Civil and Environmental Research
ISSN 2224-5790 (Paper) ISSN 2225-0514 (Online)
Vol.8, No.9, 2016

# Derived Models for the Prediction of Cole's and Dip Parameters for Velocity Gradients Determination in Open Natural Channels

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**Abstract**:
Measurement of vertical velocity in open channel hydraulics is a very reasonable exercise that attracts peculiar results in stream flow. Observations were made on set of values obtained from calibration of models on continuous longitudinal stream flow. They showed that, there emanated effects of eddy currents. The exercise is characterized by downward trend in the velocity gradient resulting in a dip. However, at the early stage of flow development, a stage corresponding to the bed region, where little or no flow occurs, there had been what should be referred to as hypothetic zero flow region. There is vertical relationship between flow pattern developing and the position from the bed and free surface. Local scouring is induced around structural elements such as a bridge pier, an abutment at the lower region eddies and adjourning loose embankment areas. Occurrence is as a result of interaction between turbulent flow and sediment particles at the bed. The entrainment of sediment particles from the bed is probabilistic and definitely dependent upon flow characteristics and strongly influenced by instantaneous shear stresses developed along stream movement. The computed values of Cole's and Dip parameters for vertical velocities estimation were 0.77 and 0.78 respectively. For accurate measurement of vertical velocity profiles, a constant (ℵ) has been introduced in models for velocity gradient determination. (ℵ) This coefficient has the estimated value of 3.6 - using mathematical iteration methods advanced by Jacobi and Hardy Cross. The proposed models (PIDMWLEL and PIDMWLL) are recommended to be most suitable for predicting velocity profile in open channels, when aspect ratios are greater than five (5) and ten (10).

**Keywords:** Appearance; calibration; channel; flow velocity

## 1. Introduction:

Flow regime and pattern records are necessary tools for project planning, design and execution of watershed systems. In order to properly manage the abundant water resource, study of flow is essential to enable the establishment of adequate controls and mitigate flood menace, control of point and non-point sources of pollution. Essentially to maintain and improve the quality of unique environmental resource endowment and the physical characteristics of the coastal areas. A detailed baseline ecological data to guide the use of coastal areas for the diverse and often conflicting industrial and social needs of the nation so that continued viability of all aspects of the ecosystems will be ensured. In specific terms, local scouring is induced around structural elements such as bridge pier, abutment lower region and loose embankment areas. This occurs as a result of complex interaction between turbulent flow and sediment particles at the bed.

1.1 **Study Area*:*** Ikpa River is situated about sixty (60km) kilometers from the coast of the Atlantic Ocean and is located approximately three kilometers (3km) from Uyo Central Area and relatively positioned on a stretch across four distinct local government areas of Ibiono, Itu, Uruan and Uyo. Starting from Ediene in Ibiono through Itam it passes through Ntak Inyang in Itu Local Government Area; Nduetong Oku, Ikot Inyang, Use Offot, Anua Offot and Ekpri Nsukkara in Uyo Local Government Area, then Mbiakong, Ibiaku Uruan and Ikpa terminating at Nwaniba beach, where it empties into the Cross River Estuaries. At Ntak Inyang, two bridges run across the Ikpa River Channel. The old bridge is a two-span bridge approximately 40m length while the newly constructed four span skewed across the channel about 60m – 80m length Ikpa River is situated about sixty (60km) kilometers from the coast of the Atlantic Ocean and is located approximately three kilometers (3km) from Uyo Central Area and relatively positioned on a stretch across four distinct local government areas of Ibiono, Itu, Uruan and Uyo. Starting from Ediene in Ibiono through Itam it passes through Ntak Inyang in Itu Local Government Area; Nduetong Oku, Ikot Inyang, Use Offot, Anua Offot and Ekpri Nsukkara in Uyo Local Government Area, then Mbiakong, Ibiaku Uruan and Ikpa terminating at Nwaniba beach, where it empties into the Cross River Estuaries. At Ntak Inyang, two bridges run across the Ikpa River Channel. The old bridge is a two-span bridge approximately 40m length while the newly constructed four span skewed across the channel about 60m – 80m length Ikpa River is situated about sixty (60km) kilometers from the coast of the Atlantic Ocean and is located approximately three kilometers (3km) from Uyo Central Area and relatively positioned on a stretch across four distinct local government areas of Ibiono, Itu, Uruan and Uyo. Starting from Ediene in Ibiono through Itam it passes through Ntak Inyang in Itu Local Government Area; Nduetong Oku, Ikot Inyang, Use Offot, Anua Offot and Ekpri Nsukkara in Uyo Local Government Area, then Mbiakong, Ibiaku Uruan and Ikpa terminating at Nwaniba beach, where it empties into the Cross River Estuaries. At Ntak Inyang, two bridges run across the Ikpa River Channel. The old bridge is a two-span bridge approximately 40m length while the newly constructed four span skewed across the channel about 60m – 80m length.

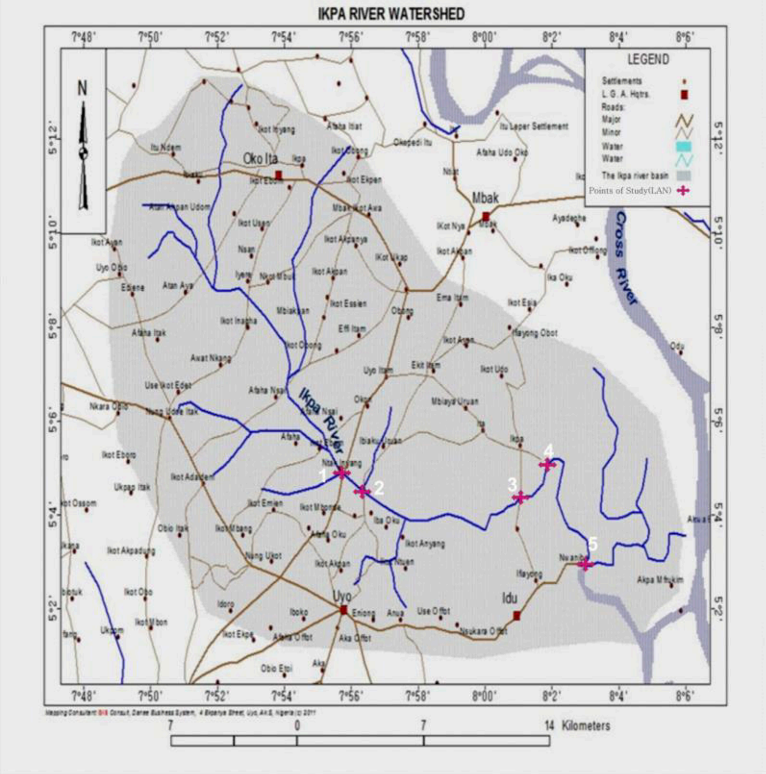


Plate AI.1 Map of Ikpa River Project Area

## 1.2 Literature Review

### 1.2.1 Velocity Profile Estimation: Recent works and Contributors

Absi (2009) made views analytically for determinations of velocity distribution profile and dips in open channels. This position agreed with the analysis of the Reynolds – Averaged Navier - Stokes equations and a log – wake modified eddy viscosity distribution. The equation developed enabled the prediction of velocity- dip-phenomenon. The maximum velocity below the free surface became the primary focus. Two different degrees of approximations were: a semi analytical solution of the ordinary differential equation being full dip – modified – log wake law and a simple dip – modified – log wake law. The velocity profiles of the two laws and the numerical solution of the ordinary differential equations were made to compare with his experimental data. The equation proposed to represent the assertion in the law is:

$$\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}=v\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^2 u}{\partial z^2}+\frac{\partial -uv}{\partial y}+\frac{\partial -uw}{\partial z}+g sin\theta \qquad (1)$$

Yang et al (2004), earlier worked on Velocity Distribution and Dip – Phenomenon in smooth uniform open channels. The investigation centered on the mechanism of dip phenomenon touching on location of the maximum velocity below the free surface vis-a-vis the secondary currents in open channel flows. This research authenticated that, making the classical log law a partial solution to the description of the velocity distribution in the inner region. The local shear velocity was introduced into the dimensionless distance, $U_*$ (V) y/h. It added that the velocity deviation from the log law possesses linear proportionality to the logarithmic distance, ln(1-y/h) from the free surface. That new dip modified log law again gave two logarithmic distances, one from the bed ln$U_*$ (V)y/h, and the other from the free surface ln(1-y/h), respectively, and the dip – correction factor, α introduced.

