

Markov Chain Model Application on Share Price Movement in Stock Market

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Abstract

The success of an investor especially in a stock market hinges much on the choice of decision made which in turn depends to a large extent on how well informed one is in stock analysis. The Markov chain model was used to analyse and to make predictions on the three states that exist in stock price change which are share prices increase, decrease or remain unchanged. The two top banks used to illustrate are Guarantee Trust bank of Nigeria and First bank of Nigeria. The six years data used were obtained from 2005 to 2010. The Transition matrix was derived using the Microsoft Excel. Obtaining powers of transition matrices and probability vector analysis using the R Statistical software, equilibrium was attained in about twenty years. It was realized that regardless of a bank current share price, in the long run we could predict that its share price will depreciate with a probability of 0.4229, remain unchanged with probability of 0.2072 and appreciate with a probability of 0.3699. The probability of each of the two banks appreciating is also on the increase, with GTB taking the lead with a probability of 0.4614 at equilibrium, and then FBN with a probability of 0.3799. For a company price to remain the same over a period of time is not a good sign for the company's performance and considering the probability of unchanged for this sector, we will notice that at equilibrium, the probability of GTB shares remaining the same is 0.0688 which is lower to that of First bank which implies that GTB shares change hands more than the FBN.

Keywords: Stock Market, Transition Matrix, Equilibrium, Probability Vector, Stock Prediction

1. Introduction

A stock exchange is an established legal framework surrounding the trading of shares of many companies or organisations. When a stock market is on the rise it is considered to be an up and coming economy. It is often considered as the primary indicator of a country's economic strength and development. A rise in share prices is usually associated with increased business investments and vice versa, and this also affects the wealth of households and their consumption. It is therefore imperative that the central bank of a country keeps in check the control and behavior of the stock market and in general on the smooth operation of financial system functions.

The success or failure of the individuals, corporate bodies or organisations in the stock market depends on the choice of decision made which in turn depends to a large extent on how well informed you are in stocks analysis. It is then useful to come up with statistical models that obtain estimates with a view to predict the share price movement of stocks. Stocks are the most difficult components of national income to estimate accurately in Nigeria and the worldwide. Stock prices skyrocket with little reason, and then plummet quickly and people/investors are concerned about the future of stock of which this research intends to proffer solution (Anvwar and Philip, 1997).

2. Determinants that lead to Share Price Movement

Careful observations and studies of share prices of stocks especially the banking sector in the Nigerian Stock Exchange have shown that the share prices appreciates, depreciates or remains the same over a period of time (days, weeks or months) and factors that determine these changes are as follows:

- **Wrong Information:-** Economists and finance experts have long studied price in speculative markets. Wrong information in their findings on a bank/sector may lead to an increase in share price as many investors may demand for more shares no matter the price at which the shares will be sold if the

wrong information favours the bank/sector or may lead to a decrease in share price as many investors may sell their shares no matter the price at which the shares will be bought if the wrong information does not favour the bank/sector.

- **Sector Participation:-** The sector participation e.g. that banks or insurance is the number of deals it makes before the closure of the trading of a particular day. When the number of trading of a bank or insurance on the floor of the stock exchange is observed to be high, the bank or insurance will have an increase in share price and vice versa. However when a bank does not participate in trading, its share price remains the same.
- **Business Situations:-** Business situations are the circumstances and things that are happening at a particular time in a business. When business is slow or bad, stock price do fall and since investors do not want to own shares that will constantly fall, they sell their shares no matter the price, causing the stock price to fall further. If they anticipate a recession in the future they may push stock prices down even though the current state of business is good, thus making share price of a particular bank to decrease at that period. If they anticipate a revival of business they may push stock prices up by demanding for more shares even though the current state of the sector is poor this will then lead to an increase in share prices.
- **Government Policy:-** The economic policy of the government changes, for example change in interest rates by CBN or banks can have great effects on people's willingness to buy or sell shares.

