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Common Fixed Point of Occasionally Weakly Compatible Maps on Intuitionistic Fuzzy Metric Spaces for Integral Type Inequality

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Abstract
In this paper, We obtain common fixed point theorems in intuitionistic fuzzy metric spaces using Occasionally weakly compatible maps for integral type inequality. Our results are the intuitionistic fuzzy version of some fixed point theorems for weakly compatible mappings on different metric spaces.
Key words and phrases: Intuitionistic fuzzy metric space, Occasionally weakly Compatible mappings, Common fixed point.

Introduction and Preliminaries

The first result on fixed points for contractive type mapping was the much celebrated Banach's contraction principle by S. Banach [12] in 1922. In the general setting of complete metric space, this theorem runs as the follows, **Theorem 1(Banach's contraction principle)** Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y ∈ X,
$d(fx, fy) \leq c\, d(x, y)$ Then f has a unique fixed point a ∈ X, such that for each x ∈ X $\lim_{n\to\infty} f^n x = a$. In 2002, A. Branciari [1] analyzed the existence of fixed point for mapping f defined on a complete metric space (X , d) satisfying a general contractive condition of integral type.

Theorem 2(Branciari) Let (X, d) be a complete metric space, c∈ (0, 1) and let f: X→X be a mapping such that for each x, y ∈ X, $\int_0^{d(fx,fy)} \varphi(t)dt \leq c \int_0^{d(x,y)} \varphi(t)dt$. Where φ: [0,+∞) →[0,+∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+∞) , non negative, and such that for each ε >o, $\int_0^{\varepsilon} \varphi(t)dt,$ then f has a unique fixed point
a ∈ X such that for each x ∈ X, $\lim_{n\to\infty} f^n x = a$. A fine work has been done by B.E.Rhoades [3] extending the result of Brianciari by replacing the condition [1] by the following $\int_0^{d(fx,fy)} \varphi(t)dt \leq \int_0^{max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \varphi(t)dt$. K.Atanassov [8] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, J. H. Park [7] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conforms. Recently, in 2006, Alaca et al.[2] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-Conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [9].Intuitionistic fuzzy set theory has been used to extract information by reflecting and modeling the hesitancy present in real-life situations. In this paper, we obtain common fixed point theorems in intuitionistic fuzzy metric spaces using occasionally weakly compatible maps for integral type inequality. These concepts were originally introduced by K. Menger [8] in study of statistical metric spaces. D. Coker [5], Turkoglu [6], S. Manro [11] studied in Common fixed point theorems in intuitionistic fuzzy metric spaces. Zadeh [13] An introduction to intuitionistic fuzzy topological spaces

Definition 1.1[10]. A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions:
(1) * is associative and commutative,
(2) * is continuous,
(3) a * 1 = a for all a ∈ [0, 1],
(4) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0, 1],
Two typical examples of continuous t-norm are a * b = ab and a * b = min (a, b).

Definition 1.2[10]. A binary operation ◊: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions:
(1) ◊ is associative and commutative,
(2) ◊ is continuous,

(3) a $\diamond$1 = a for all a ∈ [0, 1],
(4) a $\diamond$ b ≤ c $\diamond$ d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0, 1],
Two typical examples of continuous t-norm are a $\diamond$ b = ab and a $\diamond$ b = min (a, b).
Alaca et al. [2] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [9] as :

Definition 1.3[1]. A 5-tuple (X, M, N, $*$, $\diamond$) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-co norm and M,N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

(i) $M(x,y,t) + N(x,y,t) \le 1 for\ all\ x,y \in X\ and\ t > 0;$

(ii) $M(x,y,0) = 0\ for\ all\ x,y \in X;$

(iii) $M(x,y,t) = 1\ for\ all\ x,y \in X\ and\ t > 0\ if\ and\ only\ if\ x = y;$

(iv) $M(x,y,t) = M(y,x,t)\ for\ all\ x,y \in X\ and\ t > 0;$

(v) $M(x,y,t) * M(y,z,s) \le M(x,z,t+s) for\ all\ x,y \in X\ and\ s,t > 0;$

(vi) $for\ all\ x,y \in X, M(x,y,\ .): [0,\infty) \to [0,1]\ is\ left\ continuous;$

