

Analytical Study of Reinforced Concrete Horizontally Curved Beam of Rectangular Hollow Section

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Abstract

This research is devoted to study the behavior of Horizontally Curved Reinforced Concrete Beam (HCRCB) of hollow and solid section theoretically by finite element method, the 20-node isoparametric brick element has been used to represent the concrete and the reinforcement idealized as an axial members imbedded within the concrete elements, a parametric study of 210 beams with different cross sections had been done included the effect of wall thickness, and the effect of flange depth on the behavior of HCRCB as well as two techniques of rearrangement the concrete in the hollow core to strengthen the beam. From the analytical results it was concluded that rearranging the core area improved the ultimate load capacity for beams with shear span to effective depth ratio (a/d) more than 2 and the effect is reversed for beams with (a/d) less than 2. Also the technique of adding the hollow core area to the top and bottom flange represent the optimal and produce the maximum increment in the ultimate load which equal to 57%. While the technique of adding the hollow core area to the outside perimeter produced 20% increase in the ultimate load.

Keywords: Three Dimensional Analysis, Reinforced Concrete Horizontally Curved Beam, Hollow Section

1.Introduction

Since the start of using concrete and reinforced concrete structures engineers have found their way to create a new types of element to meet the architectural and structural requirement of a certain building or general geometry condition. And for that engineers needed to analyze each part to properly design it or to understand its behavior. One of these parts is the horizontally curved beam that is rather increasing in the modern world construction for various reasons mostly architectural. And one of its common uses is bridge curved girders, as shown in Figure (1).



Figure (1): Horizontally curved girder (Gateshead Millennium Bridge spanning the River Tyne in England)

2.Literature Review

In 1977, Badawy et al^[1], presented a theoretical study to calculate a collapse load for reinforced concrete horizontally curved beams. In this study, the effect of shear on ultimate load, failure modes, and internal moments and forces in the curved beams were investigated. In 1999, Sennah, and Kennedy^[2] tested four (1/12) liner scale simply supported curved composite concrete deck-steel multi-cell bridge model to substantiate and verify the analytical model presented using finite element method. In 2008 Al-Mutairee^[3] conducted a theoretical study on the behavior of horizontally curved beams under static and dynamic loads using nonlinear finite element method by writing two computer programs NFHCBSL and NFHCBDL for static and dynamic analyses respectively. The subject of HCRCB was addressed in several researches and papers, In 2013 Al-Mutairee^[4] studied the effects of non-uniform distribution of longitudinal reinforcements on the behavior of reinforced concrete (RC) horizontally curved beams with fixed-ends under static loads The results showed that the effect of non-uniform distributions of longitudinal reinforcement of RC horizontally curved beams with fixed-ends is effective and can be used to improve the strength of this type of beams, and the increment in the ultimate load obtained was equal to 23.5%. In 2014 Al-Shaarbaf and Abbas^[5] presented nonlinear analysis using finite element method to study a pre-stressed concrete box section beam. The authors concluded that the ultimate load increases as the wall thickness of the box section increased.

3.Numerical analysis

Numerical analysis had been done by using nonlinear finite element analysis. The analysis was performed by an application of a computer programs NFHCBSL (Nonlinear Finite element analysis of Horizontally Curved Beam under Static Load) taken from [3]. A convergence study where done to verify the solution and element mesh, the mesh chosen divided the width to 12element and the depth to 18 elements with approximately 50 sectional segments (varies between examples) and the number of element for each section were 216 and 192 for

solid and hollow sections respectively, the total number of element were 10800 for solid and 9600 for hollow. Parametric studies were done to study the effect of various parameters on the behavior of the HCRCB of hollow section. To prepare the input data for the NFHCBSL program mesh, thus generation program was written by authors using FORTRAN compiler.

4. Section details

All hollow sections for curvature equal to 135° angle curve were reinforced according to the (ACI-code 2008)^[6] requirement for bending, shear, and torsion. In addition, the other samples were reinforced with the same amount of steel for comparison purposes. The distance between ends still constant for any θ .

