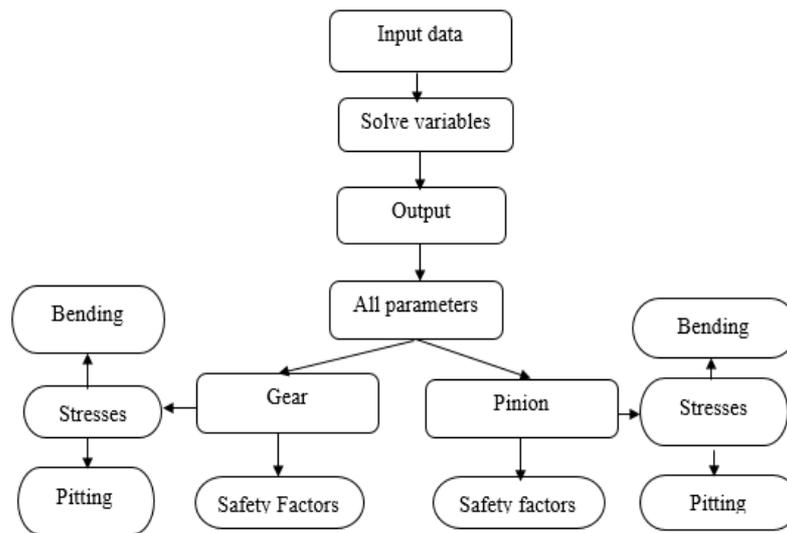


Fig.20 Flow Chart of the Application Architecture

A look at the system architecture above shows that the only input required by the application is the data given which are loaded via the application interface. A click on “Solve variables” on the application interface displays all the results.

3.2.1 Data Input for ASCT

The minimum input data required are: power (kW), pinion speed (rev/min), Brinell hardness material selection, quality of gears, pinion life, reliability, diameter gears (mm) and teeth, module (mm), face width (mm) and type of mesh. Fig. 21 shows the part of the software for loading data

3.2.2 Output of ASCT

The ASCT displays two main results. Firstly, it displays all the stress parameters which comprise the transmitted load, reliability factor, dynamic factor, geometry factor, pitch-line velocity, etc. as shown on the left hand side in Fig.21 below. Then finally, the pinion and the gear parameters which comprise of the bending and pitting stresses and their corresponding factors of safety are also displayed.

3.3 Development of the Application Interface

Microsoft word was used to design the interface of the AGMA stress calculator tool (ASCT) as shown in Fig.21. It was then developed with Matlab programming language using the GUIDE tools. The basic controls used for the interface includes, Edit text, List boxes and button controls. The User Interface controls were bound to series of script and functions that solve for all the parameters that will be used to determine the bending and pitting stresses and their corresponding factors of safety.