(vii) $\lim_{t\to\infty} M(x,y,t) = 1\ for\ all\ x,y \in X\ and\ t > 0;$

(viii) $N(x,y,0) = 1\ for\ all\ x,y \in X;$

(ix) $M(x,y,t) = 1\ for\ all\ x,y \in X\ and\ t > 0\ if\ and\ only\ if\ x = y;$

(x) $N(x,y,t) = N(y,x,t)\ for\ all\ x,y \in X\ and\ t > 0;$

(xi) $N(x,y,t) \diamond N(y,z,s) \ge N(x,z,t+s) for\ all\ x,y \in X\ and\ s,t > 0;$

(xii) $for\ all\ x,y \in X, N(x,y,\ .): [0,\infty) \to [0,1]\ is\ right\ continuous;$

(xiii) $\lim_{t\to\infty} N(x,y,t) = 0\ for\ all\ x,y \in X\ and\ t > 0.$

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 1.1. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \diamond)$ such that t-norm $*$ and t-co norm $\diamond$ are associated as $x \diamond y = 1 - ((1-x)*(1-y)) for\ all\ x,y \in X.$

Remark 1.2. In intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, $M(x,y,*)$ is non-decreasing and $N(x,y,\diamond)$ is non-increasing for all $x,y \in X$.

Alaca, Turkoglu and Yildiz [2] introduced the following notions:

Definition 1.4[2]. Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is called Cauchy-sequence if, for all t > 0 and P > 0, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$

and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0,$

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if , for all $t > 0$,

$\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$.

Definition 1.5[1]. An intuitionistic fuzzy metric space (X, M, N, $*$, $\diamond$) is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 1.6[2]. A pair of self mappings (f, g) of a metric space is said to be weakly compatible if they commute at the coincidence points i.e. $fu = gu$ *for some* $u \in X$, then $fgu = gfu$.

Definition 1.7[2]. Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space. f *and* g be self maps on X. A point x in X is called a coincidence point of f *and* g iff $fx = gx$.

Definition 1.8[2]. A pair of self mappings (f, g) intuitionistic fuzzy metric space is said to be weakly compatible if they commute at the coincidence points i.e. $fu = gu$ *for some* $u \in X$, then $fgu = gfu$.
Definition 1.9[3]. A pair of self mappings (f, g) intuitionistic fuzzy metric space is said to be occasionally weakly compatible iff there is a point x in X which is coincidence point of they commute at the coincidence points f *and* g at which f *and* g commute.
Lemma 1.1[3]. Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space. f *and* g be self maps on X and f *and* g have a unique point of coincidence, $w = fx = gx$, then w is the unique common fixed point of f *and* g.

Lemma 1.2[1]. Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space and *for all* $x, y \in X, t > 0$ and if for a number $k \in (0, 1)$ such that $M(x, y, kt) \geq M(y, x, t)$ *and* $N(x, y, kt) \leq N(y, x, t)$ then $x = y$.

2. Main results

Theorem 2.1. Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space with continuous t-norm * and continuous t-co norm $\diamond$. Let P, Q, S and T be self mappings of X. Let the pairs (P, S) and (Q, T) be owc. If there exist k $\in$(0, 1) such that

$$\int_0^{M(Px,Qy,kt)} \zeta(t)\,dt \geq \int_0^{\min\left\{\begin{matrix} M(Sx,Ty,t), M(Qy,Sx,t), M(Sx,Px,t), M(Qy,Ty,t), \\ M(Px,Ty,t).\left(\frac{M(Sx,Px,t)}{M(Qy,Ty,t)}\right) \end{matrix}\right\}} \zeta(t)\,dt$$

And

$$\int_0^{N(Px,Qy,kt)} \zeta(t)\,dt \leq \int_0^{\max\left\{\begin{matrix} N(Sx,Ty,t), N(Qy,Sx,t), N(Sx,Px,t), N(Qy,Ty,t), \\ N(Px,Ty,t).\left(\frac{N(Sx,Px,t)}{N(Qy,Ty,t)}\right) \end{matrix}\right\}} \zeta(t)\,dt$$

for all $x, y \in X$ *and* $t > 0$. Then, there is a unique fixed point of P , Q, S and T.

