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A novel hybrid fuzzy MCDM approach for the selection of building materials

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Abstract
The selection of building materials is one of the most critical activities in the design of a building and is often observed to be a multi-criterion decision-making problem with conflicting and different objectives. This paper proposes a building material selection model based on a hybrid fuzzy MCDM techniques, a multi-criterion decision analysis approach. The fuzzy MCDM is used to prioritize and assign important weightings for evaluating criteria. Ratings of alternatives versus qualitative criteria and the importance weights of all the criteria are assessed in linguistic values represented by fuzzy numbers. Ranking formulae and membership functions for the final fuzzy evaluation values can be clearly developed for better executing the decision making. A numerical is used to demonstrate the feasibility of the proposed approach.

Keywords: Fuzzy MCDM, fuzzy logic, building materials selection, ranking, maximizing set and minimizing set.

1. Introduction

Construction is an area of study wherein making decisions adequately can mean the difference between success and failure. In building design stages, construction materials are often grouped together to form what is called building assemblies. Moreover, most of the activities belonging to this sector involve taking into account a large number of conflicting aspects, which hinders their management as a whole.

One of the most important tasks in the design development stage of building design is the selection of the appropriate building assemblies to be used in the various elements of the building, e.g. walls, roofs, floors and so on. The selection of building materials is regarded as a multi-criteria decision problem largely based on trusting experience rather than using numerical approach due to lack of formal and availability of measurement methods and strategies [Akadiri, 2013]. This decision will have a significant impact on the performance of the building with respect to the various design criteria. Although this decision cannot be entirely separated from other design stages, the use of decision-making techniques can render this a more rational decision [Nassar et al. 2003].

Therefore, there is need for developing a systematic and holistic material selection process of identifying and evaluating trade-offs. The characterisation of material selection process as an essentially multifaceted problem involving numerous, variegated considerations, often with complex trade-offs among them, implied that a suitable solution might be found among the family of multi-criteria decision analysis (MCDA) methods [Shapira and Goldenberg, 2005; Barker and Zabinky, 2011; Zavadskas et al. 2014a; Alibaba and Ozdeniz, 2004]. Further analysis and profiling of the selection problem and the identification of the solution methods desirable capabilities, triggered the consideration of a fuzzy multiple criteria decision making (MCDM) model.

2. **Fuzzy set theory**

Fuzzy set theory has proven advantages within vague, imprecise and uncertain contexts and it resembles human reasoning in its use of approximate information and uncertainty to generate decisions [Nieto-Morote and Ruz-Vila, 2012]. Initially proposed by Zadeh [1965], It was specially designed to mathematically represent uncertainty and vagueness and provide formalized tools for dealing with the imprecision intrinsic to many decision problems. The fuzzy set theory has been extensively applied to objectively reflect the ambiguities in human judgment and effectively resolve the uncertainties in the available information in an ill-defined multiple criteria decision making environment. Numerous approaches have been proposed to solve fuzzy MCDM problems. A review and comparison of many of these methods can be found in Akadiri et al. [2013], Mardani et al. [2015], Karbir et al. [2014] and Zavadskas et al. [2014b]. Some recent applications on materials evaluation and selection can be found in [Akadiri et al. 2013; Xue et al; 2016; Khoshnava et al. 2016]. Despite the merits, most of the above papers cannot present membership functions for the final fuzzy evaluation values, nor can they clearly develop defuzzification formulae from the membership functions of the final fuzzy evaluation values, limiting the applicability of the fuzzy MCDM methods available. Many fuzzy number ranking methods have been studied. Some recent approaches can be seen in [Zhang and Xu, 2012; Kucukvar et al; 2014; Li and Yang; 2015]. However, in spite of the merits, some of these methods are computational complex and difficult to implement and none of them can satisfactorily rank fuzzy numbers in all situations and cases.

To resolve these limitations, this work suggests a maximizing set and minimizing set based fuzzy MCDM approach for the evaluation and selection of building materials. Herein, the ranking approach of maximizing set and minimizing set of Chen [1985] is applied for defuzzification due to its simplicity of implementation. Furthermore, defuzzification procedure can be clearly presented and formulae can be developed. Finally, a numerical example demonstrates the computational process of the proposed model.

2.1 Fuzzy Sets

$A=\{(x, f_A(x)) \mid x \in U\}$, where U is the universe of discourse, x is an element in U, A is a fuzzy set in U, $f_A(x)$ is the membership function of A at x [Kaufmann and Gupta, 1991]. The large $f_A(x)$, the stronger the grade of membership for x in A.

