

## Fixed Point Theory on a Soft Banach Space

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**Abstract:-** In this present paper some soft point and common soft point results are provided which generalized some well-known results.

Selection and Peer –review under responsibility of the Conference Committee Members of Functional Nonmaterial’s in Industrial Application.

**Keywords :-** soft point ,contractive mapping , soft Banach space, normed linear space.

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**2. Introduction and Preliminaries:-** In 1999, Molodtsov [10] proposed a completely new approach, which is called soft set theory for modeling uncertainty. Then Maji et al. (2003) [8] introduced several operations on soft sets. Aktas and Cagman (2007) [1] compared soft set with fuzzy sets and rough sets. Recently studies on soft vector spaces and soft normed linear space have been initiated by Das and Samanta [3, 4, 5] and later on studied by Yazar et al [19]. Maji et al [9], Chen [2] introduced a new definition of soft set theory.

We introduced soft contractive mapping on soft Banach space and section 1 study some of its properties. In section 2 preliminary results are given. In section 3 show that concept of soft Banach space and Related theorem proved.

**Definition 2.1:-** Let  $X$  be an initial universe set and  $E$  be a set of parameters. A pair  $(F, E)$  is called a soft set over  $X$  if and only if  $F$  is a mapping from  $E$  into the set of all subsets of the set  $X$  i.e.  $F: E \rightarrow P(X)$  is the power set of  $X$ .

**Definition 2.2:-** The intersection of two sets  $(A, D)$  and  $(B, C)$  over  $X$  is the soft set  $(F, G)$ , where

$C = D \cap C$  and  $\forall \varepsilon \in C, H(\varepsilon) = A(\varepsilon) \cap B(\varepsilon)$ . This is denoted by  $(A, D) \cap (B, C) = (F, G)$ .

**Definition 2.3:-** The union of two sets  $(A, D)$  and  $(B, C)$  over  $X$  is the soft set, where  $C = A \cup B$  and  $\forall \varepsilon \in C,$

$$H(\varepsilon) = \begin{cases} A(\varepsilon) & \text{if } \varepsilon \in D - C \\ B(\varepsilon) & \text{if } \varepsilon \in C - D \\ A(\varepsilon) \cup B(\varepsilon) & \text{if } \varepsilon \in D \cap C \end{cases}$$

This relationship is denoted by  $(A, D) \cup (B, C) = (F, G)$ .

**Definition 2.4:-** The soft set  $(A, D)$  over  $X$  is said to be a null soft set denoted by  $\emptyset$  if for all  $\varepsilon \in D, A(\varepsilon) = \emptyset$  (null set).

**Definition 2.5:-** A soft set  $(A, D)$  over  $X$  is said to be an absolute soft set, if for all  $\varepsilon \in D, A(\varepsilon) = X$ .

**Definition 2.6:-** The difference  $(F, E)$  of two soft sets  $(F, E)$  and  $(F, E)$  over  $X$  denoted by  $(F, E)/(F, E)$ , is defined as  $F(e) = A(e)/B(e)$  for all  $e \in E$

**Definition 2.7:-** The complement of a soft set  $(A, D)$  is denoted by  $(A, D)^c$  and is defined by  $(A, D)^c = (A^c, D)$  where  $A^c: D \rightarrow S(X)$  mapping given by  $A^c(\alpha) = A(\alpha)^c, \forall \alpha \in D$ .

**Definition 2.8:-** Let  $\mu$  be the set of real number and  $B(\mu)$  be the collection of all nonempty bounded subsets of  $\mu$  and  $E$  taken set of parameters. Then a mapping  $A: E \rightarrow B(\mu)$  is called a soft real set. It is denoted by  $(A, E)$ . If specifically  $(A, E)$  is a singleton soft set, then identifying  $(A, E)$  with the corresponding soft element, it will be called a soft real number and denoted  $\tilde{r}, \tilde{s}, \tilde{t}$  etc.  $\bar{0}, \bar{1}$  are the soft real number where  $\bar{0}(e) = 0, \bar{1}(e) = 1$  for all  $e \in E$

E, respectively.

**Definition 2.9:-** for two soft real numbers

- I.  $\tilde{r} \leq \tilde{s}$  if  $\tilde{r}(e) \leq \tilde{s}(e)$ , for all  $e \in E$ .
- II.  $\tilde{r} \geq \tilde{s}$  if  $\tilde{r}(e) \geq \tilde{s}(e)$ , for all  $e \in E$ .
- III.  $\tilde{r} < \tilde{s}$  if  $\tilde{r}(e) < \tilde{s}(e)$ , for all  $e \in E$ .
- IV.  $\tilde{r} > \tilde{s}$  if  $\tilde{r}(e) > \tilde{s}(e)$ , for all  $e \in E$ .

**Definition 2.10:-** A soft set over X is said to be a soft point if there is exactly one  $e \in E$ , such that  $P(e) = \{x\}$  for some  $x \in X$  and  $P(e) = \emptyset, \forall e \in E \setminus \{e\}$ . It will be denoted by  $\tilde{x}_\lambda$ .

**Definition 2.11:-** Two soft point  $\tilde{x}_\lambda, \tilde{y}_\lambda$  are said to be equal if  $e=e'$  and  $P(e)=P(e')$  i.e.  $x=y$ . Thus  $\tilde{x}_\lambda \neq \tilde{y}_\lambda \Leftrightarrow x \neq y$  or  $e \neq e'$ .

**Definition 2.12:-** A mapping  $\tilde{d}: SP(\tilde{X}) * SP(\tilde{X}) \rightarrow \tilde{R}(E)^*$ , is said to be a soft metric on the soft set  $\tilde{X}$  if d satisfies the following condition:

- (M1)  $\tilde{d}(\tilde{x}_{\lambda_1}, \tilde{y}_{\lambda_2}) \succeq \tilde{0}$  for all  $\tilde{x}_{\lambda_1}, \tilde{y}_{\lambda_2} \in \tilde{X}$ ,
- (M2)  $\tilde{d}(\tilde{x}_{\lambda_1}, \tilde{y}_{\lambda_2}) = \tilde{0}$  if and only if  $\tilde{x}_{\lambda_1} = \tilde{y}_{\lambda_2}$ ,
- (M3)  $\tilde{d}(\tilde{x}_{\lambda_1}, \tilde{y}_{\lambda_2}) \succeq \tilde{d}(\tilde{y}_{\lambda_2}, \tilde{x}_{\lambda_1})$  for all  $\tilde{x}_{\lambda_1}, \tilde{y}_{\lambda_2} \in \tilde{X}$ ,
- (M4)  $\tilde{d}(\tilde{x}_{\lambda_1}, \tilde{z}_{\lambda_3}) \succeq \tilde{d}(\tilde{x}_{\lambda_1}, \tilde{y}_{\lambda_2}) + \tilde{d}(\tilde{y}_{\lambda_2}, \tilde{z}_{\lambda_3})$  for all  $\tilde{x}_{\lambda_1}, \tilde{y}_{\lambda_2}, \tilde{z}_{\lambda_3} \in \tilde{X}$ .

The soft set  $\tilde{X}$  with a soft metric  $\tilde{d}$  on  $\tilde{X}$  is called a soft metric space and denoted by  $(\tilde{X}, \tilde{d}, E)$ .

**Definition 2.13:- (Cauchy Sequence):** A sequence  $\{\tilde{x}_{\lambda_n}\}_n$  of soft point in  $(\tilde{X}, \tilde{d}, E)$  is considered as a Cauchy Sequence in  $\tilde{X}$  if corresponding to every  $\tilde{\epsilon} \succeq \tilde{0}, \exists m \in \mathbb{N}$  such that  $\tilde{d}(\tilde{x}_{\lambda_i}, \tilde{x}_{\lambda_j}) \preceq \tilde{\epsilon}, \forall i, j \geq m$ , i.e.  $\tilde{d}(\tilde{x}_{\lambda_i}, \tilde{x}_{\lambda_j}) \rightarrow \tilde{0}$  as  $i, j \rightarrow \infty$ .

