# Plane Strain Deformation In Thermoelastic Microelongated Solid

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**Abstract**

The purpose of this paper is to study the two dimensional deformation in a thermoelastic microelongated solid. A mechanical force is applied along the interface of fluid half space and thermoelastic microelongated half space. The normal mode analysis has been applied to obtain the exact expressions for displacement component, force stress and temperature distribution. The effect of microelongation on the displacement component, force stress and temperature distribution has been depicted graphically for Green-Lindsay (GL) theory of thermoelasticity.

**Keywords:** Thermoelasticity, Microelongation, Normal mode analysis

## 1. Introduction

The dynamical interaction between the thermal and mechanical has great practical applications in modern aeronautics, astronatics, nuclear reactors, and high-energy particle accelerators. Classical elasticity is not adequate to model the behavior of materials possessing internal structure. Furthermore, the micropolar elastic model is more realistic than the purely elastic theory for studying the response of materials to external stimuli. (Eringen and Suhubi 1964) and (Suhubi and Eringen 1965) developed a nonlinear theory of micro-elastic solids. Later (Eringen 1965,1967) developed a theory for the special class of micro-elastic materials and called it the "linear theory of micropolar elasticity". Under this theory, solids can undergo macro-deformations and micro-rotations. (Eringen 1971)extended his work to include the axial stretch during the rotation of molecules and developed the theory of micro-polar elastic solid with stretch. The micropolar theory was extended to include thermal effects by(Nowacki 1966), (Eringen 1970), (Tauchert et al. 1968), (Tauchert 1971), (Nowacki and Olszak 1974). One can refer to (Dhaliwal and Singh 1983) for a review on the micropolar thermoelasticity and a historical survey of the subject, as well as to (Eringen and Kafadar 1976) in "Continuum Physics" series in which the general theory of micromorphic media has been summed up.

There are two important generalized theories of thermoelasticity. The first is due to (Lord and Shulman 1967). The second generalization of the coupled theory of elasticity is known as the theory of thermoelasticity with two relaxation time or the theory of temperature-rate-dependent thermoelasticity. (Muller 1971), in the review of thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by (Green and Laws 1972). (Green and Lindsay 1972) obtained another version of the constitutive equations. These equations were also obtained independently and more explicitly by (Suhubi 1975). This theory contains two constants that act as relaxation times and modify all the equations of coupled theory, not only the heat equations. The classical Fourier law of heat conduction is not violated if the medium under consideration has a centre of symmetry.

(Barber 1984) studied thermoelastic displacements and stresses due to a heat source moving over the surface of a half plane. (Sherief 1986) obtained components of stress and temperature distributions in a thermoelastic medium due to a continuous source. (Dhaliwal et al. 1997) investigated thermoelastic interactions caused by a continuous line heat source in a homogeneous isotropic unbounded solid. (Chandrasekharaiah and Srinath 1998) studied thermoelastic interactions due to a continuous point heat source in a homogeneous and isotropic unbounded body. (Sharma et al. 2000) investigated the disturbance due to a time-harmonic normal point load in a homogeneous isotropic thermoelastic half-space. (Sharma and Chauhan 2001) discussed mechanical and thermal sources in a generalized thermoelastic half-space. (Sharma et al. 2004) investigated the steady-state response of an applied load moving with constant speed for infinite long time over the top surface of a homogeneous thermoelastic layer lying over an infinite half-space. (Sarbani and Amitava 2004) studied the transient disturbance in half-space due to moving internal heat source under L-S model and obtained the solution for displacements in the transform domain. (Aouadi 2006) studied thermomechanical interaction in a generalized thermo-microstretch elastic half space. (Deswal and Choudhary 2008) studied a two-dimensional problem due to moving loads in generalized thermoelastic solid with diffusion. (El. Maghraby 2010) considered two dimensional problem of generalized thermoelastic half space under the action of body forces and subjected to thermal shock. (Youssef 2010) solved the problem on a generalized thermoelastic infinite medium with a spherical cavity subjected to a moving heat source. (S. Shaw and B. Mukhopadhyay 2012, 2013) discussed a couple of problems in a thermoelactic microelongated medium subjected to heat source.

