

Fixed point Results with Soft G- Metric spaces

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Abstract

In the present paper, we prove fixed point results of mapping defined on soft G-metric space.

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2 INTRODUCTION & PRELIMINARIES

In the year 1999, Molodtsov [8] initiated a novel concept of soft sets theory as a new mathematical tool for dealing with uncertainties. A soft set is a collection of approximate descriptions of an object. Soft systems provide a very general framework with the involvement of parameters. Since soft set theory has a rich potential, applications of soft set theory in other disciplines and real life problems are progressing rapidly. Maji et al. [5,6] worked on soft set theory and presented an application of soft sets in decision making problems.

Guler et. Al. [4] introduced the concept of soft G-metric space according to a soft element and obtained some of its properties. Then, they defined soft G-convergence and soft G-continuity, they proved existence and uniqueness of fixed points in soft G-metric spaces.

Our aim of this article is to present fixed point theorems in soft G-metric space satisfying a new rational contractive condition.

Definition 2.1: Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if F is a mapping from E into the set of all subsets of the set X , i.e. $F: E \rightarrow P(X)$, where $P(X)$ is the power set of X .

Definition 2.2: The intersection of two soft sets (F, A) and (G, B) over X is the soft set (H, C) , where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. This is denoted by $(F, A) \cap (G, B) = (H, C)$.

Definition 2.3: The union of two soft sets (F, A) and (G, B) over X is the soft set, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \varepsilon \in A \cap B \end{cases}$$

This relationship is denoted by $(F, A) \cup (G, B) = (H, C)$.

Definition 2.4: The soft set (F, A) over X is said to be a null soft set denoted by Φ if for all $\varepsilon \in A, F(\varepsilon) = \phi$ (null set)

Definition 2.5: A soft set (F, A) over X is said to be an absolute soft set, if for all $\varepsilon \in A, F(\varepsilon) = X$.

Definition 2.6: The difference (H, E) of two soft sets (F, E) and (G, E) over X denoted by $(H, E) \setminus (G, E)$, is defined as $H(\varepsilon) = F(\varepsilon) \setminus G(\varepsilon)$ for all $\varepsilon \in E$.

Definition 2.7: The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by

$(F, A)^c = (F^c, A)$ where $F^c: A \rightarrow P(X)$ is mapping given by $F^c(\alpha) = X - F(\alpha), \forall \alpha \in A$.

Definition 2.8: Let \mathfrak{R} be the set of real numbers and $\mathcal{B}(\mathfrak{R})$ be the collection of all nonempty bounded subsets of \mathfrak{R} and E taken as a set of parameters. Then a mapping $F: E \rightarrow \mathcal{B}(\mathfrak{R})$ is called a soft real set. It is denoted by (F, E) . If specifically (F, E) is a singleton soft set, then identifying (F, E) with the corresponding soft element, it will be called a soft real number and denoted $\tilde{r}, \tilde{s}, \tilde{t}$ etc.

$\bar{0}, \bar{1}$ are the soft real numbers where $\bar{0}(e) = 0, \bar{1}(e) = 1$ for all $e \in E$, respectively.

Definition 2.9: For two soft real numbers

- (i) $\tilde{r} \leq \tilde{s}$, if $\tilde{r}(e) \leq \tilde{s}(e)$, for all $e \in E$.
- (ii) $\tilde{r} \geq \tilde{s}$, if $\tilde{r}(e) \geq \tilde{s}(e)$, for all $e \in E$.
- (iii) $\tilde{r} < \tilde{s}$, if $\tilde{r}(e) < \tilde{s}(e)$, for all $e \in E$.
- (iv) $\tilde{r} > \tilde{s}$, if $\tilde{r}(e) > \tilde{s}(e)$, for all $e \in E$.

Definition 2.10: A soft set over X is said to be a soft point if there is exactly one $e \in E$, such that $P(e) = \{x\}$ for some $x \in X$ and $P(e') = \emptyset, \forall e' \in E \setminus \{e\}$. It will be denoted by \tilde{x}_e .

Definition 2.11: Two soft points $\tilde{x}_e, \tilde{y}_{e'}$ are said to be equal if $e = e'$ and $P(e) = P(e')$ i.e. $x = y$. Thus $\tilde{x}_e \neq \tilde{y}_{e'} \Leftrightarrow x \neq y$ or $e \neq e'$.

