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Fixed Point Theorems in Random Fuzzy Metric Space Through Rational Expression

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Abstract
In the present paper we will find some fixed point theorems in random fuzzy metric space, random fuzzy 2-metric space and random fuzzy 3-metric space through rational expression.Also we will find the results for integral type mappings.

2.1 Introduction

In 1965, the concept of fuzzy set was introduced by Zadeh [39]. After him many authors have developed the theory of fuzzy sets and applications. Especially, Deng [9], Erceg [11], Kaleva and Seikkala [26]. Kramosil and Michalek [28] have introduced the concept of fuzzy metric spaces by generalizing the definition of probabilistic metric space. Many authors have also studied the fixed point theory in these fuzzy metric spaces are [1], [7], [13], [19], [21], [24], [25], [32] and for fuzzy mappings [2], [3], [4], [5], [22], [31].

In 1994, George and Veeramani [18] modified the definition of fuzzy metric spaces given by Kramosil and Michalek [28] in order to obtain Hausdroff topology in such spaces. Gregori and Sapena [20] in 2002 extended Banach fixed point theorem to fuzzy contraction mappings on complete fuzzy metric space in the sense of George and Veeramani [18]. It is remarkable that Sharma, Sharma and Iseki [34] studied for the first time contraction type mappings in 2-metric space. Wenzhi [38] and many others initiated the study of Probabilistic 2-metric spaces. As we know that 2-metric space is a real valued function of a point triples on a set X, whose abstract properties were suggested by the area of function in Euclidean spaces.

Now it is natural to expect 3-metric space which is suggested by the volume function. The method of introducing this is naturally different from 2-metric space theory from algebraic topology.

The concept of Fuzzy-random-variable was introduced as an analogous notion to random variable in order to extend statistical analysis to situations when the outcomes of some random experiment are fuzzy sets. But in contrary to the classical statistical methods no unique definition has been established before the work of Volker [37]. He presented set theoretical concept of fuzzy-random-variables using the method of general topology and drawing on results from topological measure theory and the theory of analytic spaces. No results in fixed point are introduced in random fuzzy spaces. In [17] paper authors Gupta, Dhagat, Shrivastava introduced the fuzzy random spaces and proved common fixed point theorem.

In the present paper we will find some fixed point theorems in random fuzzy metric space, random fuzzy 2-metric space and random fuzzy 3-metric space through rational expression.Also we will find the results for integral type mappings.

To start the main result we need some basic definitions.

2.2 Preliminaries:

2.2.1 Definitions

Definition2.2.1.1: (Kramosil and Michalek 1975)

A binary operation $*$: $[0,1]\times[0,1] \rightarrow [0,1]$ is a t-norm if it satisfies the following conditions :

(i) $*(1,a) = a$, $*(0,0) = 0$
(i) $*(a,b) = *(b,a)$
(ii) $*(c,d) \geq *(a,b)$ whenever $c \geq a$ and $d \geq b$
(iii) $*(*(a,b),c) = *(a, *(b,c))$ where $a,b, c,d \in [0,1]$

Definition 2.2.1.2: (Kramosil and Michalek 1975)

The 3-tuple $(X,M, *)$ is said to be a fuzzy metric space if X is an arbitrary set $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions:

(i) $M(x,y,0) = 0$

(ii) M(x,y,t) = 1 for all t > 0 iff x = y,
(iii) M(x,y,t) = M(y,x,t) ,
(iv) M(x,y,t) ∗ M(y,z,s) ≤ M(x,z,t + s) ,
(v) M(x,y, .) : [0,∞[→[0,1] is left-continuous,

Where x, y, z ∈ X and t, s > 0.
In order to introduce a Hausdroff topology on the fuzzy metric space, in (Kramosil and Michalek 1975) the following definition was introduced.

Definition 2.2.1.3: (George and Veermani 1994)
The 3-tuple (X, M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on X^2 ×]0,∞[satisfying the following conditions :

(i) M(x,y,t) > 0
(ii) M(x,y,t) = 1 iff x = y,
(iii) M(x,y,t) = M(y,x,t) ,
(iv) M(x,y,t) ∗ M(y,z,s) ≤ M(x,z,t + s) ,
(v) M(x,y, .) :]0,∞[→[0,1] is continuous,

Where x, y, z ∈ X and t, s > 0.

Definition 2.2.1.4: (George and Veermani 1994)
In a metric space (X, d) the 3-tuple (X, Md,*) where Md(x, y, t) = t / (t + d(x, y)) and a*b = ab is a fuzzy metric space. This Md is called the standard fuzzy metric space induced by d.

Definition 2.2.1.5: (Gregori and Sepene 2002)
Let (X, M,∗) be a fuzzy metric space. A mapping f: X→ X is fuzzy contractive if there exists 0 < k < 1 such that

$$\frac{1}{M(f(x),f(y),t)} - 1 \le k\left(\frac{1}{M(x,y,t)} - 1\right)$$

For each x,y ϵ X and t > 0 .

Definition 2.2.1.6: (Gregori and Sepene 2002)
Let **(X,** M, **∗)** be a fuzzy metric space . We will say that the sequence $\{x_n\}$ in X is fuzzy contractive if there exists k ϵ (0, 1) such that

$$\frac{1}{M(x_{n+1},x_{n+2},t)} - 1 \le k\left(\frac{1}{M(x_n,x_{n+1},t)} - 1\right) \quad \text{for all } t > 0 , n \in N.$$

We recall that a sequence $\{x_n\}$ in a metric space (X, d) is said to be contractive if there exist 0 < k < 1 such that d $(x_{n+1}, xn_{+2}) \le kd(x_n, x_{n+1})$ for all n ϵ N.

Definition 2.2.1.7: (Kumar and Chugh 2001)
Let (X, τ) be a topological space. Let f and g be mappings from a topological space (X, τ) into itself. The mappings f and g are said to be compatible if the following conditions are satisfied**:**

(i) fx = gx , x ∈ X Implies fgx = gfx ,
(ii) The continuity of f at a point x in X implies lim gfx_n=fx whenever $\{x_n\}$ is a sequence in X such that lim gx_n=lim fx_n=fx for some x in X.

Definition 2.2.1.8 : A binary operation ∗ : [0,1] x [0,1] x [0,1]→[0,1] is called a continuous t-norm if ([0,1], ∗) is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 \le a_2 * b_2 * c_2$ whenever $a_1 \le a_2$, $b_1 \le b_2$, $c_1 \le c_2$ for all a_1, a_2, b_1, b_2 and c_1, c_2 are in [0,1].

Definition 2.2.1.9 : The 3-tuple (X,M,∗) is called a fuzzy 2-metric space if X is an arbitrary set, ∗ is a continuous t-norm and M is a fuzzy set in X^3 x [0,∞) satisfying the following conditions for all x, y, z, u ∈ X and t_1, t_2, t_3 > 0.

(FM'-1) M(x, y, z, 0) = 0,
(FM'-2) M(x, y, z, t) = 1, t > 0 and when at least two of the three points are equal,
(FM'-3) M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t),
(Symmetry about three variables)
(FM'-4) $M(x, y, z, t_1+t_2+t_3) \ge M(x, y, u, t_1)*M(x, u, z, t_2)* M(u, y, z, t_3)$
(This corresponds to tetrahedron inequality in 2-metric space)

The function value M(x, y, z, t) may be interpreted as the probability that the area of triangle is less than t.
(FM'-5) M(x, y, z, .): [0, 1)→[0,1] is left continuous.

Definition 2.2.1.10: Let (X, M,∗) is a fuzzy 2-metric space:

(1) A sequence $\{x_n\}$ in fuzzy 2-metric space X is said to be convergent to a point x ∈ X, if

$$\lim_{n\to\infty} M(x_n, x, a, t) = 1$$

for all a∈ X and t > 0.

(2) A sequence $\{x_n\}$ in fuzzy 2-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, a, t) = 1$$

for all a∈ X and t > 0, p > 0.

(3) A fuzzy 2-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.2.1.11: A function M is continuous in fuzzy 2-metric space iff whenever $x_n \to x$, $y_n \to y$, then

$$\lim_{n\to\infty} M(x_n, y_n, a, t) = M(x, y, a, t)$$

for all a∈ X and t > 0.

Definition 2.2.1.12: Two mappings A and S on fuzzy 2-metric space X are weakly commuting iff

$$M(ASu, SAu, a, t) \geq M(Au, Su, a, t)$$

for all u, a ∈ X and t > 0.

Definition 2.2.1.13: A binary operation $\ast : [0,1]^4 \to [0,1]$ is called a continuous t-norm if ([0,1], ∗) is an abelian topological monoid with unit 1 such that $a_1 \ast b_1 \ast c_1 \ast d_1 \leq a_2 \ast b_2 \ast c_2 \ast d_2$ whenever $a_1 \leq a_2$, $b_1 \leq b_2$, $c_1 \leq c_2$ and $d_1 \leq d_2$ for all $a_1, a_2, b_1, b_2, c_1, c_2$ and d_1, d_2 are in [0,1].

Definition 2.2.1.14 : The 3-tuple (X, M, ∗) is called a fuzzy 3-metric space if X is an arbitrary set, ∗ is a continuous t-norm and M is a fuzzy set in $X^4 \times [0,\infty)$ satisfying the following conditions : for all x, y, z, w, u ∈X and $t_1, t_2, t_3, t_4 > 0$.

(FM''-1) M(x, y, z, w, 0) = 0,

(FM''-2) M(x, y, z, w, t) = 1 for all t > 0,
(only when the three simplex ⟨ x, y, z, w ⟩ degenerate)

(FM''-3) M(x, y, z, w, t) =M(x, w, z, y, t) = M(y, z, w, x, t) = M (z, w, x, y, t) = …

(FM''-4) $M(x, y, z, w, t_1 + t_2 + t_3 + t_4) \geq M(x, y, z, u, t_1) \ast M(x, y, u, w, t_2) \ast M(x, u, z, w, t_3) \ast M(u, y, z, w, t_4)$

(FM''-5) M(x, y, z, w, .) : [0,1)→[0,1] is left continuous.

