

# Mathematical Modeling of Wastewater Aeration Efficiency using Natural Stepped Cascades

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## Abstract

The concentration of dissolved oxygen is very important in wastewater whether before or after treatment. It is a prime indicator in determining how satisfactory a biological wastewater treatment is occurring. In this paper, the aeration performance of stepped cascade was investigated. The natural land formation of one of the mountains in Lebanon is shaped in several cascade structures with different heights. The aeration efficiency, wind speed, step height and flow rate were measured. An empirical correlation predicting the effect of the wind speed, step height and flow rate on the aeration efficiency ( $E_{20}$ ) was developed. The results indicated that the step height and the flow rate had a significant effect on the aeration efficiency. For low wind speed there is no significant wind effect on the aeration efficiency, while high wind speed improves the aeration efficiency due to higher turbulence. It was noticed that the aeration efficiency is shown to depend on the biochemical oxygen demand.

**Keywords:** Aeration efficiency, dissolved oxygen, stepped cascade, mathematical modeling

## 1. Introduction

Aeration is the primary means by which the wastewater replenishes its oxygen content. It is often required in water bodies that suffer from anoxic conditions, such as sewage discharges, agricultural run-off or in wastewater treatment plants. Aeration can be achieved through the infusion of air into the bottom of the water bodies or by surface agitation. Aeration provides wastewater with its requirement from oxygen to support aerobic treatment and stabilization of wastewater. Dissolved oxygen (DO) is a major contributor to wastewater quality, where aerobic bacteria breathe oxygen to decompose the organic matters. According to adopted treatment processes, the proposed aeration system may differ from extensive use of mechanical systems for oxygen transfer to biological reactors in large wastewater treatment facilities, to simple use of cascades for increasing oxygen concentrations in wastewater where appropriate. The construction and operation costs of wastewater treatment plants are extremely sensitive to the changes in the aeration rate. Regarding the operational costs for biological treatment, approximately 70% of the total energy consumption is used for the aeration system (Bischof et al. 1992). That shows how important the use of natural and environmental friendly systems is.

During the flow of wastewater in gravity aeration, the wastewater and air are in close contact where gas exchange occurs between the air and the wastewater. Oxygen diffuses from the air into the wastewater and helps to increase the dissolved oxygen content of the wastewater (Sanjib et al. 2010). Many investigators have studied the parameters affecting the aeration efficiency for stepped cascades. Most of them concluded that it depends on the step height, flow discharge and temperature. In this paper, a mathematical model was developed to determine the effect of the wind speed, step height, flow rate and biochemical oxygen demand on the stepped cascade aeration efficiency.

As the wind speed increases, the water turbulence increases. The strong turbulent mixing increases the coefficient of transfer in comparison with a quiescent fluid, and the large amount of entrained air bubbles increases the air-water interface area due to the cumulative bubble surface areas.

## 2. Background

The hydraulic aspect of stepped cascade was addressed by a number of researchers. Chanson (1993, 1994, 1995, 1996), Chamani & Rajaratnam (1994), Chanson & Toombes (1997, 2002) investigated a stepped cascade for various flow rates and different step geometries. Stepped cascade flows can be classified into nappe flow, transition flow and skimming flow. Baylar et al. (2006) found that nappe flow occurs with low discharges, a transition flow regime takes place for intermediate discharges. Baylar et al. (2007) confirmed that nappe flow leads to larger aeration efficiency compared to transition or skimming flows.

The aeration efficiency of stepped cascade structures has been studied experimentally by several researchers. Gameson (1957) was the pioneer of studying the aeration potential of weirs in rivers. Apted and Novak (1973), Avery and Novak (1978), and Nakasone (1987) have conducted several laboratory investigations into weir aeration. Wilhelms et al. (1992), Chanson (1995), Ervine (1998), and Gulliver et al. (1998) reviewed aeration efficiency studies of hydraulic structures. A number of empirical equations limited to certain boundary conditions were developed (Koduri & Barkdoll 2003). The aeration efficiency of any hydraulic structure,  $E$ , may be defined as (Gameson 1957):

$$E = \frac{C_d - C_u}{C_s - C_u} = \frac{r - 1}{r}$$

Where;

$C_d$  &  $C_u$  are the downstream and upstream dissolved oxygen concentration respectively;

$C_s$  is the oxygen saturation concentration;

$r$  is the oxygen deficit ratio.