In a similar approach, Guo and Julien (2007) noted that the understanding of velocity dip phenomenon was poor. He showed that, the dip could hardly be modelled with log-wake law velocity profiles because as opined, this would impose a velocity increase with distance from the boundary. Earlier, Guo and Julien (2003) and Guo et al**.** (2005) proposed a modified log-wake law (MLWL) that well represented experimental data in pipes and zero-pressure gradient (ZPG) boundary layers. In open channel, flows associated with the dip phenomenon are similar to those in pipes and boundary layers, where a zero velocity gradient exists at the maximum velocity. The representation of their equations Guo and Julien (2003) and Guo et al. (2005) gave the modified log-wake like this:

$$\frac{U}{U_*} = \left(\frac{1}{K}\ln\frac{yu_*}{v} + B\right) + \frac{2\Pi}{K}sin^2\pi\frac{\xi}{2} - \frac{\xi^3}{3K} \qquad (2)$$

Where, *U* is the time-averaged velocity in the flow direction,
*U*∗, is the shear velocity; *y,* is the relational distance from the bed of the river to a position along the stage,
*K*, is Karman constant,
*v*, is the kinematic viscosity of the fluid,
*B,* is the additive constant, that relates to the wall roughness,
$\Pi$, is Coles wake strength and
$\xi$, is the normalized distance relative to the dip position, $\delta$.

The terms in parentheses are the logarithmic law of the wall; the sine square term is the wake law that expresses the effects of the constant pressure gradient in pipes or the convective inertia in ZPG boundary layers, and the cubic function forces the log law gradient to be zero at the maximum velocity.

The study of fluid flows is subdivided into classical hydrodynamics and experimental hydraulics. Navier- Stokes and other hydro - dynamicians all contributed to the development of a formidable array of mathematical equations and methods. Prandtl and Karman published series of papers in 1920s and 1930s, covering various aspects of boundary layer theories and turbulence. Several other works relating to the determination of the Dips Phenomenon were also published as one that appeared in Yan et al. (2011), showing that the dip is an intrinsic feature of open channel flows; and that it describes the location where the maximum velocity occurs. Based on these studies, a double – spiral flow was proposed to describe the secondary flow (Wang et al., 1988). The occurrence of the secondary flow structure was also confirmed by other research work (Nezu and Nakagawa, 1993). Based on a wide range of experimental studies, Nezu and Nakagawa (1993) demonstrated the essential processes governing the velocity dip: low momentum fluid parcels are transported by the secondary motion from the near – bank to center, while high momentum fluid parcels are moved by this motion from free surface toward the bed. Iwagaki (1954) also made very positive contributions on the law of resistance to turbulent flow in open rough channels.

The real breakthrough came with the work of Prandtl (1901) that flow was divided into two independent parts. On one hand the free fluid considered, could be treated as in viscid (which obey the law of hydrodynamics) and on the other hand, the transition layer on the fixed boundary was used. In 1930s, the efforts of Nikurádse (1933), Moody (1944), Colebrook (1939) and others resulted in clear understanding of pipe flow in particular. This led directly to modern methods for estimating flows in pipes and channels. Theoretical investigations of Prandtl and Karman on flow through pipes and the experimental studies of Nikurádse (1933) have led to rationalize formulas for velocity distribution for turbulent flows over flat plates and circular pipes. The formula paved the way for further development of formulas to open channel flows. Although similarities exist between the flow through pipes and flow through open channel but certain basic factors like dimension, cross-section of channel, and non-uniform distribution of shear along wetted perimeter, distinguishes the open channel from pipe flow. The spatial distribution of the longitudinal velocity component in a cross section is one of the basic properties of an open channel flow. It is directly inter related, and interacts with other flow properties such as the shear stress distribution and secondary flows. The velocity distribution in pipe flow is initiated by the well-known universal law of velocity distribution in the turbulent boundary layer deduced by Prandtl (1925) using the mixing-length hypothesis and by von Karman (1930). Prandtl (1932) developed the general form of velocity distribution, which is generally considered as P-vK law. This law was derived by assuming that “shear stress remains constant”, and can be applied.

Vanoni (1946) and Einstein and Chen (1955) modified von Karman constant in universal Prandtl - von Karman(P-vK) law for sediment laden flow and noticed that von Karman constant becomes smaller with increase of sediment concentration and greater for clear water (k>0.4). The P-vK law was derived by taking shear stress as constant whereas the shear stress is not constant in turbulent layer (outer zone) in open channel flow.

Millikan (1939) suggested that actual velocity distribution consists of logarithmic part and correction part, where the correction part takes into account, the outer layer. Coles (1956) proposed a semi-empirical equation of velocity distribution, which can be applied to outer region and wall region of plate and open channel. He generalized the logarithmic formula of the wall with tried wake function, w(y/8).This formula is asymptotic to the logarithmic equation of the wall as the distance, y approaches the wall. This is basic formulation towards outer layer region. Zagustin and Zagusin (1969) proposed an analytical solution for turbulent flow in smooth pipes based on a new concept of balance of pulsation energy.

Daily et al. (1966) assumed the P-vK law for smooth boundaries to get the surface velocity. They used experimental data to arrive at two-form of velocity defect law, one of which applied to y/Yw<0.15 and other y/Yw>0.15, where y/Yw is scaled length. But later Monin and Yaglom (1971) discovered from experimental results that velocity distribution have some important aspects in both rough and smooth walls. The experimentation in turbulent boundary layer region shows that results deviate completely from logarithmic equation in outer region.

Coleman (1981) proposed that the velocity equation for sediment-laden flow consists of two parts, as originally discussed by Coles for clear-water flow. In addition, he had revealed that the von Karman coefficient is independent of sediment concentration and the wake strength Π (Pee) is a function of the global Richardson number, which is the ratio between potential energy and kinetic energy.

Naot (1982) demonstrated the use of an algebraic stressed model for the calculation of secondary current. From early sand experiment by Nikurádse (1933), it is evident that rough wall are associated with high turbulent shear stress and hence P-vK law is completely different from that of suitable smooth wall. Wang and Nickerson (1972) showed that transition from rough wall characteristics to smooth wall is completely with in a distance smaller than ten times the roughness height step. Since the turbulent normal stress and turbulent energy near the wall also follow the transition, strong gradients in the normal stresses is formed which is responsible for secondary current.

Coleman and Alonso (1983) developed an equation, which predicts the velocity profile in the viscous sub-layer, the buffer zone, the logarithmic or inertial zone, and the outer or wake region in conduit. The channel flows provided that secondary or cross flow is weak or absent.

Sarma et al. (1983) studied velocity distribution in a smooth rectangular channel by dividing the channel into four regions, thus:

- Region 1 comprises the inner region of the bed and the outer region of the sidewall;
- Region 2 belongs to the inner regions of both the bed and the sidewall;
- Region 3 consists of the inner region of the sidewall and the outer region of the bed;
- Region 4 forms the outer regions of both the bed and the sidewall. The aspect ratio varying from 2.0 to 8.0 and the Froude number from 0.2 to 0.7 is evident. Experiments included the aspect ratio of 1.0 and also for Froude numbers of 0.2 and 0.3.