Definition 2.2.1.15: Let (X, M, ∗) be a fuzzy 3-metric space:

(1) A sequence $\{x_n\}$ in fuzzy 3-metric space x is said to be convergent to a point x ∈ X, if

$$\lim_{n\to\infty} M(x_n, x, a, b, t) = 1$$

for all a, b ∈ X and t > 0.

(2) A sequence $\{x_n\}$ in fuzzy 3-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, a, b, t) = 1$$

for all a, b ∈ X and t > 0, p > 0.

(3) A fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.2.1.16: A function M is continuous in fuzzy 3-metric space iff whenever $x_n \to x$, $y_n \to y$

$$\lim_{n\to\infty} M(x_n, y_n, a, b, t) = M(x, y, a, b, t)$$

for all a, b ∈ X and t > 0.

Definition 2.2.1.17: Two mappings A and S on fuzzy 3-metric space X are weakly commuting iff

$$M(ASu, SAu, a, b, t) \geq M(Au, Su, a, b, t)$$

for all u, a, b ∈ X and t > 0.

Definition2.2.1.18: Throughout this chapter, (Ω,Σ) denotes a measurable space. $\xi:\Omega\to X$ is a measurable selector. X is any non empty set. $\star$ is continuous t-norm, **M** is a fuzzy set in $X^2\times[0,\infty)$

A binary operation $*$:[0,1]x[0,1]→[0,1] is called a continuous t-norm if ([0,1],$*$) is an abelian Topological monodies with unit 1 such that $a * b \geq c * d$ whenever

$a \geq c$ and $b \geq d$, For all a, b, c, d, $\in$ [0, 1]

Example of t-norm are $a * b = a\, b$ and $a * b = \min\{a, b\}$

Definition2.2.1.18 (a): The 3-tuple (X, M, Ω $*$) is called a **Random fuzzy metric space**, if X is an arbitrary set,* is a continuous t-norm and M is a fuzzy set in X^2 x $[0,\infty)$ satisfying the following conditions: for all

ξ x, ξ y, ξ z $\in$ X and s, t > 0,

$$(RFM-1): M(\xi x,\xi y,0)=0$$

$$(RFM-2): M(\xi x,\xi y,t)=1,\forall t\succ 0,\Leftrightarrow x=y$$

$$(RFM-3): M(\xi x,\xi y,t)=M(\xi y,\xi x,t)$$

$$(RFM-4): M(\xi x,\xi z,t+s)\geq M(\xi x,\xi y,t)*M(\xi z,\xi y,s)$$

$$(RFM-5): M(\xi x,\xi y,\xi a):[0,1)\to[0,1]\, is\ left\ continuous$$

In what follows, (X, M, Ω ,*) will denote a random fuzzy metric space. Note that M (ξ x, ξ y, t) can be thought of as the degree of nearness between ξ x and ξ y with respect to t. We identify ξ x = ξ y with M (ξ x, ξ y, t) = 1 for all t > 0 and M (ξ x, ξ y, t) = 0 with ∞.In the following example, we know that every metric induces a fuzzy metric.

Example Let (X, d) be a metric space.

Define $a * b = a\, b$, or ab =min {a, b}) and for all x, y, $\in$ X and t > 0,

$$M(\xi x,\xi y,t)=\frac{t}{t+d(\xi x,\xi y)}$$

Then (X, M, Ω, *) is a fuzzy metric space. We call this random fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition2.2.1.18 (b): Let (X, M, Ω, *) is a random fuzzy metric space.

(i)A sequence {ξx_n} in X is said to be convergent to a point ξ x $\in$ X,

$$\lim_{n\to\infty} M(\xi x_n,\xi x,t)=1$$

(ii) A sequence {ξx_n} in X is called a Cauchy sequence if

$$\lim_{n\to\infty} M(\xi x_{n+p},\xi x_n,t)=1,\forall t\succ 0\, and\ p\succ 0$$

(iii) A random fuzzy metric space in which every Cauchy sequence is convergent is said to be Complete.

Let (X.M,*) is a fuzzy metric space with the following condition.

(RFM-6) $$\lim_{t\to\infty} M(\xi x,\xi y,t)=1,\forall \xi x,\xi y\ \varepsilon\ X$$

. **Definition2.2.1.18 (c)**: A function M is continuous in fuzzy metric space iff whenever

$$\xi x_n \to \xi x, \xi y_n \to \xi y \Rightarrow \lim_{n\to\infty} M(\xi x_n, \xi y_n, t) \to M(\xi x, \xi y, t)$$

Definition2.2.1.18 (d): Two mappings A and S on fuzzy metric space X are weakly commuting iff

M (AS ξ u, SA ξ u, t) $\geq$ M (A ξ u, S ξ u, t)

Some Basic Results 2.2.1.18 (e):

Lemma (i) [Motivated by 19] for all ξ x, ξ y, $\in$ X, M(ξx, ξy) is non -decreasing.

Lemma (ii) Let {ξy_n} be a sequence in a random fuzzy metric space (X, M, Ω,*) with the condition

(RFM -6) If there exists a number q $\in$ (0,1) such that

$$M(\xi y_{n+2}, \xi y_{n+1}, qt) \geq M(\xi y_{n+1}, \xi y_n, t) \;, \forall t \succ 0 \; and \; n = 1, 2, 3......,$$
$$then \; \{\xi y_n\} \; is \; a \; cauchy \; sequence \; in \; X.$$

Lemma (iii) [Motivated by 32] If, for all ξx, ξy $\in$ X, t > 0 and for a number q $\in$ (0,1),

$$M(\xi x, \xi y, qt) \geq M(\xi x, \xi y, t), then \; \xi x = \xi y$$

Lemmas 1, 2, 3 of **2.2.1.18 (e)**:) hold for random fuzzy 2-metric spaces and random fuzzy 3-metric spaces also.

Definition2.2.1.18 (f): A binary operation *: [0, 1] x [0,1] x [0,1] $\to$ [0,1] is called a continuous t-norm if ([0,1],*) is an abelian topological monodies with unit 1 such that $a_1 * b_1 * c_1 \geq a_2 * b_2 * c_2$ whenever $a_1 \geq a_2$, $b_1 \geq b_2$, $c_1 \geq c_2$ for all a_1, a_2, b_1, b_2 and c_1, c_2 are in [0,1].

Definition2.2.1.18 (g): The 3-tuple (X, M, Ω,*) is called a random fuzzy 2-metric space if X is an arbitrary set, * is continuous t-norm and M is fuzzy set in X^3 x [0,∞) satisfying the followings

$$(RFM^{'}-1): M\left(\xi x, \xi y, \xi z, 0\right) = 0$$
$$(RFM^{'}-2): M\left(\xi x, \xi y, \xi z, t\right) = 1, \forall t \succ 0, \Leftrightarrow x = y$$
$$(RFM^{'}-3): M\left(\xi x, \xi y, t\right) = M(\xi x, \xi z, \xi y, t) = M(\xi y, \xi z, \xi x, t), symmetry \; about \; three \; \mathrm{var} \; riable$$
$$(RFM^{'}-4): M\left(\xi x, \xi y, \xi z, t_{1,} \, t_{2,}, t_{3,}\right) \geq M(\xi x, \xi y, \xi u, t_{1,}) * M(\xi x, \xi u, \xi z, t_{2,}) * M(\xi u, \xi y, \xi z, t_{3,})$$
$$(RFM^{'}-5): M(\xi x, \xi y, \xi z): [0,1) \to [0,1] \; is \; left \; continuous, \forall \xi x, \xi y, \xi z, \xi u \; \varepsilon \; X, t_1, t_2, t_3 \succ 0$$

.

Definition2.2.1.18 (h): Let (X, M, Ω,*) be a random fuzzy 2-metric space. A sequence { ξ x_n} in fuzzy 2-metric space X is said to be convergent to a point ξ x $\in$ X,

$$\lim_{n\to\infty} M(\xi x_n, \xi x, \xi a, t) = 1, \; for \; all \; \xi a \; \varepsilon \; X \; and \; t \succ 0$$

(2) A sequence { ξ x_n} in random fuzzy 2-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M(\xi x_{n+p}, \xi x_n, \xi a, t) = 1, \; for \; all \; \xi a \; \varepsilon \; X \; and \; t, p \succ 0$$

(3) A random fuzzy 2-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition2.2.1.18 (i): A function M is continuous in random fuzzy 2-metric space, iff whenever

For all ξ a $\in$ X and t > 0.

$$\xi x_n \to \xi x, \xi y_n \to \xi y, then \lim_{n\to\infty} M(\xi x_n, \xi y_n, \xi a, t) = M(\xi x, \xi y, \xi a, t), \forall \xi a \,\varepsilon\, X \; and \; t \succ 0$$

Definition2.2.1.18 (j): Two mappings A and S on random fuzzy 2-metri space X are weakly commuting iff

$$M(AS\xi u, SA\xi u, \xi a, t) \geq M(A\xi u, S\xi u, \xi a, t), \forall \xi u, \xi a \,\varepsilon\, X and \; t \succ 0$$

Definition2.2.1.18 (k): A binary operation *: $[0, 1]^4 \to [0, 1]$ is called a continuous t-norm if ([0, 1], *) is an abelian topological monoid with unit 1 such that

$a_1 * b_1 * c_1 * d_1 \geq a_2 * b_2 * c_2 * d_2$ Whenever $a_1 \geq a_2$, $b_1 \geq b_2$, $c_1 \geq c_2$ and $d_1 \geq d_2$ for all a_1, a_2, b_1, b_2, c_1, c_2 and d_1, d_2 are in [0,1].

