

Analysis of Uniformly Loaded Simply Supported Rectangular Plates with Lifting Corners Using Strip Moment Ratio (SMR) method.

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Abstract

The application of the SMR to rectangular plates with corners allowed to lift is presented in this paper. The classical plate bi-harmonic equation for rectangular plates was adapted to the problem. It is assumed that the load on the plate is supported by the joint effort of transverse x-x strips and the longitudinal y-y strips only. The effect of the two diagonal x-y strips required for holding down is neglected. The effect of Poisson ratio on the span moments and plate's deflection is also presented. The indication is that an increase in Poisson ratio brings about an increase in the longer span moments of the plate and has got no significant effect on the shorter span moment and deflection of the plate. The results of practical application show close agreement with the exact classical results.

Keywords: Plate, strip moment ratio, SMR, plate deflection and Plate moments, Lifting Corners.

1.0 Introduction

Thin plates when subjected to transverse loading suffer bending and undergo transverse deflections which are usually small compared with the thickness of the plate. The governing equation for rectangular plate is familiar and has the form.

$$\frac{\partial^4 w}{\partial x^4 2} \ + \frac{2 \ \partial^4 w}{\partial x^2 \ \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}. \ \dots \ 1$$

Where q and D is the load on the plate and the plate stiffness respectively.

However, the solution of the above equation has been possible using the infinite trigonometric series and that is only for a limited class of problems. The absence of known trigonometric functions to satisfy some load and displacement functions and boundary conditions is a major drawback of the series solutions. This has prompted the introduction of several approximate methods including the finite element methods (1, 2, 3, and 4). The need to simplify the finite element method and yet obtain good results informed the introduction of the finite strip method [5] and the difference-based finite element method [6]. In the same vein, the grillage analysis (7) and finite difference methods have also been developed, all in an effort to resolve the plate governing equation with relative ease and accuracy. The yield line theory (8, 9, 10, 11, 12) and the strip method (13, 14, 15, 16,), which are plastic methods have been developed and applied predominantly for the analysis of reinforced concrete slabs. This paper applies the strip moment ratio theory SMR(17) to rectangular plates if the corners are allowed to lift.

1.1 The basic assumption

The development of the SMR method is hinged on the following concepts and assumptions.

- The validity of the circular beam theorem.
- The validity of hypothesis of plane section is assumed
- The thickness of plate is considered small compared to its other dimensions.

The load q on the plate is supported by the combined effort of the longitudinal x- strips and the transverse ystrips (Figs. 1). Thus, if α , β are used to denote the contributory load ratio for each type of strips respectively, then the sum of the ratio must be equal to unity. This in effect is a direct interpretation of the governing equation and hence

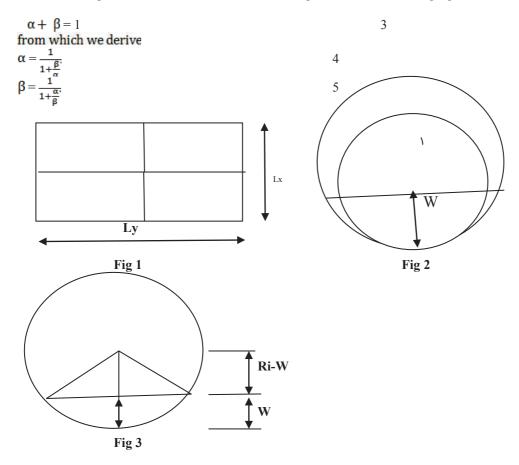
• represents an expression of compatibility of deflection at any given point (x, y) of the plate. See figures 2,3 and 4 also

For corners allowed to lift the above equation 1 becomes

$$\frac{\partial^4 w}{\partial x^4 2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$



The relationships between the load fractions can be expressed in the following equation



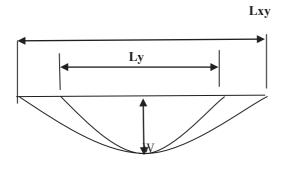


Fig 4

Fig 1 shows the authors' interpretation of the modified plate bi-harmonic equation as strips in the x-direction and y-direction.

Fig 2 shows the circular beam theory. Each of the beam strips bend as a segment of complete circle. The deflection W shown here is a constant for all the strips.

Fig 3 shows the compatibility of each of the beams of the plate. This introduces the concept of eccentric circles. Here the radii is different for each strip circle but the deflection is always the same

Fig 4 shows an exaggerated section of fig 3. Here actual lengths of each of the beam strips are indicated.