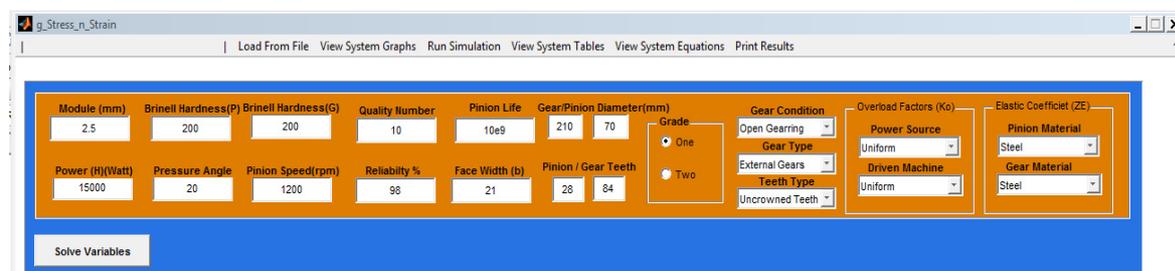


Fig. 21 Part of the Software for Loading Data

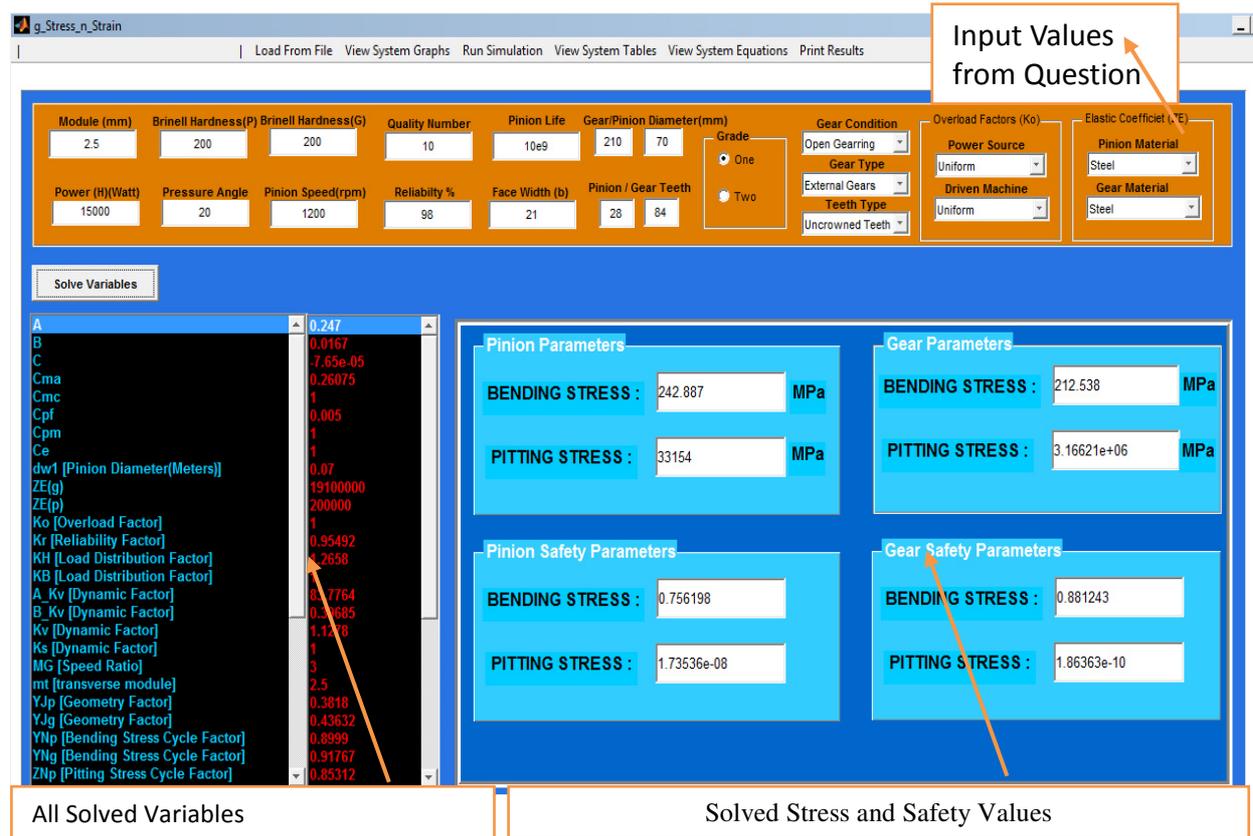


Fig.22 The Software Interface

3.4 Installation of the Application Interface

As introduced earlier, the application (ASCT) was engineered in Matlab using the Matlab functions. After the development of the application, it was built and packaged into windows executable file (i.e. .exe) which can be run on any windows OS. The application however cannot run on non-windows OS like Mac OSX, Linux, etc. The steps for using the application are outlined as follows;

- The setup file must be run by the user for the installation of the software to be executed.
- Upon installing it, an executable file and an icon is placed on the desktop which when opened, launches the executable file from the Application folder of the target machine.
- The application is called *forth* by the user by running the application icon file found on the desktop after installing the application. On running it, the application launches the interface with the input data for loading as shown in Fig 22.

4. Calculation Analyses

4.1 Solution of a Sample Question to be Compared with that of the Software

4.1.1 Sample Question

A 20° spur pinion with 70 mm diameter and a module of 2.5 mm transmits 15 kW to a 210 mm diameter gear. The pinion speed is 1200 rev/min, number of teeth of pinion and gear are 28 and 84 respectively and the gears are grade 1, 21 mm face width, through-hardened steel at 200 Brinell, uncrowned, manufactured to a No. 10 quality standard, and considered to be of open gearing quality installation. Find the AGMA bending and contact stresses and the corresponding factors of safety for a pinion life of 10⁹ cycles and a reliability of 0.98.