3. Markov Chain Model

A Markov chain is a sequence of experiments that consists of a finite number of states with some known probabilities P_{ij} , where P_{ij} is the probability of moving from state i to state j or simply put is stochastic process which depends on immediate outcome and not on history. It may be regarded as a series of transitions between different states, such that the probabilities associated with each transition depends only on the immediate proceeding state and not on how the process arrived at that state and the probabilities associated with the transitions between the states are constant with time.

When the present outcome is known, information about earlier trials does not affect probabilities of future events. The Markov chain model can then be said to be a sequence of consecutive trials such that

$$P\{X_n = j / X_{n-1} = i_{n-1}, \dots, X_0 = i_0\} = P\{X_n = j / X_{n-1} = i_{n-1}\}$$

$P\{x_n = j\} = P_j^{(n)}$ is the absolute probability of outcome P_j , $j = 1, 2, 3, \dots$ is a system of events (actually set of outcomes at any trial) that are mutually exclusive (Voskoglou, 1994; Hoppensteadt, 1992).

An important class of Markov chain model is that of which the transition probabilities are independent of n , we have $P\{x_n = j / x_{n-1} = i\} = P_{ij}$ which is a homogenous Markov chain where the order of the subscripts in P_{ij} corresponds to the direction of the transition i.e. $i \rightarrow j$. Hence we have $\sum_{i=1}^n P_{ij} = 1$ and $P_{ij} \geq 0$, Since for any fixed i , the transition probability P_{ij} will form a probability distribution. If the limiting distribution of x_n as $n \rightarrow \infty$ exist, the transition probabilities are most conveniently handled in matrix form as $P = P_{ij}$ i.e

$$P = \begin{pmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \dots & P_{nn} \end{pmatrix}$$

And this is referred to as the transition matrix, which depends on the number of states involved and may be finite or infinite (Hamilton, 1989; Michael, 2005).

3.1 n-Step Transition Probabilities

The absolute probabilities at any stage where n is greater than unity is determined by the used of n-step transition probabilities i.e.

In matrix terms, let p be the transition matrix of the Markov chain, then

$$P^1 = PP^{(0)} \text{ (for } n=1\text{)}$$

$$\text{Also } P^2 = PP^1 = P(PP^{(0)}) = P^2P^{(0)} \text{ (for } n=2\text{)}$$

$$\text{And in general } P^{(n)} = P^nP^{(0)}$$

3.2 Fixed Point Probability Vectors.

The definition of a regular chain states in terms of the powers of P, has the following important consequence. For each j and for k sufficiently large, each of the transition probabilities $P_{1j}^{(k)}$ $P_{2j}^{(k)}$... $P_{nj}^{(k)}$ is close to the same number. That is each of the entries in the jth column of the k-step transition matrix $P^{(k)}$ is close to W_j . Another way of saying these is that for large values of k, the k-step transition matrix.

$$P^{(k)} = \begin{pmatrix} P_{11}^{(k)} & P_{12}^{(k)} & \dots & \dots & P_{1n}^{(k)} \\ P_{21}^{(k)} & P_{22}^{(k)} & \dots & \dots & P_{2n}^{(k)} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ P_{n1}^{(k)} & P_{n2}^{(k)} & \dots & \dots & P_{nn}^{(k)} \end{pmatrix}$$

is very close to a matrix H that has all row identical

$$\begin{pmatrix} W \\ W \\ \dots \\ W \end{pmatrix} = \begin{pmatrix} W_1 & W_2 & \dots & \dots & W_n \\ W_1 & W_2 & \dots & \dots & W_n \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ W_1 & W_2 & \dots & \dots & W_n \end{pmatrix}$$

Where $W = (W_1 \ W_2 \ \dots \ W_n)$

The vector W is called the fixed point or stationary vector and this observation is supported by calculating higher powers of P. That is the rows $(W_1 \ W_2 \ \dots \ W_n)$

The rows are supposed to add up to 1 but do not sometimes due to round-up errors.