**Definition 2.14:- (Complete Metric Space):** A soft metric space  $(\tilde{X}, \tilde{d}, E)$  is called complete, if every Cauchy Sequence in  $\tilde{X}$  converges to some point of  $\tilde{X}$ .

**Definition 2.15:-** Let  $\tilde{X}$  be the absolute soft vector space i.e.  $\tilde{x}_\lambda = x, \forall \lambda \in A$ . Then a mapping  $\|\cdot\|: SE \rightarrow \tilde{R}(A)^*$  is said to be soft norm on the soft vector space  $\tilde{X}$  if  $\|\cdot\|$  satisfies the following condition.

1.  $\|\tilde{x}\| \succeq \tilde{0}$ , for all  $\tilde{x} \in \tilde{X}$ .
2.  $\|\tilde{x}\| = \tilde{0}$ , if and only if  $\tilde{x} = \tilde{0}$
3.  $\|\alpha \tilde{x}\| \succeq |\tilde{\alpha}| \|\tilde{x}\|$ , for all  $\tilde{x} \in \tilde{X}$  and for every soft scalar  $\tilde{\alpha}$ .

**Definition 2.16:-** A sequence of soft element  $\{\tilde{x}_n\}$  in a normed linear space  $(\tilde{x}, \|\cdot\|, A)$  is said to be convergent and converges to a soft element  $\tilde{x}$  if  $\|\tilde{x}_n - \tilde{x}\| \rightarrow \tilde{0}$  as  $n \rightarrow \infty$ . This means for every  $\tilde{\epsilon} \succeq \tilde{0}$ , choose arbitrary, there exists a natural number  $N(\tilde{\epsilon})$ , such that  $\tilde{0} \leq \|\tilde{x}_n - \tilde{x}\| \leq \tilde{\epsilon}$ , whenever  $n > N$ . we denoted this by  $\tilde{x}_n \rightarrow \tilde{x}$  as  $n \rightarrow \infty$  or by  $\lim_{n \rightarrow \infty} \tilde{x}_n = \tilde{x}$  is said to be the limit of the sequence  $\tilde{x}_n$  as  $n \rightarrow \infty$ .

**Definition 2.17:-** Let  $(\tilde{x}, \|\cdot\|, A)$  be a soft normed linear space. Then  $\tilde{x}$  is said to be complete if every of Cauchy sequence in  $\tilde{x}$  converges to a soft element of  $\tilde{x}$ . Every complete soft normed linear space is called a soft Banach space.

**Definition 2.18:-** A sequence of soft real number  $\{\tilde{s}_n\}$  is said to be convergent if for arbitrary  $\tilde{\epsilon} \gtrsim \tilde{0}$ , there exists a natural number N such that for all  $n \geq N$ ,  $|\tilde{s}_n - \tilde{s}| \leq \tilde{\epsilon}$ . we denoted it by  $\lim_{n \rightarrow \infty} \tilde{s}_n = \tilde{s}$ .

### 3. MAIN RESULT

**THEORAM 3.1:** Let  $(f, \varphi)$  be a soft mapping of Banach space  $\tilde{X}$  in to itself. If F satisfies the following contractive conditions.

$(f, \varphi)^2 = I$ , Where I is the identity mapping ( 3.1.1)

$$\begin{aligned} & \| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \| \\ & \leq \mu \max \left\{ \frac{\| (\tilde{x}_\lambda - (f, \varphi)(\tilde{x}_\lambda)) (\tilde{y}_\lambda - (f, \varphi)(\tilde{y}_\lambda)) \| + \| (\tilde{x}_\lambda - (f, \varphi)(\tilde{y}_\lambda)) (\tilde{y}_\lambda - (f, \varphi)(\tilde{x}_\lambda)) \|}{\| (\tilde{x}_\lambda - \tilde{y}_\lambda) \|}, \right. \\ & \left. \frac{\| (\tilde{x}_\lambda - (f, \varphi)(\tilde{x}_\lambda)) (\tilde{x}_\lambda - (f, \varphi)(\tilde{y}_\lambda)) \| + \| (\tilde{y}_\lambda - (f, \varphi)(\tilde{y}_\lambda)) (\tilde{y}_\lambda - (f, \varphi)(\tilde{x}_\lambda)) \|}{\| (\tilde{x}_\lambda - \tilde{y}_\lambda) \|} \right\} + \\ & \rho \{ \| (\tilde{x}_\lambda) - (f, \varphi)(\tilde{x}_\lambda) \| + \| (\tilde{y}_\lambda - (f, \varphi)(\tilde{y}_\lambda)) \| \} + \omega \tilde{d} \| (\tilde{x}_\lambda - \tilde{y}_\lambda) \| \end{aligned}$$

For Every  $\tilde{x}_\lambda, \tilde{y}_\lambda \in SP(\tilde{X})$ . Where  $\mu, \rho, \omega > 0$  and  $\mu + \omega < 1$ . Then  $(f, \varphi)$  has a soft point, if  $4\mu + 3\rho + \omega < 2$ . then  $(f, \varphi)$  has a unique soft point.

**PROOF:** Suppose  $\tilde{x}_\lambda$  in a point in the Banach sapace.

$$\begin{aligned} \tilde{y}_\lambda &= \frac{1}{2} [(f, \varphi) + I] \tilde{x}_\lambda \\ \tilde{z}_\lambda &= (f, \varphi) (\tilde{y}_\lambda) \text{ and} \\ \tilde{w} &= 2\tilde{y}_\lambda - \tilde{z}_\lambda \end{aligned}$$

We have

$$\begin{aligned} \| \tilde{z}_\lambda - \tilde{x}_\lambda \| &= \| ((f, \varphi)(\tilde{y}_\lambda) - (f, \varphi)^2(\tilde{x}_\lambda)) \| = \| ((f, \varphi)(\tilde{y}_\lambda) - (f, \varphi)((f, \varphi)(\tilde{x}_\lambda)) \| \\ &\leq \mu \max \left\{ \frac{\| ((f, \varphi)(\tilde{y}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \| \| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)^2(\tilde{x}_\lambda)) \| + \| ((f, \varphi)(\tilde{y}_\lambda) - (f, \varphi)^2(\tilde{x}_\lambda)) \| \| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \|}{\| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \|}, \right. \\ & \left. \frac{\| ((f, \varphi)(\tilde{y}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \| \| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)^2(\tilde{x}_\lambda)) \| + \| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)^2(\tilde{x}_\lambda)) \| \| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \|}{\| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \|} \right\} + \\ & \rho \{ \| ((f, \varphi)(\tilde{y}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \| + \| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)^2(\tilde{x}_\lambda)) \| \} \\ & + \omega \| ((f, \varphi)(\tilde{y}_\lambda) - (f, \varphi)(\tilde{x}_\lambda)) \| \\ &\leq \mu \max \left\{ \frac{\| ((f, \varphi)(\tilde{y}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \| \| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)^2(\tilde{x}_\lambda)) \| + \| \frac{1}{2} ((f, \varphi) + I) \tilde{x}_\lambda - \tilde{x}_\lambda \| \| ((f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)) \|}{\| \frac{1}{2} ((f, \varphi) + I) \tilde{x}_\lambda - (f, \varphi)(\tilde{x}_\lambda) \|} \right\} \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{\| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| \frac{1}{2} ((f, \varphi) + I) \bar{x}_\lambda - \bar{x}_\lambda \right\| + \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \right\| + \left\| (f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda) \right\|}{\left\| \frac{1}{2} ((f, \varphi) + I) \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \right\|} \right\} + \\
 & \rho \{ \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \right\| \} + \omega \left\| \frac{1}{2} ((f, \varphi) + I) \bar{x}_\lambda - \bar{x}_\lambda \right\| \\
 \leq & \mu \max \left\{ \frac{2 \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| \| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \| + \| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \| \| (f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda) \|}{\| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \|}, \right. \\
 & \left. \frac{\| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| \| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \| + 2 \| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \| \| (f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda) \|}{\| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \|} \right\} + \\
 & \rho \{ \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \right\| + \frac{\omega}{2} \| (f, \varphi) \bar{x}_\lambda - \bar{x}_\lambda \| \} \\
 \leq & \mu \max \{ 2 \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda) \right\|, \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \\
 & 2 \left\| (f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda) \right\| \} \\
 & \rho \{ \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \right\| + \frac{\omega}{2} \| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \| \} \\
 & \dots \dots \dots
 \end{aligned}$$