**2. Fundamental Model**

The constitutive equation for a homogeneous, isotropic, microelongated, thermoelastic solid are

$$\sigma_{kl} = -\beta_0(1+t_1\delta_{2k}\frac{\partial}{\partial t})T\delta_{kl} + \lambda_0\delta_{kl}\varphi + \lambda\delta_{kl}u_{r,r} + \mu(u_{k,l}+u_{l,k}) \tag{1}$$

$$m_k = a_0\varphi_{,k} \tag{2}$$

$$s-t = -\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T + \lambda_1\varphi + \lambda_0 u_{k,k} \tag{3}$$

$$q_k = \frac{K}{T_0}T_{,k} \tag{4}$$

where $t = \sigma_{kk}$ is microelongational stress tensor, $s = s_{kk}$ are component of stress tensor, $\beta_0 = (3\lambda + 2\mu)\alpha_{t_1}, \beta_1 = (3\lambda + 2\mu)\alpha_{t_2}$ ; $a_0, \lambda_0, \lambda_1$ are microelongational constants, $C_E$ is the specific heat at constant strain, $K$ is the thermal conductivity, $m_k$ is the component of microstretch vector $\alpha_{t_1}$ and $\alpha_{t_2}$ are coefficent of linear thermal expansion where T is temperature above reference temperature $T_0$ , q is heat flux, $\varphi$ is microelongational scalar, $\vec{u}$ is displacement vector. $k = 1$ and 2 for L-S and G-L theories respectively.The field equation of motion according to[33, 34] and heat conduction equation according to [35] for the displacement, microelongation and temperature changes are,

$$-\beta_0(1+t_1\delta_{2k}\frac{\partial}{\partial t})T_{,i} + \lambda_0\varphi_{,i} + (\lambda+\mu)u_{j,ij} + \mu u_{i,jj} = \rho\ddot{u}_i \tag{5}$$

$$a_0\varphi_{,ii} + \beta_1(1+t_1\delta_{2k}\frac{\partial}{\partial t})T + \lambda_1\varphi - \lambda_0 u_{j,j} = \frac{1}{2}\rho j_0\ddot{\varphi} \tag{6}$$

$$K\nabla^2 T - \rho C_E(1+t_0\delta_{1k}\frac{\partial}{\partial t})\dot{T} = \beta_0 T_0(1+t_0\delta_{1k}\frac{\partial}{\partial t})\dot{u}_{k,k} + \beta_1 T_0\dot{\varphi} \tag{7}$$

Here we have considered a homogenous, microelongated, isotropic, infinite, thermoelastic body at a uniform reference temperature $T_0$ in $xy$-plane with displacement vector $\vec{u} = (u, v, 0)$, i.e two dimensional disturbance of medium is assumed. Hence equations (5)-(7) become

$$-\beta_0(1+t_1\delta_{2k}\frac{\partial}{\partial t})\frac{\partial T}{\partial x} + \lambda_0\frac{\partial\varphi}{\partial x} + (\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y} + \mu\frac{\partial^2 u}{\partial y^2} = \rho\frac{\partial^2 u}{\partial t^2} \tag{8}$$

$$-\beta_0(1+t_1\delta_{2k}\frac{\partial}{\partial t})\frac{\partial T}{\partial y} + \lambda_0\frac{\partial\varphi}{\partial y} + \mu\frac{\partial^2 v}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y} + (\lambda+2\mu)\frac{\partial^2 v}{\partial y^2} = \rho\frac{\partial^2 v}{\partial t^2} \tag{9}$$

$$a_0\nabla^2\varphi + \beta_1(1+t_1\delta_{2k}\frac{\partial}{\partial t})T + \lambda_1\varphi - \lambda_0\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{1}{2}\rho j_0\frac{\partial^2\varphi}{\partial t^2} \tag{10}$$

$$K\nabla^2 T - \rho C_E(1+t_0\delta_{1k}\frac{\partial}{\partial t})\frac{\partial T}{\partial t} = \beta_0 T_0(\frac{\partial}{\partial t} + t_0\delta_{1k}\frac{\partial^2}{\partial t^2})\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \beta_1 T_0\frac{\partial\varphi}{\partial t} \tag{11}$$