Thus,

Suppose P is a transition matrix of a regular Markov chain then

- (i) P^n approaches a stochastic matrix H as $n \rightarrow \infty$
- (ii) Each row of H is the same probability vector $W = (W_1 \ W_2 \ \dots \ W_n)$. The components of W are all positive.

The practical calculation of the fixed probability vector of a transition matrix P is done by using $WP_1 = W$ which gives rise to n equations in n unknowns and $\sum_{i=1}^n W_i = 1$ (Calvet and Adlai, 2004; Kemeny and Thompson, 1974; Kulkarni, 1995;).

3.3 Derivation of the Three State Transition Matrix

The transition matrix we would require involves three states only as the stock (Banking) assumes basically three states. The states are the chances that a stock decreases, that it remains the same (unchanged) and that it increases. We state the three states as follows:

D = Bank share price decreases

U = Bank share price remains the same

I = Bank share price increases.

Matrix of transition probabilities provides a precise description of the behavior of a Markov chain. Each element in the matrix represents the probability of the transition from a particular state to the next state. The transition probabilities are usually determined empirically, that is based solely on experiment and observation rather than theory. In another way, relying or based on practical experience without reference to scientific principles. Historical data collected can be translated to probability that constitute the Markov matrix of probabilities.

To compute the probability matrix for Markov process with three (3) states, one can compile a table as shown in the table below.

Table1. Transition Matrix

State	1	2	3	Sum of row
1	P_{11}	P_{12}	P_{13}	T_1
2	P_{21}	P_{22}	P_{23}	T_2
3	P_{31}	P_{32}	P_{33}	T_3

Each entry P_{ij} in the table refers to the number of times a transition has occurred from state i to state j . The probability transition matrix is formed by dividing each element in every row by the sum of each row.

This research covered the two top banks quoted on the Nigerian Stock exchange. The data on share price of the two banks namely first bank of Nigeria Plc (FBN), and Guaranty trust bank plc (GTB), were collected from the daily list published by the Nigeria stock exchange (Cashcraft Asset Management Limited) from 2005 to 2010.

The transition from one state to another (that is the share price movement pattern, which could be that a decrease in price can be followed by another decrease or a decrease is followed by unchanged or a decrease followed by an increase etc) was observed from the data collected and the result for each bank for the period (6 years) under study was compiled using Microsoft Excel as follows;

Table 2. The Share Price Movement of FBN from 2005-2010

	Decrease in Share price (D)	Unchanged in Share price (U)	Increase in Share price (I)
Decrease in share price (D)	290	20	175
Unchanged in share price (U)	17	182	23
Increase in share price (I)	178	24	249

Table 3. The Share Price Movement of GTB from 2005-2010

	Decrease in Share price (D)	Unchanged in Share price (U)	Increase in Share price (I)
Decrease in share price (D)	297	26	222
Unchanged in share price (U)	38	18	25
Increase in share price (I)	204	35	280

Table 4. The Share Price Movement of the Two Top Banks combined from 2005-2010.

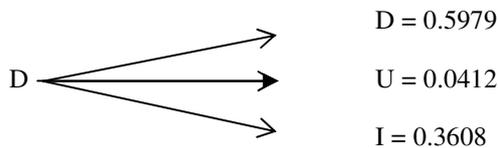
	Decrease in Share price (D)	Unchanged in Share price (U)	Increase in Share price (I)
Decrease in share price (D)	587	46	397
Unchanged in share price (U)	55	200	48
Increase in share price (I)	382	59	529

Consequently, from the share price movement compiled and the transition probabilities computed, the transition matrix for each bank is therefore as shown below;

Transition Matrix for FBN

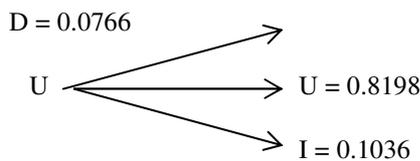
$$P = \begin{pmatrix} 0.5979 & 0.0412 & 0.3608 \\ 0.0766 & 0.8198 & 0.1036 \\ 0.3947 & 0.0532 & 0.5521 \end{pmatrix}$$

With values for each vector movement of the FBN as follows.