Definition2.2.1.18 (l): The 3-tuple (X, M, Ω, *) is called a fuzzy 3-metric space if X is an arbitrary set, * is a continuous t-norm monoid and M is a fuzzy set in X^4 x [0, ∞] satisfying the following conductions:

$$(RFM'' - 1) : M\left(\xi x, \xi y, \xi z, \xi w, 0\right) = 0$$

$$(RFM'' - 2) : M\left(\xi x, \xi y, \xi z, \xi w, t\right) = 1, \forall t \succ 0,$$

$$Only \; when \; the \; three simplex \left\langle x, y, z, w \right\rangle \deg enerate$$

$$(RFM'' - 3) : M\left(\xi x, \xi y, \xi z, \xi w, t\right) = M(\xi x, \xi w, \xi z, \xi y, t) = M(\xi z, \xi w, \xi x, \xi y, t) = ----$$

$$(RFM'' - 4) : M\left(\xi x, \xi y, \xi z, \xi w, t + t_2 + t_3\right) \geq M(\xi x, \xi y, \xi z, \xi u, t_{1,}) *$$
$$M(\xi x, \xi y, \xi u, \xi w, t_{2,}) * M(\xi x, \xi u, \xi z, \xi w, t_{3,}) * M(\xi u, \xi y, \xi z, \xi w, t_4)$$

$$(RFM'' - 5) : M(\xi x, \xi y, \xi z, \xi w) : \left[0,1\right) \to \left[0,1\right] is \; left \; continuous,$$
$$\forall \xi x, \xi y, \xi z, \xi u, \xi w \;\varepsilon\; X, t_1, t_2, t_3, t_4 \succ 0$$

Definition2.2.1.18 (m): Let (X, M, Ω,*) be a Random fuzzy 3-metric space:

(1)A sequence {ξXn} in fuzzy 3-metric space X is said to be convergent to a point ξx ∈ X, if

$$\lim_{n\to\infty} M(\xi x_n, \xi x, \xi a, \xi b, t) = 1, for \; all \; \xi a, \xi b \,\varepsilon\, X \; and \; t \succ 0$$

(2)A sequence { ξ x_n} in random fuzzy 3-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M(\xi x_{n+p}, \xi x_n, \xi a, \xi b, t) = 1, for \; all \; \xi a, \xi b \,\varepsilon\, X \; and \; t, p \succ 0$$

(3)A random fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition2.2.1.18 (n): A function M is continuous in random fuzzy 3-metric space if

$$\xi x_n \to \xi x, \xi y_n \to \xi y, then \lim_{n\to\infty} M(\xi x_n, \xi y_n, \xi a, \xi b, t) = M(\xi x, \xi y, \xi a, t), \forall \xi a, \xi b \;\varepsilon\; X \; and \; t \succ 0$$

Definition2.2.1.18 (o): Two mappings A and S on random fuzzy 3-metric space X are weakly commuting iff,

$$M(AS\xi u, SA\xi u, \xi a, \xi b, t) \geq M(A\xi u, S\xi u, \xi a, \xi b, t) \; \forall u, a, b \,\varepsilon\, X and \; t \succ 0$$

2.2.2 Prepositions.

Preposition 2.2.2.1 (Gregori and Sepene 2002)

Let (X, d) be a metric space. The mapping f: X → X is a contractive (a contraction) on the metric space (X, d) with contractive constant k iff f is fuzzy contractive, with contractive constant k, on the standard fuzzy metric space (X, Md, ∗), induced by d.

Preposition 2.2.2.2 (Gregori and Sepene 2002)
Let (X, M, $*$) be a complete fuzzy metric space in which fuzzy contractive sequences are Cauchy. Let T: X → X be a fuzzy contractive mapping being k the contractive constant. Then T has a unique fixed point.
Preposition 2.2.2.3 (Gregori and Sepene 2002)
Let (X, Md, $*$) be the standard fuzzy metric space induced by the metric d on X. The sequence $\{x_n\}$ in X is contractive in (X, d) iff $\{x_n\}$ is fuzzy contractive in (X, Md, $*$).
Preposition 2.2.2.1 and 2.2.2.3 imply that Preposition 2.2.2.2 is a generalization of Banach fixed point theorem to fuzzy metric spaces as defined by George and Veermani.

It is to be noted that all the prepositions are true for (RFM)

Now, we state and prove our main theorem as follows,

2.3 Main Results
Theorem 2.3.1: Let $(X, \Omega, M, \star)$ be a complete Random fuzzy metric space in which fuzzy contractive sequences are Cauchy and T , R and S be mappings from $(X, \Omega, M, \star)$ into itself $\xi : \Omega \to X$ is a measurable selector satisfying the following conditions :

$$T(X) \subseteq R(X) \text{ and } T(X) \subseteq S(X) \tag{2.3.1.1}$$

$$\frac{1}{M(T(\xi x),T(\xi y),t)} - 1 \leq k\left(\frac{1}{Q(\xi x,\xi y,t)} - 1\right) \tag{2.3.1.2}$$

with $0 < k < 1$ and

$$Q(\xi x, \xi y, t) = \min\left\{\begin{matrix} M(R\xi x,S\xi y,t), M(S\xi x,R\xi y,t), M(R\xi x,T\xi x,t), \\ \frac{M(S\xi x,R\xi y,t)M(R\xi x,T\xi x,t)}{M(R\xi x,S\xi y,t)}, \frac{M(S\xi x,T\xi x,t)M(S\xi y,T\xi y,t)}{M(R\xi y,\xi Ty,t)} \end{matrix}\right\}$$

The pairs T, S and T, R are compatible. R, T and S are w-continuous. (2.3.1.3)
Then R, T and S have a unique common fixed point. (2.3.1.4)
Proof: Let $\xi x_0 \in X$ be an arbitrary point .Since $T(X) \subseteq R(X)$ and $T(X) \subseteq S(X)$, we can construct a sequence $\{x_n\}$ in X such that

$$T\xi x_{n-1} = R\xi x_n = S\xi x_n \tag{2.3.1.5}$$

Now,

$$Q(\xi x_n, \xi x_{n+1}, t) = \min\left\{\begin{matrix} M(R\xi x_n, S\xi x_{n+1}, t), M(S\xi x_n, R\xi x_{n+1}, t), M(R\xi x_n, T\xi x_n, t), \\ \frac{M(Sx\xi_n,R\xi x_{n+1},t)M(R\xi x_n,T\xi x_n,t)}{M(R\xi x_n,S\xi x_{n+1},t)}, \frac{M(S\xi x_n,T\xi x_n,t)M(S\xi x_{n+1},\xi Tx_{n+1},t)}{M(R\xi x_{n+1},T\xi x_{n+1},t)} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} M(T\xi x_{n-1}, T\xi x_n, t), M(T\xi x_{n-1}, T\xi x_n, t), M(T\xi x_{n+1}, T\xi x_n, t), \\ \frac{M(T\xi x_{n-1},T\xi x_n,t)M(T\xi x_{n+1},T\xi x_n,t)}{M(T\xi x_{n-1},T\xi x_n,t)}, \\ \frac{M(T\xi x_{n-1},T\xi x_n,t)M(T\xi x_n,T\xi x_{n+1},t)}{M(T\xi x_n,T\xi x_{n+1},t)} \end{matrix}\right\}$$

$$= \min\{M(T\xi x_{n-1}, T\xi x_n, t), M(T\xi x_n, T\xi x_{n+1}, t)\}$$

We now claim that $M(T\xi x_{n-1}, T\xi x_n, t) < M(T\xi x_n, T\xi x_{n+1}, t)$
Otherwise we claim that $M(T\xi x_{n-1}, T\xi x_n, t) \geq M(T\xi x_n, T\xi x_{n+1}, t)$
i.e $Q(\xi x_n, \xi x_{n+1}, t) = M(T\xi x_n, T\xi x_{n+1}, t)$ (2.3.1.6)

$$\therefore \quad \frac{1}{M(T\xi x_n,T\xi x_{n+1},t)} - 1 \leq k\left(\frac{1}{M(T\xi x_n,T\xi x_{n+1},t)} - 1\right) \qquad \text{[by 2.3.1.2]}$$

which is a contradiction.

$$\text{Hence,} \quad \frac{1}{M(T\xi x_n,T\xi x_{n+1},t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1},T\xi x_n,t)} - 1\right) \tag{2.3.1.7}$$

∴ $\{T\xi x_n\}$ is a fuzzy contractive sequence in $(X, \Omega, M, \star)$ So $\{T\xi x_n\}$ is a Cauchy sequence
As X is a complete fuzzy metric space, $\{T\xi x_{n-1}\}$ is convergent. So, $\{T\xi x_{n-1}\}$ converges to some point z in X.
∴ $\{T\xi x_{n-1}\}$, $\{R\xi x_n\}$, $\{S\xi x_n\}$ converges to z. By w-continuity of R, S and T, there exists a point ξu in ξX such that $\xi x_n \to \xi u$ as $n \to \infty$ and so $\lim R\xi x_n = \lim S\xi x_n = \lim T\xi x_{n-1} = z$ implies

$$R\xi u = S\xi u = T\xi u = \xi z \tag{2.3.1.8}$$

Also by compatibility of pairs T, S and T, R and Tu = Ru = Su = z implies

$$T\xi z = TR\xi u = RT\xi u = R\xi z \text{ and } T\xi z = TS\xi u = ST\xi u = S\xi z$$

Therefore, $T\xi z = R\xi z = S\xi z$ (2.3.1.9)

We now claim that $T\xi z = \xi z$.

If not $\frac{1}{M(T\xi x_n, T\xi x_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, t)} - 1\right)$

$$Q(\xi z, \xi u, t) = \min\left\{\begin{array}{l} M(R\xi z, S\xi u, t), M(S\xi z, R\xi u, t), M(R\xi z, T\xi z, t), \\ \frac{M(S\xi z, R\xi u, t) M(R\xi z, T\xi z, t)}{M(R\xi z, S\xi u, t)}, \frac{M(S\xi z, T\xi z, t) M(S\xi u, T\xi u, t)}{M(R\xi u, T\xi u, t)} \end{array}\right\}$$

$$= \min\left\{\begin{array}{l} M(T\xi z, \xi z, t), M(T\xi z, \xi z, t), M(T\xi z, T\xi z, t), \\ \frac{M(T\xi z, \xi z, t) M(T\xi z, T\xi z, t)}{M(T\xi z, \xi z, t)}, \frac{M(T\xi z, T\xi z, t) M(\xi z, \xi z, t)}{M(\xi z, \xi z, t)} \end{array}\right\}$$

$$= \min\{M(T\xi z, \xi z, t), M(T\xi z, \xi z, t), 1, 1, 1\}$$

$$= M(T\xi z, \xi z, t)$$

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, t)} - 1\right)$$

which is a contradiction.

