 

# Color image analyses using four deferent transformations (FFT-DCT-DWT-DMWT)

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**Abstract**

The transformation is the process that converts information from the spatial domain of the signal and translating it to another domain. the aim of this paper is to compeer between four transformations are( discrete Fourier transform ,Discrete Cosine Transforms, Wavelet transform and discrete Multiwavelet transform). And there effective with color image. We determined and apply each transform on the image alone and study the effectiveness such as the noise, enhanesment, brightness, compretion, resolution beside the analyses then retrieving the image by applying the inverse of each transform. The performance of this technique has been done by computer using visual basic 6package.

**Keyword**: image processing, spatial domain ,DCT ,FFT ,DWT ,DMWT

## Introduction

imaging science, image processing is any form of signal processing for which the input is an image, such as a photograph or video frame; the output of image processing may be either an image or a set of characteristics or parameters related to the image. Most image-processing techniques involve treating the image as a two-dimensional signal and applying standard signal-processing techniques to it.[1]

Image processing usually refers to digital image processing, bu toptical and analog image processing also are possible. This article is about general techniques that apply to all of them. The acquisition of images (producing the input image in the first place) is referred to asimaging.

Image processing refers to processing of a 2D picture by a computer. Basic definitions:

An image defined in the "real world" is considered to be a function of two real variables, for example, a(x,y) with a as the amplitude (e.g. brightness) of the image at the real coordinate position (x,y). .[2]

Modern digital technology has made it possible to manipulate multi-dimensional signals with systems that range from simple digital circuits to advanced parallel computers.

Image processing systems require that the images be available in digitized form, that is, arrays of finite length binary words. For digitization, the given Image is sampled on a discrete grid and each sample or pixel is quantized using a finite number of bits. The digitized image is processed by a computer. To display a digital image, it is first converted into analog signal, which is scanned onto a display. .[6]

## Fourier Transform

TheForier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components. The output of the transformation represents the image in the Fourier or frequency domain, while the input image is the spatial domain equivalent. In the Fourier domain image, each point represents a particular frequency contained in the spatial domain image.The Fourier Transform is used in a wide range of applications, such as image analysis, image filtering, image reconstruction and image compression. As we are only concerned with digital images, we will restrict this discussion to the Discrete Fourier Transform (DFT). the DFT is the sampled Fourier Transform and therefore does not contain all frequencies forming an image, but only a set of samples which is large enough to fully describe the spatial domain image. The number of frequencies corresponds to the number of pixels in the spatial domain image, i.e. the image in the spatial and Fourier domain are of the same size.

For a square image of size N×N, the two-dimensional DFT is given by:

$$F(k,l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i,j)\, e^{-\iota 2\pi \left(\frac{ki}{N} + \frac{lj}{N}\right)}$$

where f(a,b) is the image in the spatial domain and the exponential term is the basis function corresponding to each point F(k,l) in the Fourier space. The equation can be interpreted as: the value of each point F(k,l) is obtained by multiplying the spatial image with the corresponding base function and summing the result.The basic functions are sine and cosine waves with increasing frequencies, i.e. F(0,0) represents the DC-component of the image which corresponds to the average brightness and F(N-1,N-1) represents the highest frequency.In a similar way, the Fourier image can be re-transformed to the spatial domain. The inverse Fourier transform is given by:

$$f(a,b) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F(k,l)\, e^{\iota 2\pi \left(\frac{ka}{N} + \frac{lb}{N}\right)}$$

Note the $\frac{1}{N^2}$ normalization term in the inverse transformation. This normalization is sometimes applied to the forward transform instead of the inverse transform, but it should not be used for both.$$

To obtain the result for the above equations, a double sum has to be calculated for each image point. However, because the Fourier Transform is separable, it can be written as

$$F(k,l) = \frac{1}{N} \sum_{b=0}^{N-1} P(k,b)\, e^{-\iota 2\pi \frac{lb}{N}}$$

where

$$P(k,b) = \frac{1}{N} \sum_{a=0}^{N-1} f(a,b)\, e^{-\iota 2\pi \frac{ka}{N}}$$

Using these two formulas, the spatial domain image is first transformed into an intermediate image using N one-dimensional Fourier Transforms. This intermediate image is then transformed into the final image, again using N one-dimensional Fourier Transforms. Expressing the two-dimensional Fourier Transform in terms of a series of2N one-dimensional transforms decreases the number of required computations.