Further, Samaga et al. (1986) developed a model in alluvial channel using two layer models, where in the region, $d_a<y/\delta<0.2$, where, $d_a$ is arithmetic mean size sediment of bed particles, velocity distribution is assumed to be parabolic. However, for region $y/\delta>0.2$ it was logarithmic. In the approach, velocity and eddy viscosity remain constant at $y/\delta = 0.2$ but shear stress is discontinuous at $y/\delta = 0.2$. Nezu and Nakagawa (1984) investigated experimentally the turbulent structure and their currents in air conduit by considering an essential interaction between secondary currents and bed form. The hot-wire anemometers were used to measure all three components of velocity. The structure of secondary currents was examined through the equations of mean flow vortices and mean flow energy.

Naot (1985) designed eight cases to study the hydrodynamic response of open channel flow to wall roughness and lateral homogeneity. The response of the secondary current to the wall roughness, heterogeneity was done by Willis (1985) based on the equations of motion for uniform flow and a parabolic distribution of eddy viscosity over the turbulent portion of the boundary layer. Effects of increased roughness are accounted for by a shift of the distribution of eddy viscosity toward the flow boundary. The resulting velocity distribution agrees with published data and has an advantage over the classical logarithmic distribution for flow over rough surfaces in that the lower velocity limit is zero at distance measurements near the boundary. Chen (1991) represented turbulent velocity profile by Power law relations and recommended using mPL =1/7 for hydraulically smooth flows and mPL=1/6 for hydraulically rough flows, where mPL is power law index. Tominaga and Nezu (1992) measured velocity with a fibre-optic laser-Doppler anemometer in steep open-channel flows over smooth and in completely rough beds. Velocity distribution in steep open channel is necessary for solving the problems of soil erosion and sediment transport, and he observed the integral constant A in the log law coincided with the usual value of 5.29 in subcritical flows. Swamee (1993) presented a generalized equation for velocity distribution in the inner law region of a turbulent boundary layer. The equation includes linear and logarithmic velocity distributions and it is valid for hydraulically smooth and rough boundaries and the transition range in between.

Kirkgoz et al. (1998) conducted experiments in 12 different test conditions with Reynolds number ranging from 28,026 to 136,842 in a rectangular laboratory channel. From the experiment they observed that the fully developed turbulent boundary layer along the center line of the channel, developing up to the free surface for a flow aspect ratio, b/h > = 3, where b/h is width ratio in turbulent inner regions of developing and fully developed boundary flows. The measured velocity profiles agree well with the logarithmic 'law of the wall' distribution when the coefficients in expression are 2.44 and 5.5, respectively.

Sarma et al. (2000) tried to formulate the velocity distribution law in open channel flows by taking generalized version of binary version of velocity distribution, which combines the logarithmic law of the inner region and parabolic law of the outer region. The law developed by taking velocity-dip in to account.

Wilkerson et al. (2005) using data from three previous studies, developed two models for predicting depth-averaged velocity distributions in straight trapezoidal channels that are not wide. The data used for developing the model are free from the effect of secondary current. Yang et al. (2005) derived dip modified log law taking into account the negative Reynolds shear stress near the free surface. The new law consists of a combination of two logarithmic distances; one from bed and the other from free surface and a dip correction factor α. This law is able to give explanation on the occurrence of velocity near a corner. In addition, it incorporates dip- phenomenon both in central and corner portion.

Cheng (2007) derived power law as first order approximation of power law, and its index is computed as a function of Reynolds number as well as relative roughness height. As log law is generally applied in near bed region, also it is assumed that shear velocity is global velocity scale and can be applicable to both inner and outer region, when Reynolds number increases there is a region of overlap between power law and log law and both the condition of both region holds. The range of overlap is quite narrow about 20% of the flow depth. Power law not only applicable to the overlap region but also be applied in outer region as explained by Hinze (1975), Bergstrom et al. (2001). Knight et al. (2007) used Shiono and Knight Method (SKM), which is a new approach to calculating the lateral distributions of depth-averaged velocity and boundary shear stress for flows in straight prismatic channels, also accounted for secondary flow effect. This method (SKM) had been used to analyse straight trapezoidal open channel flow. The Mach number for secondary current varies with the aspect ratio. Mach number for secondary current is three (3) for aspect ratio less than or equal to 2.2 and four (4) for aspect ratio greater than or equal to four (4). Afzal et al. (2007) analyzed the power - law velocity distribution in fully developed turbulent pipe and channel flows in terms of the envelope of the friction factor. This model gives good approximation for low Reynolds number in designed process of actual system compared to log law.

### 1.3 Methodology

Three study stations were established out of five along the river course within the water shed to give even spread – Ikpa River Water shed. The measurements in the study area were carried out under exploration system using both modern and local engineering techniques. The contemplation was to establish adequate number of hydrological and hydro-geological monitoring stations for the purpose of collection of data. The stations were to function as Local Area Network - LAN. The stations were selected based on major roads crossing on the channel and specific landmarks then water front changes. The following five (5) points were chosen for data collection: Ntak Inyang, Nduetong/Ibiaku Uruan, Mbiakong, Ikpa Town Beach, Nwaniba Beach point (Refer to PlateAI.1). Water samples were collected from the five points for water quality analysis. Flow measurement took place at the first three points only. Stream wise flow measurement along the channel executed from Ntak Inyang to Nwaniba beach covering a length well over 19.5 kilometers. Vertical flow measurement was done only from the first three stations. Data for water quality and longitudinal (streamline) flow were collected from all the five points. The data for water quality although thorough analyses were carried out, did not form essentially the objective of the work at its present status.

Ikpa River is situated about sixty (60 km) kilometers from the coast of the Atlantic Ocean and is located approximately three kilometers (3km) from Uyo Central Area and relatively positioned on a stretch across four distinct local government areas of Ibiono, Itu, Uruan and Uyo. Starting from Ediene in Ibiono through Itam it passes through Ntak Inyang in Itu Local Government Area; Nduetong Oku, Ikot Inyang, Use Offot, Anua Offot and Ekpri Nsukkara in Uyo Local Government Area, then Mbiakong, Ibiaku Uruan and Ikpa terminating at Nwaniba beach, where it empties into the Cross River Estuaries. At Ntak Inyang, two bridges run across the Ikpa River Channel. The old bridge is a two-span bridge approximately 40m length while the newly constructed four span skewed across the channel about 60m – 80m length (Plate AI.1).

At the early stage of flow, corresponding to the bed region, there is little or no flow and can be referred to as the hypothetic flow region. Nezu and Rodi hypothesis seem to give a picture of the vertical relationship to flow developing from almost static to quasi preponderance of hyperbolic order.

Plate A I.2 Image Capture of Ikpa River from Ntak Inyang Reach

**2.0 Discussion and Results**:

Absi (2009) in his work which was in line with the analysis of the Reynolds – Averaged Navier - Stokes equations and a log – wake modified eddy viscosity distribution. The proposed equation he developed enabled the prediction of velocity- dip- phenomenon. The maximum velocity below the free surface became his primary focus. In it, two different degrees of approximations were: a semi analytical solution of the ordinary differential equation being full dip – modified – log wake law and a simple dip – modified – log wake law. The velocity profiles of the two laws and the numerical solution of the ordinary differential equations were made to compare with his experimental data. The equation proposed to represent the assertion in the law is:

$$\frac{\partial UV}{\partial y} + \frac{\partial UW}{\partial z} = v\frac{\partial^2 U}{\partial y^2} + v\frac{\partial^2 U}{\partial z^2} + \frac{\partial -uv}{\partial y} + \frac{\partial -uw}{\partial z} + g\sin\theta \qquad (1)$$

Where, $\frac{\partial UV}{\partial y}$ is the product term of the first order derivative of the partial differential equation in the y - plane.

$\frac{\partial UW}{\partial z}$ is the velocity gradient product term of partial differential equation in the z - plane

$v\frac{\partial^2 U}{\partial y^2}$ is the second order derivative of the component of velocity in the y - direction

$v\frac{\partial^2 U}{\partial z^2}$ is the second order derivative of velocity component in the z direction. The last two expressions arises from the stream flow experiment conducted by Reynolds, Blasius, Karman and Pohlhausen.