Definition 2.12: A mapping $\tilde{d}: SP(\tilde{X}) \times SP(\tilde{X}) \rightarrow \mathbb{R}(E)^+$, is said to be a soft metric on the soft set \tilde{X} if \tilde{d} satisfies the following conditions:

- (M1) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \geq \bar{0}$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \in \tilde{X}$,
- (M2) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \bar{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$,
- (M3) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \tilde{d}(\tilde{y}_{e_2}, \tilde{x}_{e_1})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \in \tilde{X}$,
- (M4) $\tilde{d}(\tilde{x}_{e_1}, \tilde{z}_{e_3}) \geq \tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) + \tilde{d}(\tilde{y}_{e_2}, \tilde{z}_{e_3})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \in \tilde{X}$.

The soft set \tilde{X} with a soft metric \tilde{d} on \tilde{X} is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Definition 2.13 (Cauchy Sequence): A sequence $\{\tilde{x}_{\lambda, m}\}_n$ of soft points in $(\tilde{X}, \tilde{d}, E)$ is considered as a Cauchy sequence in \tilde{X} if corresponding to every $\tilde{\epsilon} \geq \bar{0}, \exists m \in \mathbb{N}$ such that $\tilde{d}(\tilde{x}_{\lambda, i}, \tilde{x}_{\lambda, j}) \leq \tilde{\epsilon}, \forall i, j \geq m$, i.e. $\tilde{d}(\tilde{x}_{\lambda, i}, \tilde{x}_{\lambda, j}) \rightarrow \bar{0}$, as $i, j \rightarrow \infty$.

Definition 2.14 (Soft Complete Metric Space): A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called complete, if every Cauchy Sequence in \tilde{X} converges to some point of \tilde{X} .

Definition 2.15[4]: Let X be a nonempty set and E be the nonempty set of parameters. Let $\tilde{G}: SE(\tilde{X}) \times SE(\tilde{X}) \times SE(\tilde{X}) \rightarrow \mathbb{R}(E)^+$ be a function satisfying the following axioms:

- (G₁) $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = 0$ if $\tilde{x} = \tilde{y} = \tilde{z}$
- (G₂) $\tilde{G}(\tilde{x}, \tilde{x}, \tilde{y}) > 0$ for all $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x} \neq \tilde{y}$
- (G₃) $\tilde{G}(\tilde{x}, \tilde{x}, \tilde{y}) \leq \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})$ for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$ with $\tilde{y} \neq \tilde{z}$
- (G₄) $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = \tilde{G}(\tilde{x}, \tilde{z}, \tilde{y}) = \tilde{G}(\tilde{y}, \tilde{z}, \tilde{x}) = \dots$ (Symmetry in all three variables)

(G₅) $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \leq \tilde{G}(\tilde{x}, \alpha, \alpha) + \tilde{G}(\alpha, \tilde{y}, \tilde{z})$ for all $\tilde{x}, \tilde{y}, \tilde{z}, \alpha \in X$ (Rectangle inequality)

Then the function \tilde{G} is called a soft generalized metric or soft G-metric on \tilde{X} and $(\tilde{X}, \tilde{G}, E)$ is called a soft G-metric space.

Definition 2.16: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space, let $\{\tilde{x}_n\}$ be a sequence of soft points of \tilde{X} , a soft point $\tilde{x} \in \tilde{X}$ is said to the limit of the sequence $\{\tilde{x}_n\}$, if $\lim_{n \rightarrow \infty} \tilde{G}(\tilde{x}, \tilde{x}_n, \tilde{x}_m) = 0$. Then $\{\tilde{x}_n\}$ is G-convergent to \tilde{x} .

Proposition 2.17[4]: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space, then for a sequence $\{\tilde{x}_n\} \subseteq \tilde{X}$ and a soft point $\tilde{x} \in \tilde{X}$. The following are equivalent

- (i) $\{\tilde{x}_n\}$ is soft G-convergent to \tilde{x} .
- (ii) $\tilde{G}(\tilde{x}_n, \tilde{x}_n, \tilde{x}) \rightarrow 0$ as $n \rightarrow \infty$
- (iii) $\tilde{G}(\tilde{x}_n, \tilde{x}, \tilde{x}) \rightarrow 0$ as $n \rightarrow \infty$
- (iv) $\tilde{G}(\tilde{x}_m, \tilde{x}_n, \tilde{x}) \rightarrow 0$ as $m, n \rightarrow \infty$.

Definition 2.18: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space, then the sequence $\{\tilde{x}_n\}$ is said to be soft G-Cauchy if for every $\varepsilon > 0$ there exists a positive integer N such that $\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_l) < \varepsilon$ for all $n, m, l \geq N$ i.e. $\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_l) \rightarrow 0$ as $n, m, l \rightarrow \infty$.