Even with these computational savings, the ordinary one-dimensional DFT has $N^2$ complexity. This can be reduced to $N\ log_2 N$ if we employ the Fast Fourier Transform (FFT) to compute the one-dimensional DFTs. This is a significant improvement, in particular for large images. There are various forms of the FFT and most of them restrict the size of the input image that may be transformed, often to $N = 2^n$ where n is an integer. The mathematical details are well described in the literature.

The Fourier Transform produces a complex number valued output image which can be displayed with two images, either with the real and imaginary part or with magnitude and phase. In image processing, often only the magnitude of the Fourier Transform is displayed, as it contains most of the information of the geometric structure of the spatial domain image. However, if we want to re-transform the Fourier image into the correct spatial domain after some processing in the frequency domain, we must make sure to preserve both magnitude and phase of the Fourier image.

The Fourier domain image has a much greater range than the image in the spatial domain. Hence, to be sufficiently accurate, its values are usually calculated and stored in float values.

The Fourier Transform is used if we want to access the geometric characteristics of a spatial domain image. Because the image in the Fourier domain is decomposed into its sinusoidal components, it is easy to examine or process certain frequencies of the image, thus influencing the geometric structure in the spatial domain.

In most implementations the Fourier image is shifted in such a way that the DC-value (i.e. the image mean) F(0,0) is displayed in the center of the image. The further away from the center an image point is, the higher is its corresponding frequency.

**Discrete Cosine Transforms**

The discrete cosine transform (DCT) is a technique for converting a signal into elementary

frequency components. It is widely used in image compression **discrete cosine transform** (**DCT**) expresses a finite sequence of data points in terms of a sum of cosine functions oscillating at different frequencies. DCTs are important to numerous applications in science and engineering, from lossy compression of audio (e.g. MP3) and images (e.g. JPEG) (where small high-frequency components can be discarded), tospectral methods for the numerical solution of partial differential equations. The use of cosine rather than sine functions is critical in these applications: for compression, it turns out that cosine functions are much more efficient (as described below, fewer functions are needed to approximate a typical signal), whereas for differential equations the cosines express a particular choice of boundary conditions.

In particular, a DCT is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. DCTs are equivalent to DFTs of roughly twice the length, operating on real data with even symmetry (since the Fourier transform of a real and even function is real and even), where in some variants the input and/or output data are shifted by half a sample. There are eight standard DCT variants, of which four are common.

The most common variant of discrete cosine transform is the type-II DCT, which is often called simply "the DCT",[1][2] its inverse, the type-III DCT, is correspondingly often called simply "the inverse DCT" or "the IDCT". Two related transforms are the discrete sine transform (DST), which is equivalent to a DFT of real and odd functions, and the modified discrete cosine transform (MDCT), which is based on a DCT of overlapping data.

evaluation is completed in the case no computer failure errors injected into the transform system. When the fault to lerance design, an efficient concurrent error detection scheme for DCT While DCT algorithms that employ an unmodified FFT often have some theoretical overhead compared to the best specialized DCT algorithms, the former also have a distinct advantage: highly optimized FFT programs are widely available. Thus, in practice, it is often easier to obtain high performance for general lengths N with FFT-based algorithms. (Performance on modern hardware is typically not dominated simply by arithmetic counts, and optimization requires substantial engineering effort.) Specialized DCT algorithms, on the other hand, see widespread use for transforms of small, fixed sizes such as the $8 \times 8$ DCT-II used in JPEG compression, or the small DCTs (or MDCTs) typically used in audio compression. (Reduced code size may also be a reason to use a specialized DCT for embedded-device applications.)

DCT performed along a single dimension followed by a one-dimensional DCT in
the other dimension [7] .The definition of the two-dimensional DCT for an input image A and
output image C is

$$C(u,v)=\alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}f(x,y)\cos\left[\frac{\pi(2x+1)u}{2N}\right]\cos\left[\frac{\pi(2y+1)v}{2N}\right].$$

for u, v = 0,1,2,…,N −1 and α(u) and α(v) are defined as follows:

$$\alpha(u)=\begin{cases}\sqrt{\frac{1}{N}} & for \quad u=0\\ \sqrt{\frac{2}{N}} & for \quad u\neq 0.\end{cases} \quad \ldots\ldots\ldots..(2$$

WAVELETS

The wavelet transform is a time-frequency transform that provides both the frequency as well as time localization in the form of a multi resolution decomposition of the signal

**W**AVELETS are a useful tool for signal processing applications such as image compression and denoising.Until recently, only scalar wavelets were known: wavelets generated by one scaling function. But one can imagine a situation when there is more than one scaling function [16].