Sin θ depicts the slope condition of the stream bed, while (g) is acceleration due to gravity.

Yang et al (2004), investigated the mechanism of dip phenomenon where by the location of the maximum velocity appearing below the free surface vis-a-vis the secondary currents in open channel flows, authenticated the fact. It was found in this research that, the classical log law gave a clue to the solution, of the velocity distribution in the inner region if the local shear velocity was introduced into the dimensionless distance, $U_*$ (V) y/h. It added that the velocity deviation from the log law possesses linear proportionality to the logarithmic distance, ln(1-y/h) from the free surface. That new dip modified log law again gave two logarithmic distances, one from the bed ln$U_*$ (V)y/h, and the other from the free surface ln(1-y/h), respectively, and the dip – correction factor, α introduced.

In a similar approach, Guo and Julien (2007) noted that the understanding of velocity dip phenomenon was poor. He showed that the dip could hardly be modelled with log-wake law velocity profiles because as opined in that publication, it imposes a velocity increase with distance from the boundary. Guo and Julien (2003) and Guo et al**.** (2005) proposed a modified log-wake law (MLWL) that well represented experimental data in pipes and zero-pressure gradient (ZPG) boundary layers. In open channel, flows associated with the dip phenomenon are similar to those in pipes and boundary layers, where a zero velocity gradient exists at the maximum velocity.

A lead equation for the development of secondary equations that enable determination of other functions which can predict vertical flow accurately is given like this as opined by Absi (2009), Guo (2008) and Tang (2011):

$$U_a = \frac{1}{k}\left[ln\frac{\xi}{\xi_0} + \propto \ln(1-\xi)\right] \qquad (2)$$

Where,

$$U_a = \frac{U}{U_*} \qquad (3)$$

$$\xi = \frac{y}{h} \qquad (4)$$

Equation (3), is a relative distance, which is the ratio of the distance from stage point y, to the water depth h; Ua is velocity ratio, a non-dimensionless quantity

U = flow velocity, $U_*$ = friction velocity; the quotient of these two velocities is velocity gradient

α = f (aspect ratio, channel slope, friction velocity)

ξ is a depth ratio

h = water depth

A quantity being also a ratio river width to the overall depth, h which will be mentioned later in the work is $A_r = w/h$, called "aspect ratio"

The dip – modified log law predicts the velocity dip – phenomenon by the introduction of the term $\ln(1-\xi)$ of the equation (2), and (α) as dip-correction parameter (Yang et al. 2004). This law contains only (α) and reverts to the classical log law for (α) = 0. Instead of the parabolic profile for eddy viscosity equation (2), a more appropriate approximation in accordance with the log – wake law given by Nezu and Rodi (1986) is achieved by adding yet another term. Putting annotations as stated above and rearranging the terms then introduce another parameter coefficient as suggested by Coles. In rigorous iterations in the research work, the equation below is obtained as a reformed derivative of equation (2).

## 3.0 Derived Equations

$$\frac{dUa}{d\varepsilon} = \frac{1}{k}\left(1-\alpha\frac{\xi}{1-\xi}\right) + \left(\frac{1}{\xi} \pm \pi\prod Sin\,\pi\,\xi\right) \qquad (5)$$

∏= Coles parameter expressing the strength of the wake function = f (Re)

$$\frac{h}{u_*}\frac{dU}{dy} = \frac{1}{k}\left(1-\frac{\alpha y}{h-y}\right) \quad \left(\frac{h}{y} \pm \pi \prod \; Sin\,\pi\,\frac{y}{h}\right)$$

$$\text{i.e.}\frac{dU}{dy} = \frac{U_*}{kh}\left(1-\frac{\alpha y}{h-y}\right)\left(\frac{h}{y} \pm \pi \prod \; Sin\,\pi\,\frac{y}{h}\right) \qquad (6)$$

Differentiating equation (5) we obtain:

$$\text{i.e. } \frac{d^2U}{dy_2} = -\frac{U_*}{kh}\left(\frac{\alpha(h-y)+\alpha y}{(h-y)^2}\right)\left(\frac{h}{y} + \pi\prod Sin\,\pi\frac{y}{h}\right) + \left(\frac{U_*}{kh}\left[1-\frac{\alpha y}{h-y}\right]\right)\left(-\frac{h}{y^2} + \frac{\pi^2\Pi_1}{h}\,Cos\,\frac{\pi y}{h}\right) \qquad (7)$$

But using numerical method on the first and second order equation solutions, equations of the form (8) and (9) are obtained;

$$\frac{dU}{dy} = \frac{U_{i+1}-U_{i-1}}{2\Delta y} = M_1 \qquad (8)$$

$$\frac{d^2U}{dy^2} = \frac{U_{i+1}-2U_i+U_{i-1}}{(\Delta y)^2} = M_2 \qquad (9)$$

Substituting equations (8) and (9) in equations (6) and (7) respectively we obtain:

$$M_1 = \frac{U_*}{hk}\left(1-\frac{\alpha y_1}{h-y_i}\right)\left(\frac{h}{y} + \pi \prod \; Sin\,\frac{\pi y_1}{h}\right) \qquad (10)$$

$$M_2 = -\frac{U_*}{kh}\left(\frac{\alpha(h-y_i)+\alpha y_i}{(h-y_i)^2}\right)\left(\frac{h}{y} + \pi \prod \; Sin\,\frac{\pi y_i}{h}\right)$$

$$+ \frac{U_*}{kh}\left(1-\frac{\alpha y_i}{(h-y_i)}\right)\left(-\frac{h}{y^2} + \frac{\pi^2\Pi_1}{h}Cos\frac{\pi y}{h}\right) \qquad (11)$$

These equations can be expressed as:

$$M_1 = \beta_0(1-\alpha\beta_1)\;(\beta_2+\beta_3\Pi) \qquad (12)$$

$$M_2 = \alpha\beta_4(\;\beta_2 - \beta_3\;) + \beta_0\beta_1(1-\alpha)\;(y_1+\Pi y_2) \qquad (13)$$

Where,

$$\beta_0 = \frac{U_*}{kh} \qquad (14)$$

$$\beta_1 = \frac{y_i}{h-y_i} \qquad (15)$$

$$\beta_2 = \frac{h}{y_i} \qquad (16)$$

$$\beta_3 = \pi\,Sin\,\frac{\pi y_i}{h} \qquad (17)$$

$$\beta_4 = -\frac{U_*}{k(h-y_i)^2} \qquad (18)$$

$$y_1 = -\frac{h}{y^2} \qquad (19)$$

$$y_2 = \frac{\pi^2}{h}\cos\frac{\pi y_i}{h} \qquad (20)$$

Equations (12) and (13) are re-arranged into:

$$M_1 = a_0 + \alpha a_1 + \prod(a_2 + a_3\alpha) \qquad (21)$$

$$M_2 = b_0 + \alpha b_1 + \prod(b_2 + b_3\alpha) \qquad (22)$$

Where,

$$a_0 = \beta_0\beta_2 \qquad (23)$$

$$a_1 = -\beta_0\beta_1\beta_2 \qquad (24)$$

$$a_2 = \beta_0\beta_3 \qquad (25)$$

$$a_3 = -\beta_0\beta_1\beta_3 \qquad (26)$$

$$b_0 = \beta_0 y_1 \qquad (27)$$

$$b_1 = -\beta_0\beta_1 y_1 + \beta_2\beta_4 \qquad (28)$$

$$b_2 = \beta_0 y_2 \qquad (29)$$

$$b_3 = \beta_3\beta_4 + \beta_0\beta_1 y_2 \qquad (30)$$

Hence, eliminating the product term '$\alpha \prod$' in (21) by multiplying by $b_3$ and equation (22) by $a_3$ and obtaining another set of pair equations (31) and (32) emerged. The difference of the two equations yields equation (33). Solving for α gives equation (34) in terms the parameter, Π and results of the first and second order differential equations symbols.