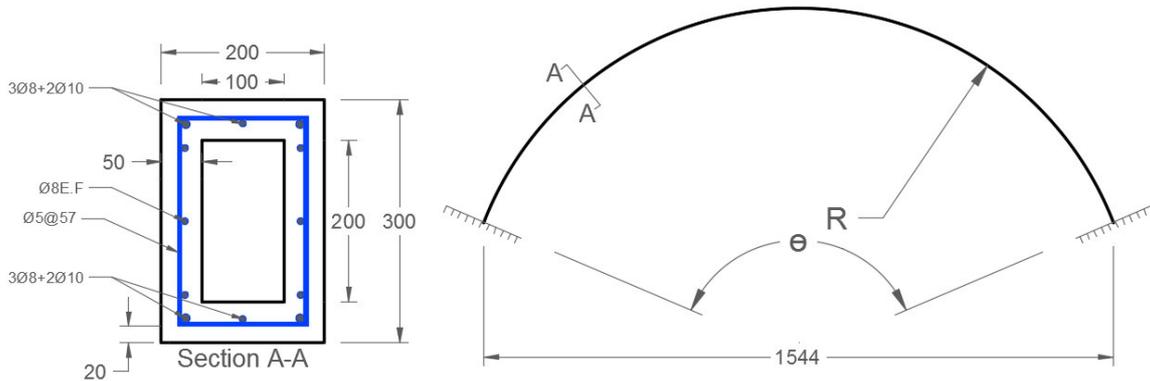


Figure (2): Reinforcement and cross section details of the 135° angle sample.

5. Parametric study

To investigate the effect of the cross section dimension on the behavior of the hollow HCRCB a total number of 210 beams were analyzed using NFHCBSL computer program, each beam required 24552 input data

5.1 Effect of wall thickness

To study the effect of the section thicknesses on HCRCB, wall with thickness (30, 40, 50, 60, 70, 80, and 90 mm) were analyzed. The analysis is done for angles equal to 90° , 135° , and 180° . The cross sectional area of the beams increases with the increase in the thickness therefore it is not constant, but the outer dimension still constant, i.e. (200mm*300mm). The details of the cross sections taken are tabulated in table (1).

The ACI-code 318-08 (11.6.4.4) recommended that the effective wall thickness should be taken equal to A_{oh}/P_h where A_{oh} is the area enclosed by the stirrups and P_h is the perimeter of the stirrups, and the closed stirrups provided for torsion to be located in the outer half of the effective wall thickness. For the current section dimensions the effective thickness is equal to $(155 * 255 / (2 * (155 + 255))) = 48.2mm$

the effect of the wall thickness as shown in Figure (3) where the sections with thickness less than (48.2mm) provided less ultimate load capacity than the sections with wall thickness greater than 48.2mm) by approximately 50%. Furthermore, increasing the thickness beyond the effective thickness improve the ultimate capacity of the beams by (17%, 9%, and 3%) for angles 90° , 135° , and 180° respectively. For beam with angle equal to 90° the ratio of the shear force to the bending moment ($V/M=2.2$) which was the highest among the three beams and the ratio of shear to torsion ($V/T=13.05$) and since the increase in the thickness beyond (48.2mm) did not improve the torsional capacity, but the wall thickness increment improved shear carrying capacity by increasing the value of (b_w), therefore increase the ultimate load of the beams by 17% which was the highest among the three beams. In the other two beams with angles (135° , and 180°) the two ratios (V/M , and V/T) where (1.70, and 8.35) for angle equal to 135° , and (1.30, and 3.56) for angle equal to 180° , which implies that the two beams depend more on moment and torsional capacity than on shear capacity thus, the increase in the ultimate load improved by (9%, and 3%) respectively.

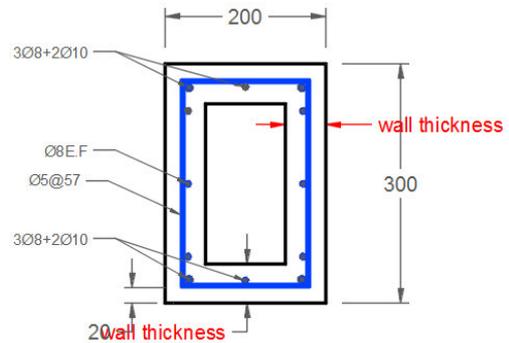


Figure (3): Section details for changing wall thickness parametric study.

Table (1): Cross sections details for the effect of wall thickness.