(A)

**CASE I:** When

$$\begin{aligned}
 & \max \{ 2 \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda) \right\|, \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + 2 \left\| (f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda) \right\| \} \\
 & = 2 \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda) \right\|
 \end{aligned}$$

Then

$$\begin{aligned}
 & \| \bar{x}_\lambda - \bar{x}_\lambda \| \leq \mu \{ 2 \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda) \right\| \} \\
 + P \{ & \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \right\| \} + \frac{\omega}{2} \| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \| \\
 & \leq \mu \{ 2 \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{y}_\lambda \right\| + \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| \} \\
 & + p \{ \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \right\| \} + \frac{\omega}{2} \| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \| \\
 & \leq \mu \{ 3 \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{y}_\lambda \right\| \} + p \{ \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \\
 & \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \right\| \} + \frac{\omega}{2} \| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \| \\
 & \leq (3\mu + P) \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \mu \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{y}_\lambda \right\| + P \{ \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \\
 & \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \right\| \} + \frac{\omega}{2} \| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \| \\
 & \leq (3\mu + P) \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \mu \left\| (f, \varphi)(\bar{x}_\lambda) - \frac{1}{2} ((F + I) \bar{x}_\lambda) \right\| + P \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| \\
 & + \left\| (f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda \right\| \} + \frac{\omega}{2} \| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \| \\
 & \leq (3\mu + P) \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \left( \frac{P}{2} + P + \frac{\omega}{2} \right) \| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \| \\
 & \dots \dots \dots
 \end{aligned}$$

(B)

Also,

$$\begin{aligned}
 & \| \bar{x} - \bar{x}_\lambda \| \leq \| 2 \bar{y}_\lambda - \bar{x}_\lambda - \bar{x}_\lambda \| = \left\| (f, \varphi) + I \right\| \bar{x}_\lambda - (f, \varphi)(\bar{y}_\lambda) - \bar{x}_\lambda \| \\
 & = \left\| (f, \varphi)(\bar{x}) - (f, \varphi)(\bar{y}_\lambda) \right\| \\
 \leq & \mu \max \left\{ \frac{\| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \| \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \|}{\| \bar{x}_\lambda - \bar{y}_\lambda \|}, \right. \\
 & \left. \frac{\| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \| \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| + \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \| \| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \|}{\| \bar{x}_\lambda - \bar{y}_\lambda \|} \right\} + \\
 & \rho \{ \| \bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda) \| + \left\| \bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda) \right\| \} + \omega \| \bar{x}_\lambda - \bar{y}_\lambda \|
 \end{aligned}$$

$$\begin{aligned}
 &\leq \mu \max \left\{ \frac{\|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|x_{\lambda} - (f, \varphi)(y_{\lambda})\| \left\| \frac{1}{2} (f, \varphi) + I \right\| \|y_{\lambda} - (f, \varphi)(x_{\lambda})\|}{\|x - \frac{1}{2} (f, \varphi) + I\| \|x_{\lambda}\|}, \right. \\
 &\quad \left. \frac{\|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \|x_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \left\| \frac{1}{2} (f, \varphi) + I \right\| \|y_{\lambda} - (f, \varphi)(x_{\lambda})\|}{\|x - \frac{1}{2} (f, \varphi) + I\| \|x_{\lambda}\|} \right\} + \\
 &\quad \rho \{ \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \} + \omega \left\| x_{\lambda} - \frac{1}{2} (f, \varphi) + I \right\| \|x_{\lambda}\| \\
 &\leq \mu \max \left\{ \frac{2 \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|x_{\lambda} - (f, \varphi)(y_{\lambda})\| \|y_{\lambda} - (f, \varphi)(x_{\lambda})\|}{\|x - (f, \varphi)\| \|x_{\lambda}\|}, \right. \\
 &\quad \left. \frac{2 \|y_{\lambda} - (f, \varphi)(x_{\lambda})\| \|x_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \|x_{\lambda} - (f, \varphi)(x_{\lambda})\|}{\|x - (f, \varphi)\| \|x_{\lambda}\|} \right\} \\
 &\quad + \rho \{ \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \} + \frac{\omega}{2} \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \\
 &\leq \mu \max \{ 2 \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|x_{\lambda} - (f, \varphi)(y_{\lambda})\|, 2 \|x_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \} \\
 &\quad + \rho \{ \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \} + \frac{\omega}{2} \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \quad \dots\dots\dots (C)
 \end{aligned}$$

CASE I When

$$\begin{aligned}
 &\max \{ 2 \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|x_{\lambda} - (f, \varphi)(y_{\lambda})\|, 2 \|x_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \} \\
 &\quad = 2 \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|x_{\lambda} - (f, \varphi)(y_{\lambda})\|
 \end{aligned}$$

Then

$$\begin{aligned}
 \|\tilde{u} - \tilde{x}_{\lambda}\| &\leq \mu \{ 2 \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|x_{\lambda} - (f, \varphi)(y_{\lambda})\| \} + \rho \{ \\
 &\|x_{\lambda} - (f, \varphi)(x_{\lambda})\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \} + \frac{\omega}{2} \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \\
 &\leq \mu \{ 2 \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|x_{\lambda} - y_{\lambda}\| \} + \rho \{ \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \} \\
 &\quad + \frac{\omega}{2} \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \\
 &\leq \mu \{ 3 \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \|x_{\lambda} - y_{\lambda}\| + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| \} + \rho \{ \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \\
 &\quad + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \frac{\omega}{2} \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \} \\
 &\leq \mu \{ 3 \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \frac{1}{2} \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \} + \rho \{ \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \\
 &\quad + \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \frac{\omega}{2} \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \} \\
 &\leq (3\mu + \rho) \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \left( \frac{\mu}{2} + \rho + \frac{\omega}{2} \right) \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \\
 &\quad \dots\dots\dots (D)
 \end{aligned}$$

Now by equations (B) and (D)

$$\begin{aligned}
 \|\tilde{x}_{\lambda} - \tilde{u}\| &\leq \|\tilde{x}_{\lambda} - \tilde{x}_{\lambda}\| + \|\tilde{x}_{\lambda} - \tilde{u}\| \\
 &\leq (3\mu + \rho) \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \left( \frac{\mu}{2} + \rho + \frac{\omega}{2} \right) \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \\
 &\quad + (3\mu + \rho) \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + \left( \frac{\mu}{2} + \rho + \frac{\omega}{2} \right) \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \\
 &\leq 2(3\mu + \rho) \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + 2 \left( \frac{\mu}{2} + \rho + \frac{\omega}{2} \right) \|x_{\lambda} - (f, \varphi)(x_{\lambda})\| \\
 &\leq 2(3\mu + \rho) \|y_{\lambda} - (f, \varphi)(y_{\lambda})\| + (\mu + 2\rho + \omega) \|x_{\lambda} - (f, \varphi)(x_{\lambda})\|
 \end{aligned}$$

