Tchebichef Moment Based Hilbert Scan for Image Compression

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Abstract

Image compression is now essential for applications such as transmission and storage in data base, so we need to compress a vast amount of information whereas, the compressed ratio and quality of compressed image must be enhanced, for this reason, this paper develop a new algorithm that used a discrete orthogonal Tchebichef moment based Hilbert curve for image compression. The analyzed image was divided into 8×8 image sub-blocks, the Tchebichef moment has been applied to each one, and then the transformed coefficients 8×8 sub-block shall be reordered in Hilbert scan into a linear array, at this step Huffman coding is implemented. Experimental results show that this algorithm improves the coding efficiency on the one hand; and on the other hand the quality of reconstructed image is also not significantly decreased.

Keywords: Huffman Coding, Tchebichef Moment Transforms, Orthogonal Moment Functions, Hilbert, zigzag scan.

1. Introduction

Image Transform methods using orthogonal kernel functions are commonly used in image compression. One of the most widely known image transform method is Discrete Cosine Transform (DCT), used in JPEG compression standard [1].The computing devices such as Personal digital assistants (PDAs), digital cameras and mobile phones require a lot of image transmission and processing. Therefore, it is essential to have efficient image compression techniques, which could be scalable and applicable to these smaller portable devices. A new class of transform called Discrete Tchebichef Transform (DTT), which is derived from a discrete class of popular Tchebichef polynomials, is a novel orthonormal version of orthogonal transform. It has found applications on image analysis and compression [2][3]. The Tchebichef moment compression that is proposed in this paper is meant for smaller computing devices owing to its low computational complexity [4]. R.Mukundan [5] has proposed orthonormal version of Tchebichef moments and analysed some of their computational aspects. Mukundan has also shown details of various computational aspects of Tchebichef moments and their feature representation capability using methods of image reconstruction [6]. A block wise moment computation scheme which avoids numerical instabilities to yield a perfect reconstruction has been introduced in the literature [7]. Mukundan and Hunt [8] have shown that, for natural images, DTT and DCT exhibit similar energy compactness performance. It was very difficult to determine which of the two is better. Nur Azman et al. [9] made a comparison between Tchebichef moment transform and DCT. They have shown that, there is a significant advantage for Tchebichef moments in terms of error reconstruction and average length of Huffman codes. And they use zigzag scan in Huffman coding. And they show the Tchebichef moment provides a compact support to sub-block reconstruction for image compression. Furthermore they argue that the Tchebichef Moment Compression clearly performs potentially better for broader domains on real digital images and graphically generated images.

The present work is an investigation about the use of Hilbert scan for image compression, instead of zigzag scan. To carry out this investigation, first of all, apply TMT, the image is divided into 8×8 blocksof pixels. The 8×8 blocks are processed from left-toright and from top-to-bottom. After the transformation, two main issues occur, which are: the quantization process and the entropy coding. After the transformation and quantization over a 8×8 image sub-blocks, the new 8×8 sub-block shall be reordered in a Hilbert scan into a linear array.

2. Tchebichef Moments

The Tchebichef moments (also called Chebyshev) is a novel set of orthogonal moment applied in the fields of image analysis and Image compression. and to illustrate how to apply it image processing field, we must understand its work first, let T_{mn} be Tchebichef moments based on a discrete orthogonal polynomial set $\{t_n(x)\}$ defined directly on the image space [0, N–1], thus satisfying all the required analytical properties without any numerical approximation errors, T_{mn} will be:

$$T_{mn} = \frac{1}{\alpha(m,N)\alpha(n,N}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} t_m(i)t_n(j)img(i,j) \quad \text{............(1)}$$

For $m, n = 0,1,2,3, \dots \dots \dots, N-1$.