No.	wall thickness		cross section		hole		Opening area mm ²
	flange mm	web mm	b mm	h mm	b mm	h mm	
1	30	30	200	300	140	240	33600
2	40	40	200	300	120	220	26400
3	50	50	200	300	100	200	20000
4	60	60	200	300	80	180	14400
5	70	70	200	300	60	160	9600
6	80	80	200	300	40	140	5600
7	90	90	200	300	20	120	2400

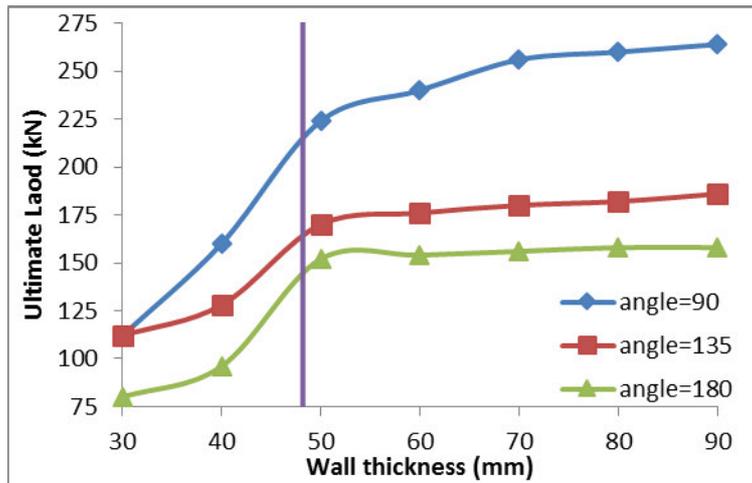


Figure (4): Effect of wall thickness on the ultimate load of hollow section HCRCB with outer dimension (200×300mm).

5.2 Effect of flange depth on the ultimate load

To study the effect of different flange thicknesses on the ultimate load of the beam cross section with 200×300 mm and opening width of 100mm as shown in Figure (5). Different flange thicknesses (30, 40, 50, 60, 70, 80, 90, 100, and 110) were analyzed and the results were as shown in Figure (6) and the details of the analyzed beams were tabulated in Table (2). The effect of the effective thickness is also clear in the investigated cases of different flange depth (with web thickness equal to 50mm as recommended), for beams with low V/M, and V/T ratios (beams depend more on bending and torsion than on shear). For sections with flange less than 50mm there was decrease in the ultimate load by 25%, 20%, and 37% for beams with angles 90°, 135°, and 180° respectively. While for flanges have depth more than 50mm the ultimate load increased by 23%, 12% and 5%. Also the decrease in the ultimate load for beams with hollow sections become lesser with the increase of θ .

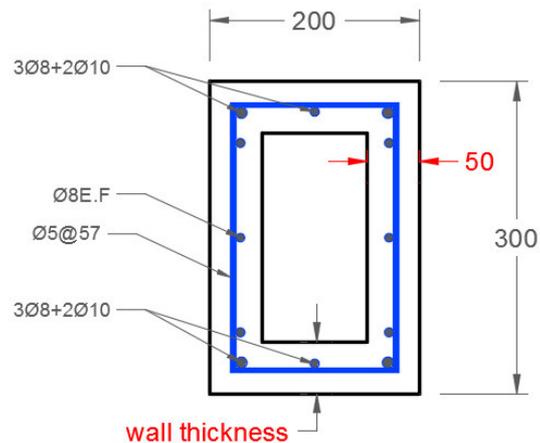


Figure (5): Section details for changing flange depth parametric study.

Table (2): Cross sections details for the effect of flange depth.

No.	wall thickness		cross section		hole		Opening area <i>mm</i> ²
	flange mm	web mm	b mm	h mm	b mm	h mm	
1	30	50	200	300	100	240	24000
2	40	50	200	300	100	220	22000
3	50	50	200	300	100	200	20000
4	60	50	200	300	100	180	18000
5	70	50	200	300	100	160	16000
6	80	50	200	300	100	140	14000
7	90	50	200	300	100	120	12000
8	100	50	200	300	100	100	10000
9	110	50	200	300	100	80	8000