$$M_1b_3 = a_0 b_3 + a_1\alpha\, b_3 + a_2b_3\prod + a_3b_3\alpha\prod \qquad (31)$$

$$M_2a_3 = a_3b_0 + a_3b_1\alpha + a_3b_2\prod + a_3b_3\alpha\prod \qquad (32)$$

$$M_1b_3 - M_2a_3 = a_0 b_3 - a_3b_0 + \alpha\,(a_1b_3 - a_3b_1) + \prod\,(a_2b_3 - a_3b_2) \qquad (33)$$

$$\alpha = \frac{M_1b_3 - M_2a_3 - a_0b_3 + a_0b_3 + b_0a_3 - (a_2b_3 - a_3b_2)\prod}{a_1b_3 - a_1b_1} \qquad (34)$$

$$M_1\,(b_1 + b_3\prod) = a_0\,(b_1 + b_3\prod) + a_2\prod\,(b_1 + b_3\prod) + \alpha\,(a_1 + a_3\prod)\,(b_1 + b_3\prod) \qquad (35)$$

$$M_2\,(a_1 + a_3\prod) = b_0\,(a_1 + a_3\prod) + b_2\prod\,(a_1 + a_3\prod) + \alpha\,(b_1 + b_3\prod)\,(a_1 + a_3\prod) \qquad (36)$$

Grouping similar terms and re arranging as appropriate, the following equations emanated:

$$M_1(b_1 + b_3\prod) - a_0(b_1 + b_3\prod) - a_2b_1\prod - a_2b_3\prod^2 - \{M_2(a_1 + a_3\prod) - b_0(a_1 + a_3\prod) - b_2\prod(a_1 + a_3\prod)\} = 0 \qquad (37)$$

$$M_1b_1 + M_1b_3\prod - a_0b_1 - a_0b_3\prod - a_2b_1\prod - a_2\, b_3\prod^2 - M_2\, a_1 -$$
$$- M_2a_3\prod + b_0a_1 + b_0\, a_3\prod + a_1b_2\prod + b_2a_3\prod^2 = 0 \qquad (38)$$

$$(a_3b_2 - a_2b_3)\prod^2 + (M_1b_3 - a_0b_3 - a_2b_1 - M_2a_3 + b_0a_3 + a_1b_2)\prod +$$
$$+ (M_1 - a_0)\, b_1 + (b_0 - M_2)\, a_1 \qquad (39)$$

$$A_0 = (a_3b_2 - a_2b_3) \qquad (40)$$

$$A_1 = (M_1b_3 - a_0b_3 - a_2b_1 - M_2a_3 + b_0a_3 + a_1b_2) \qquad (41)$$

$$A_2 = (M_1 - a_0)\, b_1 + (b_0 - M_2)\, a_1 \qquad (42)$$

In that same manner, putting annotations for parametric coefficients and then solving for ∏, the quadratic equation below is obtained.

$$A_0\prod^2 + A_1\prod + A_2 = 0 \qquad (43)$$

$$\prod = \frac{-A_1 \pm \sqrt{A_1{}^2 - 4A_0A_2}}{2A_0} \qquad (44)$$

and

$$\alpha = \frac{b_3M_1 - M_2a_3 - a_0b_3 - b_0a_3) - (a_2b_3 - b_2a_3)\prod}{a_1b_3 - b_1a_3} \qquad (45)$$

$$\alpha_1 = \frac{M_1 - a_0 - a_2\prod}{a_1 + a_3\prod} \qquad (46)$$

$$\alpha_2 = \frac{M_2 - b_0 - b_2\Pi}{b_1 - b_3\Pi} \qquad (47)$$

Then, $\alpha = (\alpha_1 + \alpha_2)/2$

(48)

These formulae are, the Cole's and the averaged – weighted Dip parameters models highly improved to take care of cross-stream flow, friction, bed roughness vis-à-vis aspect ratio. Tables of values for calibration of these models are then presented.

### 2.2 Proposed Improved Dip Modified Wake Law (PIDMWL) for the Prediction of Vertical Velocity Profile in Open Channels.

**Table 1: Data for y/h vs U/U* for Ntak Inyang River**

| S/N | U/U* | y/h |
|---|---|---|
| 1 | 0.0226 | 0.0702 |
| 2 | 0.0922 | 0.1053 |
| 3 | 1.1985 | 0.1404 |
| 4 | 3.1463 | 0.1754 |
| 5 | 4.8338 | 0.2105 |
| 6 | 6.1737 | 0.2456 |
| 7 | 7.4806 | 0.2807 |
| 8 | 8.5580 | 0.3158 |
| 9 | 9.6527 | 0.3509 |
| 10 | 9.3127 | 0.3860 |
| 11 | 8.9292 | 0.4211 |
| 12 | 8.6819 | 0.4561 |
| 13 | 8.3802 | 0.4912 |
| 14 | 8.1943 | 0.5263 |
| 15 | 7.8845 | 0.5614 |
| 16 | 7.6016 | 0.5965 |
| 17 | 7.3419 | 0.6316 |
| 18 | 7.1024 | 0.6667 |
| 19 | 6.8805 | 0.7018 |
| 20 | 6.3519 | 0.7368 |
| 21 | 5.8502 | 0.7719 |
| 22 | 5.3725 | 0.8070 |
| 23 | 5.1018 | 0.8281 |
| 24 | 4.7542 | 0.8421 |
| 25 | 3.6498 | 0.8772 |
| 26 | 3.8051 | 0.9123 |

**Table 2 Correlations**

| | | u_u | p1 | p2 | alpha1 | alpha2 |
|---|---|---|---|---|---|---|
| u_u | Pearson Correlation | 1 | .110 | .578$^{**}$ | -.608$^{**}$ | -.297 |
| | Sig. (2-tailed) | | .601 | .002 | .001 | .150 |
| | N | 25 | 25 | 25 | 25 | 25 |
| p1 | Pearson Correlation | .110 | 1 | .221 | -.255 | .045 |
| | Sig. (2-tailed) | .601 | | .278 | .209 | .826 |
| | N | 25 | 26 | 26 | 26 | 26 |
| p2 | Pearson Correlation | .578$^{**}$ | .221 | 1 | -.354 | -.253 |
| | Sig. (2-tailed) | .002 | .278 | | .076 | .213 |
| | N | 25 | 26 | 26 | 26 | 26 |
| alpha1 | Pearson Correlation | -.608$^{**}$ | -.255 | -.354 | 1 | .250 |
| | Sig. (2-tailed) | .001 | .209 | .076 | | .219 |
| | N | 25 | 26 | 26 | 26 | 26 |
| alpha2 | Pearson Correlation | -.297 | .045 | -.253 | .250 | 1 |
| | Sig. (2-tailed) | .150 | .826 | .213 | .219 | |
| | N | 25 | 26 | 26 | 26 | 26 |

**. Correlation is significant at the 0.01 level (2-tailed).

The value of alpha, $\alpha$ is 0.78. This is the calibrated value of $\alpha$ from the Table 3 arising from Nezu and Rodi equation (1986), which tallies with the average value of $\alpha$ from the derived table 3 $\alpha_1$= 0.64. It is necessary to adopt the former figure 0.78 as $\alpha$.

The highest value of U/U* was 5.84, while calibrated value from the model yields 0.103; Hence,

$$\frac{U}{U_*} = 0.103/K \qquad (49)$$

Putting = 5.84/3.43 =1.7027The value of Π was 0.45.