$E$  equals to zero corresponds to no aeration and 1 to total downstream saturation which means that the full transfer up to saturation value has occurred. For  $E$  greater than one means the downstream water is supersaturated.

Avery and Novak (1978) developed the following equation to calculate the overall deficit ratio ( $r_{tot}$ ) and efficiency ( $E_{tot}$ ) for a number of cascade steps ( $n$ ):

$$r_{tot} = r^n$$

$$E_{tot} = 1 - (1 - E)^n$$

Several factors affect the aeration efficiency of stepped cascades. According to Henry's Law, liquid temperature has a direct proportional effect on the solubility of gas in liquid. Liquid temperature is one of the most important parameters to which the oxygen transfer efficiency is sensitive. Dissolved oxygen levels fluctuate hourly, daily, weekly and seasonally. They vary with temperature and altitude. Cold water holds more oxygen than warm water and water holds less oxygen at higher altitudes. Several researchers examined the temperature effect on the aeration efficiency. The most often used temperature correction factor is the one developed by Gulliver et al. (1990).

$$E_{20} = 1 - (1 - E)^{\frac{1}{f}}$$

Where  $E$  is the aeration efficiency at the water temperature of measurement ( $T$ ) in Celsius,  $E_{20}$  the aeration efficiency at the 20°C and  $f$  is described by:

$$f = 1.0 + 0.021(T - 20) + 8.26 \times 10^{-5}(T - 20)^2$$

Gameson (1957) considered the effect of water quality on the aeration efficiency by the use of "water quality factor" in equations for the deficit ratio. Avery and Novak (1978) used a similar constant in several experimental studies. Khalifa et al. (2011) conducted several studies that account for the chemical oxygen demand (COD) in the aeration efficiency.

Baylar and Bagatur (2000) found that as the drop height increases, the surface of the nappe first becomes rough and then begins to oscillate during the drop, entraining the air.

Studies done by Apted and Novak (1973), Avery and Novak (1978), and Van der Kroon and Schram (1969a, 1969b) show a constant increase in the aeration efficiency with a decrease in discharge. Several models and equations have been developed for the oxygen transfer and aeration efficiency as mentioned below:

Gameson et al. (1958)

$$r = 1 + 0.34 a_w b_w H_d (1 + 0.046T)$$

where  $a_w = 0.85$  for sewage water and  $b_w = 1.3$  for stepped weirs

Water Research Laboratory (1973)

$$E = 1 - [1 + 0.38 a_w b_w H_d (1 + 0.046T)(1 - 0.11H_d)]^{-1}$$

Nakasone (1987)

$$E_{20} = 1 - e^{-aH_d^b q_w^c d_s^d}$$

where:

$q_w$  is the flow rate per unit width

$d_s$  is the pool depth

$H_d = D + 1.5 H_c$

$D$  = Weir drop height above the downstream water level

$H_c$  = Critical water depth on the weir

$a, b, c$  &  $d$  are constants that vary with the flow rate and cascade geometry

Imhoff and Fair (1956) were among the first to discuss wind effects in general terms. Oxygen can be driven into the flow when the wind disturbs the surface of the water body.

Most of the available literatures focused on the effect of the hydraulic loading rate and step height on the aeration efficiency. However, no study has been attempted to take into consideration the wind speed and organic loading rate effects. The objective behind this paper is to find a mathematical model for the computation of the aeration efficiency of stepped cascade based on the flow rate, wind speed, step height & the biochemical oxygen demand  $BOD_5$ .