The above definition uses the following scale factor [6] for the polynomial of degree n:

$$\beta(n,N) = N^n \quad \text{..(2)}$$

The set $\{t_n(x)\}$, has a squared-norm given by:

$$\alpha(n,N) = \sum_{i=0}^{N-1}\{t_i(x)\}^2$$

$$= \frac{N\times\left(1-\frac{1^2}{N^2}\right)\times\left(1-\frac{2^2}{N^2}\right)\times\left(1-\frac{3^2}{N^2}\right)\cdots\cdots\left(1-\frac{n^2}{N^2}\right)}{2n+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The Tchebichef orthogonal polynomials set $\{t_n(x)\}$ can be generated iteratively with initial conditions:

$$t_0(x) = 1$$

$$t_1(x) = \frac{2x+1-N}{N}$$

and the general Tchebichef orthogonal polynomial equation is represented as:

$$t_n(x) = \frac{(2n-1)\times t_1(x)\times t_{n-1}(x) - (n-1)\left(1-\frac{(n-1)^2}{N^2}\right)\times t_{n-2}(x)}{n} \quad \ldots(4)$$

for $n = 2,3,\ldots\ldots,N-1$

Discrete orthogonal Tchebichef moment has its own advantage in image processing which has not been fully investigated. Since computer image data operates on integers, discrete orthogonal Tchebichef moment is suitable for computer image processing.

The polynomial domain is discrete over natural numbers. Unlike continuous orthogonal transform, discrete orthogonal Tchebichef moment is capable of performing image reconstruction exactly without any numerical errors [7].

Since a digital image can be considered as a matrix whose row and column indices identify a point in the image and the corresponding matrix element value identifies the grey level at that point, so there is a necessary to represent the Tchebichef moment equations as a matrix, because the matrix based framework makes the problem description more amenable to mathematical programming languages and the code is less prone to errors when processing large images.

In the following discussion, from (1) the moment set consists of all orders of moments with the values in range $0 < m, n < N$ of block size $0<N<M$, where the image size is MxM pixels. The image matrix was subdivided into 8x8 pixels where the orthogonal moment on each block was computed independently.

The block size N is taken to be 8 and extendable to 16 or 32.

For simplicity, consider the discrete orthogonal moment definition (1) above, and define a kernel matrix $\boldsymbol{A}_{(N\times N)}$ as follows:

$$A = \begin{bmatrix} t_0(0) & t_1(0) & \cdots & t_{M-1}(0) \\ t_0(1) & t_1(1) & \cdots & t_{M-1}(1) \\ t_0(2) & t_1(2) & \cdots & t_{M-1}(2) \\ \vdots & \vdots & \ddots & \vdots \\ t_0(M-1) & t_1(M-1) & \ldots & t_{M-1}(M-1) \end{bmatrix} \ldots\ldots\ldots(5)$$

Let the image block intensity matrix $\boldsymbol{B}_{(M\times M)}$ with $b()$ denoting the intensity values be

$$B = \begin{bmatrix} b(0,0) & b(0,1) & \cdots & b(0,M-1) \\ b(1,0) & b(1,1) & \cdots & b(1,M-1) \\ b(2,0) & b(2,1) & \cdots & b(2,M-1) \\ \vdots & \vdots & \ddots & \vdots \\ b(M-1,0) & b(M-1,1) & \ldots & b(M-1,M-1) \end{bmatrix} \ldots..(6)$$

The matrix $\boldsymbol{T}_{(N\times N)}$ of moments defined according to (2) can now be formed as:

$$\boldsymbol{T} = \boldsymbol{A}^{T}\boldsymbol{B}\boldsymbol{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

The inverse moment relation used to reconstruct the image block from the above moment set is now simply calculated by:

$$\boldsymbol{I}=\boldsymbol{A}\boldsymbol{T}\boldsymbol{A}^{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

where $\boldsymbol{I}_{(N\times N)}$ denotes the matrix (image) of the reconstructed intensity values $img(i, j)$.[9]

3. Quantization

For each color band, the moment coefficients shall be quantized separately. The quantization process has the key role in JPEG compression which removes the high frequencies present in the original image. This is done due to the fact that the eye is much more sensitive to lower spatial frequencies than to higher frequencies. The quantization process (in this research) done by dividing values at high indexes in the vector (the amplitudes of higher frequencies) with larger values than the values by which the amplitudes of lower frequencies are divided by. The Standard JPEG luminance and chrominance quantization tables *QL* and *QR* are given below, respectively [1].