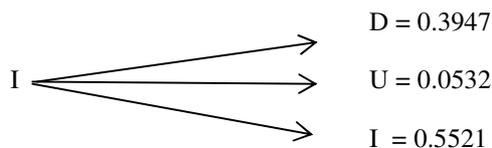
This means that

- (i) 0.5979 banks share price that decrease will still decrease
- (ii) 0.0412 bank share price that decreases will remain the same
- (iii) 0.3608 bank share price that decreases will increase.



This means that

- (i) 0.0766 banks shares price that remains the same will decrease
- (ii) 0.8198 banks share price that remains the same will still remain the same.
- (iii) 0.1036 banks share price that remain the same will increase.



This also means that

- (i) 0.3947 banks share price that increases will decrease
- (ii) 0.0532 banks share price that increase will remain unchanged

(iii) 0.5521 bank share price that increase will still increase

Transition Matrix for GTB

$$p = \begin{pmatrix} 0.5450 & 0.0477 & 0.4073 \\ 0.4691 & 0.2222 & 0.3086 \\ 0.3931 & 0.0674 & 0.5395 \end{pmatrix}$$

Transition Matrix of the Two Banks Combined.

$$P = \begin{pmatrix} 0.5699 & 0.0447 & 0.3854 \\ 0.1815 & 0.6601 & 0.1584 \\ 0.3938 & 0.0608 & 0.5454 \end{pmatrix}$$

In order to shed more light on the share price movement of this banks the transition diagram of this banks were drawn as seen below;