### 3. Materials & Methods

The study area is located at 33°44.40' N latitude and 35°27.71' E longitude at an elevation of 300 m asl. The study area has a hot-summer Mediterranean climate (Köppen Climate Classification) characterized by warm days and nights. The mean monthly temperature ranged from 08 °C (January) to 32 °C (August). The prevailing wind during the morning and the afternoon is from the west; at night it reverses, blowing from the land out to sea. Raw wastewater was obtained from one of the manholes located on the main sewer line of the campus of Beirut Arab University in Debbieh in Lebanon. The sewer line collects wastewater from toilets and kitchens of the Engineering, Architecture and Science faculties, student's dorms and administration buildings. Figure 2 shows the location of the project.



Figure 2: Project Location

Figure 3 shows the schematic representation of the stepped cascade. The wastewater was pumped from the sewer manhole through a plastic pipe to an upstream basin 1.5m long, 0.6m wide and a variable depth (40cm, 60cm & 75cm). The discharge was measured by means of a flowmeter followed by a flow control valve installed on the discharge pipe. The number of steps is five. Variations in flow rate and step cascade height were tested to find their effects on the level of aeration along with different BOD<sub>5</sub> concentrations and wind speed. Wastewater samples were collected from the upstream and downstream ends for 90 days in order to determine the characteristics of the wastewater which includes dissolved oxygen (DO), biochemical oxygen demand (BOD<sub>5</sub>), chemical oxygen demand (COD), and pH. All samples were analyzed according to the APHA standards. Dissolved oxygen concentrations were measured using the portable meter CyberScan DO100 Model. The DO meter was calibrated daily, prior to use, as recommended by the manufacturer. Three cascade heights H (40cm, 60cm and 75 cm) were tested, each for a set of flow rates of (10, 20, 30, 40, 50 & 60 m<sup>3</sup>/hr). The physical and chemical characteristics of sampled wastewater are shown in Table 1.

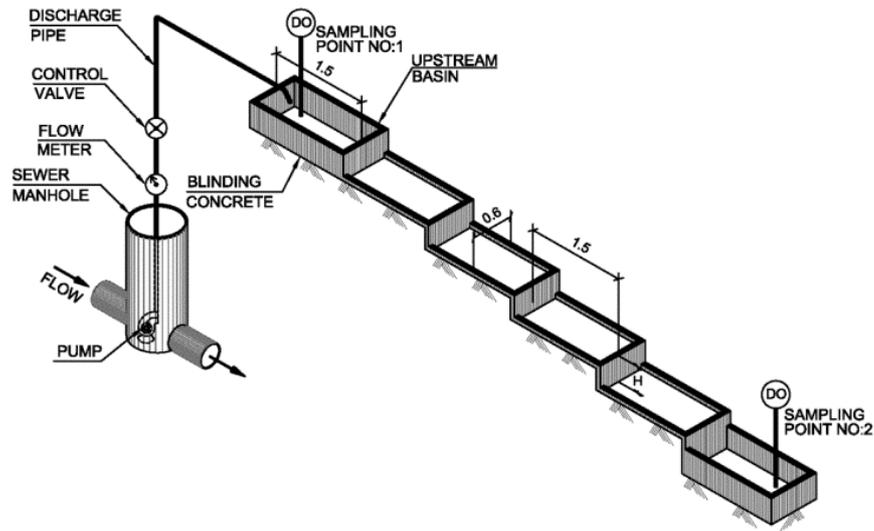


Figure 3: A Schematic representation of the field work

Table 1: Physical & chemical characteristics of sampled wastewater

Parameters	Average $\pm$ Standard Deviation
BOD <sub>5</sub> (mg/l)	252 $\pm$ 15
COD (mg/l)	326 $\pm$ 18
pH	7.24 $\pm$ 0.22

#### 4. Results & Discussions

In this study, the values of the aeration efficiency of stepped cascades were obtained depending on the flow rate (Q), drop height (h), wind speed ( $V_{wind}$ ) and biochemical oxygen demand (BOD<sub>5</sub>). Figures 3 to 5 show the observed aeration efficiency values as a function of wind speed for the six different flow rates (10, 20, 30, 40, 50 & 60 m<sup>3</sup>/hr) and different drop height (40cm, 60cm and 75cm). The results indicated that drop height, discharge and wind speed are important parameters influencing the aeration efficiency. The aeration efficiency increased with the increase of the drop height, wind speed and discharge. This can be explained that larger drop height and higher wind speed increase the turbulence which lead to greater bubble penetration into the wastewater.