Definition 2.19: A soft G-metric space $(\tilde{X}, \tilde{G}, E)$ is said to be soft G-complete space if every soft G-Cauchy sequence in $(\tilde{X}, \tilde{G}, E)$ is G-convergent in $(\tilde{X}, \tilde{G}, E)$.

Proposition 2.20[4]: Let $(\tilde{X}, \tilde{G}, E), (\tilde{X}', \tilde{G}', E')$ be two soft G-metric spaces, then a function $f: \tilde{X} \rightarrow \tilde{X}'$ is soft G-continuous at a soft point $\tilde{x} \in SE(\tilde{X})$ if and only if it is soft G-sequentially continuous at $\tilde{x} \in SE(\tilde{X})$; i.e. whenever $\{\tilde{x}_n\}$ is soft G-convergent to \tilde{x} , $\{f(\tilde{x}_n)\}$ is soft G-convergent to $f(\tilde{x})$.

3 MAIN RESULTS

Our main results of this article are as follows.

Theorem 3.1: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space and $R: (\tilde{X}, \tilde{G}, E) \rightarrow (\tilde{X}, \tilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$

$$a \tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}) \leq b \left\{ \begin{array}{l} \tilde{G}(\tilde{x}, R\tilde{y}, R\tilde{y}) \\ + \tilde{G}(\tilde{y}, R\tilde{z}, R\tilde{z}) \\ + \tilde{G}(\tilde{z}, R\tilde{x}, R\tilde{x}) \end{array} \right\} + c \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})$$

...(3.1.1)

Where $a, b, c > 0$ and $3b + c < a$. Then R has a unique fixed point \tilde{u} and R is G-continuous at \tilde{u} .

Proof: Let $\tilde{x}_0 \in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$T\tilde{x}_0 = \tilde{x}_1, T\tilde{x}_1 = \tilde{x}_2, T\tilde{x}_2 = \tilde{x}_3, \dots, T\tilde{x}_n = \tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n \neq \tilde{x}_{n+1}$

Substituting $\tilde{x} = \tilde{x}_n, \tilde{y} = \tilde{x}_{n+1}$ and $\tilde{z} = \tilde{x}_{n+1}$ in (3.1.1) then we get

$$a \tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \leq b \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \\ + \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \\ + \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_n, R\tilde{x}_n) \end{array} \right\} + c \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq b \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \\ + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \\ + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \end{array} \right\} + c \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{(b+c)}{(a-2b)} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq K \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \quad \text{Where } \frac{(b+c)}{(a-2b)} < 1.$$

On continuing this process $(n + 1)$ times

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq K^{n+1} \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Similarly we can show that

$$\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq K^n \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Next we show that $\{\tilde{x}_n\}$ is Soft G-Cauchy sequence.

Then for all $n, m \in \mathbb{N}, n < m$ we have

$$\begin{aligned} \tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_m) &\leq \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \dots + \tilde{G}(\tilde{x}_{m-1}, \tilde{x}_m, \tilde{x}_m) \\ &\leq (K^n + K^{n+1} + \dots + K^{m-1}) \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \\ &\leq \frac{K^n}{1-K} \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X}, \tilde{G}, E)$ is soft G-complete, there exists $\tilde{u} \in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to \tilde{u} .

Next we will show that \tilde{u} is fixed point of R .

For this we take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{z} = \tilde{u}$ in (3.1.1) then

$$\begin{aligned} a \tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}) &\leq b \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_n, R\tilde{u}, R\tilde{u}) \\ + \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \\ + \tilde{G}(\tilde{u}, R\tilde{x}_n, R\tilde{x}_n) \end{array} \right\} + c \tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u}) \\ a \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) &\leq b \left\{ \begin{array}{l} \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \\ + \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \\ + \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) \end{array} \right\} + c \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) \\ \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) &\leq \frac{2b}{a} \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \end{aligned}$$

This is the contraction, so $R\tilde{u} = \tilde{u}$ i.e. \tilde{u} is fixed point of R .