The equations of motion and stress components in fluid are:

$$(\lambda^f)\nabla(\nabla.\vec{u}^f) = \rho^f\frac{\partial^2\vec{u}^f}{\partial t^2} \tag{12}$$

$$\sigma_{ij}^f = \lambda^f u_{r,r}^f\delta_{ij} \tag{13}$$

where, $\vec{u}^f$ is displacement vector, $\lambda^f$ is Lame's constant and $\rho^f$ is density of fluid.

For convenience the following non-dimensional variables are used:

$$\{x',y'\}=\frac{\omega^*}{c_1}\{x,y\}\ ,\ \{u',v'\}=\frac{\omega^*\rho c_1}{\beta_0T_0}\{u,v\}\ ,\ \{t',t_0',t_1'\}=\omega^*\{t,t_0,t_1\}\ ,\ \sigma_{ij}'=\frac{\sigma_{ij}}{\beta_0T_0}\ ,\ \varphi'=\frac{\lambda_0}{\beta_0T_0}\varphi\ ,$$

$$\sigma_{ij}^{f'}=\frac{\sigma_{ij}^{f}}{\beta_0T_0},P_1'=\frac{P_1}{\beta_0T_0},T'=\frac{T}{T_0}\ \text{Where,}\ \omega^*=\frac{\rho c_1^2C_E}{K}\ ,\ c_1^2=\frac{\lambda+2\mu}{\rho}$$

Using above non dimensional variables, the equations (8)-(11) reduces to (after dropping superscripts)

$$\frac{\partial^2u}{\partial t^2}=-(1+t_1\delta_{2k}\frac{\partial}{\partial t})\frac{\partial T}{\partial x}+\frac{\partial\varphi}{\partial x}+h_1\frac{\partial^2u}{\partial x^2}+h_2\frac{\partial^2v}{\partial x\partial y}+h_3\frac{\partial^2u}{\partial y^2} \tag{14}$$

$$\frac{\partial^2v}{\partial t^2}=-(1+t_1\delta_{2k}\frac{\partial}{\partial t})\frac{\partial T}{\partial y}+\frac{\partial\varphi}{\partial y}+h_3\frac{\partial^2v}{\partial x^2}+h_2\frac{\partial^2u}{\partial x\partial y}+h_1\frac{\partial^2v}{\partial y^2} \tag{15}$$

$$\nabla^2\varphi+h_4(1+t_1\delta_{2k}\frac{\partial}{\partial t})T-h_5\varphi-h_6\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=h_7\frac{\partial^2\varphi}{\partial t^2} \tag{16}$$

$$\nabla^2T-h_8(1+t_0\delta_{1k}\frac{\partial}{\partial t})\frac{\partial T}{\partial t}=h_9(\frac{\partial}{\partial t}+t_0\delta_{1k}\frac{\partial^2}{\partial t^2})\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+h_{10}\frac{\partial\varphi}{\partial t} \tag{17}$$