Multiplying this by 1.7027; that is 0.45 x 1.7027 = 0.765 = 0.77 approximately. Π = 0.77; this justifies the value derived earlier above. This is still within reasonable estimate as suggested by other authors. In the conduct of this research, the average value of an array of Π is 0.77 and the calibrated value of alpha, α is 0.78. Subsequent iterations using Jacobi method and Hardy Cross techniques on the Nezu and Rodi equation then putting the equation on side by side with the velocity gradient on the left brings the value of the coefficient, K to 3.6; so the modified value of K is 3.6 The Model proposed is for wide channels with flow characteristics well outlined in the work. The model is presented below in bold equation 50.

$$U/U_* = \aleph((1-y/h)(y/h+{}^{\pi}\Pi \mathit{sin}(y/2h)) - \alpha \exp(-y/h) + \ln(y/yo) \qquad (50)$$

This Equation applies most when aspect ratio of channel is above 10 and beyond. From the data plot, it removes the discrepancy noticed from other Existing Equations where the calibrated values of U/U∗ were low, although the shape of graphs plotted resembled measured.

**Table 3: Computation of Predictive Equation Function Coefficients for the Estimation of Cole's and Dip Parameters**

| A1 | a1(b0-m2) | b1(m1-a0) | A2 | A1*A1 | 4*A0*A2 | Sqrt(A1*A1-(4A0*A2)) | p1 | p2 |
|---|---|---|---|---|---|---|---|---|
| 0.014 | 0.122 | 0.009 | 0.132 | 0.28 | 0.000 | 0.003 | −14.093 | −21.478 |
| 0.051 | 0.454 | 0.002 | 0.456 | 0.003 | 0.003 | 0.003 | −16.726 | −19.048 |
| 0.060 | 0.574 | −0.021 | 0.553 | 0.004 | 0.008 | 0.065 | 0.723 | −17.567 |
| −0.1064 | 0.094 | −0.042 | 0.052 | 0.011 | 0.002 | 0.099 | 14.038 | 0.506 |
| −0.202 | -0.044 | −0.051 | −0.094 | 0.041 | −0.005 | 0.214 | 15.798 | −0.454 |
| −0.1436 | 0.146 | −0.062 | 0.084 | 0.021 | 0.007 | 0.115 | 5.982 | 0.651 |
| −0.3466 | -0.077 | −0.074 | −0.151 | 0.120 | −0.020 | 0.375 | 10.835 | −0.420 |
| −0.1869 | 0.167 | −0.089 | 0.078 | 0.035 | 0.015 | 0.141 | 3.377 | 0.475 |
| −2.382 | -2.002 | −0.049 | −2.051 | 5.672 | −0.556 | 2.495 | 36.011 | −0.841 |
| −0.078 | -0.129 | 0.018 | −0.110 | 0.006 | −0.040 | 0.215 | 1.607 | −0.749 |
| 0.402 | 0.214 | 0.021 | 0.235 | 0.162 | 0.112 | 0.223 | −0.751 | −2.624 |
| −0.138 | -0.169 | 0.023 | −0.145 | 0.019 | −0.088 | 0.325 | 1.536 | −0.625 |
| 0.575 | 0.258 | 0.027 | 0.286 | 0.331 | 0.216 | 0.340 | −0.625 | −2.426 |
| −0.536 | -0.424 | 0.041 | −0.383 | 0.288 | −0.352 | 0.800 | 2.911 | −0.573 |
| 0.3535 | 0.034 | 0.073 | 0.107 | 0.125 | 0.117 | 0.089 | −0.482 | −0.808 |
| 0.405 | 0.035 | 0.09 | 0.125 | 0.164 | 0.161 | 0.058 | −0.539 | −0.719 |
| 0.465 | 0.037 | 0.113 | 0.149 | 0.216 | 0.222 | 0.076 | −0.523 | −0.728 |
| 0.538 | 0.0391 | 0.145 | 0.185 | 0.290 | 0.314 | 0.151 | −0.458 | −0.814 |
| −3.023 | -2.264 | 0.372 | −1.892 | 9.141 | −3.593 | 3.568 | 6.943 | −0.574 |
| 1.063 | 0.048 | 0.779 | 0.827 | 1.123 | 1.739 | 0.780 | −0.269 | −1.754 |
| 1.229 | 0.0556 | 1.181 | 1.237 | 1.511 | 2.843 | 1.1539 | −0.065 | −2.074 |
| 3.857 | 2.297 | 1.662 | 3.958 | 14.875 | 9.821 | 2.248 | −1.297 | −4.921 |
| −3.067 | -5.072 | 3.383 | −1.689 | 9.408 | −4.361 | 3.7108 | 5.249 | −0.498 |
| −77.765 | -97.425 | 15.316 | −82.108 | 6047.379 | −217.289 | 79.165 | 118.589 | −1.047 |
| 26.990 | 45.965 | 12.284 | 58.249 | 728.4782 | 162.367 | 23.793 | -2.294 | -36.431 |
| 5.6836 | -11.545 | -15.885 | -27.430 | 32.303 | -79.053 | 10.553 | 3.3789 | -11.267 |

This is in line with main objective of the work. The proposed equation forms equation (51)

$$U/U_* = \aleph((1-y/h)\ (y/h + {}^{\pi}\Pi\ \sinh(y/h)) \qquad (51)$$

Where, $\aleph = 3.6$

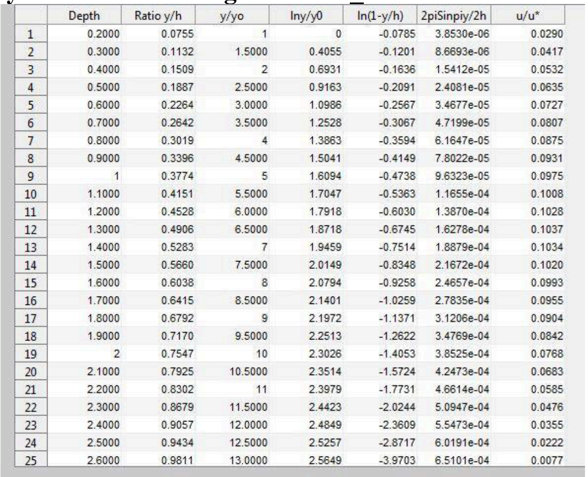
The product: ${}^{\pi}\Pi$; that is 3.142x 0.77 = 2.44

The product of pie and adjusted value of the parameter.

The value 2.44 is equivalent of inverse of the Cole's parameter, Π and replaces it in the equation.