To prove uniqueness, suppose that \tilde{u} and \tilde{v} are two fixed points of R . Then by inequality (3.1.1) we have

$$\begin{aligned} a \tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}) &\leq b \left\{ \begin{array}{l} \tilde{G}(\tilde{u}, R\tilde{v}, R\tilde{v}) \\ + \tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v}) \\ + \tilde{G}(\tilde{v}, R\tilde{u}, R\tilde{u}) \end{array} \right\} + c \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \\ a \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) &\leq b \left\{ \begin{array}{l} \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \\ + \tilde{G}(\tilde{v}, \tilde{v}, \tilde{v}) \\ + \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u}) \end{array} \right\} + c \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \end{aligned}$$

So we deduct that $(a - b - c) \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq b \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$. This implies that $\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq \frac{b}{(a-b-c)} \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$ and by repeated use of the same argument we will find

$\tilde{G}(\tilde{v}, \tilde{u}, \tilde{u}) \leq \frac{b}{(a-b-c)} \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$. Therefore we get $\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq \left\{ \frac{b}{(a-b-c)} \right\}^2 \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$. Since $3b + c < a$, this contradiction implies that $\tilde{u} = \tilde{v}$.

To show that R is soft G-continuous at \tilde{u} . Let $\{\tilde{y}_n\}$ be a sequence of soft elements in \tilde{X} such that $\{\tilde{y}_n\} \rightarrow \tilde{u}$ then we can deduce that

Using (3.3.1), we have

$$\begin{aligned} a \tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) &\leq b \left\{ \begin{array}{l} \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \\ + \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) \\ + \tilde{G}(\tilde{y}_n, R\tilde{u}, R\tilde{u}) \end{array} \right\} + c \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n) \\ a \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) &\leq b \left\{ \begin{array}{l} \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \\ + \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) \\ + \tilde{G}(\tilde{y}_n, \tilde{u}, \tilde{u}) \end{array} \right\} + c \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n) \end{aligned}$$

Taking the limit as $n \rightarrow \infty$ from which we see that $(a - 2b) \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \rightarrow 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G - convergent to $R\tilde{u} = \tilde{u}$ therefore

proposition (2.20) implies that R is G-continuous at \tilde{u} .

Theorem 3.2: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space and $R: (\tilde{X}, \tilde{G}, E) \rightarrow (\tilde{X}, \tilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$

$$a\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}) + b \min \left\{ \begin{array}{l} \tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}), \tilde{G}(\tilde{x}, R\tilde{x}, R\tilde{x}), \\ \tilde{G}(\tilde{y}, R\tilde{y}, R\tilde{y}), \tilde{G}(\tilde{z}, R\tilde{z}, R\tilde{z}) \end{array} \right\} \leq c \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \quad \dots (3.2.1)$$

Where $a, b, c > 0$ with $c - b < a$. Then R has a unique fixed point \tilde{u} and R is G-continuous at \tilde{u} .

Proof: Let $\tilde{x}_0 \in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$T\tilde{x}_0 = \tilde{x}_1, T\tilde{x}_1 = \tilde{x}_2, T\tilde{x}_2 = \tilde{x}_3, \dots, T\tilde{x}_n = \tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n \neq \tilde{x}_{n+1}$ for each $n \in N \cup \{0\}$.

Substituting $\tilde{x} = \tilde{x}_n, \tilde{y} = \tilde{x}_{n+1}$ and $\tilde{z} = \tilde{x}_{n+1}$ in (3.2.1) then we get

$$a\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) + b \min \left\{ \begin{array}{l} \tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}), \\ \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \\ \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}), \\ \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \end{array} \right\} \leq c \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b \min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\} \leq c \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b \min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \end{array} \right\} \leq c \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

Case I: If we take $\min\{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})\} = \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})$

Then $a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq c \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{c}{a+b} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

Case II: If we take $\min\{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})\} = \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

Then $a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq c \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{c-b}{a} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

From **Case- I, II**, we have

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq k\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

Similarly we can show that

$$\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq k \tilde{G}(\tilde{x}_{n-1}, \tilde{x}_n, \tilde{x}_n)$$

And $\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq k^n \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$

Next we show that $\{\tilde{x}_n\}$ is Soft G-Cauchy sequence.

Then for all $n, m \in N, n < m$ we have

$$\begin{aligned} \tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_m) &\leq \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \dots + \tilde{G}(\tilde{x}_{m-1}, \tilde{x}_m, \tilde{x}_m) \\ &\leq (k^n + k^{n+1} + \dots + k^{m-1}) \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \\ &\leq \frac{k^n}{1-k} \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X}, \tilde{G}, E)$ is soft G-complete, there exists $\tilde{u} \in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to \tilde{u} .

Next we will show that \tilde{u} is fixed point of R .