It is further assumed that the plate is undergoing circular bending (Fig. 2 and fig.3) from which the curvatures



are derived in the form

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$$R_x = \frac{L_x^2 + 4w^2}{8w}$$
 and $R_y = \frac{L_y^2 + 4w^2}{8w}$ 6
Where Rx and R_y are the radii of circle of the x-x and y-y beam strips.

 L_x , L_y are the length the x-x and y-y beam strips.

W is the deflection of the plate common to x-x and y-y strips. This is the compartibility condition of the plate model here described.

A limiting value of maximum plate deflection for the small deflection theory of plates is assumed to vary between 0 and 0.005 of the shortest span of plate.

2.0 Strip Moment Ratio (SMR)

The governing equation of thin rectangular plate is given by equation 1. From the equation, the bending and twisting moments per unit length have the following expressions.

twisting moments per unit length have the following expressions.

$$M_{x} = D_{x} \frac{\partial^{2} w}{\partial x^{2}} + D_{1} \frac{\partial^{2} w}{\partial y^{2}} = \frac{D_{x}}{R_{x}} + \frac{D_{1}}{R_{y}}.$$

$$M_{y} = D_{1} \frac{\partial^{2} w}{\partial x^{2}} + D_{y} \frac{\partial^{2} w}{\partial y^{2}} = \frac{D_{1}}{R_{x}} + \frac{D_{y}}{R_{y}}.$$

$$Where Dx = \frac{Et^{2}}{12(1-v^{2})} D_{y} = \frac{Et^{2}}{12(1-v^{2})}$$

$$D_{1} = \frac{vE_{x}t^{2}}{12(1-v^{2})}$$
If the primitive beam moments m, in the longitudinal (x, x) and m, in the

If the primitive beam moments m_x in the longitudinal (x - x) and m_y in the transverse (y - y) strips are factored by load ratios α , β respectively, then the plate moments become

$$M_{x} = \alpha m_{x}; M_{y} = \beta m_{y}$$

Using the expression for simple beam curvature equations 7, 8 can be represented as follows

Osing the expression for simple beam curvature equations 7, 8 can be represented by
$$M_x = \alpha m_x = \frac{D_x}{R_x} + \frac{D_1}{R_y}$$
 10
$$M_y = \beta m_y = \frac{D_y}{R_y} + \frac{D_1}{R_x}$$
 11

Substituting equations 6 into equations 10, 11 and 12, the following values of strip moment ratios are determined $\alpha = \frac{8w}{m_x} \frac{D_x}{4w^2 + L_x^2} + \frac{D_1}{4w^2 + L_y^2}$

$$\alpha = \frac{8W}{m_x} \frac{D_x}{4W^2 + L_x^2} + \frac{D_1}{4W^2 + L_y^2}$$

$$\beta = \frac{8w}{m_v} \frac{(D_y)}{4w^2 + L_y^2} + \frac{D_1}{4w^2 + L_x^2}$$

Where $L_{\boldsymbol{x}},$ and $\,L_{\boldsymbol{y}}$ are the width $\,$ and $\,$ length dimensions of the plate.

Denoting the aspect ratios $\frac{L_y}{L_x} = n$ and the deflection to shortest span ratio $\frac{w}{L_y} = h$ We have

$$L_v = n L_v$$

 $L_y = n L_x$ The ratio $\frac{\beta}{\alpha}$ can be established thus

$$\frac{\beta}{\alpha} = \frac{m_x}{m_y} \frac{\left(D_y}{4h^2 + n^2} + \frac{D_1}{4h^2 + 1^2} \right) \frac{\left(D_x}{4h^2 + 1^2} + \frac{D_1}{4h^2 + n^2}$$

2.1 Moments

It is clear that these ratios depend only on the mechanical and geometrical properties of the plate for a given value of h substituting these ratios into equations 3, 4 and 5, the strip moment ratios are determined as soon as the primitive strip moments (mx and my) are substituted. These will eventually yield α and β which are needed for the plate analysis for moments. For Isotropic plate

$$D_{x} = D_{y} = D; D_{1} = vD$$

The plate moments are

$$M_x = \alpha m_x$$
 and

$$M_v = \beta m_v$$

3.0 Deflections

In a similar manner as with the moments, the plate deflections are determined by factoring the primitive beam deflection Δ of the x strip by the modified x-x strip moment factor α . The effect of Poisson ratio ν increases the values of α . Therefore the deflection factor α_x is obtained where ν is made equal to zero in equations 7,8 and 9.