**DISCRETE WAVELETS TRANSFORM**

The discrete wavelet transform (DWT) is a linear transformation that operates on adata vector whose length is an integer power of two, transforming it into a numericallydifferent vector of the same length. It is a tool that separates data into different frequencycomponents, and then studies each component with resolution matched to its scale The DWT of a signal $x$ is calculated by passing it through a series of filters. First the samples are passed through a low pass filter with impulse response $g$ resulting in a convolution of the two:

$$y[n] = (x * g)[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-k].$$

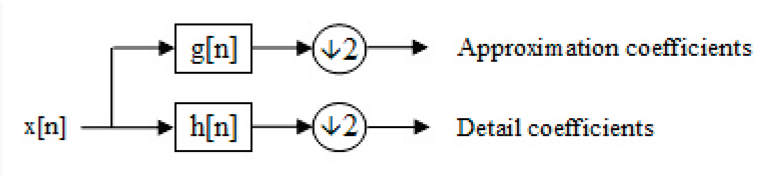
The signal is also decomposed simultaneously using a high-pass filter $h$. The outputs giving the detail coefficients (from the high-pass filter) and approximation coefficients (from the low-pass). It is important that the two filters are related to each other and they are known as a quadrature mirror filter.

However, since half the frequencies of the signal have now been removed, half the samples can be discarded according to Nyquist's rule. The filter outputs are then subsampled by 2 (Mallat's and the common notation is the opposite, g- high pass and h- low pass):

$$y_{\mathrm{low}}[n] = \sum_{k=-\infty}^{\infty} x[k]h[2n-k]$$

$$y_{\mathrm{high}}[n] = \sum_{k=-\infty}^{\infty} x[k]g[2n-k]$$

This decomposition has halved the time resolution since only half of each filter output characterises the signal. However, each output has half the frequency band of the input so the frequency resolution has been doubled.




Block diagram of filter analysis

With the subsampling operator $\downarrow$

$$(y \downarrow k)[n] = y[kn]$$

the above summation can be written more concisely.

$$y_{\mathrm{low}} = (x * g) \downarrow 2$$
$$y_{\mathrm{high}} = (x * h) \downarrow 2$$

However computing a complete convolution $x * g$ with subsequent downsampling would waste computation time.

The Lifting scheme is an optimization where these two computations are interleaved.

The main feature of DWT is multiscale representation of function. By using the wavelets, given function can be analyzed at various levels of resolution. The DWT is also invertible and can be orthogonal. .[9]

Wavelets seem to be effective for analysis of textures recorded with different resolution. It is very important problem in NMR imaging, because high-resolution images require long time of acquisition. This causes an increase of artifacts caused by patient movements, which should be avoided. will provide a tool for fast, low resolution NMR medical diagnostic.

**Motivation of Multiwavelet**

Wavelets are useful tool for signal processing applications such as signal compression and denoising. Until recently, only scalar wavelets were known, these are wavelets generated by one scaling function. But one can imagine a situation when there is more than one scaling function. This leads to the notation of **Multi wavelets**, which have several advantages in comparison to scalar wavelets. Such features as short support, orthogonality, symmetry, and vanishing moments are known to be important in signal processing. A scalar wavelet cannot possess all these properties at the same time. On the other hand, a Multiwavelet system can simultaneously provide perfect reconstruction while preserving length (orthogonality), good performance at boundaries (via linear-phase symmetry), and a high order of approximation (vanishing moments). Thus, Multiwavelets offer the possibility of superior performance for signal processing applications compared with scalar wavelets.

Multi wavelets transform domain there are first and second low-pass coefficients followed by first and second high-pass coefficients. Therefore, if we separate these four coefficients, there are four subbands in the transform domain.

Since multi wavelet decomposition produce two low-pass subbands and two high-pass subbands in each dimension, the organization of Multi wavelet subbands differ from the scalar wavelet case. During a single level of decomposition using a scalar wavelet transform, the 2-D signal data is replaced with four blocks corresponding to the subbands representing either low pass or high pass in both dimensions.

These subbands are illustrated in **Fig. (1-1a)**. The subband labels in this figure indicate how the subband data was generated. For example, the data in subband LH was obtained from high pass filtering of the rows and then low pass filtering of the columns.

The multi wavelets used here have two channels, so there will be two sets of scaling coefficients and two sets of wavelet coefficients. Since multiple iterations over the low pass data are desired, the scaling coefficients for the two channels are stored together.