2.2 Fuzzy Numbers

A real fuzzy number A is described as any fuzzy subset of the real line R with membership function f_A which possesses the following properties [Dubois and Prade, 1978]:

(a) f_A is a continuous mapping from R to [0,1];

(b) $f_A(x)=0, \forall x \in (-\infty, a]$;

(c) f_A is strictly increasing on $[a, b]$;

(d) $f_A(x)=1, x \in [b, c]$;

(e) f_A is strictly decreasing on $[c, d]$;

(f) $f_A(x)=0, \forall x \in [d, \infty)$;

where $a \le b \le c \le d$, A can be denoted as $[a, b, c, d]$. The membership function f_A of the fuzzy number A can also be expressed as:

$$f_A(x)=\begin{cases} f_A^L(x), & a \le x \le b \\ 1, & b \le x \le c \\ f_A^R(x), & c \le x \le d \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where $f_A^L(x)$ and $f_A^R(x)$ are left and right membership functions of A, respectively [Kaufmann and Gupta, 1991]. A fuzzy triangular number can be denoted as (a, b, c) [Larrhoven and Pedrycz, 1983].

2.3 α-cuts

The α-cuts of fuzzy number A can be defined as $A^\alpha=\{x \mid f_A(x) \ge \alpha\}, \alpha \in [0,1]$, where A^α is a non-empty bounded closed interval contained in R and can be denoted by $A^\alpha=[A_l^\alpha, A_u^\alpha]$, where A_l^α and A_u^α are its lower and upper bounds, respectively [Kaufmann and Gupta, 1991].

2.4 Arithmetic Operations on Fuzzy Numbers

Given fuzzy numbers A and B, $A, B \in R^+$, the α-cuts of A and B are $A^\alpha = \left[A_l^\alpha, A_u^\alpha\right]$ and $B^\alpha = \left[B_l^\alpha, B_u^\alpha\right]$, respectively. By the interval arithmetic, some main operations of A and B can be expressed as follows [20]:

$$(A \oplus B)^\alpha = \left[A_l^\alpha + B_l^\alpha, A_u^\alpha + B_u^\alpha\right] \tag{2}$$

$$(A \ominus B)^\alpha = \left[A_l^\alpha - B_u^\alpha, A_u^\alpha - B_l^\alpha\right] \tag{3}$$

$$(A \otimes B)^\alpha = \left[A_l^\alpha \cdot B_l^\alpha, A_u^\alpha \cdot B_u^\alpha\right] \tag{4}$$

$$(A \oslash B)^\alpha = \left[\frac{A_l^\alpha}{B_u^\alpha}, \frac{A_u^\alpha}{B_l^\alpha}\right] \tag{5}$$

$$(A \otimes r)^\alpha = \left[A_l^\alpha \cdot r, A_u^\alpha \cdot r\right], r \in R^+ \tag{6}$$

2.5 Linguistic Values

A linguistic variable is a variable whose values are expressed in linguistic terms. Linguistic variable is a very helpful concept for dealing with situations which are too complex or not well-defined to be reasonably described by traditional quantitative expressions [Zadeh, 1975]. For example, "importance" is a linguistic variable whose values include UI (unimportant), LI (less important), I (important), MI (more important) and VI (very important). These linguistic values can be further represented by triangular fuzzy numbers such as UI=(0.0,0.0,0.25), LI=(0.0,0.25,0.5), I=(0.25,0.5,0.75), MI=(0.50,0.75,1.00) and VI=(0.75,1.00,1.00).

3. Model development

Suppose decision makers D_t, $t = 1, 2, ..., l$, are responsible for evaluating alternatives A_i, $i = 1, 2, ..., m$, under selected criteria, C_j, $j = 1, 2, ..., n$. Criteria are categorized into three groups such as benefit qualitative criteria C_j, $j = 1, ..., g$, benefit quantitative criteria C_j, $j = g+1, ..., h$, and cost quantitative criteria C_j, $j = h+1, ..., n$. The proposed model is developed as the following steps.