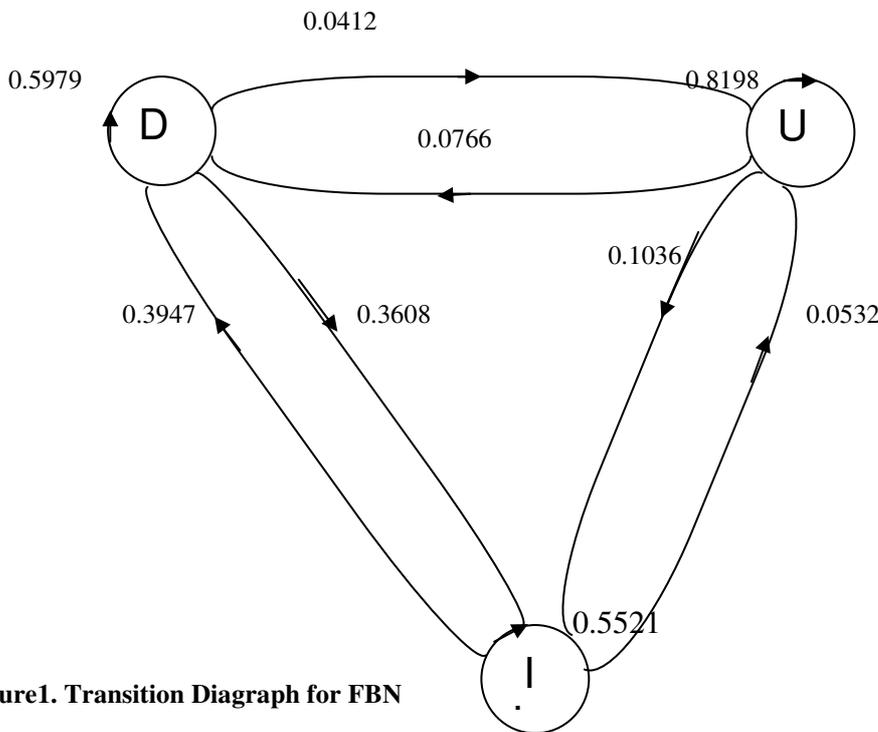


Figure1. Transition Diagram for FBN

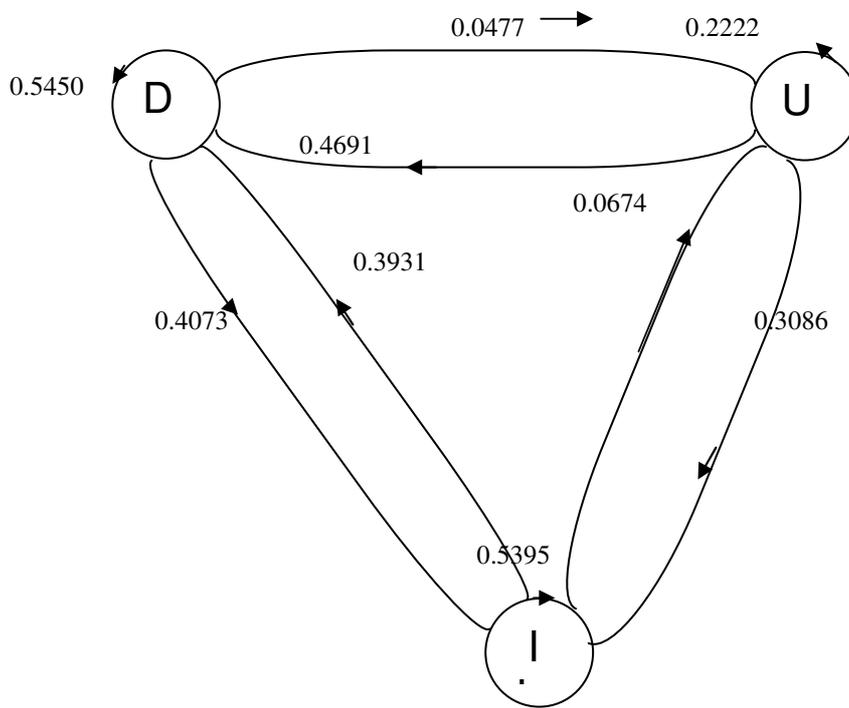


Figure 2. Transition Diagram for GTB

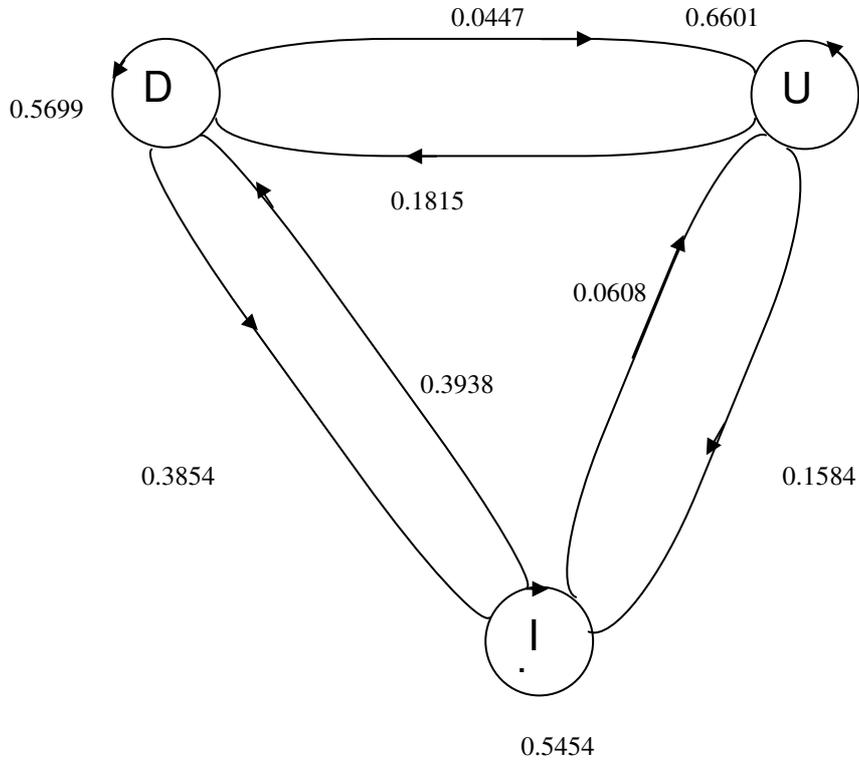


Figure 3. Transition Diagram for the two banks combined

Behavior of Share Price Movement.