$$Q_L = \begin{bmatrix} 4 & 4 & 4 & 8 & 16 & 24 & 40 & 64 \\ 4 & 4 & 8 & 16 & 24 & 40 & 64 & 128 \\ 4 & 8 & 16 & 24 & 40 & 64 & 128 & 128 \\ 8 & 16 & 24 & 40 & 64 & 128 & 128 & 256 \\ 16 & 24 & 40 & 64 & 128 & 128 & 256 & 256 \\ 24 & 40 & 64 & 128 & 128 & 256 & 256 & 128 \\ 40 & 64 & 128 & 128 & 256 & 256 & 128 & 128 \\ 64 & 128 & 128 & 256 & 256 & 128 & 128 & 64 \end{bmatrix}$$

$$Q_R = \begin{bmatrix} 4 & 4 & 4 & 8 & 16 & 32 & 64 & 128 \\ 4 & 4 & 8 & 16 & 32 & 64 & 128 & 256 \\ 4 & 8 & 16 & 32 & 64 & 128 & 256 & 256 \\ 8 & 16 & 32 & 164 & 128 & 256 & 256 & 256 \\ 16 & 32 & 64 & 128 & 256 & 256 & 256 & 256 \\ 32 & 64 & 128 & 256 & 256 & 256 & 256 & 256 \\ 64 & 128 & 256 & 256 & 256 & 256 & 256 & 256 \\ 128 & 256 & 256 & 256 & 256 & 256 & 256 & 256 \end{bmatrix}$$

4. Hilbert Scan of an Image

Hilbert Curves are named after the German mathematician *David Hilbert*. They were first described in 1891.Hilbert curve is one of the space filling curves, and has a one-to-one mapping between an n dimensional space and a one-dimensional space. Because the curve can keep the relevancy of neighboring points in the original space as far as possible, it has been used in image compression extensively [10][11]. The process of constructing a Hilbert curve scan can be seen in (fig. 2). First of all, the square is divided into four quarters, and the 2-order curve H0 is obtained by connecting the centers of the quadrants with three line segments. In the next step, four copies of H0, reduced by half, are placed into the four quarters. Thereby, the lower left copy is rotated clockwise by 90̊ while the lower right copy is rotated counterclockwise. Then the start and the end points of the four copies are connected with three line segments, and the resulting 4-order curve is called H1. In the next step, four copies of H1, reduced by half, are placed into the four quarters as in the back step, and the 8-order curve H2 can be obtained by connecting the start and the end points of the four copies. As a general rule, the 2^n-order Hilbert curve can be obtained.