3.1 Aggregate ratings of alternatives versus qualitative criteria

Assume $x_{ijt} = (a_{ijt}, b_{ijt}, c_{ijt})$, $i = 1,\dots,m,\ j = 1,\dots,g,\ t = 1,\dots,l,$

$$x_{ij} = \frac{1}{l} \otimes (x_{ij1} \oplus x_{ij2} \oplus \dots \oplus x_{ijl}), \tag{7}$$

where $a_{ij} = \frac{1}{l}\sum_{t=1}^{l} a_{ijt}$, $b_{ij} = \frac{1}{l}\sum_{t=1}^{l} b_{ijt}$, $c_{ij} = \frac{1}{l}\sum_{t=1}^{l} c_{ijt}$. x_{ijt} denotes ratings assigned by each decision maker for each alternative versus each qualitative criterion. x_{ij} denotes averaged rating of each alternative versus each qualitative criterion.

3.2 Normalize values of alternatives versus quantitative criteria

Herein, Zhang and Xu [2012] method is applied to normalize values of alternatives versus quantitative criteria, including benefit and cost, in order to make data dimensionless for calculation rationale. Benefit quantitative data has the characteristics: the larger the better; whereas cost quantitative data has the characteristics: the smaller the better. Suppose $y_{ij} = (o_{ij}, p_{ij}, q_{ij})$ denotes evaluation value of alternative i versus benefit quantitative criteria $j,\ j = g+1,\dots,h,$ as well as cost quantitative criteria $j,\ j = h+1,\dots,n.$ And x_{ij} denotes the normalized value of y_{ij},

$$x_{ij} = \left(\frac{o_{ij}}{q_{ij}^*}, \frac{p_{ij}}{q_{ij}^*}, \frac{q_{ij}}{q_{ij}^*}\right),\ q_{ij}^* = \max q_{ij},\ j \in B, \tag{8}$$

$$x_{ij} = \left(\frac{o_{ij}^*}{q_{ij}}, \frac{o_{ij}^*}{p_{ij}}, \frac{o_{ij}^*}{o_{ij}}\right),\ o_{ij}^* = \min o_{ij},\ j \in C. \tag{9}$$

For calculation convenience, assume $x_{ij} = (a_{ij}, b_{ij}, c_{ij})$, $j = g+1,\dots,n$.

3.3 Average importance weights

Assume $w_{jt} = (d_{jt}, e_{jt}, f_{jt}),\ w_{jt} \in R^+,\ j = 1,\dots,n,\ t = 1,\dots,l,$

$$w_j = \frac{1}{l} \otimes (w_{j1} \oplus w_{j2} \oplus \dots \oplus w_{jl}), \tag{10}$$

where $d_j = \frac{1}{l}\sum_{t=1}^{l} d_{jt}$, $e_j = \frac{1}{l}\sum_{t=1}^{l} e_{jt}$, $f_j = \frac{1}{l}\sum_{t=1}^{l} f_{jt}$. w_{jt} represents the weight assigned by each decision maker for each criterion and w_j represents the average importance weight of each criterion.

3.4 Develop membership functions

The membership function of the final fuzzy evaluation value, $G_i,\ i=1,\ldots,n$, of each alternative can be developed as Eq. (11). In Eq. (1), the first two parts are additive weighted ratings under benefit criteria. The third part is under cost criteria but given a negative sign. Therefore, the larger the G_i value, the better performance A_i will have.

$$G_i = \sum_{j=1}^{g} w_j \otimes x_{ij} + \sum_{g+1}^{h} w_i \otimes x_{ij} - \sum_{j=h+1}^{n} w_j \otimes x_{ij} \ , \tag{11}$$

The membership functions are developed as:

$$G_i^{\alpha} = \sum_{j=1}^{g} w_j^{\alpha} \otimes x_{ij}^{\alpha} + \sum_{j=g+1}^{h} w_j^{\alpha} \otimes x_{ij}^{\alpha} - \sum_{j=h+1}^{n} w_j^{\alpha} \otimes x_{ij}^{\alpha}, \tag{12}$$

$$w_j^{\alpha} = [(e_j - d_j)\alpha + d_j, (e_j - f_j)\alpha + f_j], \tag{13}$$

$$x_{ij}^{\alpha} = [(b_{ij} - a_{ij})\alpha + a_{ij}, (b_{ij} - c_{ij})\alpha + c_{ij}]. \tag{14}$$

From Eqs. (13) and (14), we can develop Eq. (15) as follows:

$$\sum w_j^{\alpha} \otimes x_{ij}^{\alpha} = \Big[\sum (e_j - d_j)(b_{ij} - a_{ij})\alpha^2 + \sum (a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij}))\alpha + \sum a_{ij} d_j,$$

$$\sum (b_{ij} - c_{ij})(e_j - f_j)\alpha^2 + \sum (c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij}))\alpha + \sum c_{ij} f_j \Big] \tag{15}$$