Also

$$\begin{aligned} \|z - u\| &\leq \|(f, \varphi)(\bar{y}_\lambda) - (2\bar{y}_\lambda) - \bar{z}\| = \|(f, \varphi)(\bar{y}_\lambda) - 2\bar{y}_\lambda - \bar{z}\| \\ &= 2\|(f, \varphi)(\bar{y}_\lambda) - \bar{y}_\lambda\| \end{aligned}$$

So

$$2\|(f, \varphi)(\bar{y}_\lambda) - \bar{y}_\lambda\| \leq 2(3\mu + \rho)\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + (\mu + 2\rho + \omega)\|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\|$$

$$\|(f, \varphi)(\bar{y}_\lambda) - \bar{y}_\lambda\| \leq (3\mu + \rho)\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \left(\frac{\mu + 2\rho + \omega}{2}\right)\|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\|$$

$$(1 - 3\mu - \rho)\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \leq \frac{(\mu + 2\rho + \omega)}{2}\|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\|$$

$$\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \leq \frac{(\mu + 2\rho + \omega)}{2(1 - 3\mu - \rho)}\|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\|$$

Since

$$4\mu + 3\rho + \omega < 2$$

CASE II :- When

$$\begin{aligned} \max \{ &2\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \|(f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda)\|, \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + 2\|(f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda)\| \} \\ &= \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + 2\|(f, \varphi)(\bar{x}_\lambda) - (f, \varphi)(\bar{y}_\lambda)\|. \end{aligned}$$

Then

$$\|\bar{x}_\lambda - \bar{x}_\lambda\| \leq \mu \{ 2\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + 2\|(f, \varphi)(\bar{x}_\lambda) - \bar{y}_\lambda\| + \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \}$$

$$p \{ \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \|(f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda\| \} + \frac{\omega}{2} \|(f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda\|$$

$$\leq \mu \{ 3\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \|(f, \varphi)(\bar{x}_\lambda) - \frac{1}{2}((f, \varphi) + I)\bar{x}_\lambda\| \}$$

$$+ \rho \{ \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \|(f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda\| \} + \frac{\omega}{2} \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\|$$

$$\leq (3\mu)\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \mu\|(f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda\| + P \{ \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\|$$

$$+ \|(f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda\| \} + \frac{\omega}{2} \|(f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda\|$$

$$\leq (3\mu + P)\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + (\mu + P \frac{\omega}{2})\|(f, \varphi)(\bar{x}_\lambda) - \bar{x}_\lambda\|$$

..... (E)

CASE II :- By equation (C) When

$$\max \{ 2\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \|\bar{x}_\lambda - (f, \varphi)(\bar{y}_\lambda)\|, 2\|\bar{x}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \}$$

$$= 2\|\bar{x}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\|$$

Then

$$\begin{aligned}
 \|\bar{y} - \bar{x}_\lambda\| &\leq \mu \left\{ 2\|\bar{x}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \right\} + P \left\{ \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| + \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \right\} + \frac{\omega}{2} \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| \\
 &\leq \mu \left\{ 2\|\bar{x}_\lambda - \bar{y}_\lambda\| + \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \right\} + P \left\{ \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| + \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \right\} + \frac{\omega}{2} \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| \\
 &\leq \mu \left\{ \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| + 3\|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \right\} + P \left\{ \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| + \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| \right\} \\
 &\quad + \frac{\omega}{2} \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| \\
 &\leq (3\mu + P) \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \left(\mu + P + \frac{\omega}{2}\right) \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| \\
 &\dots\dots\dots (F)
 \end{aligned}$$

Now by equations (E) and (F)

$$\begin{aligned}
 \|\bar{z}_\lambda - \bar{u}\| &\leq \|\bar{z}_\lambda - \bar{x}_\lambda\| + \|\bar{x}_\lambda - \bar{u}\| \\
 &\leq (3\mu + P) \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \left(\mu + P + \frac{\omega}{2}\right) \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| \\
 &+ \leq (3\mu + P) \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \left(\mu + P + \frac{\omega}{2}\right) \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| \\
 &\leq 2(3\mu + P) \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + 2\left(\mu + P + \frac{\omega}{2}\right) \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\|
 \end{aligned}$$

Also

$$\|\bar{z}_\lambda - \bar{u}\| \leq \|(f, \varphi)(\bar{y}_\lambda) - 2\bar{y}_\lambda - \bar{z}_\lambda\| = \|(f, \varphi)(\bar{y}_\lambda) - 2\bar{y}_\lambda + \bar{z}_\lambda\| = 2\|(f, \varphi)(\bar{y}_\lambda) - \bar{y}_\lambda\|$$

So

$$\begin{aligned}
 2\|(f, \varphi)(\bar{y}_\lambda) - \bar{y}_\lambda\| &\leq 2(3\mu + P) \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + 2\left(\mu + 2P + \frac{\omega}{2}\right) \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\| \\
 &\leq (3\mu + P) \|\bar{y}_\lambda - (f, \varphi)(\bar{y}_\lambda)\| + \left(\mu + 2P + \frac{\omega}{2}\right) \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\|
 \end{aligned}$$

$$(1 - 3\mu - P) \|(f, \varphi)(\bar{y}_\lambda) - \bar{y}_\lambda\| \leq \left(\mu + P + \frac{\omega}{2}\right) \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\|$$

$$\|(f, \varphi)(\bar{y}_\lambda) - \bar{y}_\lambda\| \leq \frac{\left(\mu + P + \frac{\omega}{2}\right)}{(1 - 3\mu - P)} \|\bar{x}_\lambda - (f, \varphi)(\bar{x}_\lambda)\|$$

Since

$$\begin{aligned}
 \frac{\left(\mu + P + \frac{\omega}{2}\right)}{(1 - 3\mu - P)} &< 1 \\
 4\mu + 2P + \frac{\omega}{2} &< 2
 \end{aligned}$$

On taking

$$F = \frac{1}{2}((f, \varphi) + I) \text{ then for every } \bar{x}_\lambda \in \bar{X}$$

By definition of q. we claim that  $\{\bar{x}_\lambda\}$  is a Cauchy sequence in  $\bar{X}$ . There fore by the property of completeness

$\{(g, \varphi)^n(\tilde{x}_\lambda)\}$  converge to same element  $\tilde{x}_\lambda^0$  in  $\tilde{X}$ .

i.e.  $\lim_{n \rightarrow \infty} (g, \varphi)^n(\tilde{x}_\lambda) = \tilde{x}_\lambda^0$

which implice  $(g, \varphi)^n(\tilde{x}_\lambda) = \tilde{x}_\lambda^0$  hence  $(f, \varphi)(\tilde{x}_\lambda^0) = \tilde{x}_\lambda^0$

i.e.  $x_0$  is a soft point of  $(f, \varphi)$

Uniqueness :- If possible let  $\tilde{y}_\lambda^0 (\neq \tilde{x}_\lambda^0)$  be another soft point of  $(f, \varphi)$