The above method for constructing Hilbert curve is applicable to the area size of *N*×*N* [12][13]. Through the Hilbert curve scan, a two-dimensional digital image can be converted into a one-dimensional gray-level sequence. It is clear that, the order of the curve will increase with accuracy in approximation if using global approximation to the scanning data points of an image. In order to have better approximation effect, the scanning data points should be divided into several partitions, and then piecewise curves are employed to approximate the partitions. A simple method for dividing the scanning data points initially is adopted. The value, recorded as *h*, is introduced to determine the length of the initial partitions, and then the whole scanning data points are divided into several partitions according to *h* data points in every partition. Generally, the value of *h* is of *N*, *N*/2 or *N*/4 when the considered image with the size
of *N*×*N*, see the pseudo code of this process in fig. 1.

```
PROCEDURE  Right(i: INTEGER);
  BEGIN
     IF i>0 THEN
        BEGIN
           Down(i-1); x:=x-h; MoveTo(x,y);
           Right(i-1); y:=y-h; MoveTo(x,y);
           Right (i-1); x:=x+h; MoveTo(x,y);
           Up(i-1);
        END;
   END;
PROCEDURE Up(i: INTEGER);
   BEGIN
      IF i>0 THEN
         BEGIN
            Left(i-1); y:=y+h; MoveTo(x,y);
            Up (i-1); x:=x+h; MoveTo(x,y);
            Up (i-1); y:=y-h; MoveTo(x,y);
            Right (i-1);
          END;
   END;
PROCEDURE Left(i: INTEGER);
  BEGIN
     IF i>0 THEN
       BEGIN
           Up (i-1); x:=x+h; MoveTo(x,y);
           Left (i-1); y:=y+h; MoveTo(x,y);
           Left (i-1); x:=x-h; MoveTo(x,y);
           Down(i-1);
       END;
  END;
PROCEDURE Down(i: INTEGER);
  BEGIN
    IF i>0 THEN
     BEGIN
         Right (i-1); y:=y-h; MoveTo(x,y);
         Down(i-1); x:=x-h; MoveTo(x,y);
         Down(i-1); y:=y+h; MoveTo(x,y);
         Left (i-1);
      END;
  END;
PROCEDURE Hilbert(i: INTEGER);
  BEGIN
     n =3; (* n is the order of the curve*)
     h0 =512; (* h0 should be a power of 2 *)
     i := 0; h := h0; x0 := h DIV 2 + h; y0 := h DIV 2;
     k:=1;
     REPEAT (*compute index of hilbert curve OF order 3*)
       i := i+ 1; h := h DIV 2;
       x0 := x0 + (h DIV 2); y0 := y0 + (h DIV 2);
       x := x0; y := y0;
       ax[k]:=x; ay[k]:=y;
       k:=k+1;
      Right(i)
    UNTIL i = n;
  END;
```

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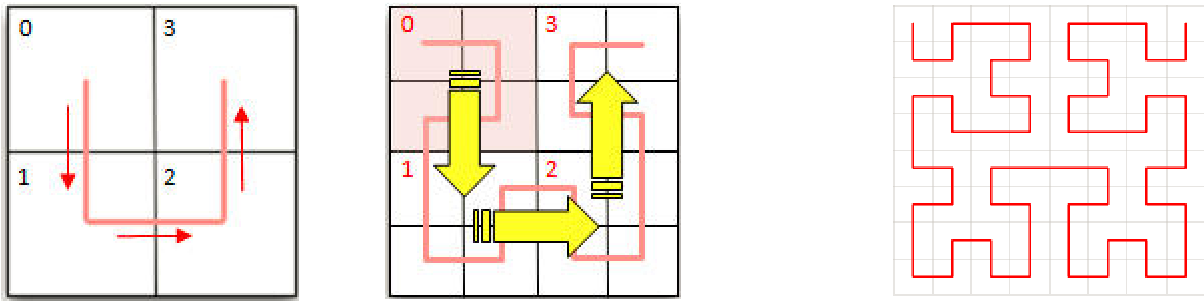


(a) The basic element of a Hilbert curve is a U-shape and called H_0

(b) The *4 x 4* grid as a nested set of *2 x 2* grids and called H_1

(c) The Hilbert Curve that fits in an *8 x 8* grid and called H_2

Fig. 2. Three Sample of Hilbert curve.

5. Huffman Coding

Proposed by DR. David A. Huffman in 1952. This is a method for the construction of minimum redundancy code. Huffman made significant contributions in several areas. Mostly information theory and coding signal design for radar and communication & design procedures for asynchronous logical circuits, and now it is using for compressing data. Huffman coding is a form of statistical coding which attempt to reduce the amounts of bits required representing the string of symbols to vary in length. Shorter codes are assigned to the most frequently used symbols & longer codes to the symbol which appear less frequently in the string. Code word length is no longer fixed like ASCII [14]**.**

6. PROPOSED COMPRESSION ALGORITHM

The proposed compression algorithm is explained through the diagram shown in Fig. 3. And the details of the steps that describe the whole work are as follows:

1. Read RGB image and convert it from RGB format into YC_bC_r format using affine transformation.
2. Partition each part(Y, C_b, and C_r) of an image using classical partition method (fixed size squares block).
3. Call a function that will compute Tchebichef moment coefficients for each block in each three parts.
4. Execute quantization process by dividing values of Tchebichef moment coefficients on quantization tables for each block in each three parts.
5. Call a function that will reorder the quantized Tchebichef moment coefficients in Hilbert order, for each block in each three parts.
6. Call a function that will build Huffman code tree based on the following steps:
    - 6.1 Extract the symbols used in the matrix of Hilbert order and their probability (number of occurences / number of total symbols)
    - 6.2 Build the Huffman dictionary.
    - 6.3 Encode your Hilbert order with the dictionary you just built.