Components of stress in dimensionless form reduces to

$$\sigma_{xx}=-T+\varphi+h_1\frac{\partial u}{\partial x}+h_{11}\frac{\partial v}{\partial y} \tag{18}$$

$$\sigma_{yy}=-(1+t_1\frac{\partial}{\partial t})T+\varphi+h_{11}\frac{\partial u}{\partial x}+h_1\frac{\partial v}{\partial y} \tag{19}$$

$$\sigma_{xy}=h_3\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \tag{20}$$

where, $(h_1,h_2,h_3)=\dfrac{(\lambda+2\mu,\lambda+\mu,\mu)}{\rho c_1^2}$, $h_4=\dfrac{\beta_1\lambda_0c_1^2}{a_0\omega^*\beta_0}$ , $h_5=\dfrac{\lambda_1c_1^2}{a_0\omega^*}$, $h_6=\dfrac{\lambda_0{}^2}{\rho a_0\omega^*}$

$$h_7=\frac{\rho j_0\omega^*c_1^2}{2a_0},\ h_8=\frac{\rho C_Ec_1^2}{K\omega^*},\ h_9=\frac{\beta_0^2T_0}{K\omega^*\rho},\ h_{10}=\frac{\beta_0\beta_1T_0c_1^2}{K\omega^*\lambda_0},\ h_{11}=\frac{\lambda}{\rho c_1^2}$$

## 3. Normal Mode Analysis

The solution of the considered physical variables can be decomposed in terms of normal mode and can be considered in the following form

$$(u,v,T,\varphi,\sigma_{ij},u^f,v^f,\sigma_{ij}^f)(x,y,t)=(u^*,v^*,T^*,\varphi^*,\sigma_{ij}^*,u^{f^*},v^{f^*},\sigma_{ij}^{f^*})(x)e^{\omega t+iby}$$

where $\omega$ is complex frequency, $b$ is wave number in $y$-direction and $u^*(x),v^*(x),T^*(x),\varphi^*(x)$, $\sigma_{ij}^*(x),u^{f^*}(x),v^{f^*}(x),\sigma_{ij}^{f^*}(x)$ are the amplitudes of field quantities.

Using Normal mode in equation (14)-(20), we get

$$(h_1D^2-A_1)u^*+ibh_2Dv^*-A_2DT^*+D\varphi^*=0 \tag{21}$$

$$ibh_2Du^*+(h_3D^2-A_3)v^*-ibA_2T^*+ib\varphi^*=0 \tag{22}$$

$$-h_6Du^*-ibh_6v^*+A_2h_4T^*+(D^2-A_4)\varphi^*=0 \tag{23}$$

$$-h_9A_6Du^*-ibh_9A_6v^*+(D^2-A_7)T^*-h_{10}\omega\varphi^*=0 \tag{24}$$

$$\sigma_{xx}^*=-T^*+\varphi^*+h_1Du^*+ibh_{11}v^* \tag{25}$$

$$\sigma_{yy}^*=-A_1T^*+\varphi^*+h_{11}Du^*+ibh_1v^* \tag{26}$$

$$\sigma_{xy}^*=h_3\left(ibu^*+Dv^*\right) \tag{27}$$

where, $A_1 = \omega^2 + h_3 b^2$, $A_2 = (1 + t_1\delta_{2k}\omega)$, $A_3 = \omega^2 + h_1 b^2$, $A_4 = b^2 + h_5 + h_7\omega^2$,

$A_5 = \omega(1 + t_0\delta_{1k}\omega)$, $A_6 = b^2 + h_8 A_5\omega$, $D = \dfrac{d}{dx}$

Eliminating $v^*(x)$, $T^*(x)$, $\varphi^*(x)$ between equations (21)-(24), we get the following eight order differential equation for $u^*(x)$ as

$$(D^8 + AD^6 + BD^4 + CD^2 + E)u^*(x) = 0 \tag{28}$$

where, $A = \dfrac{-1}{h_1 h_3}[h_1 h_3(A_4 + A_6) - h_1 A_3 + h_3 A_1 + h_3 h_6 + A_2 h_3 h_9 A_5 + b^2 h_2^2]$