For this we take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{z} = \tilde{u}$ in (3.2.1) then

$$a \tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}) + b \min \left\{ \begin{array}{l} \tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \\ \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \end{array} \right\} \leq c \tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u})$$

As $n \rightarrow \infty$, we have

$$a \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + b \min \left\{ \begin{array}{l} \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \\ \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \end{array} \right\} \leq c \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u})$$

$$a \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \leq 0 \quad \text{Since } a > 0$$

so $R\tilde{u} = \tilde{u}$. i. e. \tilde{u} is a fixed point of R .

To prove uniqueness, suppose that \tilde{u} and \tilde{v} are two fixed points of R . Then by inequality (3.2.1) we have

$$a \tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}) + b \min \left\{ \begin{array}{l} \tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \\ \tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v}), \tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v}) \end{array} \right\} \leq c \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq \frac{c}{a} \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

This contradiction implies that $\tilde{u} = \tilde{v}$.

To show that R is soft G-continuous at \tilde{u} . Let $\{\tilde{y}_n\}$ be a sequence of soft elements in \tilde{X} such that $\{\tilde{y}_n\} \rightarrow \tilde{u}$ then we can deduce that

Using (3.2.1), we have

$$a \tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) + b \min \left\{ \begin{array}{l} \tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \\ \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) \end{array} \right\} \leq c \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n)$$

$$a \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) + b \min \left\{ \begin{array}{l} \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \\ \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \end{array} \right\} \leq c \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n)$$

Taking the limit as $n \rightarrow \infty$ from which we see that $\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \rightarrow 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G - convergent to $R\tilde{u} = \tilde{u}$ therefore proposition (2.20) implies that R is G-continuous at \tilde{u} .

Theorem 3.3: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space and $R: (\tilde{X}, \tilde{G}, E) \rightarrow (\tilde{X}, \tilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$

$$a \tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}) + b \frac{\min\{\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}), \tilde{G}(\tilde{x}, R\tilde{x}, R\tilde{x}), \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}), \tilde{G}(\tilde{y}, R\tilde{y}, R\tilde{y})\}}{\min\{\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}), \tilde{G}(\tilde{x}, R\tilde{x}, R\tilde{x}), \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}), \tilde{G}(\tilde{y}, R\tilde{y}, R\tilde{y})\}} \leq c \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})$$

...(3.3.1)

Where $a, b, c > 0$ with $c - b < a$. Then R has a unique fixed point \tilde{u} and R is G-continuous at \tilde{u} .

Proof: Let $x_0 \in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$T\tilde{x}_0 = \tilde{x}_1, T\tilde{x}_1 = \tilde{x}_2, T\tilde{x}_2 = \tilde{x}_3, \dots, T\tilde{x}_n = \tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n \neq \tilde{x}_{n+1}$

Substituting $\tilde{x} = \tilde{x}_n, \tilde{y} = \tilde{x}_{n+1}$ and $\tilde{z} = \tilde{x}_{n+1}$ in (3.3.1) then we get

$$a \tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) + b \frac{\min \left\{ \begin{array}{l} \tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \end{array} \right\}}{\min \left\{ \begin{array}{l} \tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \end{array} \right\}} \leq c \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b \frac{\min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \end{array} \right\}}{\min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \end{array} \right\}} \leq c \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \left[\frac{\min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\}}{\min \{ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \}} \right] \leq c\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \quad (3.3.2)$$

Now following four cases are arise:

Case-I: If $\frac{\min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\}}{\min \{ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \}} = \frac{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})}{\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})}$

then condition (3.3.2) reduces to

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b \left[\frac{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})}{\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})} \right] \leq c\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{c}{a+b} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

Case-II: If $\frac{\min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\}}{\min \{ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \}} = \frac{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})}{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})}$

then condition (3.3.2) reduces to

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b \left[\frac{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})}{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})} \right] \leq c\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq \frac{c-b}{a} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

Case-III: If $\frac{\min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\}}{\min \{ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \}} = \frac{\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})}{\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})}$

then condition (3.3.2) reduces to

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b \left[\frac{\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})}{\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})} \right] \leq c\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{c}{a+b} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

Case-IV: If $\frac{\min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\}}{\min \{ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \}} = \frac{\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})}{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})}$

then condition (3.3.2) reduces to

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b \left[\frac{\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})}{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})} \right] \leq c\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{c-b}{a} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

From **Case- I, II, III, IV**, we have

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq k\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

On continuing this process $(n + 1)$ times

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq k^{n+1} \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Similarly we can show that

$$\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq k^n \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Next we show that $\{\tilde{x}_n\}$ is Soft G-Cauchy sequence.