Remember the actual value of v is still intact in the deflection as it is part of the flexural rigidity of the plate.

$$\alpha_x$$
 is α_w when $\nu=0$ 15
The plate deflection Δ_p is

 $\Delta_{\rm p} = \alpha_{\rm x} \Delta$ 10

Where, Δ is the primitive beam deflection of the x-x strip.

4.0 Limit of $h = \frac{w}{L_x}$

In order that the derivations are limited to small deflection theory, it is necessary to assess the limit of the deflection. The design codes often specify the value in the form of a ratio, for example $\frac{L}{360}$ (i.e. $\frac{W}{L} = 0.0028$). The model allows for the input of various values of h. The limit of h from the beam theory (see fig 2) is between h=0 and h=0.5 when large deflection is expected.

5.0 Application

For a given plate geometry, all the terms in the equation 14 are known except h. Thus, imputing the allowable h and the relevant geometric parameters of the plates, the strip moment ratios are determined and hence the plate moments and deflections are obtained. The following solution algorithm is convenient for use in a typical problem.

Step 1: Assume h (allowable $0.5 \le h \ge 0$)

Step 2: Compute plate parameters n, D_x , D_y , D_1

Step 3: Compute quantities $\frac{\beta}{\alpha}$ and (eqn 14)

Step 4: Compute \propto , and β (eqn 3 and 4).

Step 5: Compute plate moments

 $M_{x} = \mathbf{a} \mathbf{m}_{x}$ $M_{v} = \mathbf{\beta} \mathbf{m}_{v}$

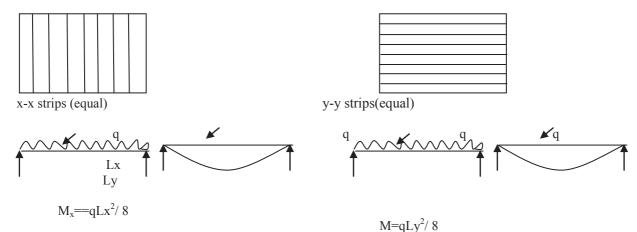
Step 6: Compute plate deflection (eqn 15 and 16)

6.0 Problems of interest

Determine the moments and maximum deflection of a simply supported rectangular Plate with a uniformly distributed load q for various aspect ratios if corners are allowed to lift. Also find the effect of varying h and Poisson ratio ν .

6.1 Solution for UDL

The primitive strip moments for this problem are as given below:



When these moment values are used in the steps listed above, the following results were produced. The average L_{xy} or L_{yx} is the average length of these unequal strips which is two-third this value.

6.2 Results

The results shown below are for various aspect ratio (n) of the plate. α , β are ratios; M_x and M_y are coefficients of qa² and Δ is coefficient of qa⁴/D. Where a, is the length of the shorter span and D Is the Plate flexural rigidity.



TABLE 1: load ratio in the x-x and y-y direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with corners allowed to lift. The deflection of the plate is also shown for h=0 and ν =0. graphical representations are also shown.

- Ir	Circutions t								
n	α	β	$M_{x SMR}/qa^2$	MxBS8110	Mxmarcus	$M_{y SMR}/qa^2$	MyBS8110	Mymarcus	Δsmr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.006510
1.2	0.675	0.3254	0.08433	0.084	0.08438	0.0407	0.059	0.0406	0.008785
1.4	0.7936	0.2065	0.09918	0.099	0.09913	0.0258	0.051	0.0259	0.010331
1.6	0.8676	0.1324	0.10845	0.1085	0.1085	0.0165	0.042	0.0165	0.011297
1.8	0.9130	0.0869	0.11413	0.114	0.11438	0.0109	0.0352	0.0121	0.011888
2	0.9412	0.0588	0.11765	0.118	0.11763	0.0074	0.029	0.0074	0.012255
3	0.9878	0.0122	0.12348	-	0.1235	0.0015	-	0.0015	-

TABLE 2: load ratio in the x-x, y-y and xy direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with lifting corners. The deflection of the plate is also shown for h=0 and