$$B = \frac{-1}{h_1 h_3}[-h_1 A_2 h_4 h_{10} h_3\omega + h_1 h_3 A_4 A_6 + h_1 A_3(A_4 + A_6) - h_1 b^2 A_2 A_5 h_9 + b^2 h_1 A_5 - A_1 h_3(A_4 + A_6) + A_1 A_3$$
$$- b^2 h_2^2(A_4 + A_6) + h_3 h_6 h_{10} A_2\omega - h_3 h_9 A_2 A_4 A_5 - h_9 A_2 A_3 A_5 - h_3 h_6 A_6 - h_3 h_4 h_9 A_2 A_5 - A_3 h_6]$$

$$C = \frac{-1}{h_1 h_3}[A_2 A_3 h_1 h_4 h_{10}\omega + A_3 A_4 A_6 h_1 - b^2 h_1 h_6 h_{10} A_2\omega + b^2 h_1 h_9 A_2 A_4 A_5 - b^2 h_1 h_6 A_6 - b^2 h_1 h_4 h_9 A_2 A_5$$
$$+ A_1 A_2 h_3 h_4 h_{10}\omega^2 - h_3 A_1 A_4 A_6 + A_1 A_3(A_4 + A_6) + b^2 A_1 A_2 A_5 h_9 - b^2 h_6 A_1 + b^2 h_2^2 A_2 h_4 h_{10}\omega + b^2 h_2^2 A_4 A_6$$
$$- 2b^2 A_6 h_2 h_6 - 2b^2 A_2 A_5 h_2 h_4 h_9 - h_6 h_{10} A_2 A_3\omega + A_2 A_3 A_4 A_5 h_9 + A_3 A_6 h_6 + A_2 A_3 A_5 h_4 h_9]$$

$$E = \frac{-1}{h_1 h_3}(-h_4 h_{10} A_1 A_2 A_3\omega - A_1 A_3 A_4 A_6 + b^2 h_6 h_{10} A_1 A_2\omega - b^2 h_9 A_1 A_2 A_4 A_5 + b^2 h_6 A_1 A_6 + b^2 h_4 h_9 A_1 A_2 A_5)$$

In a similar manner we can show that $v^*(x)$, $\theta^*(x)$, $\varphi^*(x)$ satisfies the equation

$$(D^8 + AD^6 + BD^4 + CD^2 + E)(v^*(x), \theta^*(x), \varphi^*(x)) = 0 \tag{29}$$

which can be factorized as follows, $(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)u^*(x) = 0$ (30)

The Series solution of equation (28) has the form

$$u^*(x) = \sum_{n=1}^{4}[M_n(b,\omega)e^{-k_n x}] \tag{31}$$

$$v^*(x) = \sum_{n=1}^{4}[M_n^{'}(b,\omega)e^{-k_n x}] \tag{32}$$

$$T^*(x) = \sum_{n=1}^{4}[M_n^{''}(b,\omega)e^{-k_n x}] \tag{33}$$

$$\varphi^*(x) = \sum_{n=1}^{4}[M_n^{'''}(b,\omega)e^{-k_n x}] \tag{34}$$

where $M_n(b,\omega), M_n^{'}(b,\omega), M_n^{''}(b,\omega), M_n^{'''}(b,\omega)$ are specific function depending upon $b$, $\omega$ and $k_n^2$; $n$ =1,2,3,4 are the roots of characteristic equation (30).