Then for all $n, m \in \mathbb{N}$, $n < m$ we have

$$\begin{aligned} \tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_m) &\leq \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \dots + \tilde{G}(\tilde{x}_{m-1}, \tilde{x}_m, \tilde{x}_m) \\ &\leq (k^n + k^{n+1} + \dots + k^{m-1}) \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \\ &\leq \frac{k^n}{1-k} \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X}, \tilde{G}, E)$ is soft G-complete, there exists $\tilde{u} \in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to \tilde{u} .

Next we will show that \tilde{u} is fixed point of R .

For this we take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{z} = \tilde{u}$ in (3.3.1) then

Form (3.3.1) we have

$$\begin{aligned}
 a\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}) + b \frac{\left[\min\{\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}}{\min\{\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}} \right] &\leq c\tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u}) \\
 a\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + b \frac{\left[\min\{\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}}{\min\{\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}} \right] &\leq c\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) \\
 a\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) &\leq 0 \quad \text{Since } a \geq 0
 \end{aligned}$$

This is the contraction, so $R\tilde{u} = \tilde{u}$ i.e. \tilde{u} is fixed point of R .

To prove uniqueness, suppose that \tilde{u} and \tilde{v} are two fixed points of R . Then by inequality (3.5.1) we have

$$\begin{aligned}
 a\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}) + b \frac{\left[\min\{\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v})\}}{\min\{\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v})\}} \right] &\leq c\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \\
 a\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) + b \frac{\left[\min\{\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{v}, \tilde{v})\}}{\min\{\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{v}, \tilde{v})\}} \right] &\leq c\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \\
 \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) &\leq \frac{c}{a}\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \quad \text{Since } \frac{c}{a} < 1.
 \end{aligned}$$

This contradiction implies that $\tilde{u} = \tilde{v}$.

To show that R is soft G-continuous at \tilde{u} . Let $\{\tilde{y}_n\}$ be a sequence of soft elements in \tilde{X} such that $\{\tilde{y}_n\} \rightarrow \tilde{u}$ then we can deduce that

Using (3.3.1), we have

$$\begin{aligned}
 a\tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) + b \frac{\left[\min\{\tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n)\}}{\min\{\tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n)\}} \right] &\leq c\tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n) \\
 a\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) + b \frac{\left[\min\{\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), \tilde{G}(\tilde{y}_n, \tilde{u}, \tilde{u})\}}{\min\{\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), \tilde{G}(\tilde{y}_n, \tilde{u}, \tilde{u})\}} \right] &\leq c\tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n)
 \end{aligned}$$

Taking the limit as $n \rightarrow \infty$ from which we see that $\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \rightarrow 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G-convergent to $R\tilde{u} = \tilde{u}$ therefore proposition (2.20) implies that R is G-continuous at \tilde{u} .

Theorem 3.4: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space and $R: (\tilde{X}, \tilde{G}, E) \rightarrow (\tilde{X}, \tilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$

$$\min \left\{ \begin{aligned} &\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}), [\tilde{G}(\tilde{x}, R\tilde{x}, R\tilde{x}) + \tilde{G}(\tilde{x}, R\tilde{y}, R\tilde{y})], \\ &[\tilde{G}(\tilde{y}, R\tilde{y}, R\tilde{y}) + \tilde{G}(\tilde{y}, R\tilde{x}, R\tilde{x})] \end{aligned} \right\} \leq a\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \quad \dots(3.4.1)$$

Proof: Let $\tilde{x}_0 \in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$R\tilde{x}_0 = \tilde{x}_1, R\tilde{x}_1 = \tilde{x}_2, R\tilde{x}_2 = \tilde{x}_3, \dots, R\tilde{x}_n = \tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n \neq \tilde{x}_{n+1}$

Substituting $\tilde{x} = \tilde{x}_n, \tilde{y} = \tilde{x}_{n+1}$ and $\tilde{z} = \tilde{x}_{n+1}$ in (3.4.1) then we get

$$\begin{aligned}
 \min \left\{ \begin{aligned} &\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}), \\ &[\tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n) + \tilde{G}(\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1})], \\ &[\tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_n, R\tilde{x}_n)] \end{aligned} \right\} &\leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \\
 \min \left\{ \begin{aligned} &\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \\ &[\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})], \\ &[\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+1}, \tilde{x}_{n+1})] \end{aligned} \right\} &\leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \\
 \min \left\{ \begin{aligned} &\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \\ &[\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})] \end{aligned} \right\} &\leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \quad \dots(3.4.2)
 \end{aligned}$$