Proof: As the pair (P, S) and (Q, T) are occasionally weakly compatible, so there are points $x, y \in X$ such that $Px = Sx$ *and* $Qy = Ty$. we claim that $Px = Qy$ by (2.1), we have,

$$\int_0^{M(Px,Qy,kt)} \zeta(t)\,dt \geq \int_0^{\min\left\{\begin{matrix} M(Px,Qy,t), M(Qy,Px,t), M(Px,Px,t), M(Qy,Qy,t), \\ M(Px,Qy,t).\left(\frac{1+M(Px,Px,t)}{1+M(Qy,Qy,t)}\right) \end{matrix}\right\}} \zeta(t)\,dt$$

$$\int_0^{M(Qy,Qy,kt)} \zeta(t)dt \geq \int_0^{\min\left\{ \begin{array}{l} M(Qy,Qy,t), M(Qy,Qy,t), M(Qy,Qy,t), M(Qy,Qy,t), \\ M(Qy,Qy,t).\left(\frac{M(Qy,Qy,t)}{M(Qy,Qy,t)}\right) \end{array} \right\}} \zeta(t)dt$$

$$\geq \int_0^{M(Qy,Qy,t)} \zeta(t)dt$$

and

$$\int_0^{N(Px,Qy,kt)} \zeta(t)dt \leq \int_0^{\max\left\{ \begin{array}{l} N(Px,Qy,t), N(Qy,Px,t), N(Px,Px,t), N(Qy,Qy,t), \\ N(Px,Qy,t).\left(\frac{N(Px,Px,t)}{N(Qy,Qy,t)}\right) \end{array} \right\}} \zeta(t)dt$$

$$\int_0^{N(Qy,Qy,kt)} \zeta(t)dt \leq \int_0^{\max\left\{ \begin{array}{l} N(Qy,Qy,t), N(Qy,Qy,t), N(Qy,Qy,t), N(Qy,Qy,t), \\ N(Qy,Qy,t).\left(\frac{N(Qy,Qy,t)}{N(Qy,Qy,t)}\right) \end{array} \right\}} \zeta(t)dt$$

$$\leq \int_0^{N(Qy,Qy,t)} \zeta(t)dt$$

Therefore, *by lemma 1.2*, $Px = Qy$ i.e $Px = Sx = Qy = Ty$. Suppose that, there is another point z such that $Pz = Sz$ then by inequality (2.1), we have $Pz = Sz = Qy = Ty$ so $Px = Pz$ and $w = Px = Sx$ is the unique point of P and S. By lemma 1.1, w is the only common fixed point of P and S. Similarly, there is a unique point z in X such that $z = Qz = Tz$. We now show that $z = w$. By (2.1)

$$\int_0^{M(w,z,kt)} \zeta(t)dt = \int_0^{M(Pw,Qz,kt)} \zeta(t)dt$$

$$\geq \int_0^{\min\left\{ \begin{array}{l} M(Sw,Tz,t), M(Qz,Sw,t), M(Sw,Pw,t), M(Qz,Tz,t), \\ M(Pw,Tz,t).\left(\frac{M(Sw,Pw,t)}{M(Qz,Tz,t)}\right) \end{array} \right\}} \zeta(t)dt$$

$$= \int_0^{\min\left\{ \begin{array}{l} M(w,z,t), M(z,w,t), M(w,w,t), M(z,z,t), \\ M(w,z,t).\left(\frac{M(w,w,t)}{M(z,z,t)}\right) \end{array} \right\}} \zeta(t)dt$$

$$= \int_0^{\min\{M(w,z,t), M(z,w,t), 1, 1, M(w,z,t)\}} \zeta(t)dt = \int_0^{M(w,z,t)} \zeta(t)dt$$

And

$$\int_0^{N(w,z,kt)} \zeta(t)dt = \int_0^{N(Pw,Qz,kt)} \zeta(t)dt$$

$$\leq \int_0^{\max\left\{\begin{array}{l} N(Sw,Tz,t), M(Qz,Sw,t), N(Sw,Pw,t), N(Qz,Tz,t), \\ N(Pw,Tz,t).\left(\frac{N(Sw,Pw,t)}{N(Qz,Tz,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\max\left\{\begin{array}{l} N(w,z,t), N(z,w,t), N(w,w,t), N(z,z,t), \\ N(w,z,t).\left(\frac{N(w,w,t)}{N(z,z,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\max\{N(w,z,t), N(z,w,t), 1, 1, N(w,z,t)\}} \zeta(t)dt = \int_0^{N(w,z,t)} \zeta(t)dt$$