The higher-order transition probability $P_{ij}^{(n)}$ of the transition matrix P_{ij} of each bank was calculated in order to observe the behavior of the share price and the results obtained using the R-Statistical Software are as shown below.

Table 5. Powers of the Transition Matrix for FBN Shares.

$$P = \begin{pmatrix} 0.5979 & 0.0412 & 0.3808 \\ 0.0766 & 0.8198 & 0.1036 \\ 0.3947 & 0.0532 & 0.5521 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.4463 & 0.1468 & 0.4066 \\ 0.2905 & 0.4259 & 0.2835 \\ 0.4428 & 0.1526 & 0.4044 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.4219 & 0.1901 & 0.3875 \\ 0.3789 & 0.2673 & 0.3535 \\ 0.4210 & 0.1917 & 0.3869 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.4151 & 0.2020 & 0.3822 \\ 0.4033 & 0.2234 & 0.3728 \\ 0.4149 & 0.2025 & 0.3820 \end{pmatrix}$$

$$P^{20} = \begin{pmatrix} 0.4132 & 0.2053 & 0.3806 \\ 0.4100 & 0.2112 & 0.3781 \\ 0.4131 & 0.2054 & 0.3806 \end{pmatrix}$$

$$P^{25} = \begin{pmatrix} 0.4126 & 0.2062 & 0.3801 \\ 0.4118 & 0.2079 & 0.3795 \\ 0.4126 & 0.2062 & 0.3802 \end{pmatrix}$$

$$P^{30} = \begin{pmatrix} 0.4123 & 0.2064 & 0.3799 \\ 0.4122 & 0.2069 & 0.3799 \\ 0.4124 & 0.2064 & 0.3799 \end{pmatrix}$$

Table 6. Powers of the Transition Matrix for GTB Shares.

$$P = \begin{pmatrix} 0.5450 & 0.0477 & 0.4073 \\ 0.4691 & 0.2222 & 0.3086 \\ 0.3931 & 0.0674 & 0.5395 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.4697 & 0.0688 & 0.4615 \\ 0.4698 & 0.0688 & 0.4612 \\ 0.4696 & 0.0688 & 0.4615 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.4696 & 0.0688 & 0.4615 \\ 0.4696 & 0.0688 & 0.4614 \\ 0.4696 & 0.0688 & 0.4615 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.4696 & 0.0688 & 0.4615 \\ 0.4696 & 0.0688 & 0.4614 \\ 0.4696 & 0.0688 & 0.4615 \end{pmatrix}$$

$$P^{20} = \begin{pmatrix} 0.4696 & 0.0688 & 0.4615 \\ 0.4696 & 0.0688 & 0.4614 \\ 0.4696 & 0.0688 & 0.4615 \end{pmatrix}$$

$$P^{25} = \begin{pmatrix} 0.4696 & 0.0688 & 0.4615 \\ 0.4695 & 0.0688 & 0.4614 \\ 0.4696 & 0.0688 & 0.4615 \end{pmatrix}$$

$$P^{30} = \begin{pmatrix} 0.4696 & 0.0688 & 0.4614 \\ 0.4695 & 0.0688 & 0.4614 \\ 0.4696 & 0.0688 & 0.4614 \end{pmatrix}$$

Table 7. Powers of the Transition Matrix for the Two Top Banks Combined.