Then

$$\|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\| = \|(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{y}_\lambda^0)\|$$

$$\begin{aligned} \|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\| &\leq \mu \max \left\{ \frac{\|s_{\tilde{x}_\lambda^0 - (g, \varphi)(\tilde{x}_\lambda^0)}\| \|s_{\tilde{y}_\lambda^0 - (g, \varphi)(\tilde{y}_\lambda^0)}\| + \|s_{\tilde{x}_\lambda^0 - (g, \varphi)(\tilde{y}_\lambda^0)}\| \|s_{\tilde{y}_\lambda^0 - (g, \varphi)(\tilde{x}_\lambda^0)}\|}{\|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\|}, \right. \\ &\quad \left. \frac{\|s_{\tilde{x}_\lambda^0 - (g, \varphi)(\tilde{x}_\lambda^0)}\| \|s_{\tilde{x}_\lambda^0 - (g, \varphi)(\tilde{y}_\lambda^0)}\| + \|s_{\tilde{x}_\lambda^0 - (g, \varphi)(\tilde{y}_\lambda^0)}\| \|s_{\tilde{y}_\lambda^0 - (g, \varphi)(\tilde{x}_\lambda^0)}\|}{\|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\|} \right\} \\ &\quad + \rho \{ \|\tilde{x}_\lambda^0 - (g, \varphi)(\tilde{x}_\lambda^0)\| + \|\tilde{y}_\lambda^0 - (g, \varphi)(\tilde{y}_\lambda^0)\| \} + \omega \|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\| \\ &\leq \mu \max \left\{ \frac{\|s_{\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0}\| \|s_{\tilde{y}_\lambda^0 - \tilde{x}_\lambda^0}\| + \|s_{\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0}\| \|s_{\tilde{y}_\lambda^0 - \tilde{x}_\lambda^0}\|}{\|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\|} \right. \\ &\quad \left. \frac{\|s_{\tilde{x}_\lambda^0 - \tilde{x}_\lambda^0}\| \|s_{\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0}\| + \|s_{\tilde{y}_\lambda^0 - \tilde{y}_\lambda^0}\| \|s_{\tilde{y}_\lambda^0 - \tilde{x}_\lambda^0}\|}{\|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\|} \right\} \\ &\quad + \rho \{ \|\tilde{x}_\lambda^0 - \tilde{x}_\lambda^0\| + \|\tilde{y}_\lambda^0 - \tilde{y}_\lambda^0\| \} + \omega \|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\| \\ &\leq \mu \max \{ \|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\|, 0 \} + \rho(o) + \omega \|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\| \\ &< (\mu + \omega) \|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\| \end{aligned}$$

Since  $\mu + \omega < 1$ , there for  $\|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\| = 0$

Hence  $\tilde{x}_\lambda^0 = \tilde{y}_\lambda^0$

This complete the proof.

**THEOREM 3.2:-** let K closed and convex subset of a soft Banach space  $\tilde{X}$ . Let  $(g, \varphi):K \rightarrow K$ ,  $(f, \varphi):K \rightarrow K$ , satisfy the following condition,

(3.2.1)  $(g, \varphi)$  and  $(f, \varphi)$  commute.

(3.2.2)  $(g, \varphi)^2 = I$  and  $(f, \varphi)^2 = I$ , where I denotes identity mappings.

(3.2.3)

$$\begin{aligned} \|(g, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{y}_\lambda)\| &\leq \mu \max \left\{ \frac{\|s_{(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{x}_\lambda)}\| \|s_{(f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(\tilde{y}_\lambda)}\| + \|s_{(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{y}_\lambda)}\| \|s_{(f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(\tilde{x}_\lambda)}\|}{\|(f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)\|}, \right. \\ &\quad \left. \frac{\|s_{(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{x}_\lambda)}\| \|s_{(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{y}_\lambda)}\| + \|s_{(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{y}_\lambda)}\| \|s_{(f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(\tilde{x}_\lambda)}\|}{\|(f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)\|} \right\} \\ &\quad + \rho \{ \|(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{x}_\lambda)\| + \|(f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(\tilde{y}_\lambda)\| \} \end{aligned}$$



$$+\omega\|(f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)\|$$

For every  $\tilde{x}, \tilde{y} \in \tilde{X}$ .  $\mu + \omega + \eta + \rho \leq \tilde{\delta}$  and there exist at least one soft point  $\tilde{x}_\lambda^0 = \tilde{X}$  such that

$$(g, \varphi)(\tilde{x}_\lambda^0) = (f, \varphi)(\tilde{x}_\lambda^0) = \tilde{x}_\lambda^0$$

further if  $(\mu + \omega) < 1$ .

Then  $\tilde{x}_\lambda^0$  is the unique soft point of  $(f, \varphi)$  and  $(g, \varphi)$ .

**PROOF:-**

From (3.2.1) and (3.2.2) it follows that  $[(g, \varphi)(f, \varphi)]^2 = I$  and (3.2.2) and (3.2.3) imply

$$\begin{aligned} & \|(g, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{y}_\lambda)\| \leq \\ & \leq \mu \max \left\{ \frac{\|(f, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{x}_\lambda)\| \|(f, \varphi)(f, \varphi)^2(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{y}_\lambda)\| + \right. \\ & \quad \left. \|(f, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{y}_\lambda)\| \|(f, \varphi)(f, \varphi)^2(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{x}_\lambda)\|}{\|(f, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (f, \varphi)(f, \varphi)^2(\tilde{y}_\lambda)\|} \right. \\ & \quad \left. \frac{\|(f, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{x}_\lambda)\| \|(f, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{y}_\lambda)\| + \right. \\ & \quad \left. \|(f, \varphi)(f, \varphi)^2(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{y}_\lambda)\| \|(f, \varphi)(f, \varphi)^2(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{x}_\lambda)\|}{\|(f, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (f, \varphi)(f, \varphi)^2(\tilde{y}_\lambda)\|} \right\} \end{aligned}$$

$$+ \rho \{ \|(f, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{x}_\lambda)\| + \|(f, \varphi)(f, \varphi)^2(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)^2(\tilde{y}_\lambda)\| \} \\ + \omega \|(f, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (f, \varphi)(f, \varphi)^2(\tilde{y}_\lambda)\|$$

Now we put  $(f, \varphi)(\tilde{x}_\lambda) = \tilde{z}_\lambda$  and  $(f, \varphi)(\tilde{y}_\lambda) = \tilde{v}_\lambda$ , then we get

$$\begin{aligned} & \|(g, \varphi)(f, \varphi)(\tilde{z}_\lambda) - (g, \varphi)(f, \varphi)(\tilde{v}_\lambda)\| \leq \\ & \leq \mu \max \left\{ \frac{\|(\tilde{z}_\lambda) - (g, \varphi)(\tilde{z}_\lambda)\| \|(\tilde{v}_\lambda) - (g, \varphi)(\tilde{v}_\lambda)\| \|(\tilde{z}_\lambda) - (g, \varphi)(\tilde{v}_\lambda)\| \|(\tilde{v}_\lambda) - (g, \varphi)(\tilde{z}_\lambda)\|}{\|\tilde{z}_\lambda - \tilde{v}_\lambda\|} \right. \\ & \quad \left. \frac{\|(\tilde{z}_\lambda) - (g, \varphi)(\tilde{z}_\lambda)\| \|(\tilde{z}_\lambda) - (g, \varphi)(\tilde{v}_\lambda)\| + \|(\tilde{v}_\lambda) - (g, \varphi)(\tilde{v}_\lambda)\| \|(\tilde{v}_\lambda) - (g, \varphi)(\tilde{z}_\lambda)\|}{\|\tilde{z}_\lambda - \tilde{v}_\lambda\|} \right\} \\ & + \rho \{ \|(\tilde{z}_\lambda) - (g, \varphi)(\tilde{z}_\lambda)\| + \|(\tilde{v}_\lambda) - (g, \varphi)(\tilde{v}_\lambda)\| \} \\ & + \omega \|(\tilde{z}_\lambda) - (\tilde{v}_\lambda)\| \end{aligned}$$