Therefore, by lemma 1.2, $w = z$ Hence z is a common fixed point of P, Q, S and T. For uniqueness, let u be another common fixed point of P, Q, S and T. Then by (2.1)

$$\int_0^{M(z,u,kt)} \zeta(t)dt = \int_0^{M(Pz,Qu,kt)} \zeta(t)dt$$

$$\geq \int_0^{\min\left\{\begin{array}{l} M(Sz,Tu,t), M(Qu,Sz,t), M(Sz,Pz,t), M(Qu,Tu,t), \\ M(Pz,Tu,t).\left(\frac{M(Sz,Pz,t)}{M(Qu,Tu,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\min\left\{\begin{array}{l} M(z,u,t), M(u,z,t), M(z,z,t), M(u,u,t), \\ M(z,u,t).\left(\frac{M(z,z,t)}{M(u,u,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\min\{M(z,u,t), M(u,z,t), 1, 1, M(z,u,t)\}} \zeta(t)dt = \int_0^{M(z,u,t)} \zeta(t)dt$$

And

$$\int_0^{N(z,u,kt)} \zeta(t)dt = \int_0^{N(Pz,Qu,kt)} \zeta(t)dt$$

$$\leq \int_0^{\max\left\{\begin{array}{l} N(Sz,Tu,t), N(Qu,Sz,t), N(Sz,Pz,t), N(Qu,Tu,t), \\ N(Pz,Tu,t).\left(\frac{N(Sz,Pz,t)}{N(Qu,Tu,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\max\left\{\begin{array}{l} N(z,u,t), N(u,z,t), N(z,z,t), N(u,u,t), \\ N(z,u,t).\left(\frac{N(z,z,t)}{N(u,u,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\max\{N(z,u,t), N(u,z,t), 1, 1, N(z,u,t)\}} \zeta(t)dt = \int_0^{N(z,u,t)} \zeta(t)dt$$

Therefore, *by lemma* 1.2, $z = w$. Hence z is unique common fixed point of P, Q, S and T.

Theorem 2.2. Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space with $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ for $a, b \in [0, 1]$. Let P, Q, S and T be self mappings of X. Let the pairs (P, S) and (Q, T) be owc. If there exist k ∈ (0, 1) such that

$$\int_0^{M(Px,Qy,kt)} \zeta(t)dt \geq \int_0^{\min\left\{\begin{array}{l} M(Sx,Ty,t)*M(Qy,Sx,t)*M(Sx,Px,t)*M(Qy,Ty,t)* \\ M(Px,Ty,t).\left(\frac{1+M(Sx,Px,t)}{1+M(Qy,Ty,t)}\right)\end{array}\right\}} \zeta(t)dt$$

And

$$\int_0^{N(Px,Qy,kt)} \zeta(t)dt \leq \int_0^{\max\left\{\begin{array}{l} N(Sx,Ty,t)\diamond N(Qy,Sx,t)\diamond N(Sx,Px,t)\diamond N(Qy,Ty,t)\diamond \\ N(Px,Ty,t).\left(\frac{1+N(Sx,Px,t)}{1+N(Qy,Ty,t)}\right)\end{array}\right\}} \zeta(t)dt$$

for all $x, y \in X$ *and* $t > 0$. Then, there is a unique fixed point of P , Q, S and T.

Proof: As the pair (P, S) and (Q, T) are occasionally weakly compatible, so there are points $x, y \in X$ such that $Px = Sx$ *and* $Qy = Ty$. we claim that $Px = Qy$ by (2.2), we have,

$$\int_0^{M(Px,Qy,kt)} \zeta(t)dt \geq \int_0^{\min\left\{\begin{array}{l} M(Px,Qy,t)*M(Qy,Px,t)*M(Px,Px,t)*M(Qy,Qy,t)* \\ M(Px,Qy,t).\left(\frac{1+M(Px,Px,t)}{1+M(Qy,Qy,t)}\right)\end{array}\right\}} \zeta(t)dt$$

$$\int_0^{M(Qy,Qy,kt)} \zeta(t)dt \geq \int_0^{\min\left\{\begin{array}{l} M(Qy,Qy,t)*M(Qy,Qy,t)*M(Qy,Qy,t)*M(Qy,Qy,t)* \\ M(Qy,Qy,t).\left(\frac{1+M(Qy,Qy,t)}{1+M(Qy,Qy,t)}\right)\end{array}\right\}} \zeta(t)dt$$

$$\geq \int_0^{M(Qy,Qy,t)} \zeta(t)dt$$

and

$$\int_0^{N(Px,Qy,kt)} \zeta(t)dt \leq \int_0^{\max\left\{\begin{array}{l} N(Px,Qy,t)\diamond N(Qy,Px,t)\diamond N(Px,Px,t)\diamond N(Qy,Qy,t)\diamond \\ N(Px,Qy,t).\left(\frac{1+N(Px,Px,t)}{1+N(Qy,Qy,t)}\right)\end{array}\right\}} \zeta(t)dt$$