## Tables for Ikpa River Velocity Gradient Variations (Matlab Generated)

## Table 4 Velocity Distribution Using DMLWL_ Absi 2011 for Ntak Inyang Reach



| | Depth | Ratio y/h | y/yo | lny/y0 | ln(1-y/h) | 2piSinpiy/2h | u/u* |
|---|---|---|---|---|---|---|---|
| 1 | 0.2000 | 0.0755 | 1 | 0 | -0.0785 | 3.8530e-06 | 0.0290 |
| 2 | 0.3000 | 0.1132 | 1.5000 | 0.4055 | -0.1201 | 8.6693e-06 | 0.0417 |
| 3 | 0.4000 | 0.1509 | 2 | 0.6931 | -0.1636 | 1.5412e-05 | 0.0532 |
| 4 | 0.5000 | 0.1887 | 2.5000 | 0.9163 | -0.2091 | 2.4081e-05 | 0.0635 |
| 5 | 0.6000 | 0.2264 | 3.0000 | 1.0986 | -0.2567 | 3.4677e-05 | 0.0727 |
| 6 | 0.7000 | 0.2642 | 3.5000 | 1.2528 | -0.3067 | 4.7199e-05 | 0.0807 |
| 7 | 0.8000 | 0.3019 | 4 | 1.3863 | -0.3594 | 6.1647e-05 | 0.0875 |
| 8 | 0.9000 | 0.3396 | 4.5000 | 1.5041 | -0.4149 | 7.8022e-05 | 0.0931 |
| 9 | 1 | 0.3774 | 5 | 1.6094 | -0.4738 | 9.6323e-05 | 0.0975 |
| 10 | 1.1000 | 0.4151 | 5.5000 | 1.7047 | -0.5363 | 1.1655e-04 | 0.1008 |
| 11 | 1.2000 | 0.4528 | 6.0000 | 1.7918 | -0.6030 | 1.3870e-04 | 0.1028 |
| 12 | 1.3000 | 0.4906 | 6.5000 | 1.8718 | -0.6745 | 1.6278e-04 | 0.1037 |
| 13 | 1.4000 | 0.5283 | 7 | 1.9459 | -0.7514 | 1.8879e-04 | 0.1034 |
| 14 | 1.5000 | 0.5660 | 7.5000 | 2.0149 | -0.8348 | 2.1672e-04 | 0.1020 |
| 15 | 1.6000 | 0.6038 | 8 | 2.0794 | -0.9258 | 2.4657e-04 | 0.0993 |
| 16 | 1.7000 | 0.6415 | 8.5000 | 2.1401 | -1.0259 | 2.7835e-04 | 0.0955 |
| 17 | 1.8000 | 0.6792 | 9 | 2.1972 | -1.1371 | 3.1206e-04 | 0.0904 |
| 18 | 1.9000 | 0.7170 | 9.5000 | 2.2513 | -1.2622 | 3.4769e-04 | 0.0842 |
| 19 | 2 | 0.7547 | 10 | 2.3026 | -1.4053 | 3.8525e-04 | 0.0768 |
| 20 | 2.1000 | 0.7925 | 10.5000 | 2.3514 | -1.5724 | 4.2473e-04 | 0.0683 |
| 21 | 2.2000 | 0.8302 | 11 | 2.3979 | -1.7731 | 4.6614e-04 | 0.0585 |
| 22 | 2.3000 | 0.8679 | 11.5000 | 2.4423 | -2.0244 | 5.0947e-04 | 0.0476 |
| 23 | 2.4000 | 0.9057 | 12.0000 | 2.4849 | -2.3609 | 5.5473e-04 | 0.0355 |
| 24 | 2.5000 | 0.9434 | 12.5000 | 2.5257 | -2.8717 | 6.0191e-04 | 0.0222 |
| 25 | 2.6000 | 0.9811 | 13.0000 | 2.5649 | -3.9703 | 6.5101e-04 | 0.0077 |

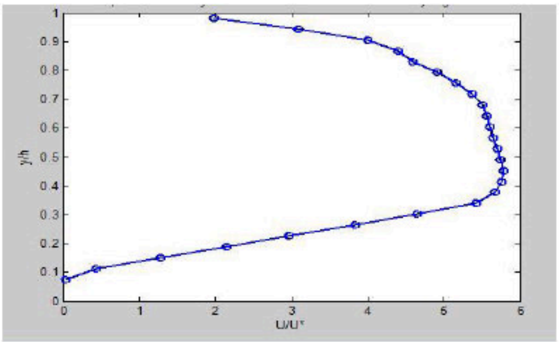


Fig 1: Variation of y/h with U/U* using Actual measurement for Ntak Inyang Reach on Ikpa River data

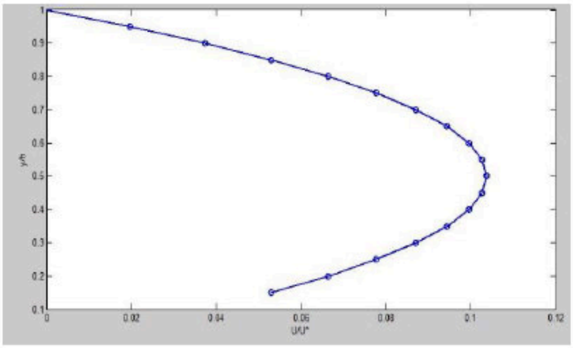


Fig 3: Variation of y/h with U/U* using Nezu and Rodi (1986) for Nduetong Reach, Ikpa River data

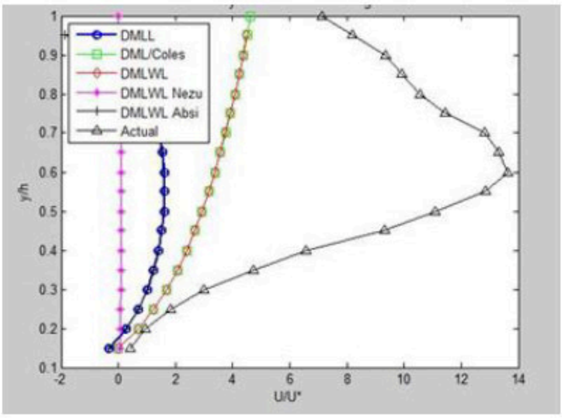


Fig. 2: Variation of y/h with U/U* for various models for Ntak Inyang Reach on Ikpa River data.

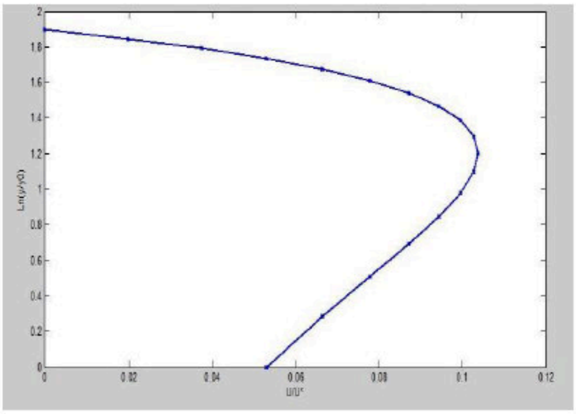


Fig 4: Variation of ln(y/y0) with U/U* using Nezu and Rodi (1986) for Nduetong Reach, Ikpa River data.

## 3. Conclusion

Stepping from the results of the research, several predictive equations have emerged. Authentically, two distinct improved models have been derived to help predict very neatly the vertical velocity profiles of river flow. Presentation of the First and Second Proposed Improved Dip Modified Log Wake Exponential Law (PIDMWLEL) and (PIDMLL) according to (Bassey, 2015) has been made in this work. Using the derivations of other cited authors, it is shown that bed roughness, slope steepness and aspect ratios do affect friction velocity and upward flow pattern. The aspect ratios of typical rivers reaches for the selected models were found to be less than 5. Above all, the developed proposed models could deal on rivers with aspect ratio greater than 10. The plots of variation of y/h with $U/U^*$ for the first three equations have close relationship with the measured. The aspect ratio of the open channel has significant influence on the velocity distribution characteristics. The plots of variation of y/h with $U/U^*$ for the first three equations have close relationship with the measured. The aspect ratio, α of the open channel has significant influence on the velocity distribution characteristics. The two Improved modified log law developed in this work are shown on equations 51 and 51. These resulting equations now have a coefficient whose reciprocal is 3.6 to complement the universal Karman's constant, k = 0.41. It is an inverse product of the constant. Appropriate adjustment and predictive estimation of Cole's and Dip parameters now emerged. The values are 0.77 and 0.78 respectively instead of 0.44 and 0.78 respectively. Dip parameters seem to have reasonable level of agreement with calibration of existing known model for determination of the dip. Taking few results of the project, the shear velocity at Ntak Inyang River reach, from the bed is 0.172 m/s and velocity gradient, $U/U_*$ has maximum value of 5.8 approximately. Maximum vertical velocity is located at y/h equals 0.56 corresponding to the depth of 1.5m for a flow depth of the open natural river, being 2.65m. One of the measurements at Odiok Itam River, where the total flow depth was 2.75m, the maximum flow occurred at y/h = 0.80 being 80% of depth with $U/U_* = 4.135$ measured