It is evident from figures 3,4 & 5 that there is no significant wind effect on the aeration efficiency for wind speed below 4 m/sec, while higher wind speed (4 to 9 m/sec) increases the aeration efficiency similar to the findings of Yu and Hamrick (1984). Figure 3 shows that for a drop height of 75cm at a flow rate of 60m<sup>3</sup>/hr, the aeration efficiency increases from 28 to 34% for a wind speed increase from 1.3 to 4 m/sec, while it increases from 34 to 56% for a wind speed increase from 4 to 7.6 m/sec. For the same drop height but for a flow rate of 10 m<sup>3</sup>/hr, the aeration efficiency increases from 5% to 12% for a wind speed increase from 1.3 m/sec to 4 m/sec, while it increases from 12% to 30% for a wind speed increase from 4 m/sec to 7.6 m/sec.

Comparing the aeration efficiency shown in figures 3 to 5, it is noticed that the higher the drop height, the higher is the aeration efficiency at the same wind speed. At wind speed of 6.3 m/sec and a discharge of 40 m<sup>3</sup>/hr, the aeration efficiencies are 8.5%, 12.1% and 42% for step height of 40 cm, 60 cm and 75 cm.

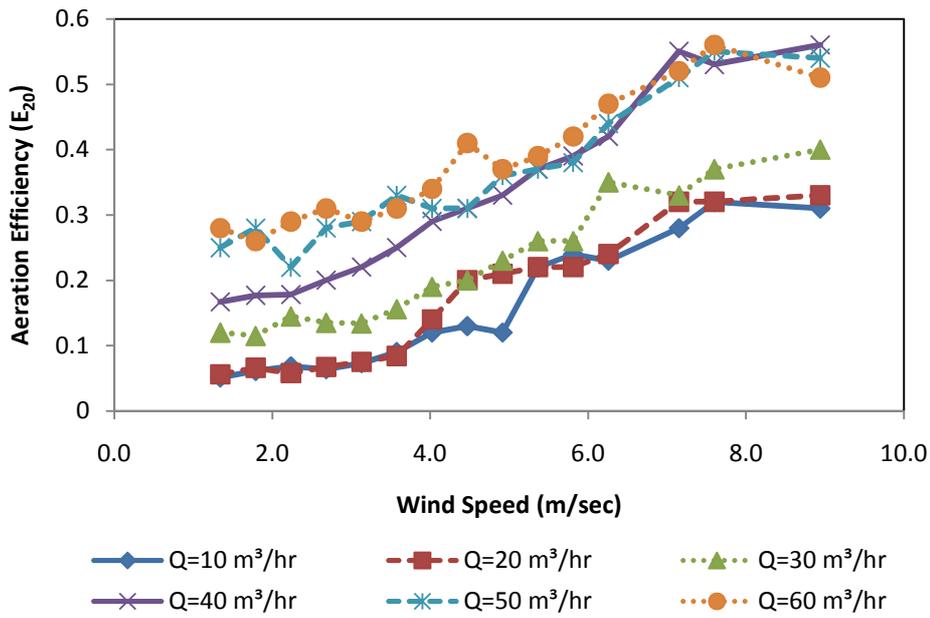


Figure 4: Variation in Aeration efficiency with wind speed for different flow rates and 75 cm step height