$$\int_0^{N(Qy,Qy,kt)} \zeta(t)dt \leq \int_0^{\max\left\{\begin{array}{l} N(Qy,Qy,t)\diamond N(Qy,Qy,t)\diamond N(Qy,Qy,t)\diamond N(Qy,Qy,t)\diamond \\ N(Qy,Qy,t).\left(\frac{1+N(Qy,Qy,t)}{1+N(Qy,Qy,t)}\right)\end{array}\right\}} \zeta(t)dt$$

$$\leq \int_0^{N(Qy,Qy,t)} \zeta(t)dt$$

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$$\int_0^{M(w,z,kt)} \zeta(t)dt = \int_0^{M(Pw,Qz,kt)} \zeta(t)dt$$

$$\geq \int_0^{\min\left\{\begin{array}{l} M(Sw,Tz,t)*M(Qz,Sw,t)*M(Sw,Pw,t)*M(Qz,Tz,t)* \\ M(Pw,Tz,t).\left(\frac{1+M(Sw,Pw,t)}{1+M(Qz,Tz,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\min\left\{\begin{array}{l} M(w,z,t)*M(z,w,t)*M(w,w,t)*M(z,z,t)* \\ M(w,z,t).\left(\frac{1+M(w,w,t)}{1+M(z,z,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\min\{M(w,z,t)*M(z,w,t)*1*1*M(w,z,t)\}} \zeta(t)dt = \int_0^{M(w,z,t)} \zeta(t)dt$$

And

$$\int_0^{N(w,z,kt)} \zeta(t)dt = \int_0^{N(Pw,Qz,kt)} \zeta(t)dt$$

$$\leq \int_0^{\max\left\{\begin{array}{l} N(Sw,Tz,t)\Diamond N(Qz,Sw,t)\Diamond N(Sw,Pw,t)\Diamond N(Qz,Tz,t)\Diamond \\ N(Pw,Tz,t).\left(\frac{1+N(Sw,Pw,t)}{1+N(Qz,Tz,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\max\left\{\begin{array}{l} N(w,z,t)\Diamond N(z,w,t)\Diamond N(w,w,t)\Diamond N(z,z,t)\Diamond \\ N(w,z,t).\left(\frac{1+N(w,w,t)}{1+N(z,z,t)}\right) \end{array}\right\}} \zeta(t)dt$$

$$= \int_0^{\max\{N(w,z,t)\Diamond N(z,w,t)\Diamond 1\Diamond 1\Diamond N(w,z,t)\}} \zeta(t)dt = \int_0^{N(w,z,t)} \zeta(t)dt$$

Therefore , by lemma 1.2, $w = z$ Hence z is a common fixed point of P, Q, S and T. For uniqueness, let u be another common fixed point of P, Q, S and T. Then by (2.1)

$$\int_0^{M(z,u,kt)} \zeta(t)dt = \int_0^{M(Pz,Qu,kt)} \zeta(t)dt$$

$$\geq \int_0^{\min\left\{\begin{matrix} M(Sz,Tu,t)*M(Qu,Sz,t)*M(Sz,Pz,t)*M(Qu,Tu,t)* \\ M(Pz,Tu,t).\left(\frac{1+M(Sz,Pz,t)}{1+M(Qu,Tu,t)}\right) \end{matrix}\right\}} \zeta(t)dt$$

$$= \int_0^{\min\left\{\begin{matrix} M(z,u,t)*M(u,z,t)*M(z,z,t)*M(u,u,t)* \\ M(z,u,t).\left(\frac{1+M(z,z,t)}{1+M(u,u,t)}\right) \end{matrix}\right\}} \zeta(t)dt$$

$$= \int_0^{\min\{M(z,u,t)*M(u,z,t)*1*1*M(z,u,t)\}} \zeta(t)dt = \int_0^{M(z,u,t)} \zeta(t)dt$$