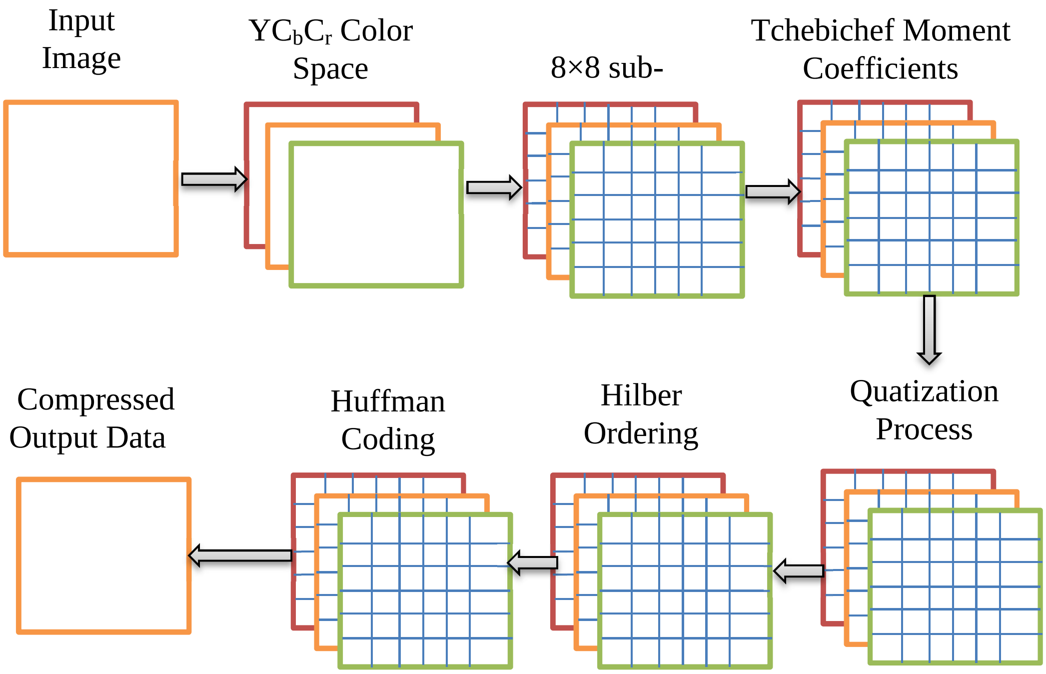


Fig. 3. Block Diagram of the proposed compression algorithm.

7. Simulation Results and Analysis

The superiority of the proposed technique is demonstrated through computer simulation running on Microsoft Window XP, Intel Core2 Duo CPU, 3 GHz Platform, using Delphi 7.0 language. To prove the performance of the presented compression algorithm, we use the peak signal-to-noise ratio (*PSNR*) as the normalized error measure to define the quality of the reconstructed image. The *PSNR* value is given by

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$MSE = \frac{1}{M \times N}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}(f(i,j) - \bar{f}(i,j))^2 \ldots\ldots\ldots\ldots\ldots (10)$$

Where *MSE* denotes the mean square error of the reconstructed image with respect to the original image, and symbols $f(i,j)$ and $\bar{f}(i,j)$ are original and reconstructed pixel-values at the location (i, j) respectively. $M \times N$ is the size of the image.

The presented compression algorithm applied to RGB images of 512×512 size. The images namely, love - birds, stripes and parrot are considered in this study. The reconstructed images of proposed method are shown in fig. 4.