$$P = \begin{pmatrix} 0.5699 & 0.0447 & 0.3854 \\ 0.1815 & 0.6601 & 0.1584 \\ 0.3938 & 0.0608 & 0.5454 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.4497039 & 0.1213776 & 0.4289185 \\ 0.4082262 & 0.2054697 & 0.3863041 \\ 0.4480810 & 0.1244000 & 0.4275189 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.4439734 & 0.1328809 & 0.4231458 \\ 0.4405546 & 0.1398236 & 0.4196219 \\ 0.4438503 & 0.1331308 & 0.4230189 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.4435056 & 0.1338308 & 0.4226636 \\ 0.4432233 & 0.1344039 & 0.4223727 \\ 0.4434954 & 0.1338514 & 0.4226532 \end{pmatrix}$$

$$P^{20} = \begin{pmatrix} 0.4434670 & 0.1339092 & 0.4226238 \\ 0.4434437 & 0.1339565 & 0.4225998 \\ 0.4434661 & 0.1339109 & 0.4226230 \end{pmatrix}$$

$$P^{25} = \begin{pmatrix} 0.4434638 & 0.1339157 & 0.4226205 \\ 0.4434619 & 0.1339196 & 0.4226186 \\ 0.4434637 & 0.1339158 & 0.4226205 \end{pmatrix}$$

$$P^{30} = \begin{pmatrix} 0.4434635 & 0.1339162 & 0.4226203 \\ 0.4434634 & 0.1339165 & 0.4226201 \\ 0.4434635 & 0.1339162 & 0.4226203 \end{pmatrix}$$

$$P^{35} = \begin{pmatrix} 0.4434635 & 0.1339162 & 0.4226203 \\ 0.4434635 & 0.1339163 & 0.4226202 \\ 0.4434635 & 0.1339163 & 0.4226203 \end{pmatrix}$$

Thus since P is the matrix

$$P = \begin{pmatrix} 0.5727 & 0.0740 & 0.3533 \\ 0.1830 & 0.6662 & 0.1508 \\ 0.3860 & 0.1024 & 0.5116 \end{pmatrix}$$

From table 7, after a period of 20 years it is noticed that equilibrium is attained.

$$P^{20} = \begin{pmatrix} 0.4435 & 0.1339 & 0.4226 \\ 0.4435 & 0.1339 & 0.4226 \\ 0.4435 & 0.1339 & 0.4226 \end{pmatrix}$$

The following comments can therefore be derived.

- (i) A bank's share price that will appreciate (I) given that it initially depreciated (D) is 0.4226
- (ii) A bank's price that will remain unchanged (U) given that its initially increased (I) is 0.1339
- (iii) A bank's price that will decrease (D) given that it's initially remain unchanged (U) is 0.4435

Other probabilities can be obtained from the equilibrium state above. Once there is movement from one state to another or even when there is no movement and suppose we do not know where the bank share price starts, when this happen we can speak of an initial probability vector.

$(P_1^{(0)}, P_2^{(0)}, P_3^{(0)})$ which leads to the n-step transition probabilities.

If the bank share price starts in a given state with probability $P^{(0)}$ (0.3333, 0.3333, 0.3333). Then the probability of the banks' share price appreciating after 30 years is given by the 3rd entry of

$$= (0.03333, 0.03333, 0.03333) \begin{pmatrix} 0.4435 & 0.1339 & 0.4226 \\ 0.4435 & 0.1339 & 0.4226 \\ 0.4435 & 0.1339 & 0.4226 \end{pmatrix}$$

$$= (0.4435 \quad 0.1339 \quad 0.4226)$$

That is to say the probability that a banks' share price will appreciate after 20 years if it starts in a state with probability of 0.3333 is 0.4226. Although this is an approximate result but we can still notice that the probability of the share price movement of the two banks continues to appreciates until the equilibrium after which it became constant. Also if each of the bank share price starts in a given state with probability $P^{(0)}$ (0.3333 0.3333 0.3333), then probability of their share prices appreciating, depreciating and remaining unchanged at a particular time can be calculated by multiplying the state vector by the higher probability at such time. For example the probabilities of FBN shares appreciating, depreciating and remaining unchanged after five years is