4.1.2 Long - hand Solution

Data summarized from the question; Pressure angle, $\Phi_f=20^\circ$; Module, $m=2.5\text{mm}$ (0.0025m) Grade 1 gears, through-hardened steel; Face width = $b = 21\text{mm}$ (0.021m); Brinell hardness, $H_B=200$; Quality number, $Q_v=10$; Open gearing installation; Pinion life= 10^9 cycles; Pinion speed, $n_p=1200\text{rpm}$; Power, $H=15\text{kW}$; Pinion diameter, $d_p=70\text{mm}$ (0.07m);

Gear diameter, $d_G=210\text{mm}$ (0.21m); Number of teeth of pinion, $N_p=28$ teeth;
 Number of teeth of gear, $N_G = 84$ teeth

General stress equations

$$\sigma(\text{Bending.Stresses}) = W^t K_O K_V K_S \frac{1}{bm_t} \frac{K_H K_B}{Y_J} \quad (5)$$

$$\sigma(\text{ccontact.stress}) = Z_E \sqrt{(W^t K_O K_V K_S \frac{K_H Z_R}{d_{w1} b} \frac{1}{Z_I})} \quad (6)$$

$$\text{Transmitted.Load, } W^t = \frac{H}{V} \quad (7)$$

$$V = \frac{\pi n_p d_p}{60} \quad (8)$$

where, V is the pitch-line velocity

$$V = \frac{\pi(1200)(0.07)}{60} \quad ; \quad V=4.3982\text{m/s}; \quad W^t = \frac{15 \times 10^3}{4.3982} ; W^t=3410.4861\text{N}; \text{Overload factors, } K_O$$

Assuming uniform power source and uniform driven machine, $K_O=1$; ;Dynamic factor, K_V

$$K_V = \left(\frac{A + \sqrt{200V}}{A} \right)^B \quad (9)$$

$$A=50+56(1-B) \text{ and } B=0.25(12-Q_v)^{\frac{2}{3}} \quad (10)$$

Substituting the quality number Q_v into the B equation and solving yields. $A=50+56(1-0.3969)$ and $B=0.25(12-10)^{\frac{2}{3}}$; $A=83.7736$ and $B=0.3969$

Then we have

$$K_V = \left(\frac{83.7736 + \sqrt{(200 \times 4.3982)}}{83.7736} \right)^{0.3969} = 1.1278$$

Size factor, K_S ; AGMA suggests $K_S=1$ (Budynas and Nisbett, 2011), therefore $K_S=1$

Load distribution factor, K_H ; $K_H=1+C_{mc}$ ($C_{pf}C_{pm}+C_{ma}C_e$) (11)

$$C_{mc}=1, \text{ (uncrowned teeth); } C_{pf} = \frac{b}{10d} = \frac{21}{10 \times 70} = 0.025 \quad b \leq 25\text{mm} = \frac{21}{10 \times 70} = 0.025 \quad (12)$$

$$C_{pf}=0.005; C_{pm}=1 \text{ (straddle mounted pinion)} C_{ma}=A+BF+CF^2; \quad (13)$$

$$A=0.247, B=0.0167, C=-0.765(10^{-4}); F = \frac{b}{25.4} = \frac{21}{25.4} = 0.8268\text{inches}$$

$$C_{ma}=0.247 + 0.0167(0.8268) - 0.765(10^{-4})(0.8268^2); C_{ma}=0.2608; C_e=1$$

$$K_H = 1 + 1(0.005 \times 1 + 0.2608 \times 1), K_H = 1.2658; \text{ Rim thickness}$$

Since the backup ratio is not known we assume $K_B=1$; Bending-strength geometry factor, Y_J

$$(Y_J)_p = 0.38 \text{ (from Fig.14-6in Budynas and Nisbett, 2011)}$$

$$(Y_J)_G = 0.44 \text{ (from Fig.14-6in Budynas and Nisbett, 2011)}$$

Elastic coefficient; $Z_E=191\sqrt{\text{MPa}}$ (from Table 14-8 in Budynas and Nisbett, 2011)

Surface condition factor

Standard surface conditions for gear teeth have not yet been established. When a detrimental surface finish effect is known to exist, AGMA specifies a value of Z_R greater than unity (Budynas and Nisbett, 2011)..Therefore, $Z_R=1$; Surface strength geometry factor, Z_I

$$\text{Speed ratio } m_G = \frac{d_G}{d_P} = \frac{210}{70} = 3; Z_1 = \frac{\cos \phi_t \sin \phi_t}{2m_n} \times \frac{m_G}{m_G + 1} \quad (\text{For external gears}) \quad (14)$$

$$= \frac{\cos 20 \sin 20}{2(1)} \times \frac{3}{3+1}; Z_1 = 0.1205$$

Bending strength, S_t (Grade 1 through –hardened steel); $S_t = 0.533\text{HB} + 88.3$ (15)