By applying Eq. (15) to Eq. (12), three equations are developed:

$$\begin{aligned}\sum_{j=1}^{g} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [&\sum_{j=1}^{g} (e_j - d_j)(b_{ij} - a_{ij})\alpha^2 + \sum_{j=1}^{g} (a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij}))\alpha + \sum_{j=1}^{g} a_{ij} d_{ij} \ ,\\ &\sum_{j=1}^{g} (b_{ij} - c_{ij})(e_j - f_j)\alpha^2 + \sum_{j=1}^{g} (c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij}))\alpha + \sum_{j=1}^{g} c_{ij} f_j].\end{aligned} \tag{16}$$

$$\begin{aligned}\sum_{j=g+1}^{h} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [&\sum_{j=g+1}^{h} (e_j - d_j)(b_{ij} - a_{ij})\alpha^2 + \sum_{j=g+1}^{h} (a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij}))\alpha + \sum_{j=g+1}^{h} a_{ij} d_{ij} \ ,\\ &\sum_{j=g+1}^{h} (b_{ij} - c_{ij})(e_j - f_j)\alpha^2 + \sum_{j=g+1}^{h} (c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij}))\alpha + \sum_{j=g+1}^{h} c_{ij} f_j].\end{aligned} \tag{17}$$

$$\sum_{j=h+1}^{n} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [\sum_{j=h+1}^{n} (e_j - d_j)(b_{ij} - a_{ij})\alpha^2 + \sum_{j=h+1}^{n}(a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij}))\alpha + \sum_{j=h+1}^{n} a_{ij} d_{ij} ,$$
$$\sum_{j=h+1}^{n} (b_{ij} - c_{ij})(e_j - f_j)\alpha^2 + \sum_{j=h+1}^{n} (c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij}))\alpha + \sum_{j=h+1}^{n} c_{ij} f_j]. \tag{18}$$

Assume:

$$A_{i1} = \sum_{j=1}^{g}(e_j - d_j)(b_{ij} - a_{ij}),\ A_{i2} = \sum_{j=g+1}^{h}(e_j - d_j)(b_{ij} - a_{ij}),\ A_{i3} = \sum_{j=h+1}^{n}(e_j - d_j)(b_{ij} - a_{ij}),$$

$$B_{i1} = \sum_{j=1}^{g}[a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij})],\ B_{i2} = \sum_{j=g+1}^{h}[a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij})],$$

$$B_{i3} = \sum_{j=h+1}^{n}[a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij})],\ C_{i1} = \sum_{j=1}^{g}(b_{ij} - c_{ij})(e_j - f_j),$$

$$C_{i2} = \sum_{j=g+1}^{h}(b_{ij} - c_{ij})(e_j - f_j),\ C_{i3} = \sum_{j=h+1}^{n}(b_{ij} - c_{ij})(e_j - f_j),$$

$$D_{i1} = \sum_{j=1}^{g}[c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij})],\ D_{i2} = \sum_{j=g+1}^{h}[c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij})],$$

$$D_{i3} = \sum_{j=h+1}^{n}[c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij})],\ O_{i1} = \sum_{j=1}^{g} a_{ij} d_j,\ O_{i2} = \sum_{j=g+1}^{h} a_{ij} d_j,\ O_{i3} = \sum_{j=h+1}^{n} a_{ij} d_j,$$

$$P_{i1} = \sum_{j=1}^{g} b_{ij} e_j,\ P_{i2} = \sum_{j=g+1}^{h} b_{ij} e_j,\ P_{i3} = \sum_{j=h+1}^{n} b_{ij} e_j,\ Q_{i1} = \sum_{j=1}^{g} c_{ij} f_j,\ Q_{i2} = \sum_{j=g+1}^{h} c_{ij} f_j,\ Q_{i3} = \sum_{j=h+1}^{n} c_{ij} f_j .$$

By applying the above assumption, Eqs. (16)-(18) can be arranged as:

$$\sum_{j=1}^{g} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [A_{i1}\alpha^2 + B_{i1}\alpha + O_{i1}, C_{i1}\alpha^2 + D_{i1}\alpha + Q_{i1}], \tag{19}$$

$$\sum_{j=g+1}^{h} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [A_{i2}\alpha^2 + B_{i2}\alpha + O_{i2}, C_{i2}\alpha^2 + D_{i2}\alpha + Q_{i2}], \tag{20}$$

$$\sum_{j=h+1}^{n} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [A_{i3}\alpha^2 + B_{i3}\alpha + O_{i3}, C_{i3}\alpha^2 + D_{i3}\alpha + Q_{i3}]. \tag{21}$$