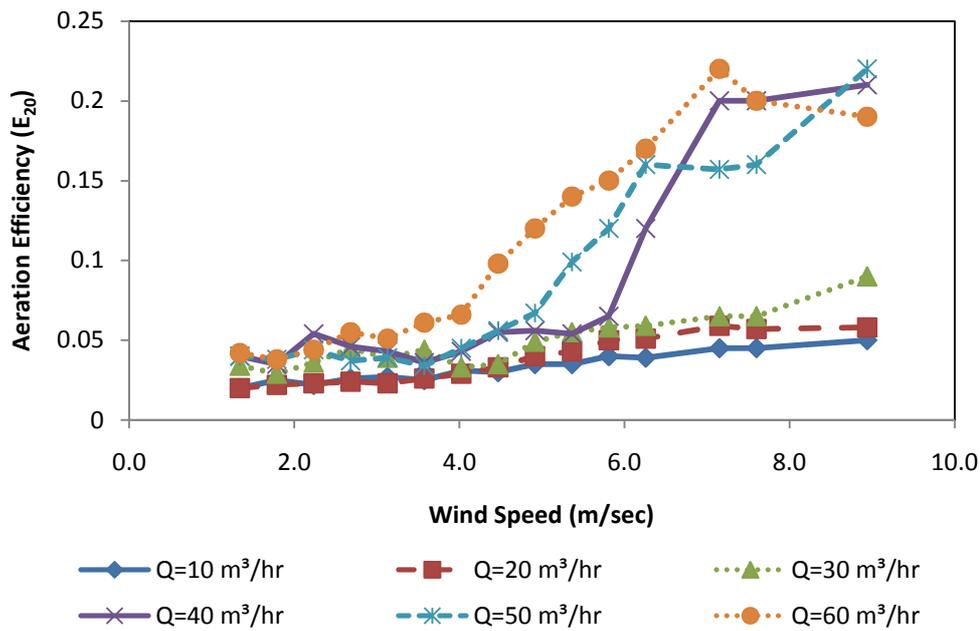


Figure 5: Variation in Aeration efficiency with wind speed for different flow rates and 60 cm step height

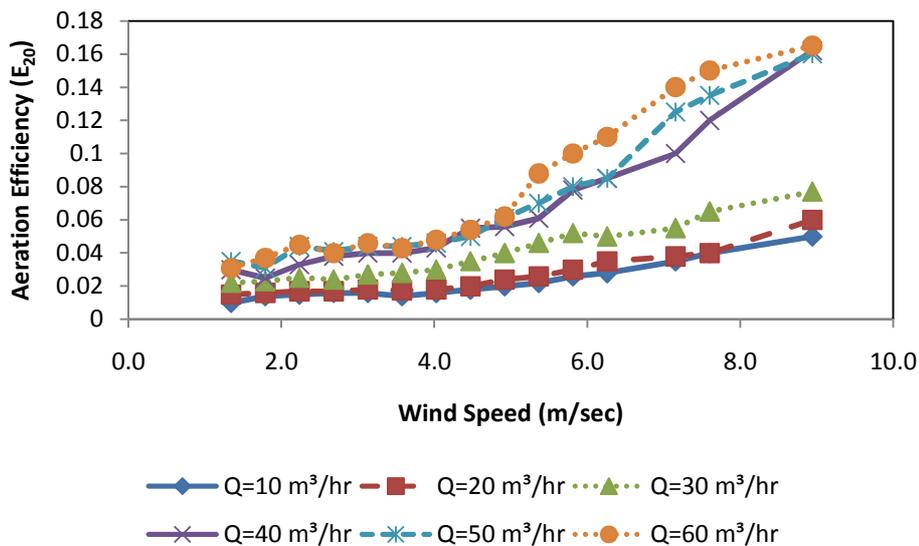


Figure 6: Variation in Aeration efficiency with wind speed for different flow rates and 45 cm step height  
 Based on the site readings for all the parameters stated above, the below equation was developed taking into account the wind speed ( $V_{wind}$ ) in m/sec, the biochemical oxygen demand ( $BOD_5$ ) in mg/l, the flow rate ( $Q$ ) in  $m^3/sec$  and the step height ( $H$ ) in meters. The correlation coefficient for the predicted equation is 0.95.