Hence $T\xi z = \xi z$

So ξz is a common fixed point of R, T and S.

Now suppose $\xi v \neq \xi z$ be another fixed point of R, T and

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, t)} - 1\right)$$

$$Q(\xi v, \xi u, t) = \min\left\{\begin{array}{l} M(R\xi v, S\xi z, t), M(S\xi v, R\xi z, t), M(R\xi v, T\xi v, t), \\ \frac{M(S\xi v, R\xi z, t) M(R\xi v, T\xi v, t)}{M(R\xi v, S\xi z, t)}, \frac{M(S\xi v, T\xi v, t) M(S\xi z, T\xi z, t)}{M(R\xi z, T\xi z, t)} \end{array}\right\}$$

$$= \min\left\{\begin{array}{l} M(\xi v, \xi z, t), M(\xi v, \xi z, t), M(\xi v, \xi v, t), \\ \frac{M(\xi v, \xi z, t) M(\xi v, \xi v, t)}{M(v, z, t)}, \frac{M(\xi v \xi, v, t) M(\xi z, \xi z, t)}{M(z, z, t)} \end{array}\right\}$$

$$= \min\{M(\xi v, \xi z, t), M(\xi v, \xi z, t), 1, 1\}$$

$$= M(\xi v, \xi z, t)$$

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, t)} - 1\right)$$

which is a contradiction. Hence $\xi v = \xi z$.

Thus R, T and S have a unique fixed point.

Theorem 2.3.2: Let $(X, \Omega, M, \star)$ be a complete Random fuzzy 2-metric space (RF-2M) in which fuzzy contractive sequences are Cauchy and T , R and S be mappings from $(X, \Omega, M, \star)$ into itself $\xi : \Omega \to X$ is a measurable selector and $a(\xi) = a > 0$satisfying the following conditions :

$$T(X) \subseteq R(X) \text{ and } T(X) \subseteq S(X) \qquad (2.3.2.1)$$

$$\frac{1}{M(T(\xi x), T(\xi y), \xi a, t)} - 1 \le k\left(\frac{1}{Q(\xi x, \xi y, \xi a, t)} - 1\right) \qquad (2.3.2.2)$$

with $0 < k < 1$ and

$$Q(\xi x, \xi y, \xi a, t) = \min\left\{\begin{array}{l} M(R\xi x, S\xi y, \xi a, t), M(S\xi x, R\xi y, \xi a, t), M(R\xi x, T\xi x, \xi a, t), \\ \frac{M(S\xi x, R\xi y, \xi a, t) M(R\xi x, T\xi x, \xi a, t)}{M(R\xi x, S\xi y, \xi a, t)}, \frac{M(S\xi x, T\xi x, \xi a, t) M(S\xi y, T\xi y, \xi a, t)}{M(R\xi y, T\xi y, \xi a, t)} \end{array}\right\}$$

The pairs T, S and T, R are compatible. R, T and S are w-continuous. (2.3.2.3)

Then R, T and S have a unique common fixed point. (2.3.2.4)

Proof: Let $\xi x_0 \in X$ be an arbitrary point of X. Since $T(X) \subseteq R(X)$ and $T(X) \subseteq S(X)$, we can construct a sequence $\{\xi x_n\}$ in X such that

$$T\xi x_{n-1} = R\xi x_n = S\xi x_n \qquad (2.3.2.5)$$

Now,

$$Q(\xi x_n, \xi x_{n+1}, \xi a, t) = \min\left\{\begin{array}{l} M(R\xi x_n, S\xi x_{n+1}, \xi a, t), M(S\xi x_n, R\xi x_{n+1}, \xi a, t), M(R\xi x_n, T\xi x_n, \xi a, t), \\ \frac{M(S\xi x_n, R\xi x_{n+1}, \xi a, t) M(R\xi x_n, T\xi x_n, \xi a, t)}{M(R\xi x_n, S\xi x_{n+1}, \xi a, t)}, \frac{M(S\xi x_n, T\xi x_n, \xi a, t) M(S\xi x_{n+1}, T\xi x_{n+1}, \xi a, t)}{M(R\xi x_{n+1}, T\xi x_{n+1}, \xi a, t)} \end{array}\right\}$$

$$=\min\left\{\begin{matrix} M(T\xi x_{n-1},T\xi x_n,\xi a,t), M(T\xi x_{n-1},T\xi x_n,\xi a,t), M(T\xi x_{n+1},T\xi x_n,\xi a,t), \\ \frac{M(T\xi x_{n-1},T\xi x_n,\xi a,t)M(T\xi x_{n+1},T\xi x_n,\xi a,t)}{M(T\xi x_{n-1},T\xi x_n,\xi a,t)}, \frac{M(T\xi x_{n-1},T\xi x_n,\xi a,t)M(\xi T x_n,T\xi x_{n+1},\xi a,t)}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} \end{matrix}\right\}$$

$$= \min\{M(T\xi x_{n-1},T\xi x_n,\xi a,t), M(T\xi x_n,T\xi x_{n+1},\xi a,t)\}$$

We now claim that $M(T\xi x_{n-1},T\xi x_n,\xi a,t) < M(T\xi x_n,T\xi x_{n+1},\xi a,t)$

Otherwise we claim that $M(T\xi x_{n-1},T\xi x_n,\xi a,t) \geq M(T\xi x_n,T\xi x_{n+1},\xi a,t)$

i.e. $$Q(\xi x_n,\xi x_{n+1},\xi a,t) = M(T\xi x_n,T\xi x_{n+1},\xi a,t) \qquad (2.3.2.6)$$

$$\therefore \quad \frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} - 1 \leq k\left(\frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} - 1\right) \qquad \text{[by (2.3.2.2)]}$$

which is a contradiction.

Hence, $$\frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1},T\xi x_n,\xi a,t)} - 1\right) \qquad (2.3.2.7)$$

$\therefore$ $\{T\,\xi\, x_n\}$ is a fuzzy contractive sequence in $(X,\Omega,M,\star)$ (X, M,*). So $\{T(X,\Omega,M,\star)\, x_n\}$ is a Cauchy sequence in $(X,\Omega,M,\star)$

As X is a complete Random fuzzy2- metric space, $\{T\,\xi x_{n-1}\}$ is convergent. So, $\{T\,\xi\, x_{n-1}\}$ converges to some point ξ z in X.

$\therefore$ $\{T\,\xi\, x_{n-1}\}$, $\{R\,\xi\, x_n\}$, $\{S\,\xi\, x_n\}$ converges to ξ z. By w-continuity of R, S and T, there exists a point ξ u in X such that $\xi\, x_n \to \xi$ u as $n\to\infty$ and so $\ln R\xi x_n = \ln S\xi x_n = \ln T\xi x_{n-1} = \xi$ z implies

$$R\,\xi\, u = S\,\xi\, u = T\,\xi\, u = \xi\, z \qquad (2.3.2.8)$$

Also by compatibility of pairs T, S and T, R and T ξ u = R ξ u = S ξ u = ξ z implies

$$T\,\xi\, z = TR\,\xi\, u = RT\,\xi\, u = R\,\xi\, z \text{ and } T\,\xi\, z = TS\,\xi\, u = ST\,\xi\, u = S\,\xi\, z$$

Therefore, $$T\,\xi\, z = R\,\xi\, z = S\,\xi\, z \qquad (2.3.2.9)$$

We now claim that T ξ z = ξ z.

If not $$\frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1},T\xi x_n,\xi a,t)} - 1\right)$$

$$Q(\xi z,\xi u,\xi a,t) = \min\left\{\begin{matrix} M(R\xi z,S\xi u,\xi a,t), M(S\xi z,R\xi u,\xi a,t), M(R\xi z,T\xi z,\xi a,t), \\ \frac{M(S\xi z,R\xi u,\xi a,t)M(R\xi z,T\xi z,\xi a,t)}{M(R\xi z,S\xi u,\xi a,t)}, \frac{M(S\xi z,T\xi z,\xi a,t)M(S\xi u,T\xi u,\xi a,t)}{M(R\xi u,T\xi u,\xi a,t)} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} M(T\xi z,\xi z,\xi a,t), M(T\xi z,\xi z,\xi a,t), M(T\xi z,T\xi z,\xi a,t), \\ \frac{M(Tz,z,a,t)M(Tz,Tz,a,t)}{M(T\xi z,\xi z,\xi a,t)}, \frac{M(Tz,Tz,a,t)M(z,z,a,t)}{M(\xi z,\xi z,\xi a,t)} \end{matrix}\right\}$$

$$= \min\{M(T\xi z,\xi z,\xi a,t), M(T\xi z,\xi z,\xi a,t), 1,1,1\}$$

$$= M(T\xi z,\xi z,\xi a,t)$$

$$\therefore \quad \frac{1}{M(T\xi x_n,T\xi x_{n+1}\xi,a,t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1},T\xi x_n,\xi a,t)} - 1\right)$$

which is a contradiction.

Hence T ξ z = ξ z

So ξ z is a common fixed point of R, T and S.

Now suppose ξ v $\neq$ ξ z be another fixed point of R, T and

$$\therefore \quad \frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1},T\xi x_n,\xi a,t)} - 1\right)$$

$$Q(\xi v,\xi u,\xi a,t) = \min\left\{\begin{matrix} M(R\xi v,S\xi z,\xi a,t), M(S\xi v,R\xi z,\xi a,t), M(R\xi v,T\xi v,\xi a,t), \\ \frac{M(S\xi v,R\xi z,\xi a,t)M(R\xi v,T\xi v,\xi a,t)}{M(R\xi v,S\xi z,\xi a,t)}, \frac{M(S\xi v,T\xi v,\xi a,t)M\xi(Sz,T\xi z,\xi a,t)}{M(R\xi z,T\xi z,\xi a,t)} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} M(\xi v,\xi z,\xi a,t), M(\xi v,\xi z,\xi a,t), M(\xi v,\xi v,\xi a,t), \\ \frac{M(\xi v,\xi z,\xi a,t)M(\xi v,\xi v,\xi a,t)}{M(v,z,a,t)}, \frac{M(\xi v,\xi v,\xi a,t)M(\xi z,\xi z,\xi a,t)}{M(\xi z,\xi z,\xi a,t)} \end{matrix}\right\}$$

$$= \min\{M(\xi v,\xi z,\xi a,t), M(\xi v,\xi z,\xi a,t), 1,,1,1\}$$

$$= M(\xi v,\xi z,\xi a,t)$$

$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, t)} - 1\right)$

This is a contradiction. Hence ξ v = ξ z.

