

Application of Time Series Models for Streamflow Forecasting

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Abstract

Precise prediction of the streamflow has a significantly importance in water resources management. In this study, two time series models, Autoregressive Moving Average model (ARMA) Autoregressive Integrated Moving Average model (ARIMA) are used for predicting streamflow. In this research, monthly streamflow from 1974 to 2010 were used. The statistics related to first 28 years were used to train the models and last 7 years were used to forecast. The prediction accuracy of both time series models is examined by comparing root mean square error (RMSE), the mean absolute percentage error (MAPE) and the Nash efficiency (NE). According to the results, ARIMA model performs better than the ARMA time series models.

Keywords: Streamflow forecasting, Time series models, ARIMA, ARMA

1. Introduction

Stochastic models are mostly used for analyzing river runoff variations. Many classes of stochastic models have been applied in operational hydrology (Koutsoyiannis, 2000; Lawrance and Kottegoda, 1977). Stochastic models widely used for times series modeling and forecasting are the deseasonalized autoregressive integrated moving average (ARIMA) and autoregressive moving average (ARMA) models. These stochastic models are also known as Box-Jenkins linear stochastic models (Box et al., 2011; Reinsel et al., 1994). Some of the key advantages of Box-Jenkins linear stochastic models are i) they have capability for modeling and forecasting a high variety of stationary and non-stationary time series (Brown and Mac Berthouex, 2002). ii) They have the ability to estimate trends and serial correlations in a time series data (Gocheva-Ilieva et al., 2014). iii) These models have been incorporated into many software packages such as SPSS, MINITAB, STATA, R, Python, Matlab, Mathematica, and With regard to cost-effectiveness, labour planning always opts for the minimum amount of workers many others (Comparison of Statistical Packages, 2015). These models are also preferable due to systematic searching at every step (identification, estimation, diagnostic) for a suitable model (Zhang, 2003). These models require long time series data for analysis, at least 50-100 observations are required for a robust result. To analyze a yearly data, this condition will present a problem (Gocheva-Ilieva et al., 2014; Milionis and Davies, 1994). Mostly hydrologic time series do not have data more than 40-50 years.

ARIMA and ARMA models have been extensively used in the field on hydrology and especially in River flow forecasting. Adhikary et al. (2012) used seasonal ARIMA model to modeling the groundwater table. They took weekly time series and concluded that SARIMA stochastic models can be applied for ground water level variations. Valipour et al. (2013) modeled the inflow of Dez dam reservoir with SARIMA and ARMA stochastic models. His research results show that SARIMA model yield better results than ARMA model. Ahlert and Mehta (1981) analyzed the stochastic process of flow data for Delaware River by ARIMA model. Yurekli et al. (2005) applied SARIMA stochastic models for modeling the monthly streamflow data of the Kelkit river. Modarres and Ouarda (2013) demonstrated heteroscedasticity of streamflow time series by ARIMA model in comparison of GARCH models. His study results show that ARIMA models perform better than GARCH models. Ahmad et al. (2001) used ARIMA model to analyse the water quality data. Kurunç et al. (2005) used the ARIMA and Thomas Fiering stochastic approach to forecast streamflow data of Yesilurmah River. Tayyab et al. (2016) used autoregressive model in comparison of neural networks to predict streamflow. The aim of this study to compare two different time series models, ARMA and ARIMA, for monthly streamflow forecasting. Data from Doyian station are used in the applications.

2. Methodology

Box and Jenkins developed ARIMA stochastic models that describe a wide class of models forecasting a univariate time series that can be made stationary by applying transformations – mainly differences for Trend and Seasonality, and power function to regulate the variance (Box and Jenkins, 1970; Box and Jenkins, 1976;

Box et al., 1967) The model, word “ARIMA” consists of three terms i.e. i) AR ii) I and iii) MA terms. Lags of differenced time series in the forecasting equations are called “autoregressive(AR)” term, whereas lags of the forecasted errors are called “moving average (MA)” term and the time series which requires differencing to become stationary should be “Integrated (I)” (Ghafoor and Hanif, 2005).