Applying Eqs. (19)-(21) to Eq. (12) to produce Eq. (22):

$$G_i^{\alpha} = [(A_{i1} + A_{i2} - C_{i3})\alpha^2 + (B_{i1} + B_{i2} - D_{i3})\alpha + (O_{i1} + O_{i2} - Q_{i3}) ,$$
$$(C_{i1} + C_{i2} - A_{i3})\alpha^2 + (D_{i1} + D_{i2} - B_{i3})\alpha + (Q_{i1} + Q_{i2} - O_{i3})] . \tag{22}$$

The right and left membership functions of G_i can be obtained as shown in Eq. (23) and Eq. (24) as follows:

$$\alpha = f_{G_i}^{L}(x)$$

$$= \frac{-(B_{i1}+B_{i2}-D_{i3})+[(B_{i1}+B_{i2}-D_{i3})^2+4(A_{i1}+A_{i2}-C_{i3})(x-(O_{i1}+O_{i2}-Q_{i3}))]^{1/2}}{2(A_{i1}+A_{i2}-C_{i3})} \quad (23)$$

If $O_{i1}+O_{i2}-Q_{i3} \le x \le P_{i1}+P_{i2}-P_{i3}$;

$$\alpha = f_{G_i}^{R}(x)$$

$$= \frac{-(D_{i1}+D_{i2}-B_{i3})+[(D_{i1}+D_{i2}-B_{i3})^2+4(C_{i1}+C_{i2}-A_{i3})(x-(Q_{i1}+Q_{i2}-O_{i3}))]^{1/2}}{2(C_{i1}+C_{i2}-A_{i3})} \quad (24)$$

If $P_{i1}+P_{i2}-P_{i3} \le x \le Q_{i1}+Q_{i2}-O_{i3}$.

3.5 Rank fuzzy numbers

In this research, Chen [1985] maximizing set and minimizing set is applied to rank all the final fuzzy evaluation values. This method is one of the most commonly used approaches of ranking fuzzy numbers in fuzzy decision making.

The maximizing set M is defined as:

$$f_M(x) = \begin{cases} \left(\frac{x_{R_i}-x_{\min}}{x_{\max}-x_{\min}}\right)^k, x_{\min} \le x_{R_i} \le x_{\max}\ , \\ 0, \text{otherwise}. \end{cases} \quad (25)$$

The minimizing set N is defined as:

$$f_N(x) = \begin{cases} \left(\frac{x_{L_i}-x_{\max}}{x_{\min}-x_{\max}}\right)^k, x_{\min} \le x_{L_i} \le x_{\max}\ , \\ 0, \text{otherwise}, \end{cases} \quad (26)$$

where $x_{\min} = \inf_x S$, $x_{\max} = \sup_x S$, $S = \cup_{i=1}^{n} S_i$, $S_i = \{x | f_{A_i}(x) > 0\}$, usually k is set to 1.

The right utility of A_i is defined as:

$$U_M(A_i) = \sup_x (f_M(x) \wedge f_{A_i}(x)), i = 1 \sim n. \tag{27}$$

The left utility of A_i is defined as:

$$U_N(A_i) = \sup_x (f_M(x) \wedge f_{A_i}(x)), i = 1 \sim n. \tag{28}$$

The total utility of A_i is defined as:

$$U_T(A_i) = \frac{1}{2}(U_M(A_i) + 1 - U_N(A_i)), i = 1 \sim n. \tag{29}$$

The total utility $U_T(A_i)$ is applied to rank fuzzy numbers. The larger the $U_T(A_i)$, the larger the fuzzy number A_i. Applying Eqs. (25)-(29) to Eqs. (23)-(24), the total utility of fuzzy number G_i can be obtained as:

$$U_T(G_i) = \frac{1}{2}(U_M(G_i) + 1 - U_N(G_i)),\ i = 1 \sim n\ ,$$

$$= \frac{1}{2}[\frac{-(D_{i1} + D_{i2} - B_{i3}) - [(D_{i1} + D_{i2} - B_{i3})^2 + 4(C_{i1} + C_{i2} - A_{i3})(x_{R_I} - (Q_{i1} + Q_{i2} - O_{i3}))]^{\frac{1}{2}}}{2(C_{i1} + C_{i2} - A_{i3})}$$