Likewise, the wavelet coefficients for the two channels are also stored together. The multi wavelet decomposition subbands are shown in **Fig.( 1-1b)**. For multi wavelets, the L and H labels have subscripts denoting the channel to which the data corresponds. For example, the subband labeled L1H2 corresponds to data from the second channel high pass filter in the horizontal direction and the first channel low pass filter in the vertical direction.

| | |
|---|---|
| *LL* | *LH* |
| *HL* | *HH* |

**(a)**

| | | | |
|---|---|---|---|
| *L1L1* | *L1L2* | *L1H1* | *L1H2* |
| *L2L1* | *L2L2* | *L2H1* | *L2H2* |
| *H1L1* | *H1L2* | *H1H1* | *H1H2* |
| *H2L1* | *H2L2* | *H2H1* | *H2H2* |

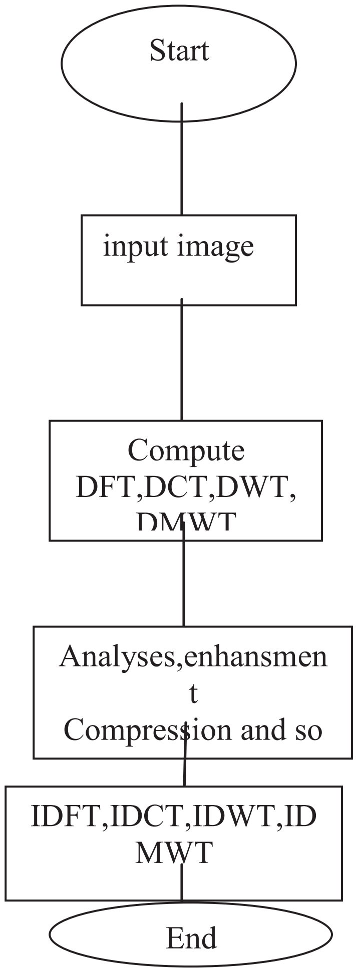
**(b)**

**Figure (1-1): Signal Subbands after a single-level decomposition, for (a) Scalar Wavelet and (b) Multi wavelets.**

The wavelet and Multi wavelet transformations are directly applicable only to one-dimensional (1-D) signals. But signals are two-dimensional (2-D) signals, so there must be a way to process them with a 1-D transform. The two main categories of methods for doing this are **separable** and **non- separable** algorithms. Separable methods simply work on each dimension in series. The typical approach is to process each of the rows in order and then process each column of the result. Non- separable methods work in both signal dimensions at the same time. While non- separable methods can offer benefits over separable methods, such as a saving in computation, they are generally more difficult to implement.

**Basic steps of compares between four transformations:**

1. input image f (x, y) to the transform,
2. Compute F (u, ʋ), the DFT of the input image.
3. Compute F (u, ʋ), the DCT of the input image.
4. Compute F (u, ʋ), the DWT of the input image.
5. Compute F (u, ʋ), the DMWT of the input image).
6. Obtain the real part (better take the magnitude) of the result in
7. Compute the inverse DFT of the result
8. Compute the inverse DCT of the result).
9. Compute the inverse DWT of the result
10. Compute the inverse DMWT of the result).




**CONCLUSIONS**

The Fourier transform and other frequency-space transforms are applied to two dimensional images for many different reasons. Some of these have little to do with the proposes of enhancing visibility and selection of features or structures of interest for measurement. For instance some of this transfom methods are used as a means of image compression to reduce the amount of data in original image for greater efficiency in transmittal or storage . in this type of application it's necessary to reconstruct the image ( bring it back from the frequency to the spatial domain ) for viewing . it's desirable to accomplish both the forward and reverse transform rapidly and with a minimum loses of image quality .

There are a number of relationships between the DFT on real inputs and the DCT. Vetterli and Nussbaumer showed that the N-point DCT can be expressed in terms of the real and imaginary parts of an N-point DFT and rotation of these DFT outputs. Harelick showed that the first N coefficients of a 2N-point DFT with appropriate symmetry of input values can be used to compute an N-point DCT.The main advantages of the DCT are that it yields a real valued output image and that it is a fast transform. A major use of the DCT is in image compression --- i.e.trying to reduce the amount of data needed to store an image. After performing a DCT it is possible to throw away the coefficients that encode high frequency components that the human eye is not very sensitive to. Thus the amount of data can be reduced, without seriously affecting the way an image looks to the human eye.wavelet transform derived features is an effective tool for texture separation. Most of examined pairs of textures can be separated by means of these features with classification error equal to zero. Only in two cases, the observed classification error