$= 0.533(200) + 88.3$; $S_t = 194.9 \text{ MPa}$; Stress cycle factors, Y_N and Z_N
 $(Y_N) = 1.3558N^{-0.0178}$ (for both pinion and gear) (16)

$(Y_N)_P = 1.3558(10^9)^{-0.0178}$
 $(Y_N)_G = 1.3558\left(\frac{10^9}{3}\right)^{-0.0178}$; $(Y_N)_G = 0.91767$
 $(Z_N) = 1.4488N^{-0.023}$ (for both pinion and gear) (17)

$(Z_N)_P = 1.4488(10^9)^{-0.023} = 0.85311$; $(Z_N)_G = 1.4488\left(\frac{10^9}{3}\right)^{-0.023} = 0.87494$; Hardness-ratio factor, Z_w

$Z_w = 1, \left(\frac{HBP}{HBG}\right) < 1.2$ (from Budynas and Nisbett, 2011)

Contact-fatigue stress (grade 1, through-hardened steel); $S_c = 2.22(\text{HB}) + 200$ (18)

$S_c = 2.22(200) + 200$; $S_c = 644 \text{ MPa}$

Reliability factor; $Y_z = 0.658 - 0.0759 \ln(1-R)$ (19)

$Y_z = 0.658 - 0.0759 \ln(1-0.98)$; $Y_z = 0.9549$

Temperature factor, Y_θ

For oil or gear-blank temperatures up to 250°F (120°C), use $KT = Y_\theta = 1.0$. For higher temperatures, the factor should be greater than unity. Heat exchangers may be used to ensure that operating temperatures are considerably below this value, as is desirable for the lubricant. (Budynas and Nisbett, 2011). Therefore $Y_\theta = 1$

Bending stresses for pinion, from equation (5)

$$(\sigma)_P = \frac{3410.4861 \times 1 \times 1.1278 \times 1 \times 1 \times 1.2658}{0.021 \times 0.0025 \times 0.38}; (\sigma)_P = 244.0454 \text{ MPa}$$

Bending stresses for Gear

$$(\sigma)_G = \frac{3410.4861 \times 1 \times 1.1278 \times 1 \times 1 \times 1.2658}{0.021 \times 0.0025 \times 0.44}; (\sigma)_G = 210.766 \text{ MPa}$$

Corresponding factor of safety for the pinion

$$S_F = \frac{S_t Y_N}{\sigma Y_\theta Y_Z} = \frac{194.9}{244.0454} \times \frac{0.9376}{0.9549} = 0.7526 \quad (20)$$

Corresponding factor of safety for the gear

$$S_F = \frac{S_t \times Y_N}{\sigma \times Y_\theta \times Y_Z} = \frac{194.9 \times 0.9177}{210.766 \times 0.9549} = 0.9262$$

Pitting stresses for the pinion

$$(\sigma_c) = 191 \times 10^3 \sqrt{\left(\frac{3410.4861 \times 1 \times 1.1278 \times 1 \times 1.2658 \times 1}{0.021 \times 0.07 \times 0.1205}\right)} = 1001.4 \text{ MPa}$$

Pitting stresses for the gear

$$(\sigma_c)_G = 191 \times 10^3 \sqrt{\left(\frac{3410.4861 \times 1 \times 1.1278 \times 1 \times 1.2658 \times 1}{0.021 \times 0.07 \times 0.1205}\right)} = 1001.4 \text{ MPa}$$

Corresponding factor of safety for the pinion

$$S_H = \frac{S_C Z_N Z_W}{\sigma_c Y_\theta Y_Z} = \frac{644}{1001.4} \times \frac{0.85311 \times 1}{1 \times 0.9549} = 0.57455$$

Corresponding factor of safety for the gear

$$S_H = \frac{S_C Z_N Z_W}{\sigma_c Y_\theta Y_Z} = \frac{644}{1001.4} \times \frac{0.877 \times 1}{1 \times 0.9549} = 0.621607 \quad (21)$$

4.1.3 ASCT Solution

The software was used to solve the same example as above; these are the results captured in Fig.22



Fig.23 The Software Displaying the Results

Table 1 Results of the Solution of the Sample Question Compared with that of the Software