using equation (31)-(34) in equation (21)-(24), we get the following relations

$$M_n^{'}(b,\omega) = H_{1n} M_n(b,\omega) \tag{35}$$

$$M_n^{''}(b,\omega) = H_{2n} M_n(b,\omega) \tag{36}$$

$$M_n^{'''}(b,\omega) = H_{3n} M_n(b,\omega) \tag{37}$$

Thus we have,

$$v^*(x) = \sum_{n=1}^{4}[H_{1n} M_n(b,\omega)e^{-k_n x}] \tag{38}$$

$$T^*(x) = \sum_{n=1}^{4}[H_{2n} M_n(b,\omega)e^{-k_n x}] \tag{39}$$

$$\varphi^*(x) = \sum_{n=1}^{4}[H_{3n} M_n(b,\omega)e^{-k_n x}] \tag{40}$$

$$\sigma^{*}_{xx} = \sum_{n=1}^{4}[H_{4n}M_n(b,\omega)e^{-k_n x}] \tag{41}$$

$$\sigma^{*}_{xy} = \sum_{n=1}^{4}[H_{5n}M_n(b,\omega)e^{-k_n x}] \tag{42}$$

$$\sigma^{*}_{yy} = \sum_{n=1}^{4}[H_{6n}M_n(b,\omega)e^{-k_n x}] \tag{43}$$

where, $H_{1n} = \dfrac{ib[(h_1 - h_2)k_n^2 - A_1]}{[(A_3 - b^2 h_2)k_n - h_3 k_n^3]}$

$$H_{2n} = \frac{[h_3 k_n^4 - (A_4 h_3 + A_3)k_n^2 + (A_3 A_4 - b^2 h_6)]H_{1n} - ib[h_2 k_n^3 - (h_2 A_4 - h_6)k_n]}{ib[A_2(k_n^2 - A_4) + A_2 h_4]}$$

$$H_{3n} = \frac{(h_1 k_n^2 - A_1 - ibh_2 k_n H_{1n} + A_2 k_n H_{2n})}{k_n} \; ; H_{4n} = ibh_{11}H_{1n} - H_{2n} + H_{3n} - h_1 k_n$$

$$H_{5n} = h_3(ib - k_n H_{1n}) \; ; \; H_{6n} = ibh_1 H_{1n} - A_1 H_{2n} + H_{3n} - h_{11}k_n$$

Similarly for medium II (i.e fluid half space), the solutions are of the form

$$u^{f^*}(x) = M_5(b,\omega)e^{-k_5(x)} \tag{44}$$

$$v^{f^*}(x) = M'_5(b,\omega)e^{-k_5(x)} \tag{45}$$

where $M_5(b,\omega)$ and $M'_5(b,\omega)$ are specific function depending upon $b$ and $\omega$ and $k_5$ is root of characteristic equation, $(D^2 - b^2 + l\omega^2)u^{f^*}(x) = 0$ (46)

Where, $l = \dfrac{\rho^f c_1^2}{\lambda^f}$ and $k_5 = \sqrt{b^2 - l\omega^2}$

Thus we have, $v^{f^*}(x) = QM_5(b,\omega)e^{-k_5(x)}$ (47)

and, $\sigma^{f^*}_{xx} = LM_5(b,\omega)e^{-k_5(x)}$ ; $\sigma^{f^*}_{xy} = 0$

where, $Q = \dfrac{k_5^2 - l\omega^2}{ibk_5}$ and $L = \dfrac{(\lambda^f)(ibQ - k_5)}{\rho c_1^2}$

**4. Applications**

In this section we determine the parameter $M_n$ ; ($n$=1,2,3,4,5). In the physical problem, we should suppress the positive exponential that are unbounded at infinity. Constants $M_1, M_2, M_3, M_4$ and $M_5$ have to be selected such that boundary condition at the surface $x = 0$ takes the form,

$$\sigma_{xx} = \sigma^f_{xx} - P_1 e^{\omega t + iby}; \; \dot{v} = \dot{v}^f \; ; \; \sigma_{xy} = \sigma^f_{xy}; \; \varphi = 0 \; ; \; \frac{\partial T}{\partial x} = 0$$

where $P_1$ is the magnitude of mechanical force.

Using the expressions of $\sigma_{xx}$, $\sigma^f_{xx}$, $v$, $v^f$, $\sigma_{xy}$, $\sigma^f_{xy}$, T, $\varphi$ into above boundary conditions, gives the following equations satisfied by the parameters.

$$\sum_{n=1}^{4}[H_{4n}M_n] - LM_5 = -P_1 \; ; \sum_{n=1}^{4}[H_{1n}M_n] - QM_5 = 0; \; \sum_{n=1}^{4}[H_{5n}M_n] = 0;$$

$$\sum_{n=1}^{4}[H_{3n}M_n] = 0; \; \sum_{n=1}^{4}[H_{2n}k_n M_n] = 0$$

Solving the above system of equations , we get the component of normal displacement, normal force stress and temperature distribution at the interface of thermoelastic microelongated half space and fluid half space.