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Original image

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 68 | 69 | 69 | 68 | 61 | 52 | 35 | 21 |
| 66 | 69 | 67 | 64 | 59 | 52 | 34 | 21 |
| 69 | 68 | 61 | 58 | 56 | 52 | 42 | 25 |
| 69 | 66 | 59 | 54 | 51 | 47 | 41 | 27 |
| 66 | 62 | 54 | 51 | 47 | 42 | 38 | 29 |
| 60 | 58 | 51 | 47 | 45 | 40 | 37 | 23 |
| 57 | 55 | 54 | 50 | 44 | 39 | 34 | 27 |
| 56 | 57 | 54 | 50 | 41 | 34 | 30 | 26 |

 

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 471009 | -411463 | 75526 | -331335 | -109227 | 183235 | 139210 |
| -425615 | 303146 | 260637 | -580735 | -222168 | -230134 | -152216 | 20085 |
| -175207 | 45431 | -62386 | 206128 | -60567 | -357733 | -175669 | -1573 |
| -19308 | -202865 | 264095 | -67143 | 81111 | 199262 | 57253 | -78391 |
| -176409 | -63850 | 42441 | 7355 | 69136 | 3902 | 83876 | 69581 |
| -163402 | -29380 | -29321 | 17623 | -10121 | 42359 | -42128 | 123913 |
| -33246 | -75190 | 43199 | -54800 | 33940 | -48016 | 37691 | -78627 |
| -63731 | 26429 | 24448 | 14853 | -34644 | 60561 | 5122 | 68059 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 11771133 | -116523 | 467993 | -140412 | -109173 | -319281 | -249272 | -171990 |
| 105655 | -635942 | -228092 | -121956 | 1446659 | -117349 | -77393 | 59940 |
| 6751 | -63997 | 120386 | -186457 | -182380 | 61563 | 150779 | 88805 |
| 164362 | -92499 | 80501 | 224750 | -94117 | -32950 | -210781 | -110405 |
| -42995 | 25836 | 105654 | -98316 | 137077 | 38869 | -108321 | -103789 |
| -54309 | -7981 | 35811 | 137757 | -67732 | 59108 | 64503 | -78273 |
| 56763 | -65128 | 46484 | 8984 | 32838 | 49891 | -3517 | -16376 |
| 74585 | -13952 | 26336 | 6811 | 57785 | 10814 | 24690 | 53894 |

The result after applying discrete Fourier transform block of size(8×8)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 68 | 69 | 69 | 68 | 61 | 52 | 35 | 21 |
| 66 | 69 | 67 | 64 | 59 | 52 | 34 | 21 |
| 69 | 68 | 61 | 58 | 56 | 52 | 42 | 25 |
| 69 | 66 | 59 | 54 | 51 | 47 | 41 | 27 |
| 66 | 62 | 54 | 51 | 47 | 42 | 38 | 29 |
| 60 | 58 | 51 | 47 | 45 | 40 | 37 | 23 |
| 57 | 55 | 54 | 50 | 44 | 39 | 34 | 27 |
| 56 | 57 | 54 | 50 | 41 | 34 | 30 | 26 |

 

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|---|---|---|---|---|---|---|---|
| 34103.6 | -8631.09 | -300.71 | -2009.59 | 3077.55 | 2695.61 | -1099.75 | 1376.24 |
| 8358.5 | -191.68 | -4914.9 | -920.38 | -931.06 | -411.17 | 2192.95 | 546.06 |
| -7341.32 | -2361.59 | 3097.95 | 1222 | -972.22 | 25.33 | 1550.58 | 1632.96 |
| -1871.03 | 488.71 | -948.14 | 837.71 | 2887.63 | 66.01 | -2084.6 | -604.93 |
| -589.97 | 2614.24 | 1632.13 | 796.37 | -363.93 | -2277.41 | -438.17 | -161.8 |
| -822.43 | 124.81 | -186.5 | 558.74 | -267.85 | -676.95 | 450.92 | 125.9 |
| -1056.98 | -702.5 | -2705.45 | 235.61 | 1412.66 | -1460.71 | -481.42 | -125.42 |
| -2748.81 | -1083.6 | 433.21 | 725.83 | 109.95 | -820.47 | -327.79 | 357.14 |

The result after applying discrete cosine transform block of size (8×8)

The result after applying discrete wavelet transform block of size (8×8)

The result after applying discrete malt wavelet transform block of size(8×8)