Solved Parameters	MANUALLY	ASCT
Transmitted Load, Wt	3410.4861	3410.4631
Pitch Line Velocity, V	4.3982	4.3982
Dynamic Factor, Kv	1.1278	1.1278
Load Distribution Factor, KH	1.2658	1.2658
Pinion Bending-Strength geometry factor, Yj	0.38	0.3818
Gear Bending-Strength geometry factor, Yj	0.44	0.43632
Bending strength, S _t (Grade 1)	194.9	194.9
Pinion Stress cycle factors, Y _N	0.9000	0.8999
Gear Stress cycle factors, Y _N	0.91767	0.91767
Pinion Stress cycle factors, Z _N	0.85311	0.85312
Bending stresses for pinion	244.0454	242.887
Bending stresses for Gear	210.766	212.538
Bending factor of safety for the pinion	0.7526	0.756198
Bending factor of safety for the gear	0.9262	0.881243
Pitting stresses for the gear	1001.4	1001.24
Pitting factor of safety for the pinion	0.57455	0.574628
Pitting factor of safety for the gear	0.621607	0.589332

5 Conclusions and Recommendations

5.1 Conclusion

In conclusion, the following has been achieved:

- An effective software for determining the bending and pitting stresses for spur and helical gears has been developed.
- Accurate results will be achieved when determining bending and pitting stresses of spur and helical gears.
- A lot of time will be saved in selecting parameters for a gear design.

- A system for studying the effects of varying one gear parameter on other parameters.

5.2 Recommendation

- The application should be further developed to include the determination of the bending and pitting stresses of bevel and worm gears since this project only focused on spur and helical gears.
- It is strongly recommended for gear designers, lecturers and the like to use this software whenever dealing with the bending and pitting stresses of spur, helical, bevel and worm gears, which are the basic gears in operation and also for more complex gears like hypoid, spiral, novikoff and others.

6 References

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APPENDIX

Code Used for the Application of the Software Interface

The code used for this project which was written in Matlab R2012a is as shown below. The code is to calculate the bending and pitting stresses of spur and helical gears and their corresponding safety factors.

```
% --- Executes on button press to solve variables.  
Function solve_variables_Call back(h Object, event data, handles)  
%0h Object handle to solve_variables (see GCBO)  
% event data reserved - to be defined in a future version of MATLAB  
% handles structure with handles and user data (see GUIDATA)  
global ZE A B C YJpYJgZEgZEp Ko YNpYNgZNpZNg mg ZI phi gear_type V Wt ...  
Kv Cmc Cma Cpm Cpf Ce F KH KB mt Zr Kr dwl Ks pBSgBSpPSgPS grade gStpSt ...  
YtgScpSc Zw pBSFgBSFpSHgSH  
% k = (get(handles.listbox1,'String'));  
K = cell(0,1);  
k2=cell(0,1);  
selected_row=get(handles.gear_condition,'Value');  
allstrings=get(handles.gear_condition,'String');  
in put value = all strings (selected_row);
```

```
% Read the CMA CONDITION TABLES
[A B C]=cma_condition(inputvalue{ 1:end});

% Read the Elastic coefficient (ZE)
mrow=get(handles.pinion_material,'Value');
mcol=get(handles.gear_material,'Value');
ZEg= Elastic_coefficient_ZE(mrow,mcol);
ZEg=ZEg * 1000;
ZEp=Elastic_coefficient_ZE(mrow,7);

% Read the Overload Factor (Ko)
mrow=get(handles.power_source,'Value');
mcol=get(handles.driven_machine1,'Value');
Ko = overlaod_factor_ko(mrow,mcol);

% Solve for the Geometry Factor of Pinion
pteeth=str2double(get(handles.pinion_teeth,'String'));
gteeth=str2double(get(handles.gear_teeth1,'String'));

YJp= yj(pteeth, gteeth );
YJg= yj(gteeth, pteeth );

% Read the mg value
pdiameter=str2double(get(handles.pinion_diameter,'String'));
gdiameter=str2double(get(handles.gear_diameter,'String'));

mg=mg_value(gteeth,pteeth,gdiameter,pdiameter);

% Solve for Stress Cycle Factors (bending) of Gear n Pinion
P Load Cycle=str2double(get(handles.pinion_life,'String'));
hb=str2double (get (handles.hb_p,'String'));
YNp=Stress_Yn (hb, pLoad Cycle);
YNg=Stress_Yn (hb, pLoad Cycle/mg );

%Solve for Pitting Stress Cycle Factor of Gear and Pinion
ZNp = Stress_ZN(pLoadCycle );
ZNg = Stress_ZN(pLoadCycle/mg );

% Solving for Surface Strength Geometry factor (ZI)
gear_type = get (handles.gear_type,'Value');
phi=str2double(get (handles.pressure_angle,'String'));
ZI=ZI value (gear type, mg, phi);
```