AR(p), MA(q) and Auto regressive moving average (ARMA(p,q)) models are some special cases of Box and Jenkins ARIMA model. In this study, ARMA and ARIMA models are used to evaluate the performances of time series of streamflow. Time series can be made deseasonalized by many methods. In this study, streamflow time series is deseasonalized by seasonal-trend decomposition procedure based on loess (STL) (Cleveland et al., 1990). Many features of STL make it a preferable choice for times series deseasonalization, like the capability to handle seasonal time series with any seasonal period greater than one (Theodosiou, 2011). In addition, STL can be easily incorporated in statistical software packages. After deseasonalizing the time series, the non-seasonal ARIMA and ARMA models can be fitted.

2.1 ARIMA Model

A non seasonal ARIMA model is denoted by ARIMA(p,d,q) whereas “p” is the number of non seasonal autoregressive term, “q” is the number of non seasonal moving average term and “d” represents the number of non seasonal differences applied to the time series. The general form of non seasonal ARIMA (p,d,q) model can be written as follows;

$$S_t = c + \Phi_1 S_{t-1} + \Phi_2 S_{t-2} + \dots + \Phi_p S_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q} \quad (1)$$

Or in backshift notation,

$$(1 - \Phi_1 B - \Phi_2 B^2 - \dots - \Phi_p B^p) S_t = c + (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q) e_t \quad (2)$$

Where c=constant term (called intercept), $\Phi_i = i^{th}$ autoregressive parameter, $\theta_j = j^{th}$ moving average parameter $e_t =$ the error term at time t

2.2 ARMA Model

The auto-regressive moving average (ARMA) model ARMA (p, q) can be expressed as;

$$S_t = c + \sum_{i=1}^p \Phi_i S_{t-i} + \sum_{j=1}^q \theta_j e_{t-j} + e_t \quad (3)$$

Where c is the constant term of the ARMA model, Φ_i indicates the i^{th} auto-regressive coefficient, θ_j is the j^{th} moving average coefficient, e_t shows the error term at time period t, and S_t refers the value of forecasted streamflow at time period t.

2.3 Time Series Models Building Procedure

In the estimation stage, several models are tentatively chosen and then computes the values of Akaike Information Criterion corrected (AIC_c) (Hurvich and Tsai, 1989) or/and the Bayesian Information Criteria (BIC) suggested by Schwarz (1978). The model structure which has the minimum AIC_c and BIC values, among others model structures, will be chosen as best model. The eq. (4) describes the formula to compute AIC_c whereas eq. (5) describes the formula for BIC.

$$AIC_c = -2 \ln(\text{Max.Likelihood}) + \frac{2 T_p}{n - T_p - 1} \quad (4)$$

$$BIC = -2 \ln(\text{Max.Likelihood}) + T_p \ln(n), \quad (5)$$

with T_p the number of AR, I and MA parameters.

After selecting the best model, third stage of diagnostic check is performed. In diagnostic check, the modeler can analyze the goodness of fit of selected model. Mostly, researchers perform two tests to check goodness of fit of selected model. First test is to check the residuals by using ACF and PACF graph. If the selected model is appropriate, the residuals graphs of both correlation functions should be white noise, indicating no remaining correlation. The second test is Ljung-Box test (1978). If the values of p-value in this test exceed the 5%, it

indicates that residuals do not have significant departure from white noise. If the selected model residuals are very large or model fails to pass Ljung-Box test, the modeler return to select alternative model and follow the same procedure until satisfactory model results are obtained.

3. Data Distribution and Data Preprocessing

In this study, two time series models, ARMA and ARIMA models used for forecasting monthly flow of Doyian station. The implantation period covered the data values from 1974 to 2003 and used for building of ARIMA and ARMA models. The testing period covers the streamflow data values from 2004 to 2010 and has been used to evaluate the performance of both selected models. In order to select the best model between the ARMA and ARIMA models, root mean square error (RMSE), mean absolute percentage error (MAPE) and Nash efficiency (NE) indexes are used in this study. RMSE is one of the most used statistical index for measuring error in the prediction with respect to original data (Lin et al., 2006). It is defined as:

$$RMSE = \sqrt{\frac{1}{N} \sum (S_o - S_f)^2} \quad (6)$$

MAPE is statistical indexes used for measuring the error in the predicting time series value. It is defined as:

$$MAPE = \frac{1}{N} \sum \left| \frac{(S_o - S_f)}{S_o} \right| \quad (7)$$

The Nash efficiency (NE) is a model evaluation criteria suggested by Nash and Sutcliffe (McCuen et al., 2006). A model efficiency of 90 % represents satisfactory performance whereas a value in range of 80 to 90 % represents fairly good performance. This statistical index helps in determining how much the predicted data match the original data. It has been used for streamflow time series forecasting evaluation (McCuen et al., 2006). It can be calculated as:

$$NE = 1 - \frac{\sum (S_o - S_f)^2}{\sum (S_o - \bar{S}_o)^2} \quad (8)$$

Where N is the total number of observations, S_o is observed flow, S_f is forecasted streamflow, \bar{S}_o is average of streamflow and \bar{S}_f is average forecasted flow.