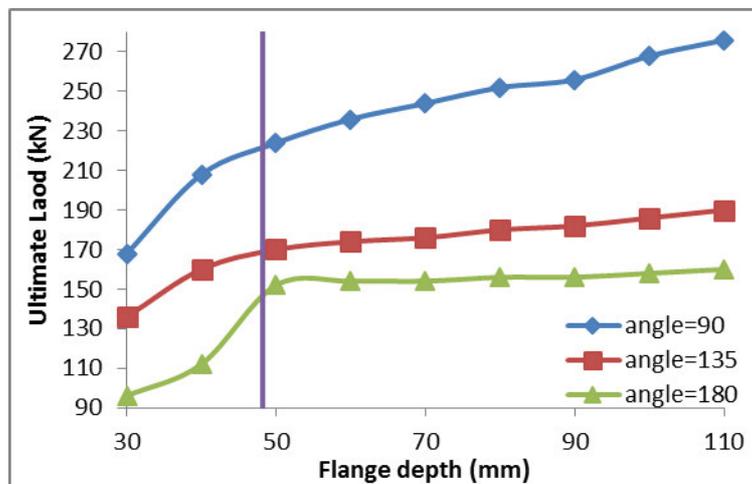


Figure (6): Effect of flange thickness on the ultimate load of hollow cross section HCRCB with original dimension (200×300).

6.Rearranging the hollow core cross section

After removing the hollow part from the cross section a study is made to rearrange the concrete to investigate the more efficient cross section have the same area of (200×300=60000, 200×400=80000, and 200×600=120000) the same box sections previously mentioned is taken and analyzed after adding the area of the box opening to the cross section in two different techniques, the first is by distributing the area of the hollow core on the outside perimeter of the section uniformly, while the second is to add the area on the top and bottom of the cross section and as shown in Figure (7) and Figure (10) respectively.

6.1Rearranging the hollow core area around the outside section perimeter uniformly

If the area of the opening equal ($h_h * b_h = A_{opening}$) and t_{add} is the additional thickness added to the perimeter of the section, thus the additional area (the shaded area in Figure (7)) must be equal to the opening area

$$A_{opening} = h_h * b_h$$

$$A_{add} = (b + 2 * t_{add}) * (h + 2 * t_{add}) - b * h$$

$$A_{openign} = A_{add}$$

$$h_h * b_h = (b + 2 * t_{add}) * (h + 2 * t_{add}) - b * h$$

Solve for t_{add} get

$$t_{add} = \frac{\sqrt{16 b_h h_h + (-2 b - 2 h)^2}}{8} - \frac{b}{4} - \frac{h}{4} \quad \text{--- (1)}$$

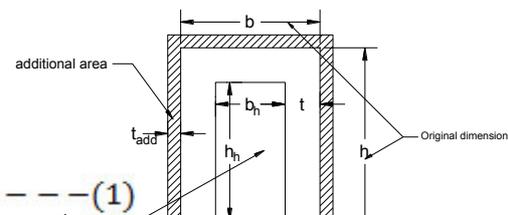


Figure (7): The opening area redistributed uniformly on the perimeter of the section.

By applying this equation to the sections taken in the parametric study the area remains constant and equal to the solid original area ($b * h$). Twenty-four sections were applied to three curves with different angles of (90°, 135°, and 180°) and the results of the beams were drawn in Figures (8). The three curves in Figure (8) shows that the ultimate load of curved beams with angle equal to (90°, and 135°) increased with the increase in the opening area because the rearrangement enhanced the

torsional and moment capacity of the section (for section with opening larger than 5600mm^2). The drop in the ultimate load for beams with opening area less than 5600mm^2 for beam with angle 180° can be explain by referring to the ACI-code equation 11-19 were the left hand terms of the equation were greater than the right hand term which indicate that there will be shear stress (τ) concentrated because of the beam dimensions, however; by increasing the opening area the beam overcome the effect and start to gain strength without reaching the value of the solid beam (the difference were -5% form the solid).

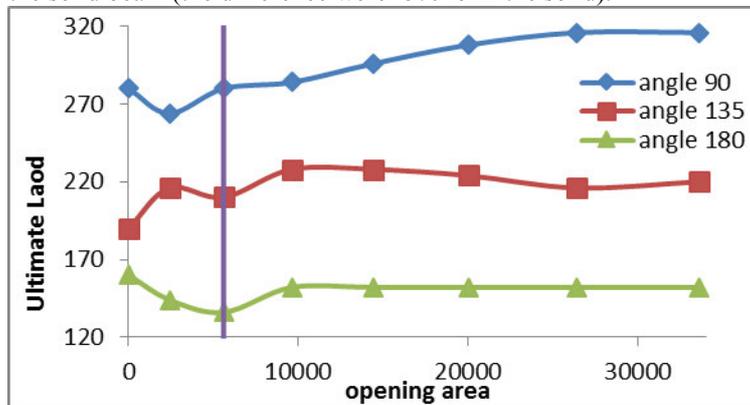


Figure (8): Effect of the cross section rearrangement (By adding the hollow core area to the outside parameter) for section dimension $200 \times 300\text{mm}$.