 $\nu = 0.15$ graphical representations are also shown

.n	α	β	M _{x SMR} /qa ²	$M_{y SMR}/qa^2$	Δ smr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.006510417
1.2	0.653126	0.34687357	0.0816408	0.0433592	0.00850425
1.4	0.761677	0.23832327	0.09520959	0.0297904	0.009917666
1.6	0.833686	0.16631417	0.10421073	0.0207893	0.010855284
1.8	0.88083	0.11916982	0.11010377	0.0148962	0.011469143
2	0.912088	0.08791209	0.11401099	0.010989	0.011876145
3	0.987805	0.02774498	0.12153188	0.0034681	0.012659571

TABLE 3: load ratio in the x-x, y-y and xy direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with lifting corners. the deflection of the plate is also shown for h=0 and \mathbf{v} =0.3. classical results from timoshenko is also shown. graphical representations are also shown.

n	α	β	$M_{x SMR}/qa^2$	$M_{y SMR}/qa^2$	Δ smr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.006510417
1.2	0.636327	0.3636733	0.07954084	0.0454592	0.008285504
1.4	0.736107	0.26389258	0.09201343	0.0329866	0.009584732
1.6	0.805492	0.19450801	0.1006865	0.0243135	0.010488177
1.8	0.853291	0.14670873	0.10666141	0.0183386	0.011110563
2	0.886598	0.11340206	0.11082474	0.0141753	0.011544244
3	0.987805	0.0423341	0.11970824	0.0052918	0.012469608

TABLE 4: load ratio in the x-x, y-y and xy direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with lifting corners. the deflection of the plate is also shown for h=0.25 and \mathbf{v} =0. graphical representations are also shown.

n	α	β	$M_{x SMR}/qa^2$	$M_{y SMR}/qa^2$	Δ smr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.006510417
1.2	0.660658	0.33934195	0.08258226	0.0424177	0.008602318
1.4	0.77605	0.22395012	0.09700623	0.0279938	0.010104816
1.6	0.851959	0.14804112	0.10649486	0.0185051	0.011093215
1.8	0.900459	0.09954131	0.11255734	0.0124427	0.011724722
2	0.931507	0.06849315	0.11643836	0.0085616	0.012128995
3	0.987805	0.0147929	0.12315089	0.0018491	0.012828217

TABLE 5: load ratio in the x-x, y-y and xy direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with lifting corners, the deflection of the plate is also shown for h=0.25 and ν =0.15, graphical representations are also shown.



n	α	β	$M_{x SMR}/qa^2$	$M_{y SMR}/qa^2$	Δ smr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.006510417
1.2	0.642628	0.35737206	0.08032849	0.0446715	0.008367551
1.4	0.748193	0.25180715	0.09352411	0.0314759	0.009742094
1.6	0.821136	0.17886379	0.10264203	0.022358	0.010691878
1.8	0.87044	0.12956036	0.10880495	0.016195	0.011333849
2	0.903883	0.09611712	0.11298536	0.0120146	0.011769308
3	0.987805	0.03011704	0.12123537	0.0037646	0.012628684

TABLE 6: load ratio in the x-x, y-y and xy direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with lifting corners. the deflection of the plate is also shown for h=0.25 and \mathbf{v} =0.3. classical results from timoshenko is also shown. graphical representations are also shown.

n	α	β	Mx SMR/qa2	My SMR/qa2	Δ smr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.006510417
1.2	0.628588	0.37141166	0.07857354	0.0464265	0.008184744
1.4	0.725916	0.27408447	0.09073944	0.0342606	0.009452025
1.6	0.795737	0.20426288	0.09946714	0.0255329	0.01036116
1.8	0.845003	0.15499744	0.10562532	0.0193747	0.011002638
2	0.879905	0.12009512	0.10998811	0.0150119	0.011457095
3	0.987805	0.04440154	0.11944981	0.0055502	0.012442688

TABLE 7: load ratio in the x-x, y-y and xy direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with lifting corners. the deflection of the plate is also shown for h=0.5 and $\mathbf{V}=0$. graphical representations are also shown.

n	α	β	$M_{x SMR}/qa^2$	$M_{y SMR}/qa^2$	Δ smr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.006510417
1.2	0.637261	0.36273941	0.07965757	0.0453424	0.008297664
1.4	0.743642	0.25635767	0.09295529	0.0320447	0.009682843
1.6	0.82004	0.17995969	0.10250504	0.022495	0.010677608
1.8	0.872916	0.12708418	0.10911448	0.0158855	0.011366091
2	0.909091	0.09090909	0.11363636	0.0113636	0.011837121
3	0.987805	0.02173913	0.12228261	0.0027174	0.012737772