Thus R, T and S have unique common fixed point. This completes our proof.

Theorem 2.3.3: Let $(X, \Omega, M, \star)$ be a complete Random fuzzy 3-metric space (RF-3M) in which fuzzy contractive sequences are Cauchy and T , R and S be mappings from $(X, \Omega, M, \star)$ into itself $\xi : \Omega \to X$ is a measurable selector and ξa ,ξb, > 0satisfying the following conditions :

$$T(X) \subseteq R(X) \text{ and } T(X) \subseteq S(X) \qquad (2.33.1)$$

$$\frac{1}{M(T(\xi x), T(\xi y), \xi a, \xi b, t)} - 1 \leq k\left(\frac{1}{Q(\xi x, \xi y, \xi a, \xi b, t)} - 1\right) \qquad (2.3.3.2)$$

with 0 < k < 1 and

$$Q(\xi x, \xi y, \xi a, t) = \min\left\{\begin{array}{l} M(R\xi x, S\xi y, \xi a, \xi b, t), M(S\xi x, R\xi y, \xi a, \xi b, t), M(R\xi x, T\xi x, \xi a, \xi b, t), \\ \frac{M(S\xi x, R\xi y, \xi a, \xi b, t) M(R\xi x, T\xi x, \xi a, \xi b, t)}{M(R\xi x, S\xi y, \xi a, \xi b, t)}, \frac{M(S\xi x, T\xi x, \xi a, \xi b, t) M(S\xi y, T\xi y, \xi a, \xi b, t)}{M(R\xi y, T\xi y, \xi a, \xi b, t)} \end{array}\right\}$$

The pairs T, S and T, R are compatible. R, T and S are w-continuous. (2.3.3.3)

Then R, T and S have a unique common fixed point. (2.3.3.4)

Proof: Let ξ $x_0 \in X$ be an arbitrary point of X. Since T(X) ⊆ R(X) and T(X) ⊆ S(X), we can construct a sequence { ξ x_n} in X such that

$$T\xi x_{n-1} = R\xi x_n = S\xi x_n \qquad (2.3.3.5)$$

Now,

$$Q(\xi x_n, \xi x_{n+1}, \xi a, \xi b, t) = min\left\{\begin{array}{l} M(R\xi x_n, S\xi x_{n+1}, \xi a, \xi b, t), M(S\xi x_n, R\xi x_{n+1}, \xi a, t), M(R\xi x_n, T\xi x_n, \xi a, \xi b, t), \\ \frac{M(S\xi x_n, R\xi x_{n+1}, \xi a, \xi b, t) M(R\xi x_n, T\xi x_n, \xi a, \xi b, t)}{M(R\xi x_n, S\xi x_{n+1}, \xi a, \xi b, t)}, \frac{M(S\xi x_n, T\xi x_n, \xi a, \xi b, t) M(S\xi x_{n+1}, T\xi x_{n+1}, \xi a, \xi b, t)}{M(R\xi x_{n+1}, T\xi x_{n+1}, \xi a, \xi b, t)} \end{array}\right\}$$

$$= \min\left\{\begin{array}{l} M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t), M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t), M(T\xi x_{n+1}, T\xi x_n, \xi a, \xi b, t), \\ \frac{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t) M(T\xi x_{n+1}, T\xi x_n, \xi a, \xi b, t)}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)}, \frac{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t) M(\xi T x_n, T\xi x_{n+1}, \xi a, \xi b, t)}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} \end{array}\right\}$$

$$= \min\{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t), M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)\}$$

We now claim that $M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t) < M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)$

Otherwise we claim that $M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t) \geq M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)$

i.e. $\quad Q(\xi x_n, \xi x_{n+1}, \xi a, \xi b, t) = M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t) \qquad (2.3.3.6)$

$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1\right) \qquad$ [by (2.3.32)]

which is a contradiction.

Hence, $\frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right) \qquad (2.3.3.7)$

∴ {T ξ x_n} is a fuzzy contractive sequence in $(X, \Omega, M, \star)$ (X, M,*). So {T$(X, \Omega, M, \star)$ x_n} is a Cauchy sequence in $(X, \Omega, M, \star)$

As X is a complete Random fuzzy3- metric space, {T ξx_{n-1}} is convergent. So, {T ξ x_{n-1}} converges to some point ξ z in X.

∴ {T ξ x_{n-1}}, {R ξ x_n}, {S ξ x_n} converges to ξ z. By w-continuity of R, S and T, there exists a point ξ u in X such that ξ $x_n \to$ ξ u as $n \to \infty$ and so $ln\, R\xi x_n = ln\, S\xi x_n = ln\, T\xi x_{n-1}$ = ξ z implies

$$R\,\xi\,u = S\,\xi\,u = T\,\xi\,u = \xi\,z \qquad (2.3.3.8)$$

Also by compatibility of pairs T, S and T, R and T ξ u = R ξ u = S ξ u = ξ z implies

$$T\,\xi\,z = TR\,\xi\,u = RT\,\xi\,u = R\,\xi\,z \text{ and } T\,\xi\,z = TS\,\xi\,u = ST\,\xi\,u = S\,\xi\,z$$

Therefore, $\quad T\,\xi\,z = R\,\xi\,z = S\,\xi\,z \qquad (2.3.39)$

We now claim that T ξ z = ξ z.

If not $\frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$

Q (ξ z, ξ u, ξ a, ξb, t) =

$$\min\begin{Bmatrix} M(R\xi z, S\xi u, \xi a, \xi b, t), M(S\xi z, R\xi u, \xi a, \xi b, t), M(R\xi z, T\xi z, \xi a, \xi b, t), \\ \frac{M(S\xi z, R\xi u, \xi a, \xi b, t) M(R\xi z, T\xi z, \xi a, \xi b, t)}{M(R\xi z, S\xi u, \xi a, \xi b, t)}, \frac{M(S\xi z, T\xi z, \xi a, t) M(S\xi u, T\xi u, \xi a, \xi b, t)}{M(R\xi u, T\xi u, \xi a, \xi b, t)} \end{Bmatrix}$$

$$= \min\begin{Bmatrix} M(T\xi z, \xi z, \xi a, \xi b, t), M(T\xi z, \xi z, \xi a, \xi b, t), M(T\xi z, T\xi z, \xi a, \xi b, t), \\ \frac{M(Tz, z, \xi a, \xi b, t) M(Tz, Tz, \xi a, \xi b, t)}{M(T\xi z, \xi z, \xi a, \xi b, t)}, \frac{M(T\xi z, T\xi z, \xi a, \xi b, t) M(\xi z, \xi z, \xi a, \xi b, t)}{M(\xi z, \xi z, \xi a, \xi b, t)} \end{Bmatrix}$$

$$= \min\{M(T\xi z, \xi z, \xi a, \xi b, t), M(T\xi z, \xi z, \xi a, \xi b, t), 1,1,1\}$$

$$= M(T\xi z, \xi z, \xi a, \xi b, t)$$

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1} \xi, a, \xi b, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$$

which is a contradiction.

Hence T ξ z = ξ z

So ξ z is a common fixed point of R, T and S.

Now suppose ξ v ≠ ξ z be another fixed point of R, T and

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$$

$$Q(\xi v, \xi u, \xi a, \xi b, t) = \min\begin{Bmatrix} M(R\xi v, S\xi z, \xi a, \xi b, t), M(S\xi v, R\xi z, \xi a, \xi b, t), M(R\xi v, T\xi v, \xi a, \xi b, t), \\ \frac{M(S\xi v, R\xi z, \xi a, \xi b, t) M(R\xi v, T\xi v, \xi a, \xi b, t)}{M(R\xi v, S\xi z, \xi a, \xi b, t)}, \frac{M(S\xi v, T\xi v, \xi a, \xi b, t) M\xi(Sz, T\xi z, \xi a, \xi b, t)}{M(R\xi z, T\xi z, \xi a, \xi b, t)} \end{Bmatrix}$$

$$= \min\begin{Bmatrix} M(\xi v, \xi z, \xi a, \xi b, t), M(\xi v, \xi z, \xi a, \xi b, t), M(\xi v, \xi v, \xi a, \xi b, t), \\ \frac{M(\xi v, \xi z, \xi a, \xi b, t) M(\xi v, \xi v, \xi a, \xi b, t)}{M(v, z, \xi a, \xi b, t)}, \frac{M(\xi v, \xi v, \xi a, \xi b, t) M(\xi z, \xi z, \xi a, \xi b, t)}{M(\xi z, \xi z, \xi a, \xi b, t)} \end{Bmatrix}$$

$$= \min\{M(\xi v, \xi z, \xi a, \xi b, t), M(\xi v, \xi z, \xi a, \xi b, t), 1,1,1\}$$

$$= M(\xi v, \xi z, \xi a, \xi b, t)$$

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$$

This is a contradiction. Hence ξ v = ξ z.

Thus R, T and S have unique common fixed point. This completes our proof.

2.4 Integral type mapping

Impact of fixed point theory in different branches of mathematics and its applications is immense. In 2002, A. Branciari [3] analyzed the existence of fixed point for mapping f defined on a complete metric space (X, d) satisfying a general contractive condition of integral type.

THEOREM 2.4.1: (Branciari)

Let (X, d) be a complete metric space, c ∈(0, 1) and let f: X→X be a mapping such that for each x, y ∈ X,

$$\int_0^{d(fx,fy)} \xi(t)dt \le c \int_0^{d(x,y)} \xi(t)dt \qquad (2.4.1.1)$$

where $\xi : [0,+\infty)\to[0,+\infty)$ is a lebesgue integrable mapping which is summable on each compact subset of $[0,+\infty)$, non-negative and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \xi(t)dt$, then f has a unique fixed point a ∈ X such that for each x ∈ X, $lim_{n\to\infty} f^n x = a$.