$$+1 - \frac{-(B_{i1} + B_{i2} - D_{i3}) + [(B_{i1} + B_{i2} - D_{i3})^2 + 4(A_{i1} + A_{i2} - C_{i3})(x_{L_i} - (O_{i1} + O_{i2} - Q_{i3}))]^{\frac{1}{2}}}{2(A_{i1} + A_{i2} - C_{i3})}]. \tag{30}$$

where

$$x_{R_i} = -(2(C_{i1} + C_{i2} - A_{i3})x_{\min} + (x_{\min} - x_{\max})(D_{i1} + D_{i2} - B_{i3} + x_{\min} - x_{\max}))$$

$$-(x_{\max} - x_{\min})[(D_{i1} + D_{i2} - B_{i3} + x_{\min} - x_{\max})^2$$

$$+4(C_{i1} + C_{i2} - A_{i3})(x_{\min} - Q_{i1} - Q_{i2} + O_{i3})]^{\frac{1}{2}} / 2(C_{i1} + C_{i2} - A_{i3}). \tag{31}$$

$$x_{L_i} = -(2(A_{i1} + A_{i2} - C_{i3})x_{\max} + (x_{\max} - x_{\min})(B_{i1} + B_{i2} - D_{i3} + x_{\max} - x_{\min}))$$

$$+ (x_{\min} - x_{\max})[(B_{i1} + B_{i2} - D_{i3} + x_{\max} - x_{\min})^2$$

$$+ 4(A_{i1} + A_{i2} - C_{i3})(x_{\max} - O_{i1} - O_{i2} + Q_{i3})]^{\frac{1}{2}} / 2(A_{i1} + A_{i2} - C_{i3}). \quad (32)$$

4. Implementation of the Selection Model

The worked example for elucidating the application of the model in practice involves the application to a hypothetical but realistic scenario of a building material selection problem. The case study used intends to provide an indication of the use of the hybrid multi-criteria decision-making model for the problem analysed (i.e., the selection of building materials). The proposed scenario taken as study case is a hypothetical design of a single family home located in a light residential area of Lagos, Nigeria. Three architects (D1, D2, D3) of an architectural firm are working with a client to select materials (in this case roofing elements) for a proposed residential building. The client tells the architects that he wants a building made from materials that are friendly to the environment. The client qualifies his specifications, however, to say that he does not want the building's functions to be compromised by the design or choice of materials. The architects decide to use multi-criteria decision analysis (MCDA) to make the material choices that will best satisfy the clients' needs. Table 1 summarizes the details for the three options of roofing elements for the proposed project. The description of the three options is based on the standard practices and construction details commonly used in Nigeria.

Table 1 Summary of roofing options for the proposed project

Description	**Option A**	**Option B**	**Option C**
Element type	Pitched Roof Timber Construction	Pitched Roof Timber Construction	Pitched Roof Timber Construction
Building type	Residential	Residential	Residential
Element	Timber trussed rafters and joists with insulation, roofing underlay, counterbattens, battens and Nigeria produced concrete interlocking tiles	Structurally insulated timber panel system with OSB/3 each side, roofing underlay, counterbattens, battens and Nigeria produced reclaimed clay tiles	Structurally insulated timber panel system with plywood (temperate EN 636-2) decking each side, roofing underlay, counterbattens, battens and Nigeria produced Fibre cement slates
Size of tile or slate	420mm x 330mm	420mm x 330mm	420mm x 330mm
Pitch of roof	22.5^0	22.5^0	22.5^0

Four benefit qualitative criteria such as environmental impact (C_1), resource efficiency (C_2), performance capability (C_3), functionality (C_4); one benefit quantitative criterion such as area size (C_5); and one cost quantitative criterion such as lifecycle cost (C_6) are chosen for evaluating the building materials. Further assume that linguistic values and their corresponding triangular fuzzy numbers shown in Table 2 are used to evaluate each building material candidate versus each qualitative criterion. Ratings of building material candidates versus qualitative criteria are given by decision makers as shown in Table 3. Through Eq. (7), averaged ratings of building material candidates versus qualitative criteria can be obtained as also displayed in Table 3. In addition, suppose values of building material candidates versus quantitative criteria are present as in Table 4. According to Eqs. (8) and (9), values of alternatives under benefit and cost quantitative criteria can be normalized as shown in Table 5. The linguistic values and its corresponding fuzzy numbers, shown in section 2.5, are used by decision makers to evaluate the importance of each criterion as displayed in Table 6. The average weight of each criterion can be obtained using Eq. (10) and can also be shown in Table 6.