Now following two cases are arise:

Case-I: If $\min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \\ [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})] \end{array} \right\} = \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})$

then condition (3.4.2) reduces to

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

CaseII: If $\min \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \\ [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})] \end{array} \right\} = [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})]$

then condition (3.4.2) reduces to

$$\begin{aligned} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2}) &\leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + [\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) - \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})] &\leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) &\leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \end{aligned}$$

From **Case- I, II**, we have

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

On continuing this process $(n + 1)$ times

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a^{n+1}\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Similarly we can show that

$$\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq a^n\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Next we show that $\{\tilde{x}_n\}$ is Soft G-Cauchy sequence.

Then for all $n, m \in N, n < m$ we have

$$\begin{aligned} \tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_m) &\leq \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \dots + \tilde{G}(\tilde{x}_{m-1}, \tilde{x}_m, \tilde{x}_m) \\ &\leq (a^n + a^{n+1} + \dots + a^{m-1})\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \\ &\leq \frac{a^n}{1-a}\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X}, \tilde{G}, E)$ is soft G-complete, there exists $\tilde{u} \in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to \tilde{u} .

Next we will show that \tilde{u} is fixed point of R .

For this we take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{z} = \tilde{u}$ in (3.4.1) then

$$\begin{aligned} \min \left\{ \begin{array}{l} \tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}), [\tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n) + \tilde{G}(\tilde{x}_n, R\tilde{u}, R\tilde{u})], \\ [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{x}_n, R\tilde{x}_n)] \end{array} \right\} &\leq a\tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u}) \\ \min \left\{ \begin{array}{l} \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), [\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})], \\ [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u})] \end{array} \right\} &\leq a\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) \\ \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) &\leq 0 \end{aligned}$$

This is the contraction, so $R\tilde{u} = \tilde{u}$ i.e. \tilde{u} is fixed point of R .

To prove uniqueness, suppose that \tilde{u} and \tilde{v} are two fixed points of R . Then by inequality (3.1.1) we have

$$\min \{ \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), [\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) + \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})], [\tilde{G}(\tilde{v}, \tilde{v}, \tilde{v}) + \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})] \} \leq a\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

Since $a < 1$, this contradiction implies that $\tilde{u} = \tilde{v}$.

To show that R is soft G-continuous at \tilde{u} . Let $\{\tilde{y}_n\}$ be a sequence of soft elements in \tilde{X} such that $\{\tilde{y}_n\} \rightarrow \tilde{u}$ then we can deduce that

Using (3.4.1), we have

$$\min \left\{ \begin{array}{l} \tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n)], \\ [\tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) + \tilde{G}(\tilde{y}_n, R\tilde{u}, R\tilde{u})] \end{array} \right\} \leq a\tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n)$$

$$\min \left\{ \begin{array}{l} \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), [\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n)], \\ [\tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) + \tilde{G}(\tilde{y}_n, \tilde{u}, \tilde{u})] \end{array} \right\} \leq a\tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n)$$

Taking the limit as $n \rightarrow \infty$ from which we see that $\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \rightarrow 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G – convergent to $R\tilde{u} = \tilde{u}$ therefore proposition (2.20) implies that R is G-continuous at \tilde{u} .

Theorem 3.5: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space and $R: (\tilde{X}, \tilde{G}, E) \rightarrow (\tilde{X}, \tilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$

$$\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}) \leq a \max \left\{ \begin{array}{l} \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}), [\tilde{G}(\tilde{x}, R\tilde{x}, R\tilde{x}) + \tilde{G}(\tilde{x}, R\tilde{y}, R\tilde{y})], \\ [\tilde{G}(\tilde{y}, R\tilde{y}, R\tilde{y}) + \tilde{G}(\tilde{y}, R\tilde{x}, R\tilde{x})] \end{array} \right\} \quad \dots(3.5.1)$$

Where $0 \leq a < 1$. Then R has a unique fixed point \tilde{u} and R is G-continuous at \tilde{u} .