After the paper of Branciari, a lot of research works have been carried out on generalizing contractive condition of integral type for a different contractive mapping satisfying various known properties.

Theorem 2.4.2: Let (X, d) be a complete metric space and f : X→X such that

$$\int_0^{d(fx,fy)} u(t)dt + \alpha \int_0^{d(x,fx)+\, d(y,fy)} u(t)dt + \beta \int_0^{d(x,y)} u(t)dt + \gamma \int_0^{max\{d(x,fx)+\, d(y,fx)\}} u(t)dt$$

For each x, y∈X with non-negative real's α, β, γ such that 2α + β + γ < 1, where u :[0, +∞)→[0, +∞) is a Lebesgue integrable mapping which is summable , non-negative and such that for each ε > 0,

$$\int_0^{\varepsilon} u(t)dt > 0$$

Then f has a unique fixed point in X.

There is a gap in the proof of theorem (3.4.2.1) . In fact, the authors [10] used the inequality $\int_0^{a+b} u(t)dt \le \int_0^{a} u(t)dt + \int_0^{b} u(t)dt$ for $0 \le a < b$, which is not true in general .The aim of research paper of H. Aydi [23] was to present in the presence of this inequality an extension of theorem 2.4.2 using altering distances .

THEOREM 2.4.3: Let (X, M, Ω *) be a complete Random Fuzzy metric space in which fuzzy contractive sequences are Cauchy and T, R and S be mappings from (X, M, Ω *) into itself(Ω, Σ) denotes a measurable space. $\xi : \Omega \to X$ is a measurable selector satisfying the following conditions:

$$T(X) \subseteq R(X) \text{ and } T(X) \subseteq S(X)$$

$$\frac{1}{M(T(\xi x),T(\xi y),t)} - 1 \leq k\left(\frac{1}{Q(\xi x,\xi y,t)} - 1\right)$$

with 0 < k < 1 and

$$\int_0^{Q(\xi x,\xi y,t)} \theta(t)dt = \int_0^{\min\left\{\begin{matrix} M(R\xi x,S\xi y,t), M(S\xi x,R\xi y,t), M(R\xi x,T\xi x,t), \\ \frac{M(S\xi x,R\xi y,t)M(R\xi x,T\xi x,t)}{M(R\xi x,S\xi y,t)}, \frac{M(S\xi x,T\xi x,t)M(S\xi y,T\xi y,t)}{M(R\xi y,\xi Ty,t)} \end{matrix}\right\}} \theta(t)dt$$

For every $\xi x, \xi y \in X$, $k \in (0,1)$ and each $t > 0$, Where $\theta : [0,+\infty] \to [0,+\infty]$ is a legesgue integrable mapping which is summable on each compact subset of $[0,+\infty]$, non negative, and such that, $\forall\ \varepsilon > 0, \int_0^{\varepsilon} \theta(t)\, dt > 0$

The pairs T, S and T, R are compatible, R, T, S are w-continuous. Then T, R and S have a unique common fixed point.

Proof: Let $\xi x_0 \in X$ be an arbitrary point .Since $T(X) \subseteq R(X)$ and $T(X) \subseteq S(X)$, we can construct a sequence $\{x_n\}$ in X such that

$$T\xi x_{n-1} = R\xi x_n = S\xi x_n$$

Now

$$\int_0^{Q(\xi x_n,\xi x_{n+1},t)} \xi(t)dt = \int_0^{\min\left\{\begin{matrix} M(R\xi x_n,S\xi x_{n+1},t), M(S\xi x_n,R\xi x_{n+1},t), M(R\xi x_n,T\xi x_n,t), \\ \frac{M(Sx\xi_n,R\xi x_{n+1},t)M(R\xi x_n,T\xi x_n,t)}{M(R\xi x_n,S\xi x_{n+1},t)}, \frac{M(S\xi x_n,T\xi x_n,t)M(S\xi x_{n+1},\xi Tx_{n+1},t)}{M(R\xi x_{n+1},T\xi x_{n+1},t)} \end{matrix}\right\}} \xi(t)dt$$

$$= \int_0^{\min\left\{\begin{matrix} M(T\xi x_{n-1},T\xi x_n,t), M(T\xi x_{n-1},T\xi x_n,t), M(T\xi x_{n+1},T\xi x_n,t), \\ \frac{M(T\xi x_{n-1},T\xi x_n,t)M(T\xi x_{n+1},T\xi x_n,t)}{M(T\xi x_{n-1},T\xi x_n,t)}, \frac{M(T\xi x_{n-1},T\xi x_n,t)M(T\xi x_n,T\xi x_{n+1},t)}{M(T\xi x_n,T\xi x_{n+1},t)} \end{matrix}\right\}} \xi(t)dt$$

$$= \int_0^{\min\{M(T\xi x_{n-1},T\xi x_n,t), M(T\xi x_n,T\xi x_{n+1},t)\}} \xi(t)dt$$

We now claim that $M(T\xi x_{n-1},T\xi x_n,t) < M(T\xi x_n,T\xi x_{n+1},t)$
Otherwise we claim that $M(T\xi x_{n-1},T\xi x_n,t) \geq M(T\xi x_n,T\xi x_{n+1},t)$

i.e. , $$\int_0^{Q(\xi x_n,\xi x_{n+1},t)} \xi(t)dt = \int_0^{M(T\xi x_n,T\xi x_{n+1},t))} \xi(t)dt$$

$\therefore$ Hence, $$\frac{1}{M(T\xi x_n,T\xi x_{n+1},t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1},T\xi x_n,t)} - 1\right)$$

$\therefore$ $\therefore$ $\{T\xi x_n\}$ is a fuzzy contractive sequence in $(X, \Omega, M, \star)$ So $\{T\xi x_n\}$ is a Cauchy sequence
As X is a complete fuzzy metric space, $\{T\xi x_{n-1}\}$ is convergent. So, $\{T\xi x_{n-1}\}$ converges to some point z in X.
$\therefore$ $\{T\xi x_{n-1}\}$, $\{R\xi x_n\}$, $\{S\xi x_n\}$ converges to z. By w-continuity of R, S and T, there exists a point ξu in ξX such that $\xi x_n \to \xi u$ as $n\to\infty$ and so $\lim R\xi x_n = \lim S\xi x_n = \lim T\xi x_{n-1}$ = z implies

$$R\xi u = S\xi u = T\xi u = \xi z$$

Also by compatibility of pairs T, S and T, R and Tu = Ru = Su = z implies

$$T\xi z = TR\xi u = RT\xi u = R\xi z \text{ and } T\xi z = TS\xi u = ST\xi u = S\xi z$$

Therefore, $T\xi z = R\xi z = S\xi z$

We now claim that $T\xi z = \xi z$.

If not $\frac{1}{M(T\xi x_n, T\xi x_{n+1}, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, t)} - 1\right)$

$$\int_0^{Q\,(\xi z, \xi u, t)} \xi(t)dt = \int_0^{\min\left\{\begin{matrix} M(R\xi z, S\xi u, t), M(S\xi z, R\xi u, t), M(R\xi z, T\xi z, t), \\ \frac{M(S\xi z, R\xi u, t)M(R\xi z, T\xi z, t)}{M(R\xi z, S\xi u, t)}, \frac{M(S\xi z, T\xi z, t)M(S\xi u, T\xi u, t)}{M(R\xi u, T\xi u, t)} \end{matrix}\right\}} \xi(t)dt$$

$$= \int_0^{\min\left\{\begin{matrix} M(T\xi z, \xi z, t), M(T\xi z, \xi z, t), M(T\xi z, T\xi z, t), \\ \frac{M(T\xi z, \xi z, t)M(T\xi z, T\xi z, t)}{M(T\xi z, \xi z, t)}, \frac{M(T\xi z, T\xi z, t)M(\xi z, \xi z, t)}{M(\xi z, \xi z, t)} \end{matrix}\right\}} \xi(t)dt$$

$$= \int_0^{\min\{M(T\xi z, \xi z, t), M(T\xi z, \xi z, t), 1, 1, 1\}} \xi(t)dt$$

$$= \int_0^{M(T\xi z, \xi z, t)} \xi(t)dt$$

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, t)} - 1\right)$$

which is a contradiction.

Hence $T\xi z = \xi z$

So ξz is a common fixed point of R, T and S.

Now suppose $\xi v \neq \xi z$ be another fixed point of R, T and

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, t)} - 1\right)$$

$$\int_0^{Q\,(\xi v, \xi u, t)} \xi(t)dt = \int_0^{\min\left\{\begin{matrix} M(R\xi v, S\xi z, t), M(S\xi v, R\xi z, t), M(R\xi v, T\xi v, t), \\ \frac{M(S\xi v, R\xi z, t)M(R\xi v, T\xi v, t)}{M(R\xi v, S\xi z, t)}, \frac{M(S\xi v, T\xi v, t)M(S\xi z, T\xi z, t)}{M(R\xi z, T\xi z, t)} \end{matrix}\right\}} \xi(t)dt$$

$$= \int_0^{\min\left\{\begin{matrix} M(\xi v, \xi z, t), M(\xi v, \xi z, t), M(\xi v, \xi v, t), \\ \frac{M(\xi v, \xi z, t)M(\xi v, \xi v, t)}{M(v, z, t)}, \frac{M(\xi v \xi, v, t)M(\xi z, \xi z, t)}{M(z, z, t)} \end{matrix}\right\}} \xi(t)dt$$

$$= \int_0^{\min\{M(\xi v, \xi z, t), M(\xi v, \xi z, t), 1, 1\}} \xi(t)dt$$

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, t)} - 1\right)$$

which is a contradiction. Hence $\xi v = \xi z$.

Thus R, T and S have a unique fixed point.