Image Name	TMT Method		Proposed Compression Method	
	PSNR (dB)	MSE	PSNR (dB)	MSE
Stripes	37.04	12.87	37.06	12.79
Love-birds	33.31	30.32	33.76	27.37
Parrota	32.72	34.73	33.30	30.43

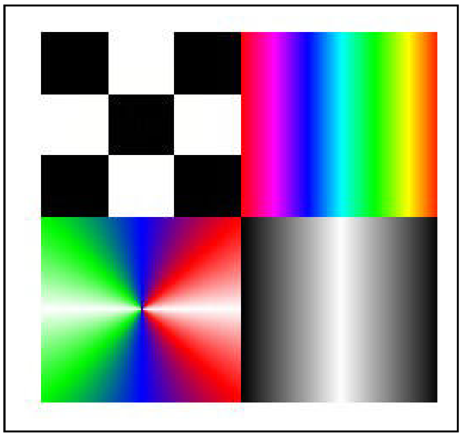
Table 1: A comparison of performance measures of the proposed method with a recently reported method for the images of size 512 x 512

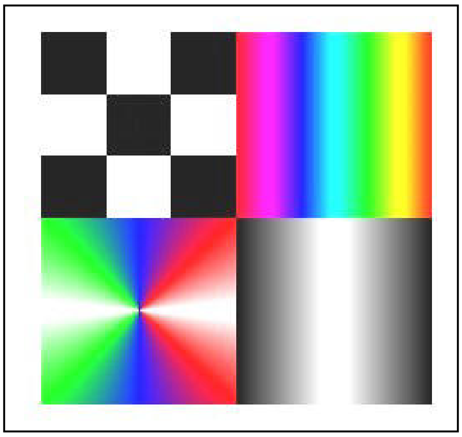
From table 1, It is not difficult to notice that the discrete Tchebichef transform gives a better reconstruction (higher PSNR value) for image 'Stripes'. The 'Stripes' image has high predictability and large intensity gradients. The images 'Love-birds' and 'Parrot' have low predictability compared to 'Stripes'. In general the discrete

Tchebichef transform gives a better performance for images with sharp boundaries, and high predictability. Furthermore the proposed algorithm uses the Hilbert curve to approximate each partition, which leads to smaller maximum absolute error, and because of the recursive nature of the Hilbert scan which allows us to use them for hierarchical indexing of higher-dimensional data. As a result, the proposed algorithm has better PSNR than the scheme based on zigzag scan.

8. Conclusions

The proposed compression scheme certainly improves the efficiency of the moments to a reasonable reconstruction. The proposed algorithm utilized from Hilbert scan, since the Hilbert scan can preserve point neighborhoods as much as possible and take advantage of the high correlation between neighboring pixels, in addition the recursive nature of the Hilbert scan which allows us to use them for hierarchical indexing of higher-dimensional data. This leads to better PSNR compared with zigzag scan. It is clearly found that for all the chosen images, the proposed method gives better results than TMT based conventional compression methods, but the all blocks of the scanned region must have same size and each size must be a power of two, which limits the application of the Hilbert scan greatly.

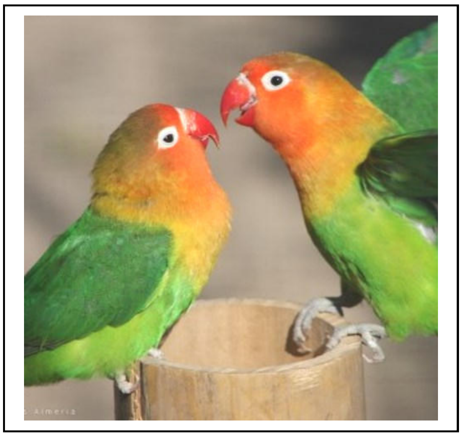




(a) Original Stripes (a) Reconstructed Stripes

(b) Original Parrot (b) Reconstructed Parrota

(c) Original Love-birds



(c) Reconstructed Love-birds

Fig. 4. The result of Proposed Compression Method on Set of RGB Images

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