Proof: Let $x_0 \in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$R\tilde{x}_0 = \tilde{x}_1, R\tilde{x}_1 = \tilde{x}_2, R\tilde{x}_2 = \tilde{x}_3, \dots, \dots, R\tilde{x}_n = \tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n \neq \tilde{x}_{n+1}$

Substituting $\tilde{x} = \tilde{x}_n, \tilde{y} = \tilde{x}_{n+1}$ and $\tilde{z} = \tilde{x}_{n+1}$ in (3.5.1) then we get

$$\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \leq a \max \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \\ [\tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n) + \tilde{G}(\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1})], \\ [\tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_n, R\tilde{x}_n)] \end{array} \right\}$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a \max \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})], \\ [\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+1}, \tilde{x}_{n+1})] \end{array} \right\}$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a \max \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})], \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\} \quad \dots(3.5.2)$$

Now following three cases are arise:

Case-I: If $\max \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \\ [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})], \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\} = \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

then condition (3.5.2) reduces to

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

Case-II: If $\max \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \\ [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})], \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\} = [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})]$

then condition (3.5.2) reduces to

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a[\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})]$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{a}{1-a} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

Case-III: If $\max \left\{ \begin{array}{l} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \\ [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})], \\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \end{array} \right\} = \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})$

then condition (3.5.2) reduces to

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})$$

Which is contradiction.

From **Case- I, II, III**, we have

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

On continuing this process $(n + 1)$ times

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a^{n+1}\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Similarly we can show that

$$\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq a^n\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Next we show that $\{\tilde{x}_n\}$ is Soft G-Cauchy sequence.

Then for all $n, m \in \mathbb{N}, n < m$ we have

$$\begin{aligned} \tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_m) &\leq \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \dots + \tilde{G}(\tilde{x}_{m-1}, \tilde{x}_m, \tilde{x}_m) \\ &\leq (a^n + a^{n+1} + \dots + a^{m-1})\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \\ &\leq \frac{a^n}{1-a}\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1) \end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X}, \tilde{G}, E)$ is soft G-complete, there exists $\tilde{u} \in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to \tilde{u} .

Next we will show that \tilde{u} is fixed point of R .

For this we take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{z} = \tilde{u}$ in (3.5.1) then

$$\begin{aligned} \tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}) &\leq a \max \left\{ \tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u}), [\tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n) + \tilde{G}(\tilde{x}_n, R\tilde{u}, R\tilde{u})], \right. \\ &\quad \left. [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{x}_n, R\tilde{x}_n)] \right\} \\ \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) &\leq a \max \left\{ \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), [\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})], \right. \\ &\quad \left. [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u})] \right\} \\ \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) &\leq a\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \\ (1-a)\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) &\leq 0 \end{aligned}$$

This is the contraction, so $R\tilde{u} = \tilde{u}$ i.e. \tilde{u} is fixed point of R .

To prove uniqueness, suppose that \tilde{u} and \tilde{v} are two fixed points of R . Then by inequality (3.5.1) we have

$$\begin{aligned} \tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}) &\leq a \max \left\{ \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{v}, R\tilde{v})], \right. \\ &\quad \left. [\tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v}) + \tilde{G}(\tilde{v}, R\tilde{u}, R\tilde{u})] \right\} \\ \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) &\leq a \max \left\{ \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), [\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) + \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})], \right. \\ &\quad \left. [\tilde{G}(\tilde{v}, \tilde{v}, \tilde{v}) + \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})] \right\} \\ \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) &\leq a \max \{ \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u}) \} \end{aligned} \quad \dots(3.5.3)$$

Case I: If $\max\{\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})\} = \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$ then (3.5.3) we get

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

This is a contradiction implies that $\tilde{u} = \tilde{v}$.

Case II: If $\max\{\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})\} = \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$ then (3.5.3) we get

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a\tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$$

So we deduct that $\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a\tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$. By repeated use of the same argument we will find $\tilde{G}(\tilde{v}, \tilde{u}, \tilde{u}) \leq a\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$. Therefore we get $\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a^2\tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$, Since $a < 1/2$, this contradiction implies that $\tilde{u} = \tilde{v}$.

To show that R is soft G-continuous at \tilde{u} . Let $\{\tilde{y}_n\}$ be a sequence of soft elements in \tilde{X} such that $\{\tilde{y}_n\} \rightarrow \tilde{u}$ then we can deduce that

Using (3.5.1)

$$\begin{aligned} \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) &= \tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \\ &\leq a \max \left\{ \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n)], \right. \\ &\quad \left. [\tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) + \tilde{G}(\tilde{y}_n, R\tilde{u}, R\tilde{u})] \right\} \end{aligned}$$

$$\begin{aligned} \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) &\leq \alpha \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) \\ (1 - \alpha)\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) &\leq 0 \end{aligned}$$

We see that $\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \rightarrow 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G – convergent to $R\tilde{u} = \tilde{u}$ therefore proposition (2.20) implies that R is G-continuous at \tilde{u} .

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