Theorem 2.4.2: Let $(X, \Omega, M, \star)$ be a complete Random fuzzy 2-metric space (RF-2M) in which fuzzy contractive sequences are Cauchy and T , R and S be mappings from $(X, \Omega, M, \star)$ into itself $\xi : \Omega \to X$ is a measurable selector and a(ξ) >0 satisfying the following conditions :

$$T(X) \subseteq R(X) \text{ and } T(X) \subseteq S(X) \tag{2.4.2.1}$$

$$\frac{1}{M(T(\xi x),T(\xi y),\xi a,t)} - 1 \le k\left(\frac{1}{Q(\xi x,\xi y,\xi a,t)} - 1\right) \tag{2.4.2.2}$$

with $0 < k < 1$ and

$$\int_0^{Q(\xi x,\xi y,\xi a,t)} \theta(t)dt = \int_0^{\min\left\{\begin{array}{c} M(R\xi x,S\xi y,\xi a,t), M(S\xi x,R\xi y,\xi a,t), M(R\xi x,T\xi x,\xi a,t), \\ \frac{M(S\xi x,R\xi y,\xi a,t)M(R\xi x,T\xi x,\xi a,t)}{M(R\xi x,S\xi y,\xi a,t)}, \frac{M(S\xi x,T\xi x,\xi a,t)M(S\xi y,T\xi y,\xi a,t)}{M(R\xi y,T\xi y,\xi a,t)} \end{array}\right\}} \theta(t)dt$$

For every $\xi x, \xi y \in X$, $k \in (0,1)$ and each $t > 0$, Where $\theta : [0,+\infty] \to [0,+\infty]$ is a legesgue integrable mapping which is summable on each compact subset of $[0,+\infty]$, non negative, and such that, $\forall\ \varepsilon > 0$, $\int_0^{\varepsilon} \theta(t)\ dt > 0$

The pairs T, S and T, R are compatible, R, T, S are w-continuous. Then T, R and S have a unique common fixed point.

PROOF: Let $x_0 \in X$ be an arbitrary point of X. Since $T(X) \subseteq R(X)$ and $T(X) \subseteq S(X)$, we can construct a sequence $\{x_n\}$ in X such that

$$Tx_{n-1} = Rx_n = Sx_n$$

Now ,

$$\int_0^{Q(\xi x_n,\xi x_{n+1},\xi a,t)=)} \theta(t)dt = \int_0^{\min\left\{\begin{array}{c} M(R\xi x_n,S\xi x_{n+1},\xi a,t), M(S\xi x_n,R\xi x_{n+1},\xi a,t), M(R\xi x_n,T\xi x_n,\xi a,t), \\ \frac{M(S\xi x_n,R\xi x_{n+1},\xi a,t)M(R\xi x_n,T\xi x_n,\xi a,t)}{M(R\xi x_n,S\xi x_{n+1},\xi a,t)}, \frac{M(S\xi x_n,T\xi x_n,\xi a,t)M(S\xi x_{n+1},T\xi x_{n+1},\xi a,t)}{M(R\xi x_{n+1},T\xi x_{n+1},\xi a,t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\left\{\begin{array}{c} M(T\xi x_{n-1},T\xi x_n,\xi a,t), M(T\xi x_{n-1},T\xi x_n,\xi a,t), M(T\xi x_{n+1},T\xi x_n,\xi a,t), \\ \frac{M(T\xi x_{n-1},T\xi x_n,\xi a,t)M(T\xi x_{n+1},T\xi x_n,\xi a,t)}{M(T\xi x_{n-1},T\xi x_n,\xi a,t)}, \frac{M(T\xi x_{n-1},T\xi x_n,\xi a,t)M(\xi T x_n,T\xi x_{n+1},\xi a,t)}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\{M(T\xi x_{n-1},T\xi x_n,\xi a,t), M(T\xi x_n,T\xi x_{n+1},\xi a,t)\}} \theta(t)dt$$

We now claim that $M(T\xi x_{n-1},T\xi x_n,\xi a,t) < M(T\xi x_n,T\xi x_{n+1},\xi a,t)$
Otherwise we claim that $M(T\xi x_{n-1},T\xi x_n,\xi a,t) \ge M(T\xi x_n,T\xi x_{n+1},\xi a,t)$

i.e. , $\frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} - 1 \le k\left(\frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} - 1\right)$ [by (2.4.2.2)]
which is a contradiction

$\therefore$ $\frac{1}{M(Tx_n,Tx_{n+1},a,t)} - 1 \le k\left(\frac{1}{L(Tx_n,Tx_{n+1},a,t)} - 1\right)$

This is a contradiction.

Hence, Hence, $\frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1},T\xi x_n,\xi a,t)} - 1\right)$

$\{T\xi x_n\}$ is a fuzzy contractive sequence in $(X, M, *)$. So $\{Tx_n\}$ is a Cauchy sequence in $(X, M, *)$.

As X is a complete fuzzy metric space, $\{T\xi x_{n-1}\}$ is convergent. So, $\{T\xi x_{n-1}\}$ converges to some point ξz in X.

$\therefore$ $\{T\xi x_{n-1}\}$, $\{R\xi x_n\}$, $\{S\xi x_n\}$ converges to ξz. By w-continuity of R, S and T, there exists a point ξu in X such that $\xi x_n \to \xi u$ as $n \to \infty$ and so $\ln R\xi x_n = \ln S\xi x_n = \ln T\xi x_{n-1} = \xi z$ implies

$$R\xi u = S\xi u = T\xi u = \xi z \tag{2.4.2.8}$$

Also by compatibility of pairs T, S and T, R and T ξ u = R ξ u = S ξ u = ξ z implies

T ξ z = TR ξ u = RT ξ u = R ξ z and T ξ z = TS ξ u = ST ξ u = S ξ z

Therefore, T ξ z = R ξ z = S ξ z (2.4.2.9)

We now claim that T ξ z = ξ z.

If not $\frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, t)} - 1\right)$

$$\int_0^{Q(\xi z, \xi u, \xi a, t)} \theta(t)dt = \int_0^{\min\left\{\begin{array}{c} M(R\xi z, S\xi u, \xi a, t), M(S\xi z, R\xi u, \xi a, t), M(R\xi z, T\xi z, \xi a, t), \\ \frac{M(S\xi z, R\xi u, \xi a, t) M(R\xi z, T\xi z, \xi a, t)}{M(R\xi z, S\xi u, \xi a, t)}, \frac{M(S\xi z, T\xi z, \xi a, t) M(S\xi u, T\xi u, \xi a, t)}{M(R\xi u, T\xi u, \xi a, t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\left\{\begin{array}{c} M(T\xi z, \xi z, \xi a, t), M(T\xi z, \xi z, \xi a, t), M(T\xi z, T\xi z, \xi a, t), \\ \frac{M(Tz\xi, \xi z, \xi a, t) M(T\xi z, Tz, a, t)}{M(T\xi z, \xi z, \xi a, t)}, \frac{M(T\xi z, T\xi z, \xi a, t) M(\xi z, z, \xi a, t)}{M(\xi z, \xi z, \xi a, t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\left\{\begin{array}{c} M(T\xi z, \xi z, \xi a, t), M(T\xi z, \xi z, \xi a, t), M(T\xi z, T\xi z, \xi a, t), \\ \frac{M(Tz, z, a, t) M(Tz, Tz, a, t)}{M(T\xi z, \xi z, \xi a, t)}, \frac{M(Tz, Tz, a, t) M(z, z, a, t)}{M(\xi z, \xi z, \xi a, t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\{M(T\xi z, \xi z, \xi a, t), M(T\xi z, \xi z, \xi a, t), 1, 1, 1\}} \theta(t)dt$$

$$= \int_0^{M(Tz, z, a, t)} \theta(t)dt$$

$$\frac{1}{M(T\xi x_n, T\xi x_{n+1} \xi, a, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, t)} - 1\right)$$

which is a contradiction.

Hence T ξ z = ξ z

So ξ z is a common fixed point of R, T and S.

Now suppose ξ v ≠ ξ z be another fixed point of R, T and S

Now suppose ξ v ≠ ξ z be another fixed point of R, T and

$$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, t)} - 1\right)$$

$$\int_0^{L(v, z, a, t \quad Q(\xi v, \xi u, \xi a, t))} \theta(t)dt = \int_0^{\min\left\{\begin{array}{c} M(R\xi v, S\xi z, \xi a, t), M(S\xi v, R\xi z, \xi a, t), M(R\xi v, T\xi v, \xi a, t), \\ \frac{M(S\xi v, R\xi z, \xi a, t) M(R\xi v, T\xi v, \xi a, t)}{M(R\xi v, S\xi z, \xi a, t)}, \frac{M(S\xi v, T\xi v, \xi a, t) M\xi(Sz, T\xi z, \xi a, t)}{M(R\xi z, T\xi z, \xi a, t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\left\{\begin{array}{c} M(\xi v, \xi z, \xi a, t), M(\xi v, \xi z, \xi a, t), M(\xi v, \xi v, \xi a, t), \\ \frac{M(\xi v, \xi z, \xi a, t) M(\xi v, \xi v, \xi a, t)}{M(v, z, a, t)}, \frac{M(\xi v, \xi v, \xi a, t) M(\xi z, \xi z, \xi a, t)}{M(\xi z, \xi z, \xi a, t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\{M(v, z, a, t), M(v, z, a, t), 1, 1, 1, 1, 1, 1\}} \theta(t)dt$$

$$\therefore \qquad \therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, t)} - 1\right)$$

which is a contradiction. Hence ξ v = ξ z.