We have

$$(g, \varphi)(f, \varphi)^2 = I: (g, \varphi)(f, \varphi) \text{ has at least one fixed point, say } \tilde{x}_\lambda^0 \text{ in } K, \text{ i.e.} \quad (3.2.4)$$

$$\begin{aligned} & (g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) = \tilde{x}_\lambda^0 \\ \text{and } & (g, \varphi)(g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) = (g, \varphi)(\tilde{x}_\lambda^0) \\ (3.2.5) \quad & (f, \varphi)(\tilde{x}_\lambda^0) = (g, \varphi)(\tilde{x}_\lambda^0) \end{aligned}$$

NOW

$$\begin{aligned} & \|(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\| = \|(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)^2(\tilde{x}_\lambda^0)\| = \|(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0)\| \\ & \leq \mu \max \left\{ \|(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\| \|(f, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0)\| + \right. \\ & \quad \frac{\|(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0)\| \|(f, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\|}{\|(f, \varphi)(\tilde{x}_\lambda^0) - (f, \varphi)(g, \varphi)(\tilde{x}_\lambda^0)\|} \right. \\ & \quad \frac{\|(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\| \|(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0)\| + \left. \right. \\ & \quad \left. \frac{\|(f, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0)\| \|(f, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\|}{\|(f, \varphi)(\tilde{x}_\lambda^0) - (f, \varphi)(g, \varphi)(\tilde{x}_\lambda^0)\|} \right\} \\ & + \rho \{ \|(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\| + \|(f, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0)\| \} \\ & + \omega \|(f, \varphi)(\tilde{x}_\lambda^0) - (f, \varphi)(g, \varphi)(\tilde{x}_\lambda^0)\| \\ & \leq \mu \max \left\{ \frac{\|(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\| \|(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0)\| + \|(g, \varphi)(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0)\| \|(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\|}{\|(g, \varphi)(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0)\|} \right\} \end{aligned}$$

$$\begin{aligned} & \left\{ \frac{\|(g, \varphi)(x_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\| \| (x_\lambda^0) - (x_\lambda^0) \| + \|(g, \varphi)(x_\lambda^0) - (x_\lambda^0)\| \| (x_\lambda^0) - (g, \varphi)(x_\lambda^0) \|}{\|(g, \varphi)(x_\lambda^0) - (x_\lambda^0)\|} \right\} \\ & + \rho \{ \|(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0)\| + \| (x_\lambda^0) - (x_\lambda^0) \| \} + \omega \|(g, \varphi)(\tilde{x}_\lambda^0) - (x_\lambda^0)\| \\ & \leq \mu \max \{ \|(g, \varphi)(\tilde{x}_\lambda^0) - (x_\lambda^0)\|, 0 \} + \rho(0) + \omega \|(g, \varphi)(\tilde{x}_\lambda^0) - (x_\lambda^0)\| \\ & \leq (\mu + \omega) \|(g, \varphi)(\tilde{x}_\lambda^0) - (x_\lambda^0)\| \end{aligned}$$

There for

$$\|(g, \varphi)(\tilde{x}_\lambda^0) - (x_\lambda^0)\| \leq (\mu + \omega) \|(g, \varphi)(\tilde{x}_\lambda^0) - (x_\lambda^0)\|$$

Since  $\mu + \omega + \eta + \rho < 1$ , it follow

$$(g, \varphi)(\tilde{x}_\lambda^0) = (x_\lambda^0) \text{ i.e}$$

$(\tilde{x}_\lambda^0)$  is the soft point of  $(g, \varphi)(\tilde{x}_\lambda^0) = (f, \varphi)(\tilde{x}_\lambda^0)$  therefor, we have

$$(f, \varphi)(\tilde{x}_\lambda^0) = (\tilde{x}_\lambda^0)$$

i.e.  $\tilde{x}_\lambda^0$  is the common soft point of  $(g, \varphi)$  and  $(f, \varphi)$ .

Uniqueness:- Now we shall prove that  $\tilde{x}_\lambda^0$  is the uniqueness common soft point of  $(g, \varphi)$  and  $(f, \varphi)$ . If possible let  $\tilde{y}_\lambda^0$  be another soft point of  $(g, \varphi)$  and  $(f, \varphi)$ .

Now by using (3.2.1), (3.2.2), (3.2.3) and (3.2.4), (3.2.5)

We have

$$\begin{aligned} \|\tilde{x}_\lambda^0 - \tilde{y}_\lambda^0\| &= \|(g, \varphi)^2(\tilde{x}_\lambda^0) - (g, \varphi)^2(\tilde{y}_\lambda^0)\| = \|(g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{y}_\lambda^0)\| \\ &\leq \mu \max \left\{ \frac{\|(f, \varphi)(g, \varphi)(x_\lambda^0) - (g, \varphi)(g, \varphi)(x_\lambda^0)\| \|(f, \varphi)(g, \varphi)(\tilde{y}_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{y}_\lambda^0)\| + \|(g, \varphi)(f, \varphi)(x_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{y}_\lambda^0)\| \|(f, \varphi)(g, \varphi)(\tilde{y}_\lambda^0) - (g, \varphi)(g, \varphi)(x_\lambda^0)\|}{\|(g, \varphi)(f, \varphi)(x_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{y}_\lambda^0)\|} \right. \\ &\quad \left. + \frac{\|(f, \varphi)(g, \varphi)(x_\lambda^0) - (g, \varphi)(g, \varphi)(x_\lambda^0)\| \|(f, \varphi)(g, \varphi)(x_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{y}_\lambda^0)\| + \|(f, \varphi)(g, \varphi)(\tilde{y}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{y}_\lambda^0)\| \|(f, \varphi)(g, \varphi)(\tilde{y}_\lambda^0) - (g, \varphi)(g, \varphi)(x_\lambda^0)\|}{\|(f, \varphi)(g, \varphi)(x_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{y}_\lambda^0)\|} \right\} \\ &\quad + \rho \{ \|(f, \varphi)(g, \varphi)(x_\lambda^0) - (g, \varphi)(f, \varphi)(x_\lambda^0)\| + \|(f, \varphi)(g, \varphi)(\tilde{y}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{y}_\lambda^0)\| \} \\ &\quad + \omega \|(f, \varphi)(g, \varphi)(x_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{y}_\lambda^0)\| \\ &\leq \mu \max \left\{ \frac{\|(x_\lambda^0) - (x_\lambda^0)\| \|(x_\lambda^0) - (x_\lambda^0)\| + \|(x_\lambda^0) - (x_\lambda^0)\| \|(x_\lambda^0) - (x_\lambda^0)\|}{\|(x_\lambda^0) - (x_\lambda^0)\|}, \frac{\|(x_\lambda^0) - (x_\lambda^0)\| \|(x_\lambda^0) - (x_\lambda^0)\| + \|(x_\lambda^0) - (x_\lambda^0)\| \|(x_\lambda^0) - (x_\lambda^0)\|}{\|(x_\lambda^0) - (x_\lambda^0)\|} \right\} \\ &\quad + \rho \{ \|(x_\lambda^0) - (x_\lambda^0)\| + \| (x_\lambda^0) - (x_\lambda^0) \| \} + \omega \|(x_\lambda^0) - (x_\lambda^0)\| \\ &\leq \mu \max \{ \|(x_\lambda^0) - (x_\lambda^0)\|, 0 \} + \rho(0) + \omega \|(x_\lambda^0) - (x_\lambda^0)\| \\ &\leq (\mu + \omega) \|(x_\lambda^0) - (x_\lambda^0)\| \end{aligned}$$

Since  $\mu + \omega + \eta + \rho < 1$ , it follow that

$$(\tilde{x}_\lambda^0) = (\tilde{y}_\lambda^0)$$

Proving the uniqueness of  $\tilde{x}_\lambda^0$ , the proof of the theorem 2 is complete.