TABLE 8: load ratio in the x-x, y-y and xy direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with lifting corners. the deflection of the plate is also shown for h=0.5 and $\mathbf{V}=0.15$. graphical representations are also shown.

n	α	β	$M_{x SMR}/qa^2$	$M_{y SMR}/qa^2$	Δ smr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.006510417
1.2	0.625135	0.37486533	0.07814183	0.0468582	0.008139774
1.4	0.723329	0.27667089	0.09041614	0.0345839	0.009418348
1.6	0.795902	0.20409807	0.09948774	0.0255123	0.010363306
1.8	0.848031	0.15196938	0.10600383	0.0189962	0.011042065
2	0.885177	0.11482255	0.11064718	0.0143528	0.011525748
3	0.987805	0.03638254	0.12045218	0.0045478	0.012547102



TABLE 9: load ratio in the x-x, y-y and xy direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with lifting corners, the deflection of the plate is also shown for h=0. 5 and v=0.3, classical results from Timoshenko is also shown, graphical representations are also shown.

n	α	β	$M_{x SMR}/qa^2$	$M_{y SMR}/qa^2$	Δ smr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.006510417
1.2	0.615731	0.38426916	0.07696636	0.0480336	0.008017329
1.4	0.707266	0.29273435	0.08840821	0.0365918	0.009209188
1.6	0.776346	0.22365428	0.09704321	0.0279568	0.010108668
1.8	0.827368	0.17263211	0.10342099	0.021579	0.010773019
2	0.864865	0.13513514	0.10810811	0.0168919	0.011261261
3	0.987805	0.0498008	0.1187749	0.0062251	0.012372385

TABLE 10: load ratio in the x-x, y-y and xy direction together with the bending moments in the x-x and y-y spans of a s-s udl rectangular plate with lifting corners, the deflection of the plate is also shown for h=0.15 and v=0.3, classical results from timoshenko is also shown, graphical representations are also shown.

n	α	β	$M_{x SMR}/qa^2$	$M_{y SMR}/qa^2$	Δ smr/qa4/D
1	0.5	0.5	0.0625	0.0625	0.006510417
1.2	0.633206	0.3667939	0.07915076	0.0458492	0.008244871
1.4	0.732083	0.26791663	0.09151042	0.0334896	0.009532336
1.6	0.801707	0.19829331	0.10021334	0.0247867	0.010438889
1.8	0.850119	0.14988147	0.10626482	0.0187352	0.011069252
2	0.884063	0.11593695	0.11050788	0.0144921	0.011511238
3	0.987805	0.04309586	0.11961302	0.005387	0.012459689

6.3 Discussion of Results:

The results in Tables 1 to 10 show the effect of h and poison ratio v on the moments and deflection of S-S rectangular Plates when corners are allowed to lift. The effect of Poisson ratio on the results can be easily observed in Tables 1, 2 and 3. Here it is clear that Poisson ratio affects the longer span moment greatly. As the aspect ratio of the plate increases the results asymptotes to $0.125~\rm qa^2$ for moment and $0.013021\rm qa^4/D$ for deflection. This is for any value of Poisson ratio v. the effect of the serviceability criterion h on moments and deflection is significant. Classical solutions are not common for rectangular plates simply supported with corners not held down and so comparison of results obtained by the SMR for this type of plate are only possible with codes such as BS8110 and for poisson ratio v=0 only .

7.0 Conclusion.

The new method for the analysis of plate SMR has been extended to cases of corners allowed to lift with great success. Several exact results were confirmed and without any modification can be used in the design office. The practical significance of the research as has been earlier observed in (17) consists in the fact that the established SMR method for the analysis of uniformly loaded simply supported rectangular plates:-

- Enhances close agreement of the theoretical prediction with code value, which is the combination of experimental values and elastic method predictions.
- Permits quantitative estimation of moments and deflections of plates in a simple way. Plates of any material can be analyzed and designed because Poisson ratio is adequately taken care of.
- Permits the use of serviceability limit of deflection which primitive value or expected value can be inputted into the model before the solution is obtained.
- May be incorporated in scientific research codes of practice the areas of steel, plastic, wood, re-inforced concrete and any other material.

The SMR method has been extended to plates of various support conditions and load types. It has also been extended to Skew plates, irregular plates, circular plates, Triangular plates with great accuracy. Plate vibration and plate buckling problems have also been solved with the method. This is the second extension of the SMR method published others shall follow subsequently.



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