4. Results and Discussion

To determine the prediction accuracy of these time series models, first determine the suitable structures of ARMA and ARIMA models on the basis of AICc and BIC criteria by using the Box and Jenkins model selection procedure. The procedure of selecting suitable ARMA and ARIMA structures is explained below.

4.1 Selecting Suitable ARIMA and ARMA Models

Monthly streamflow data (Fig. 1) indicates seasonal variation having order of 12. To fit the non seasonal ARIMA model, deseasonalization was applied to the streamflow data. The deseasonalization was performed by using STL decomposition method. Deseasonalized monthly streamflow time series (Fig. 2) shows no specific seasonal or trend, but still exhibits non stationary behavior. A non seasonal difference is applied to time series to make it stationary. The plots of ACF, PACF and deseasonalized time series after applying non seasonal difference are shown in Fig. 3. Both plots are examined in order to identify the structure of model. According to the correlations plots, the suggested ARMA model is (1, 3), because PACF plot is suggesting AR(3) term whereas ACF is suggesting MA(1) term. For ARIMA model selection, different ARIMA model structures were estimated to find the best model structure.

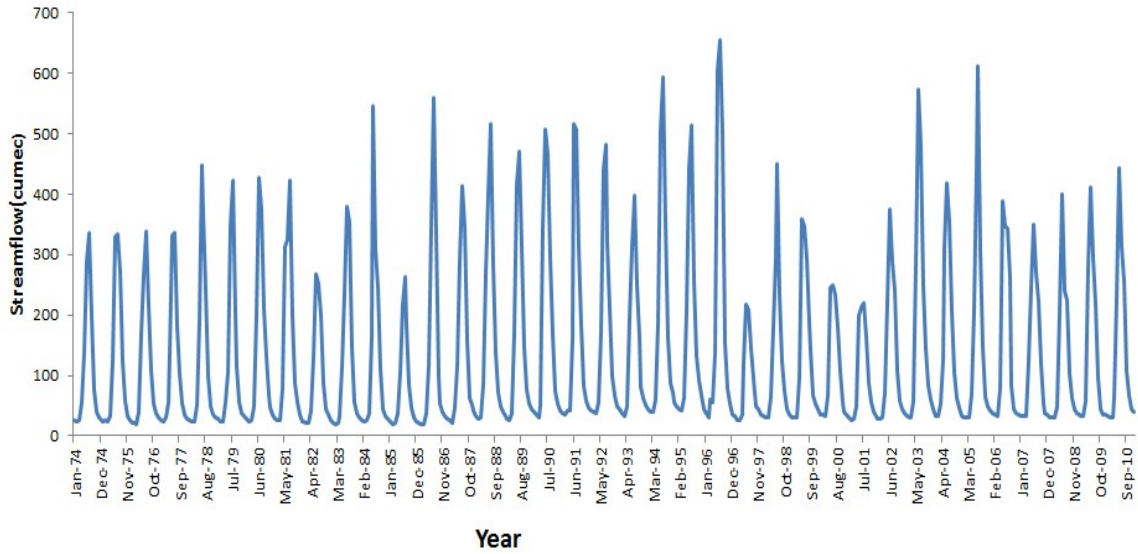