The same analysis had been done for sections 200×400 , and 200×600 and the results for all sections were as drawn in Figure (9).

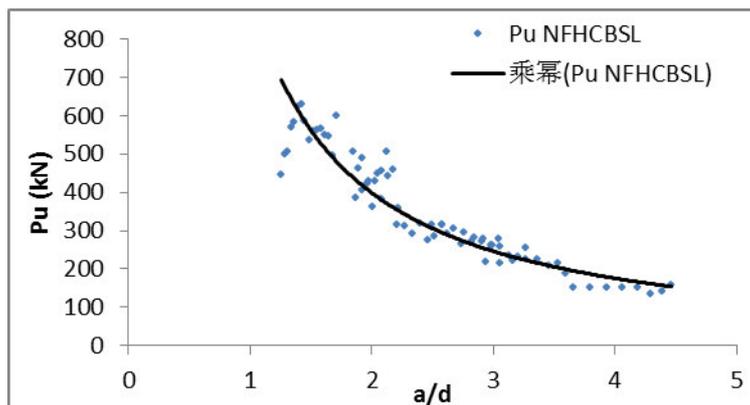


Figure (9): The ultimate load carrying capacity for different beam with different a/d ratios.

From Figure (9) which describes the result of the beams taken in the parametric study regarding the effect of rearrangement of the hollow core area on the outside perimeter of the box horizontally curved beam an equation to estimate the ultimate load can be obtained with ($R^2 = 91\%$) as follows:

$$Pu = 908.35 * \left(\frac{a}{d}\right)^{-1.187} \text{ ----- (2)}$$

Using this equation estimation value for the ultimate load for the HCRCB with hollow core area distributed on the outside perimeter can be found (the equation is limited to the range of the parametric study beams).

A reduction factor is required to compensate the point under the line of the best fit in Figure (9) the factor is suggested to be $(120 * d/a)$ thus the equation is then becomes

$$Pu = 908.35 * \left(\frac{a}{d}\right)^{-1.187} - \frac{120d}{a} \text{ ----- (3)}$$

6.2Rearranging the hollow core area on the top and bottom section flange

If the area of the opening is to be added to the top and bottom flanges the additional area (shaded area in Figure (10)) must be equal to the opening area (of the hollow core), therefore;

$$A_{\text{opening}} = h_n * b_n$$

$$A_{\text{add}} = t_{f\text{add}} * b * 2$$

$$A_{\text{add}} = A_{\text{opening}}$$

$$t_{f\text{add}} * b * 2 = h_n * b_n$$

$$t_{f_{add}} = \frac{h_n * b_n}{2b} \quad \text{--- (4)}$$

Where $t_{f_{add}}$ is the additional flange depth added to maintain constant cross section area of the solid section ($b * h$).

The general behavior of the beam with original dimension of (200×300mm) showed that with the increase in the opening area the ultimate load of the beams were also increased by (26%, 57%, and 15%) for beams with angle (90° , 135° , and 180°)

respectively as shown in Figure (11).

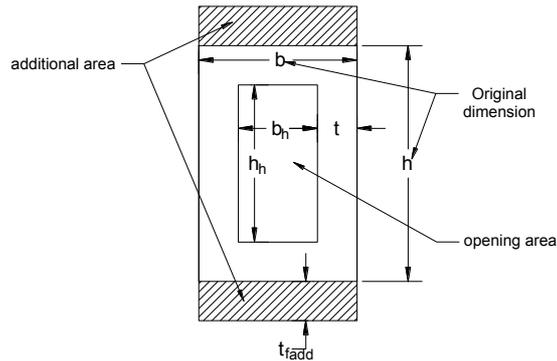


Figure (10): Representation of rearrangement of core area on the top and bottom flange.