Table 2 Linguistic values and fuzzy numbers for ratings

Very low(VL) /Very difficult(VD) /Very far(VF)	(0.00,015,0.30)
Low(L)/Difficult(D)/Far(F)	(0.15,0.30,0.50)
Medium(M)	(0.30,0.50,0.70)
High(H)/Easy(E)/Close(C)	(0.50,0.70,0.85)
Very high(VH)/Very easy(VE)/Very close(VC)	(0.70,0.85,1.00)

Table 3 Ratings of building material candidates versus qualitative criteria

Candidates	Criteria	D_1	D_2	D_3	Averaged Ratings
A_1	C_1	VH	H	VH	(0.63,0.80,0.95)
	C_2	VE	E	M	(0.50,0.68,0.85)
	C_3	C	VC	VC	(0.63,0.80,0.95)
	C_4	M	H	H	(0.43,0.63,0.80)
A_2	C_1	VH	VH	H	(0.63,0.80,0.95)
	C_2	M	M	E	(0.37,0.57,0.75)
	C_3	C	C	VC	(0.57,0.75,0.90)
	C_4	VH	VH	VH	(0.70,0.85,1.00)
A_3	C_1	L	L	H	(0.27,0.43,0.62)
	C_2	VE	E	VE	(0.63,0.80,0.95)
	C_3	M	M	C	(0.37,0.57,0.75)
	C_4	L	M	H	(0.32,0.50,0.68)

Table 3 Values of material candidates versus quantitative criteria

Criteria	Building Materials Candidates			Units
	A_1	A_2	A_3	
C_5	100	80	90	hectare
C_6	2	5	10	million

Table 4 Normalization of quantitative criteria

Criteria	Building Materials Candidates		
	A_1	A_2	A_3
C_5	1	0.8	0.9
C_6	1	0.4	0.2

Table 5 Averaged weight of each criterion

	D_1	D_2	D_3	Averaged weights
C_1	MI	VI	IM	(0.50,0.75,0.92)
C_2	IM	MI	LI	(0.25,0.50,0.75)
C_3	LI	LI	VI	(0.25,0.53,0.67)
C_4	UI	IM	VI	(0.33,0.50,0.67)
C_5	MI	VI	IM	(0.50,0.75,0.92)
C_6	VI	VI	VI	(0.75,1.00,1.00)

Apply Eqs. (11)-(22) and $g=4$, $h=5$, $n=6$ to the numerical example to produce $A_{i1}, A_{i2}, A_{i3},$ $B_{i1}, B_{i2}, B_{i3},$ $C_{i1}, C_{i2}, C_{i3},$ $D_{i1}, D_{i2}, D_{i3},$ $O_{i1}, O_{i2}, O_{i3},$ $P_{i1}, P_{i2}, P_{i3},$ Q_{i1}, Q_{i2}, Q_{i3} for each candidate as displayed in Table 7. The calculation values for $A_{i1}+A_{i2}-C_{i3},$ $B_{i1}+B_{i2}-D_{i3},$ $O_{i1}+O_{i2}-Q_{i3},$ $C_{i1}+C_{i2}-A_{i3},$ $D_{i1}+D_{i2}-B_{i3},$ $P_{i1}+P_{i2}-P_{i3}$, $Q_{i1}+Q_{i2}-O_{i3}$ are shown in Table 8.

Table 6 Values for A_{i1}, A_{i2}, A_{i3}, B_{i1}, B_{i2}, B_{i3}, C_{i1}, C_{i2}, C_{i3}, D_{i1}, D_{i2}, D_{i3}, O_{i1}, O_{i2}, O_{i3}, P_{i1}, P_{i2}, P_{i3}, Q_{i1}, Q_{i2}, Q_{i3}

	A_1	A_2	A_3
A_{i1}	0.17	0.17	0.17
A_{i2}	0.00	0.00	0.00
A_{i3}	0.00	0.00	0.00
B_{i1}	0.77	0.76	0.62
B_{i2}	0.25	0.16	0.23
B_{i3}	0.25	0.10	0.05
C_{i1}	0.12	0.12	0.12
C_{i2}	0.00	0.00	0.00
C_{i3}	0.00	0.00	0.00
D_{i1}	-1.11	-1.11	-1.08
D_{i2}	-0.17	-0.13	-0.15
D_{i3}	0.00	0.00	0.00
O_{i1}	0.74	0.78	0.49
O_{i2}	0.50	0.40	0.45
O_{i3}	0.75	0.30	0.15
P_{i1}	1.68	1.71	1.28
P_{i2}	0.75	0.60	0.68
P_{i3}	1.00	0.40	0.20
Q_{i1}	2.68	2.70	2.23
Q_{i2}	0.92	0.73	0.83
Q_{i3}	1.00	0.40	0.20