$$(0.3333 \quad 0.3333 \quad 0.3333) \begin{pmatrix} 0.4463 & 0.1468 & 0.4066 \\ 0.2905 & 0.4259 & 0.2835 \\ 0.4428 & 0.1526 & 0.4044 \end{pmatrix}$$

$$= (0.3932 \quad 0.2417 \quad 0.3648)$$

That is the probability of FBN share price appreciating after 5 years is 0.3648, depreciating is 0.3932 and the probability of the share price remaining unchanged is 0.2417. The probabilities of the two top banks in Nigeria stock exchange appreciating were computed for some period (that is some selected years from now) and the result is as shown in the table below.

Table 12: The Probabilities of the Two Top Banks in Nigeria Stock Exchange Appreciating.

Period(years from now)	FBN	GTB
5	0.3648	0.4614
10	0.3759	0.4614
15	0.3789	0.4614
20	0.3797	0.4614
25	0.3799	0.4614
30	0.3799	0.4614

The probabilities of the four banks stock remaining unchanged were also computed and the result is as shown in the table below.

Table 13. The Probabilities of the Four Top Banks in Nigeria Stock Exchange Remaining Unchanged.

Period(some selected years from now)	FBN	GTB
5	0.2417	0.0688
10	0.2163	0.0688
15	0.2092	0.0688
20	0.2073	0.0688
25	0.2067	0.0688
30	0.2065	0.0688

Similarly the probabilities of the two banks stock depreciating were computed and the result is as shown in the table below.

Table 14. The Probabilities of the two Top Banks in Nigeria Stock Exchange Depreciating.

Period(selected years from now)	FBN	GTB
5	0.3932	0.4697
10	0.4072	0.4696
15	0.4111	0.4696
20	0.4121	0.4696
25	0.4123	0.4695
30	0.4123	0.4695

The performance of the two banks combined was also computed and the result is as shown below.

Table 15. The Performance of the two Banks Combined

Period(some selected years from now)	Probabilities of banks share price Appreciating	Probabilities of banks Share price remaining unchanged	Probabilities of banks share price depreciating
5	0.3659	0.2155	0.4185
10	0.3696	0.2077	0.4226
15	0.3698	0.2072	0.4228
20	0.3699	0.2072	0.4229
25	0.3699	0.2072	0.4229
30	0.3699	0.2072	0.4229

Results and Conclusions.

As a result of this research work, the findings are;

- (i) From the derived matrices for the individual banks and the combined banks one is able to predicts the probability of moving from a given state to another state for a transition.

(ii) Regardless of a bank current share price, in the long run we could predict that its share price will depreciate with a probability of 0.4229, remain unchanged with probability of 0.2072 and appreciate with a probability of 0.3699.

(iii) From table 15 above we notice that the probability of the share price of the two top banks in Nigerian stock appreciating continue to increase until equilibrium was reached (after 20 years) and then became constant. On the other hand, regardless of a bank current price today (any of the two mentioned banks), the probability of its share price appreciating by the year 2025 is approximately 0.4. For the sake of investors or future investors to be well informed, that the probability of a bank share price depreciating is also approximately 0.4 which implies that an investor who buys a share today have equal chances of the share price appreciating or depreciating by the year 2025.

(iv) From Table 12, the probability of each of the two banks appreciating is also on the increase with GTB taking the lead with a probability of 0.4614 at equilibrium, and then followed by FBN with a probability of 0.3799.

(v) For a company price to remain the same over a period of time is not a good sign for the company's performance and considering the probability of unchanged for this sector, we will notice that at equilibrium, the probability of GTB shares remaining the same is 0.0688 which is the least compared to First bank which implies that GTB shares change hands more than the FBN.

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