And

$$\int_0^{N(z,u,kt)} \zeta(t)dt = \int_0^{N(Pz,Qu,kt)} \zeta(t)dt$$

$$\leq \int_0^{\max\left\{\begin{matrix} N(Sz,Tu,t)\lozenge N(Qu,Sz,t)\lozenge N(Sz,Pz,t)\lozenge N(Qu,Tu,t)\lozenge \\ N(Pz,Tu,t).\left(\frac{1+N(Sz,Pz,t)}{1+N(Qu,Tu,t)}\right) \end{matrix}\right\}} \zeta(t)dt$$

$$= \int_0^{\max\left\{\begin{matrix} N(z,u,t)\lozenge N(u,z,t)\lozenge N(z,z,t)\lozenge N(u,u,t)\lozenge \\ N(z,u,t).\left(\frac{1+N(z,z,t)}{1+N(u,u,t)}\right) \end{matrix}\right\}} \zeta(t)dt$$

$$= \int_0^{\max\{N(z,u,t)\lozenge N(u,z,t)\lozenge 1\lozenge 1\lozenge N(z,u,t)\}} \zeta(t)dt = \int_0^{N(z,u,t)} \zeta(t)dt$$

Therefore , by lemma 1.2, $z = w$. Hence z is unique common fixed point of P, Q, S and T.

Theorem 2.3. Let (X, M, N, $*$, $\lozenge$) be an intuitionistic fuzzy metric space with continuous t-norm * and continuous t-co norm $\lozenge$. Let P, Q, S and T be self mappings of X. Let the pairs (P, S) and (Q, T) be owc. If there exist k $\in$ (0, 1) such that

$$\int_0^{M(Px,Qy,kt)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Sx,Ty,t)*M(Qy,Sx,t)*M(Sx,Px,t)*M(Qy,Ty,t)* \\ M(Px,Ty,t).\left(\frac{1+M(Sx,Px,t)}{1+M(Qy,Ty,t)}\right) \end{matrix}\right\}\right\}} \zeta(t)dt$$

And

$$\int_0^{N(Px,Qy,kt)} \zeta(t)dt \leq \int_0^{\varphi\left\{\max\left\{\begin{matrix} N(Sx,Ty,t)\lozenge N(Qy,Sx,t)\lozenge N(Sx,Px,t)\lozenge N(Qy,Ty,t)\lozenge \\ N(Px,Ty,t).\left(\frac{1+N(Sx,Px,t)}{1+N(Qy,Ty,t)}\right) \end{matrix}\right\}\right\}} \zeta(t)dt$$

for all $x, y \in X$ *and* $t > 0$ *and* $\emptyset, \varphi: [0,1]^4 \to [0,1]$ such that $\emptyset(t,t,1,t) > t, \varphi(t,t,0,t)$.Then, there is a unique fixed point of P , Q, S and T.

Proof: The proof follows on the lines of theorem 2.1

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References:

[1] A.Branciari. A fixed point theorem for mappings satisfying a general contractive condition of integral type. Int.J.Math.Sci. 29(2002), no.9, 531 - 536.
[2] Alaca, D. Turkoglu, and C. Yildiz, Fixed points in Intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals, 29 (2006), 1073-1078.
[3]B.E. Rhoades, Two fixed point theorems for mappings satisfying a general contractive condition of integral type. International Journal of Mathematics and Mathematical Sciences, 63, (2003), 4007 - 4013.
[4] B. Schweitzer and A. Sklar, Probabilistic Metric Spaces, North Holland Amsterdam, 1983.
[5] D. Coker: An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and System, 88(1997), 81–89.
[6] D. Turkoglu, C. Alaca, Y. J. Cho and C. Yildiz: Common fixed point theorems in intuitionistic fuzzy metric spaces, J. Appl. Math. & Computing, 22(2006), 411–424.
[7] J. H. Park, Intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals, 22 (2004), 1039-1046.
[8] K. Atanassov, Intuitionistic Fuzzy sets, Fuzzy sets and system, 20 (1986), 87-96.
[9] Kramosil and J. Michalek, Fuzzy metric and Statistical metric spaces, Kybernetica 11 (1975), 326-334.
[10] K. Menger, Statistical metrics, Proc. Nat. Acad. Sci. (USA), 28 (1942), 535- 537.
[11] S. Manro, S. Kumar, S.Singh, Common Fixed Point Theorems in Intuitionistic Fuzzy Metric Spaces, Applied Mathematics, 1 (2010), 510-514.
[12] S. Banach, Sur les oprations dansles ensembles abs traits et leur application aux equations intgrales, Fund. Math.3,(1922)133-181 (French).
[13] Zadeh, Fuzzy sets, Infor. and Control, 8 (1965), 338-353.

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