Table 7 Values for $A_{i1}+A_{i2}-C_{i3}$, $B_{i1}+B_{i2}-D_{i3}$, $O_{i1}+O_{i2}-Q_{i3}$, $C_{i1}+C_{i2}-A_{i3}$, $D_{i1}+D_{i2}-B_{i3}$, $P_{i1}+P_{i2}-P_{i3}$, $Q_{i1}+Q_{i2}-O_{i3}$

	A_1	A_2	A_3
$A_{i1}+A_{i2}-C_{i3}$	0.17	0.17	0.17
$B_{i1}+B_{i2}-D_{i3}$	1.02	0.92	0.84
$O_{i1}+O_{i2}-Q_{i3}$	0.24	0.78	0.74
$C_{i1}+C_{i2}-A_{i3}$	0.12	0.12	0.12
$D_{i1}+D_{i2}-B_{i3}$	-1.53	-1.34	-1.28
$P_{i1}+P_{i2}-P_{i3}$	1.43	1.91	1.75
$Q_{i1}+Q_{i2}-O_{i3}$	2.84	3.13	2.91

Through Eqs. (23) and (24), the left, $f_{G_i}^L(x)$, and right, $f_{G_i}^R(x)$, membership functions of the final fuzzy evaluation value, G_i, $i=1,...,n$, of each building material candidates candidate can be obtained and displayed in Table 9.

Table 8 Left and right membership functions of G_i

$f_{G_1}^L(x)$	$\dfrac{-1.02+\left[(1.02)^2+4(0.17)(x-0.24)\right]^{\frac{1}{2}}}{2\times 0.17}$, $0.24\le x\le 1.43$
$f_{G_1}^R(x)$	$\dfrac{1.53+\left[(-1.53)^2+4(0.12)(x-2.84)\right]^{\frac{1}{2}}}{2\times 0.12}$, $1.43\le x\le 2.84$
$f_{G_2}^L(x)$	$\dfrac{-0.92+\left[(0.92)^2+4(0.17)(x-0.78)\right]^{\frac{1}{2}}}{2\times 0.17}$, $0.78\le x\le 1.91$
$f_{G_2}^R(x)$	$\dfrac{1.34+\left[(-1.34)^2+4(0.12)(x-3.13)\right]^{\frac{1}{2}}}{2\times 0.12}$, $1.91\le x\le 3.13$
$f_{G_3}^L(x)$	$\dfrac{-0.84+\left[(0.84)^2+4(0.17)(x-0.74)\right]^{\frac{1}{2}}}{2\times 0.17}$, $0.74\le x\le 1.75$
$f_{G_3}^R(x)$	$\dfrac{1.28+\left[(-1.28)^2+4(0.12)(x-2.91)\right]^{\frac{1}{2}}}{2\times 0.12}$, $1.75\le x\le 2.91$

By Eqs. (25)-(32), the total utilities, $U_T(G_i)$, x_{R_i} and x_{L_i} can be obtained and shown in Table 10.

Table 9 Total utilities $U_T(G_i)$, x_{R_i} and x_{L_i}

Alternatives	G_1	G_2	G_3
x_{R_i}	1.97	2.26	2.12
x_{L_i}	1.39	1.40	1.33
$U_T(G_i)$	0.315	0.551	0.517

Then according to values in Table 10, candidate A_2 has the largest total utility, $U_T(G_2)$ =0.551. Therefore A_2 becomes the most suitable building materials candidate.

5. Conclusions

A fuzzy MCDM model is proposed for the evaluation and selection of building materials candidates, where ratings of alternatives versus qualitative criteria and the importance weights of all the criteria are assessed in linguistic values represented by fuzzy numbers. Membership functions of the final fuzzy evaluation values can be developed through interval arithmetic and α-cuts of fuzzy numbers. Chen's maximizing set and minimizing set is applied to defuzzify the final fuzzy evaluation values in order to rank all the alternatives. Ranking formulae are clearly developed for better executing the decision making. Finally, a numerical has demonstrated the computational procedure of the proposed approach.

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