Theorem 2.4.3: Let $(X, \Omega, M, \star)$ be a complete Random fuzzy 3-metric space (RF-3M) in which fuzzy contractive sequences are Cauchy and T , R and S be mappings from $(X, \Omega, M, \star)$ into itself $\xi : \Omega \to X$ is a measurable selector and ξa ,ξb, > 0satisfying the following conditions :

T(X) ⊆ R(X) and T(X) ⊆ S(X) (2.43.1)

$$\frac{1}{M(T(\xi x),T(\xi y),\xi a,\xi b,t)} - 1 \leq k\left(\frac{1}{Q(\xi x,\xi y,\xi a,\xi b,t)} - 1\right) \qquad (2.4.3.2)$$

with $0 < k < 1$ and

$$\int_0^{Q(\xi x,\xi y,\xi a,t))} \theta(t)dt = \int_0^{\min\left\{\begin{array}{c} M(R\xi x,S\xi y,\xi a,\xi b,t),M(S\xi x,R\xi y,\xi a,\xi b,t),M(R\xi x,T\xi x,\xi a,\xi b,t), \\ \frac{M(S\xi x,R\xi y,\xi a,\xi b,t)M(R\xi x,T\xi x,\xi a,\xi b,t)}{M(R\xi x,S\xi y,\xi a,\xi b,t)}, \frac{M(S\xi x,T\xi x,\xi a,\xi b,t)M(S\xi y,T\xi y,\xi a,\xi b,t)}{M(R\xi y,T\xi y,\xi a,\xi b,t)} \end{array}\right\}} \theta(t)dt$$

For every $\xi x,\xi y \in X$, $k \in (0,1)$ and each $t > 0$, Where $\theta : [0,+\infty] \to [0,+\infty]$ is a legesgue integrable mapping which is summable on each compact subset of $[0,+\infty]$, non negative, and such that, $\forall\ \varepsilon > 0$, $\int_0^{\varepsilon} \theta(t)\ dt > 0$

The pairs T, S and T, R are compatible, R, T, S are w-continuous. Then T, R and S have a unique common fixed point.

Proof: Let $\xi x_0 \in X$ be an arbitrary point of X. Since $T(X) \subseteq R(X)$ and $T(X) \subseteq S(X)$, we can construct a sequence $\{\xi x_n\}$ in X such that

$$T\xi x_{n-1} = R\xi x_n = S\xi x_n \qquad (2.4.3.5)$$

Now,

$$\int_0^{Q(\xi x_n,\xi x_{n+1},\xi a,\xi b,t)} \theta(t)dt = \int_0^{\min\left\{\begin{array}{c} M(R\xi x_n,S\xi x_{n+1},\xi a,\xi b,t),M(S\xi x_n,R\xi x_{n+1},\xi a,t),M(R\xi x_n,T\xi x_n,\xi a,\xi b,t), \\ \frac{M(S\xi x_n,R\xi x_{n+1},\xi a,\xi b,t)M(R\xi x_n,T\xi x_n,\xi a,\xi b,t)}{M(R\xi x_n,S\xi x_{n+1},\xi a,\xi b,t)}, \frac{M(S\xi x_n,T\xi x_n,\xi a,\xi b,t)M(S\xi x_{n+1},T\xi x_{n+1},\xi a,\xi b,t)}{M(R\xi x_{n+1},T\xi x_{n+1},\xi a,\xi b,t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\left\{\begin{array}{c} M(T\xi x_{n-1},T\xi x_n,\xi a,\xi b,t),M(T\xi x_{n-1},T\xi x_n,\xi a,\xi b,t),M(T\xi x_{n+1},T\xi x_n,\xi a,\xi b,t), \\ \frac{M(T\xi x_{n-1},T\xi x_n,\xi a,\xi b,t)M(T\xi x_{n+1},T\xi x_n,\xi a,\xi b,t)}{M(T\xi x_{n-1},T\xi x_n,\xi a,\xi b,t)}, \frac{M(T\xi x_{n-1},T\xi x_n,\xi a,\xi b,t)M(\xi T x_n,T\xi x_{n+1},\xi a,\xi b,t)}{M(T\xi x_n,T\xi x_{n+1},\xi a,\xi b,t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\{M(T\xi x_{n-1},T\xi x_n,\xi a,\xi b,t),M(T\xi x_n,T\xi x_{n+1},\xi a,\xi b,t)\}} \theta(t)dt$$

N We now claim that $M(T\xi x_{n-1},T\xi x_n,\xi a,\xi b,t) < M(T\xi x_n,T\xi x_{n+1},\xi a,\xi b,t)$
Otherwise we claim that $M(T\xi x_{n-1},T\xi x_n,\xi a,\xi b,t) \geq M(T\xi x_n,T\xi x_{n+1},\xi a,\xi b,t)$
i.e. , $\int_0^{Q(\xi x_n,\xi x_{n+1},\xi a,\xi b,t))} \theta(t)dt = \int_0^{M(T\xi x_n,T\xi x_{n+1},\xi a,\xi b,t))} \theta(t)dt$

$$\therefore \quad \frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,\xi b,t)} - 1 \leq k\left(\frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,\xi b,t)} - 1\right) \qquad \text{[by (2.4.3.2)]}$$

This is a contradiction.

Hence, $\frac{1}{M(T\xi x_n,T\xi x_{n+1},\xi a,\xi b,t)} - 1 \leq k\left(\frac{1}{M(T\xi x_{n-1},T\xi x_n,\xi a,\xi b,t)} - 1\right)$ (2.4.3.7)

$\therefore$ $\{T\xi x_n\}$ is a fuzzy contractive sequence in $(X,\Omega,M,\star)$ (X, M,*). So $\{T(X,\Omega,M,\star)\ x_n\}$ is a Cauchy sequence in $(X,\Omega,M,\star)$

As X is a complete Random fuzzy3- metric space, $\{T\xi x_{n-1}\}$ is convergent. So, $\{T\xi x_{n-1}\}$ converges to some point ξz in X.

$\therefore$ $\{T\xi x_{n-1}\}$, $\{R\xi x_n\}$, $\{S\xi x_n\}$ converges to ξz. By w-continuity of R, S and T, there exists a point ξu in X such that $\xi x_n \to \xi u$ as $n\to\infty$ and so $\ln R\xi x_n = \ln S\xi x_n = \ln T\xi x_{n-1} = \xi z$ implies

$$R\xi u = S\xi u = T\xi u = \xi z \qquad (2.4.3.8)$$

Also by compatibility of pairs T, S and T, R and $T\xi u = R\xi u = S\xi u = \xi z$ implies

$$T\xi z = TR\xi u = RT\xi u = R\xi z \text{ and } T\xi z = TS\xi u = ST\xi u = S\xi z$$

Therefore, $T\xi z = R\xi z = S\xi z$

We now claim that $T\xi z = \xi z$.

If not $\frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$

$$\int_0^{Q(\xi z, \xi u, \xi a, \xi b, t)} \theta(t)dt = \int_0^{\min\left\{\begin{array}{c} M(R\xi z, S\xi u, \xi a, \xi b, t), M(S\xi z, R\xi u, \xi a, \xi b, t), M(R\xi z, Tz, \xi a, \xi b, t), \\ \frac{M(S\xi z, R\xi u, \xi a, \xi b, t) M(R\xi z, T\xi z, \xi a, \xi b, t)}{M(R\xi z, S\xi u, \xi a, \xi b, t)}, \frac{M(S\xi z, T\xi z, \xi a, \xi b, t) M(S\xi u, T\xi u, \xi a, \xi b, t)}{M(R\xi u, T\xi u\, \xi a, \xi b, t)} \end{array}\right\}} \theta(t)dt\, \xi$$

$$= \int_0^{\min\left\{\begin{array}{c} M(T\xi z, \xi z, \xi a, \xi b, t), M(T\xi z, \xi z, \xi a, \xi b, t), M(T\xi z, T\xi z, \xi a, \xi b, t), \\ \frac{M(Tz, z, \xi a, \xi b, t) M(Tz, Tz, \xi a, \xi b, t)}{M(T\xi z, \xi z, \xi a, \xi b, t)}, \frac{M(T\xi z, T\xi z, \xi a, \xi b, t) M(\xi z, \xi z, \xi a, \xi b, t)}{M(\xi z, \xi z, \xi a, \xi b, t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\{M(T\xi z, \xi z, \xi a, \xi b, t), M(T\xi z, \xi z, \xi a, \xi b, t), 1, 1, 1\}} \theta(t)dt$$

$$= \int_0^{M(T\xi z, \xi z, \xi a, \xi b, t)} \theta(t)dt$$

$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi, a, \xi b, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$
which is a contradiction.
Hence T ξ z = ξ z
So ξ z is a common fixed point of R, T and S.
Now suppose ξ v ≠ ξ z be another fixed point of R, T and
$\therefore \quad \frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$

$$\int_0^{Q(\xi v, \xi u, \xi a, \xi b, t)} \theta(t)dt = \int_0^{\min\left\{\begin{array}{c} M(R\xi v, S\xi z, \xi a, \xi b, t), M(S\xi v, R\xi z, \xi a, \xi b, t), M(R\xi v, T\xi v, \xi a, \xi b, t), \\ \frac{M(S\xi v, R\xi z, \xi a, \xi b, t) M(R\xi v, T\xi v, \xi a, \xi b, t)}{M(R\xi v, S\xi z, \xi a, \xi b, t)}, \frac{M(S\xi v, T\xi v, \xi a, \xi b, t) M\xi(Sz, T\xi z, \xi a, \xi b, t)}{M(R\xi z, T\xi z, \xi a, \xi b, t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\left\{\begin{array}{c} M(\xi v, \xi z, \xi a, \xi b, t), M(\xi v, \xi z, \xi a, \xi b, t), M(\xi v, \xi v, \xi a, \xi b, t), \\ \frac{M(\xi v, \xi z, \xi a, \xi b, t) M(\xi v, \xi v, \xi a, \xi b, t)}{M(v, z, \xi a, \xi b, t)}, \frac{M(\xi v, \xi v, \xi a, \xi b, t) M(\xi z, \xi z, \xi a, \xi b, t)}{M(\xi z, \xi z, \xi a, \xi b, t)} \end{array}\right\}} \theta(t)dt$$

$$= \int_0^{\min\{M(\xi v, \xi z, \xi a, \xi b, t), M(\xi v, \xi z, \xi a, \xi b, t), 1, 1, 1\}} \theta(t)dt$$

$$= \int_0^{M(\xi v, \xi z, \xi a, \xi b, t)} \theta(t)dt$$

$\frac{1}{M(T\xi x_n, T\xi x_{n+1}, \xi a, \xi b, t)} - 1 \le k\left(\frac{1}{M(T\xi x_{n-1}, T\xi x_n, \xi a, \xi b, t)} - 1\right)$
This is a contradiction. Hence ξ v = ξ z.
Thus R, T and S have unique common fixed point. This completes our proof.

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