$$E_{20} = 1 - e^{-8.8H^{5.76}Q^{0.68}V_{wind}^{0.92}BOD_5^{0.02}}$$

The measured aeration efficiencies were compared to those predicted in the above equation. Good agreement between the measured aeration efficiencies and the values computed from the predictive equation was obtained. Further confidence in the correlation can be seen from the fit line equation shown in figure 6 with a high  $R^2$  value of 0.901.

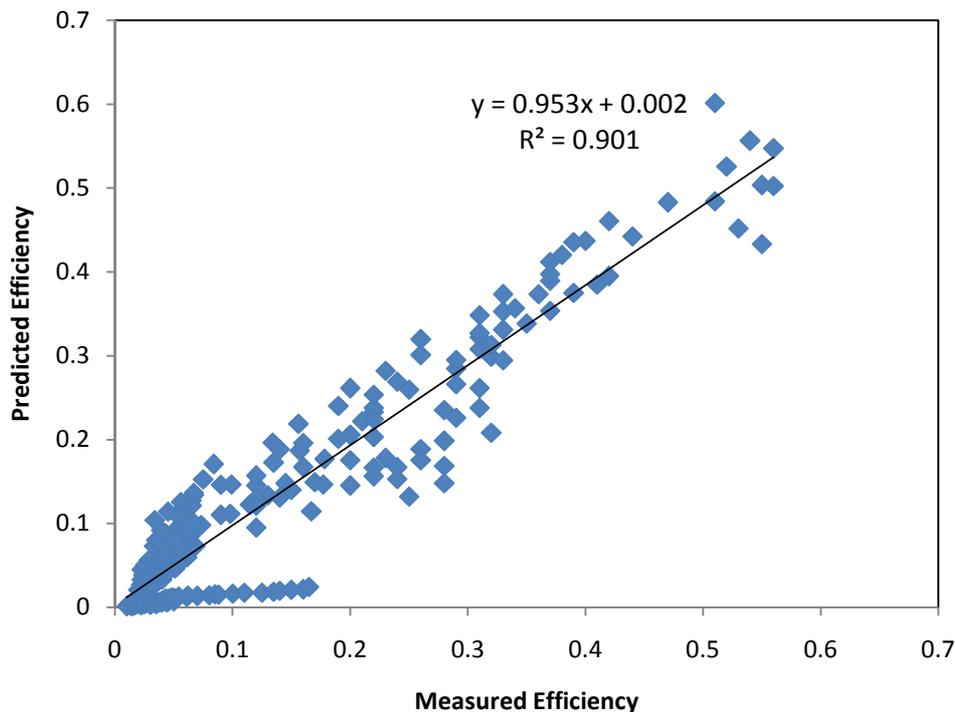


Figure 7: The comparison of measured and computed aeration efficiencies

### 5. Conclusion

A series of field experiments were run on stepped cascades in order to determine the effect of flow rate, step height, wind speed and biochemical oxygen demand on the aeration efficiency. An empirical correlation was developed that predicted the aeration efficiency of stepped cascades. It was found that stepped cascades are very

efficient in oxygen transfer because of the strong turbulent mixing. It was apparent from the results that the aeration efficiency increases with the increase of the drop height. It was observed from the results that the wind speed affects the oxygen transfer. For wind speed less than 4 m/sec, there is no significant wind effect on the aeration efficiency, while wind speed greater than 4 m/sec improve the aeration efficiency due to higher turbulence. The flow rate has shown to have a significant effect on the level of aeration. Furthermore, it was found that biochemical oxygen demand affects the level of dissolved oxygen. Further study in this area could include a sensitivity analysis to determine how sensitive the developed model is to each of the flow rate, step height, biochemical oxygen demand and the wind speed.

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