**THEOREM 3.3:-** Let  $k$  be closed and convex subset of a soft Banach space  $\tilde{X}$ . Let  $(g, \varphi)$  and  $(f, \varphi)$  and  $(h, \varphi)$  be three mapping of  $\tilde{X}$  in to it self such that

(3.3.1)

$$(g, \varphi)(f, \varphi) = (f, \varphi)(g, \varphi), \quad (f, \varphi)(h, \varphi) = (h, \varphi)(f, \varphi), \text{ and} \\ (g, \varphi)(h, \varphi) = (h, \varphi)(g, \varphi)$$

(3.3.2)  $(g, \varphi)^2 = I, (f, \varphi)^2 = I, (h, \varphi)^2 = I$ , where  $I$  denotes the identity mapping.

(3.3.3)

$$\|(g, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{y}_\lambda)\| \leq \mu \max \left\{ \frac{\|(f, \varphi)(h, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{x}_\lambda)\| \|(f, \varphi)(h, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(\tilde{y}_\lambda)\| + \|(f, \varphi)(h, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{y}_\lambda)\| \|(f, \varphi)(h, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(\tilde{x}_\lambda)\|}{\|(f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda)\|} \right\}$$

$$\frac{\| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{x}_\lambda) \| \| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{y}_\lambda) \| + \| (f, \varphi)(h, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(\tilde{y}_\lambda) \| \| (f, \varphi)(h, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(\tilde{x}_\lambda) \|}{\| (f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda) \|}} + \rho \{ \| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(\tilde{x}_\lambda) \| + \| (f, \varphi)(h, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(\tilde{y}_\lambda) \| \} + \omega \| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda) \|$$

For every  $\tilde{x}, \tilde{y} \in \tilde{K}$  and  $\mu, \omega, \rho \leq \tilde{0}$  such that  $\omega + \rho < 2$ . Then there exist at least one soft point  $\tilde{x}_\lambda^0 = \tilde{X}$ , such that  $(g, \varphi)(\tilde{x}_\lambda^0) = (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^0)$  and  $(g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) = (h, \varphi)(\tilde{x}_\lambda^0)$  further if  $(\mu + \omega + \rho) < 1$ . Then  $\tilde{x}_\lambda^0$  is the common soft point of  $(g, \varphi)$ ,  $(f, \varphi)$  and  $(h, \varphi)$ .

**Proof:-** From (3.3.1) and (3.3.2) it follows that  $[(g, \varphi)(f, \varphi)(h, \varphi)]^2 = I$ , where I is the identity mapping, from (3.3.2) and (3.3.3)

We have

$$\begin{aligned} & \| (g, \varphi)(h, \varphi)(f, \varphi)(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(h, \varphi)(f, \varphi)(f, \varphi)(\tilde{y}_\lambda) \| = \\ & \| (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{y}_\lambda) \| \leq \\ & \leq \mu \max \left\{ \frac{\| (f, \varphi)(h, \varphi)(f, \varphi)^2(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| \| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| + \| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| \| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \|}{\| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{y}_\lambda) \|} \right. \\ & \left. + \frac{\| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| \| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{y}_\lambda) \| + \| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| \| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \|}{\| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{y}_\lambda) \|} \right\} \\ & + \rho \{ \| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| + \| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{y}_\lambda) \| \} \\ & \left. + \omega \| (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(h, \varphi)^2(f, \varphi)(\tilde{y}_\lambda) \| \right\} \\ & \leq \mu \max \left\{ \frac{\| (f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| \| (f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| + \| (f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| \| (f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \|}{\| (f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda) \|} \right. \\ & \left. + \frac{\| (f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| \| (f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{y}_\lambda) \| + \| (f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| \| (f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \|}{\| (f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda) \|} \right\} \\ & + \rho \{ \| (f, \varphi)(\tilde{x}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda) \| + \| (f, \varphi)(\tilde{y}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(f, \varphi)(\tilde{y}_\lambda) \| \} \\ & + \omega \| (f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda) \| \} \end{aligned}$$

Now if we put  $(f, \varphi)(\tilde{x}_\lambda) = \tilde{z}_\lambda$  and  $(f, \varphi)(\tilde{y}_\lambda) = \tilde{v}_\lambda$ ,

$$\begin{aligned} & \| (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{z}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{v}_\lambda) \| \\ & \leq \mu \max \left\{ \frac{\| (\tilde{z}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{z}_\lambda) \| \| (\tilde{v}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{v}_\lambda) \| + \| (\tilde{z}_\lambda) - (g, \varphi)(h, \varphi)(f, \varphi)(\tilde{v}_\lambda) \| \| (\tilde{v}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{z}_\lambda) \|}{\| (f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda) \|} \right. \\ & \left. + \frac{\| (\tilde{z}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{z}_\lambda) \| \| (\tilde{z}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{v}_\lambda) \| + \| (\tilde{v}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{v}_\lambda) \| \| (\tilde{v}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{z}_\lambda) \|}{\| (f, \varphi)(\tilde{x}_\lambda) - (f, \varphi)(\tilde{y}_\lambda) \|} \right\} \\ & + \rho \{ \| (\tilde{z}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{z}_\lambda) \| + \| (\tilde{v}_\lambda) - (g, \varphi)(f, \varphi)(h, \varphi)(\tilde{v}_\lambda) \| \} \end{aligned}$$

$$+\omega\|(\tilde{z}_\lambda) - (\tilde{v}_\lambda)\|$$
  
 We have  $[(g, \varphi)(f, \varphi)(h, \varphi)]^2 = I$  and  $\mu + \omega + \rho \leq 2$ . We infer that  $(g, \varphi)(f, \varphi)(h, \varphi)$  has at least one soft point, say  $\tilde{x}_\lambda^0$  in here exist at least one soft point in  $\tilde{K}$

such that  
 (3.3.4)

$$(g, \varphi)(f, \varphi)(h, \varphi)(\tilde{x}_\lambda^0) = (\tilde{x}_\lambda^0) \quad \text{and} \\
 (f, \varphi)(h, \varphi)(f, \varphi)(g, \varphi)(h, \varphi)(\tilde{x}_\lambda^0) = (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^0) \quad (3.3.5)$$

$$(g, \varphi)(\tilde{x}_\lambda^0) = (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^0)$$

also

$$(h, \varphi)[(f, \varphi)(g, \varphi)(h, \varphi)(\tilde{x}_\lambda^0)] = (h, \varphi)(\tilde{x}_\lambda^0) \quad \text{and there for} \quad (3.3.6)$$

$$(h, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) = (h, \varphi)(\tilde{x}_\lambda^0)$$

Now by using (3.3.1), (3.3.2), (3.3.3) and (3.3.4), (3.3.5), (3.3.6) we have

$$\begin{aligned} & \| (h, \varphi)(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \| = \| (g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)^2(\tilde{x}_\lambda^0) \| = \\ & \| (g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) \| \\ & \leq \mu \max \left\{ \| (f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) \| \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) \| + \right. \\ & \frac{\| (f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) \| \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) \|}{\| (f, \varphi)(\tilde{x}_\lambda^0) - (f, \varphi)(\tilde{x}_\lambda^0) \|}, \\ & \left. \frac{\| (f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) \| \| (f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) \| +}{\| (f, \varphi)(\tilde{x}_\lambda^0) - (f, \varphi)(\tilde{x}_\lambda^0) \|} \right\} \\ & + \rho \left\{ \| (f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) \| + \right. \\ & \left. \| (f, \varphi)(h, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(f, \varphi)(\tilde{x}_\lambda^0) \| \right\} \\ & + \omega \| (f, \varphi)(\tilde{x}_\lambda^0) - (f, \varphi)(\tilde{x}_\lambda^0) \| \\ & \leq \mu \max \left\{ \frac{\| (h, \varphi)(\tilde{x}_\lambda^0) - (h, \varphi)(\tilde{x}_\lambda^0) \| \| (\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \| + \| (h, \varphi)(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \| \| (\tilde{x}_\lambda^0) - (h, \varphi)(\tilde{x}_\lambda^0) \|}{\| (h, \varphi)(\tilde{x}_\lambda^0) - (h, \varphi)(\tilde{x}_\lambda^0) \|} \right. \\ & \left. \frac{\| (h, \varphi)(\tilde{x}_\lambda^0) - (h, \varphi)(\tilde{x}_\lambda^0) \| \| (h, \varphi)(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \| + \| (\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \| \| (\tilde{x}_\lambda^0) - (h, \varphi)(\tilde{x}_\lambda^0) \|}{\| (h, \varphi)(\tilde{x}_\lambda^0) - (h, \varphi)(\tilde{x}_\lambda^0) \|} \right\} \\ & + \rho \left\{ \| (h, \varphi)(\tilde{x}_\lambda^0) - (h, \varphi)(\tilde{x}_\lambda^0) \| + \| (\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \| \right\} + \omega \| (h, \varphi)(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \| \end{aligned}$$