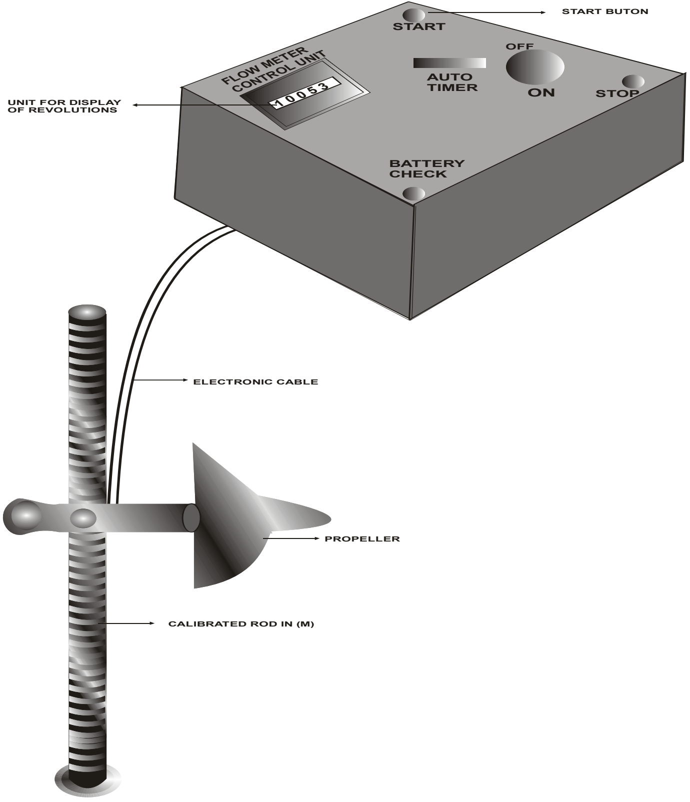


FIG. 2: LEVELING ROD

Plate A II.1 Current meter used in the flow measurement at Ntak Inyang Reach of Ikpa River

### 4. Notations and symbols used in this work:

| | |
|---|---|
| $A_0$ | coefficient of the first term of quadratic equation |
| $a_0$ | notation for the product |
| $A_1$ | coefficient of the second term of the quadratic equation |
| $a_1$ | notation for the product |
| $A_2$ | constant term of the quadratic equation |
| $a_2$ | notation for the product, |
| $a_3$ | notation for the product coefficient |
| Alpha, α | used as dip parameter |
| $A_r$ | Aspect ratio, ratio of channel width to its depth |
| $A_S$ | surface area |
| $b_0$ | notation for derived coefficient |
| $b_1$ | notation for derived coefficient term |
| $b_2$ | notation for product of velocity gradient and equation term |
| $b_3$ | notation for derived coefficient term |
| g | acceleration due to gravity |
| h | water depth from channel bed to free surface |

| | |
|---|---|
| H | head enthalpy, depth /height notation |
| $h_f$ | potential head, friction head loss |
| i | hydraulic gradient, node identifier or vertical position identifier |
| K | Karman's constant, channel conveyance |
| $k_b$ | coefficient at outer region of channel profile |
| $k_s$ | roughness thickness according to Nikurádse |
| $k_{ua}$ | roughness coefficient in turbulent region |
| l | mixing length, longitudinal distance stream wise |
| LAN | Local Area Network |
| ln | natural logarithm |
| m | mass, area ratio, hydraulic mean depth |
| $M_1$ | First order derivative of the differential equation of flow velocity |
| $M_2$ | Second order derivative of differential equation of flow velocity |
| $\eta$ | Coefficient of sidewall distance |
| n | manning's channel roughness coefficient, subscript denote number count |
| N | number |
| P | pressure, force, power |
| P –v K | Prandtl von Karman coefficient |
| q | Flow rate per unit width or unit depth, lateral channel flow |
| Q | discharge, volumetric flow rate |
| r | radius, radial distance |
| R | hydraulic radius, reaction forces |
| Re | Reynolds number |
| $R_u$ | hydraulic number |
| s | slope distance, energy gradient |
| S | entropy, channel and friction slope |
| Sin | sine |
| $S_{ox, y}$ | bed slope in x and y directions (axes) |
| t | time, thickness |
| T | temperature, torque surface width, flow surface width, Dimension of time |
| $U_*$ | friction velocity |
| u, U | flow velocity |
| $U/ U_*$ | Ratio of velocities to denote velocity gradient |
| $U_m$ | mean velocity |
| $U_{max}$ | maximum velocity |
| $V_f$ | flow friction velocity |
| $V_o$ | tangential velocity |
| $V_t$ | Vortex velocity |
| $V_x$, $V_y$, $V_z$ | velocity components in x, y and z directions |
| w | specific weight |
| W | weight, work |
| Wp | wetted perimeter |
| x, y, z | orthogonal coordinates |
| y | vertical position of particle in water channel |
| $y/\delta$ | depth ratio |
| $y_0$ | hypothetical position depth where flow is assumed to be zero |
| $y_1$ | Term in the second order differential equation |
| $y_2$ | Parameter coefficient term |
| $Y_w$ | velocity defect by Daily |
| Z | potential head |
| $\alpha_1$, $\alpha_2$ | parameters and coefficients |
| $\beta$ | in combination with other parameters to denote width or breadth in Kennedy's equation |
| $\beta_1$ | Relative vertical position from the water surface |
| $\beta_2$ | notation for parameter coefficient |
| $\beta_3$ | notation for equation term |
| $\beta_4$ | notation for parameter determination coefficient term in equation |
| $\beta_o$ | Friction Velocity constant term |
| $\delta$ | increment in width of water element in channel |
| $\Delta$ | change in |

| | |
|---|---|
| E | coefficient of velocity using energy method, absolute roughness |
| $\varepsilon_0$ | hypothetical velocity gradient by Chiu |
| $\mu$ | coefficient of dynamic viscosity |
| $\xi$ | cross velocity, relative position gradient |
| $\xi_0$ | relative hypothetic position |
| $\Pi$ | Cole’s wake strength |
| $\rho$ | fluid density |
| $\tau$ | threshold shear stress |
| $\upsilon$ | coefficient of kinematic viscosity, Poisson's ratio |
| $\mathit{\Phi}$ | shear strain, angle, function |
| $\Phi$ | velocity potential |
| $\chi$ | Cole’s parameter |
| $\Psi$ | stream function |
| $\Omega$ | angular (rotational) velocity, stage variable |
| б | distance increment longitudinally |
| e | error, base of natural number |
| $\aleph$ | Coefficient of determination of velocity gradient in the models (PIDMWLEL and PIDMLWL) |

## 5. Acknowledgment

In the work the following are gratefully acknowledged; Engr. Prof. B. A. Nwachukwu who made outstanding efforts in proposing a PhD Program for me at Federal University of Technology, Owerri (FUTO), Engr. Prof. Jonah C. Agunwamba of University of Nigeria, Nsukka (UNN), who at certain times was Adjunct Lecturer at Federal University of Technology, Owerri, provided the topic of research who modified the topic and directed the research work to attain its current outlook and Engr. Dr. B. C. Okoro of the Department of Building Technology(FUTO) for constantly reading the manuscript and patiently noting salient areas of relevance.

Dr. (Mrs.) B. U. Dike, the current indefatigable Head of Department of Civil Engineering who within a short while transformed the Department with energy and dynamism and ther staff members of FUTO are fondly remembered for the enormous contributions to like the persons of Engr. Dr. D. O. Onwuka and Engr. Dr. J. C. Ezeh and my PhD activist Engr. (Mrs.) Joan I. Arimanwa. I am indebted to Rev Dr. Eno E. Joshua and Dr. Inyang Udoinyang George of Department of Mathematics and Computer Science, University of Uyo (UNIUYO) for their assistance in the use of Matlab to develop, write and run appropriate programming for tables and graphs for models and calibrate Cole's and Dip parameters. Deep appreciations go to Dr Simeon Ozuomba and Mr. Ugonna Loveday Nwamara, staff member and final year student respectively of Electrical/Electronics/Computer Engineering Department, (UNIUYO), as they assisted in the data plots with Microsoft excel and CorelDraw 12. The Managing Director, Cross River Basin Authority, Calabar, Engr. E. U. Essien and staff members, especially Mrs. Hogan of Hydrology Unit, are commended immensely for providing equipment, the current meter free to enable fieldwork to be undertaken.

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