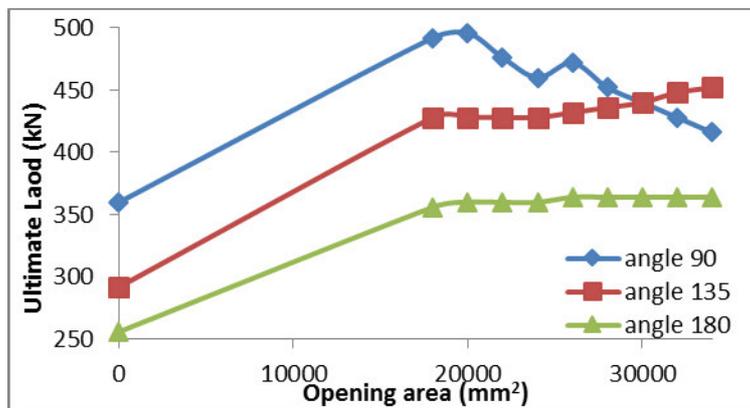


Figure (11): Effect of adding the opening area to the flanges depth on the ultimate load for section with original dimension 200×300mm.

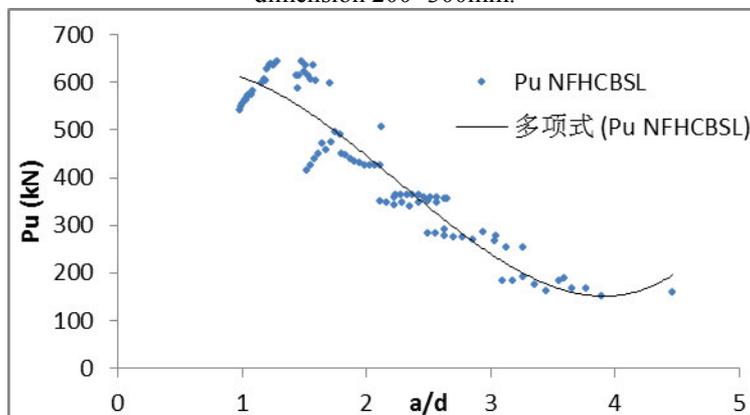


Figure (12):The ultimate load carrying capacity for different beam with different a/d ratios (for the case of redistributing the hollow core area to the top and bottom flanges).

Figure (12) shows the results of sections 200*300, 200*400, and 200*600 mm. Also from Figure (12) an estimating equation for the ultimate load of the hollow horizontally curved beam when rearranging the hollow core area on the top and bottom flange can be expressed as follows:

$$Pu = 26.63 * \left(\frac{a}{d}\right)^3 - 180.8 * \left(\frac{a}{d}\right)^2 + 190.3 * \left(\frac{a}{d}\right) + 571.8 \quad \text{--- (5)}$$

A reduction factor is required to compensate the point under the line of the best fit in Figure (12) the factor is suggested to be $(124 * d/a)$ the equation is then becomes

$$P_u = 26.63 * \left(\frac{a}{d}\right)^3 - 180.8 * \left(\frac{a}{d}\right)^2 + 190.3 * \left(\frac{a}{b}\right) + 571.8 - 124 * \frac{d}{a} \quad \text{--- (6)}$$

The equation is limited to the range of the parametric study beams.

7.Conclusion

Using the theoretical data obtained using the finite element computer program NFHCBSL the following

conclusions were made:

- 1- Due to the characteristic of the stresses on the horizontally curved beam the use of box-section beam is rather more efficient from solid beam with slightly less ultimate load capacity compared with the saving in material and self-weight.
- 2- For the case of connecting two points with constant distance between them increasing the angle of the solid HCRCB decrease its load carrying capacity by approximately (30%) when changing the angle from 90° to 180° .
- 3- Rearranging the hollow core area on the outside perimeter of the HCRCB increased the ultimate load carrying capacity by 20% for beams with a/d more than 2 (i.e. shallow beams).
- 4- Rearranging the hollow core area on the top and bottom flange increased the ultimate load of hollow HCRCB by 57% for beams with a/d more than 2.
- 5- Taken ratio of a/d close or less than 2 were the beam start to act as deep beam reversed the effect of rearranging the hollow core area from increasing to decreasing the ultimate load of the beams.
- 6- with high V/M and V/T wall thickness is more effective if increased more than the recommended by ACI-code while the increment in the thickness for beams with low ratios of V/M and V/T will not improve the ultimate load capacity by significant amount. In other words, the deficiency in the ultimate load due to the use of hollow section decrease with increasing the angle of curvature (θ).

8.Reference

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