$$\leq \mu \max \{ \| (\tilde{x}_\lambda^0) - (h, \varphi)(\tilde{x}_\lambda^0) \|, 0 \} + \rho(0) + \omega \| (h, \varphi)(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \|$$

$$\leq (\mu + \omega) \| (h, \varphi)(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \|$$

Since  $\mu + \omega + \rho \leq 1$ , it follows that

$$(h, \varphi)(\tilde{x}_\lambda^0) = (\tilde{x}_\lambda^0)$$

i.e.  $\tilde{x}_\lambda^0$  is the soft point of  $(h, \varphi)$ . Thus we have from (3.3.5)

$$(f, \varphi)(\tilde{x}_\lambda^0) = (g, \varphi)(\tilde{x}_\lambda^0)$$

Again

$$\begin{aligned} & \| (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (\tilde{x}_\lambda^0) \| = \| (g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)^2(\tilde{x}_\lambda^0) \| \\ & = \| (g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) \| \\ & \leq \mu \max \left\{ \| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0) \| \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) \| + \right. \\ & \left. \frac{\| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) \| \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) - (g, \varphi)(\tilde{x}_\lambda^0) \|}{\| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^0) - (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^0) \|} \right\} \end{aligned}$$

$$\frac{\| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(\tilde{x}_\lambda^o) \| \| (f, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \| + \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \|}{\| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^o) - (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \|} \} \\
+ \rho \{ \| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(\tilde{x}_\lambda^o) \| + \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \| \} \\
+ \omega \| (f, \varphi)(h, \varphi)(\tilde{x}_\lambda^o) - (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \| \\
\leq \mu \max \left\{ \frac{\| (g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(\tilde{x}_\lambda^o) \| \| (\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| + \| (g, \varphi)(\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| \| (\tilde{x}_\lambda^o) - (g, \varphi)(\tilde{x}_\lambda^o) \|}{\| (g, \varphi)(\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \|}, \right. \\
\left. \frac{\| (g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(\tilde{x}_\lambda^o) \| \| (g, \varphi)(\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| + \| (\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| \| (\tilde{x}_\lambda^o) - (g, \varphi)(\tilde{x}_\lambda^o) \|}{\| (g, \varphi)(\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \|} \right\} \\
+ \rho \{ \| (g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(\tilde{x}_\lambda^o) \| + \| (\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| \} + \omega \| (g, \varphi)(\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| \\
\leq \mu \max \{ \| (\tilde{x}_\lambda^o) - (g, \varphi)(\tilde{x}_\lambda^o) \|, 0 \} + \rho(0) + \omega \| (g, \varphi)(\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| \\
\leq (\mu + \omega) \| (g, \varphi)(\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \|$$

Which is contradiction.

Since  $\mu + \omega + \rho < 1$ . Hence it follows that

$$\begin{aligned} (g, \varphi)(\tilde{x}_\lambda^o) &= (\tilde{x}_\lambda^o) \\ (g, \varphi)(\tilde{x}_\lambda^o) &= (f, \varphi)(\tilde{x}_\lambda^o) \end{aligned}$$

There for  $(g, \varphi)(\tilde{x}_\lambda^o) = (f, \varphi)(\tilde{x}_\lambda^o) = (h, \varphi)(\tilde{x}_\lambda^o) = (\tilde{x}_\lambda^o)$

i.e.  $\tilde{x}_\lambda^o$  is the common soft point of  $(g, \varphi)$ ,  $(f, \varphi)$  and  $(h, \varphi)$ .

Now to confirm the uniqueness of  $\tilde{x}_\lambda^o$ . Let  $\tilde{y}_\lambda^o$  be another common soft point of  $(g, \varphi)$ ,  $(f, \varphi)$  and  $(h, \varphi)$ .

By (3.3.1), (3.3.2), (3.3.3) and (3.3.4), (3.3.5), (3.3.6)

$$\| (\tilde{x}_\lambda^o) - (\tilde{y}_\lambda^o) \| = \| (g, \varphi)^2(\tilde{x}_\lambda^o) - (g, \varphi)^2(\tilde{x}_\lambda^o) \| = \| (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \|$$

$\leq \mu \max$

$$\left\{ \frac{\| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \| \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) \| + \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) \| \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \|}{\| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (f, \varphi)(h, \varphi)(f, \varphi)(\tilde{y}_\lambda^o) \|} \right\}$$

$$\left\{ \frac{\| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \| \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) \| + \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) \| \| (f, \varphi)(f, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \|}{\| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (f, \varphi)(h, \varphi)(f, \varphi)(\tilde{y}_\lambda^o) \|} \right\}$$

+

$$\left\{ \begin{aligned} & \frac{\rho}{\| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) \| + \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) - (g, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) \|} \\ & \left\{ \right. \end{aligned} \right.$$

$$\left. \begin{aligned} & + \omega \| (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{x}_\lambda^o) - (f, \varphi)(h, \varphi)(g, \varphi)(\tilde{y}_\lambda^o) \| \\ & \leq \mu \max \left\{ \frac{\| (\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| \| (\tilde{y}_\lambda^o) - (\tilde{y}_\lambda^o) \| + \| (\tilde{x}_\lambda^o) - (\tilde{y}_\lambda^o) \| \| (\tilde{y}_\lambda^o) - (\tilde{x}_\lambda^o) \|}{\| (\tilde{x}_\lambda^o) - (\tilde{y}_\lambda^o) \|}, \right. \\ & \left. \frac{\| (\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| \| (\tilde{x}_\lambda^o) - (\tilde{y}_\lambda^o) \| + \| (\tilde{y}_\lambda^o) - (\tilde{y}_\lambda^o) \| \| (\tilde{y}_\lambda^o) - (\tilde{x}_\lambda^o) \|}{\| (\tilde{x}_\lambda^o) - (\tilde{y}_\lambda^o) \|} \right\} \end{aligned} \right\}$$

$$+ \rho \{ \| (\tilde{x}_\lambda^o) - (\tilde{x}_\lambda^o) \| + \| (\tilde{y}_\lambda^o) - (\tilde{y}_\lambda^o) \| \} + \omega \| (\tilde{x}_\lambda^o) - (\tilde{y}_\lambda^o) \| \\ \| (\tilde{x}_\lambda^o) - (\tilde{y}_\lambda^o) \| \leq (\mu + \omega) \| (\tilde{x}_\lambda^o) - (\tilde{y}_\lambda^o) \|$$

Which is contradiction.

Since  $\mu + \omega + \rho < 1$ . Hence it follows that

$$(\tilde{x}_\lambda^\rho) = (\tilde{y}_\lambda^\rho)$$

Proving the uniqueness of  $\tilde{x}_\lambda^\rho$ .

This complete of the proof of the theorem.

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