**4.1 Special Case:** Taking $\varphi = 0$, we get the Thermoelastic solid (TS).

## 5. Numerical Results and Discussions

In order to illustrate the theoretical results obtained in the preceding section , we present some numerical results for the physical constants [32]:

$\lambda = 7.59\times10^{10} N/m^2$, $\mu = 1.89\times10^{10} N/m^2$, $a_0 = 0.61\times10^{-10} N$, $\rho = 2.19\times10^3 kg/m^3$

$\beta_1 = 0.05\times10^5 N/m^2K$, $\beta_0 = 0.05\times10^5 N/m^2K$, $C_E = 966J/(kgk)$, $T_0 = 293K$

$j_0 = 0.196\times10^{-4} m^2$, $\lambda_0 = \lambda_1 = 0.37\times10^{10} N/m^2$, $\lambda^f = 2.14\times10^{10} N/m^2$,

$\rho^f = 10^3 kg/m^3$, $t_0 = 0.01$, $t_1 = 0.0001$, $K = 252J/msK$

The computations are carried out for the value of non-dimensional time $t = 0.2$ in the range $0 \le x \le 10$ and on the surface $y = 1.3$ .The numerical values for normal displacement, normal force stress and temperature distribution are shown in Figures (1)-(3) for G-L theory by taking $\delta_{1k} = 0$ , $\delta_{2k} = 1$ and $P_1 = 1.0$ , $\omega = \omega_0 + \iota\xi$ , $\omega_0 = -0.3$ , $\xi = 0.1$ and $b = 1.3$ .

(a) Thermoelastic microelongated solid(TMS) by solid line with dashed symbol ♦ .

(b) Thermoelasic solid(TS) by dashed line with centered symbol ■.

It is observed from Figure-1 that the values of normal displacement increases in the range 0 < x <2 for thermoelastic microelongated solid (TMS) whereas it decreases in the same range for thermoelastic solid(TS) and then follow oscillatory pattern in the range 2 < x < 10. From Figure-2 and Figure-3 it is clear that value of normal force stress and temperature distribution for thermoelastic microelongated solid(TMS) is more towards the point of application of mechanical source as compared to thermoelastic solid(TS) in the range 0 < x < 1.5 and then decreases to follow oscillatory pattern in the range 1.5< x <10. Hence, there is significant effect of microelongation .

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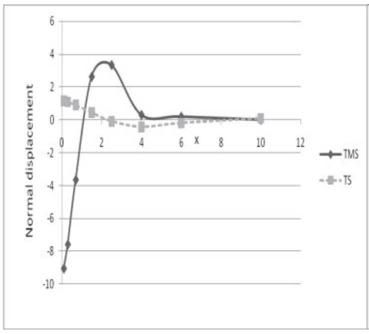


`Figure-1

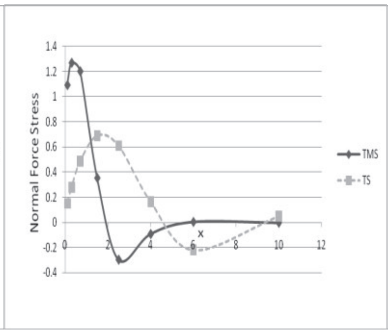


Figure-2

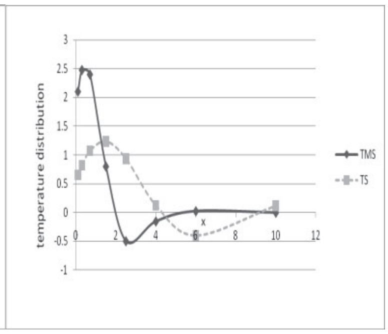


Figure-3