Figure 1. Original streamflow time series

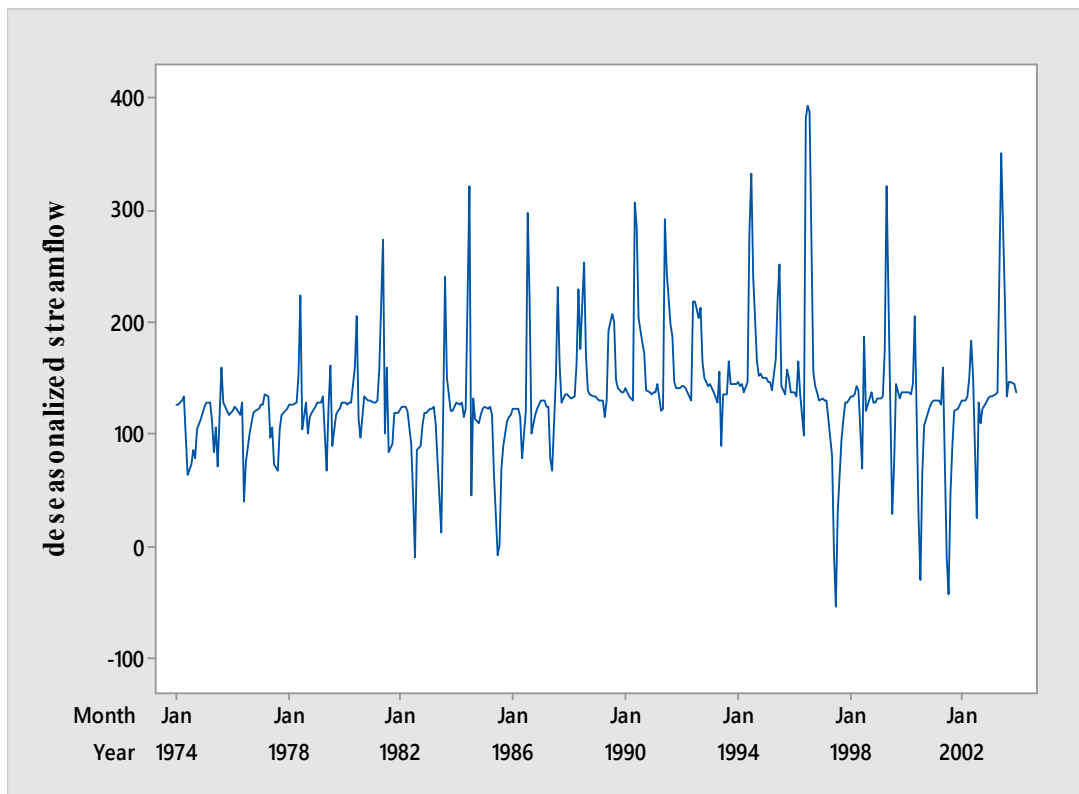


Figure 2. Deseasonalized time series plot

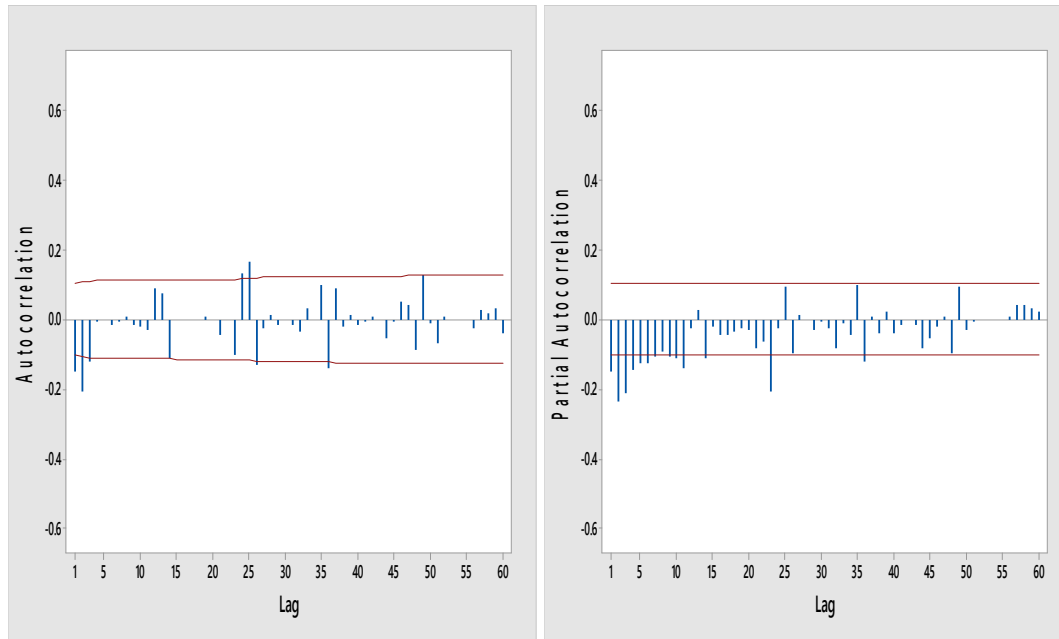


Figure 3. ACF, PACF graphs after non seasonal difference

Table 1. AIC_c and BIC values for different ARIMA model structures

Model Structure	AIC _c	BIC
ARIMA(3,1,1)	3777.5	3796.75
ARIMA(0,1,3)	3775.18	3790.6
ARIMA(3,1,3)	3781.3	3808.17
ARIMA(3,1,2)	3779.37	3802.43
ARIMA(3,1,0)	3824.78	3840.2
ARIMA(1,1,1)	3779.37	3790.95
ARIMA(1,1,2)	3776.06	3791.48
ARIMA(1,1,3)	3777.17	3796.41
ARIMA(2,1,1)	3775.47	3790.89
ARIMA(2,1,2)	3777.51	3796.75
ARIMA(2,1,3)	3779.23	3802.29
ARIMA(0,1,2)	3785.13	3796.71

The values of AIC_c and BIC of the different ARIMA models are shown in Table 5. In this case, the initial suggested model structure has the minimum AIC_c and BIC values and has been chosen as best model structure for deseasonalized streamflow time series. The selected model also passed both tests included in the diagnostic check. The plots of the ACF and PACF for the residuals are shown in Fig. 4, whereas the summary of the Ljung Box test for the deseasonalized ARIMA model is given in Table 2. The results of both tests suggest that the residuals are white noise. From Eq. (1), in back shift notation, the ARIMA(0, 1, 3) model can be written as:

$$(1-B)^d S_t = (1 - \Phi_{1NMA}(B) - \Phi_{2NMA}(B^2) - \Phi_{3NMA}(B^3))e_t \quad (9)$$

By substituting the coefficients, one obtains the model

$$S_t - S_{t-1} = e_t + 0.3987e_{t-1} + 0.356e_{t-2} + 0.183e_{t-3} \quad (10)$$

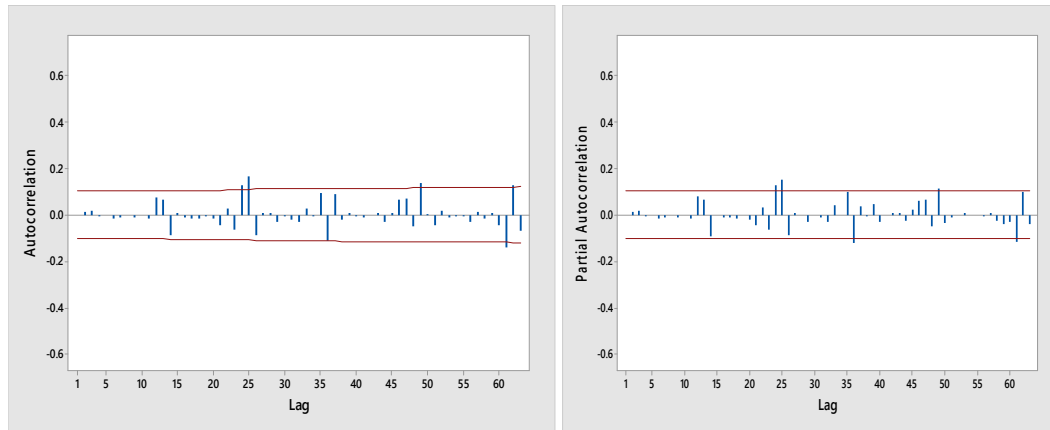


Figure 4. Residual graphs of ACF,PACF of ARIMA model

Table 2. Summary of Ljung Box test for deseasonalized ARIMA Model

lag	Lag 12	Lag 24	Lag 36	Lag 48
Chi square	1.7626	15.255	41.614	50.408
Df	9	21	33	45
p-value	0.9947	0.8099	0.1445	0.2682

4.2 Evaluation of Best Predicting Model

In order to evaluate the performance of the models, one month ahead forecasts were generated for the testing period from January 2004 to December 2010. In Table 3, the both time series models are compared with each other by using statistical indexes. The hydrograph between original forecasted streamflow using ARMA and ARIMA models in Figure 4. On the basis of results of Table 3 and Fig. 5, ARIMA model performs better than ARMA models because ARIMA model gives low value of error and high value of Nash efficiency and also in good fit with the observed data.

Table 3 Evaluation of model performance for testing period on basis of statistical indexes

Statistical Index	ARIMA ((0,1,3)	ARMA (1,3)
MAPE	17.268	19.642
RMSE	51.525	57.273
NE	0.858	0.836

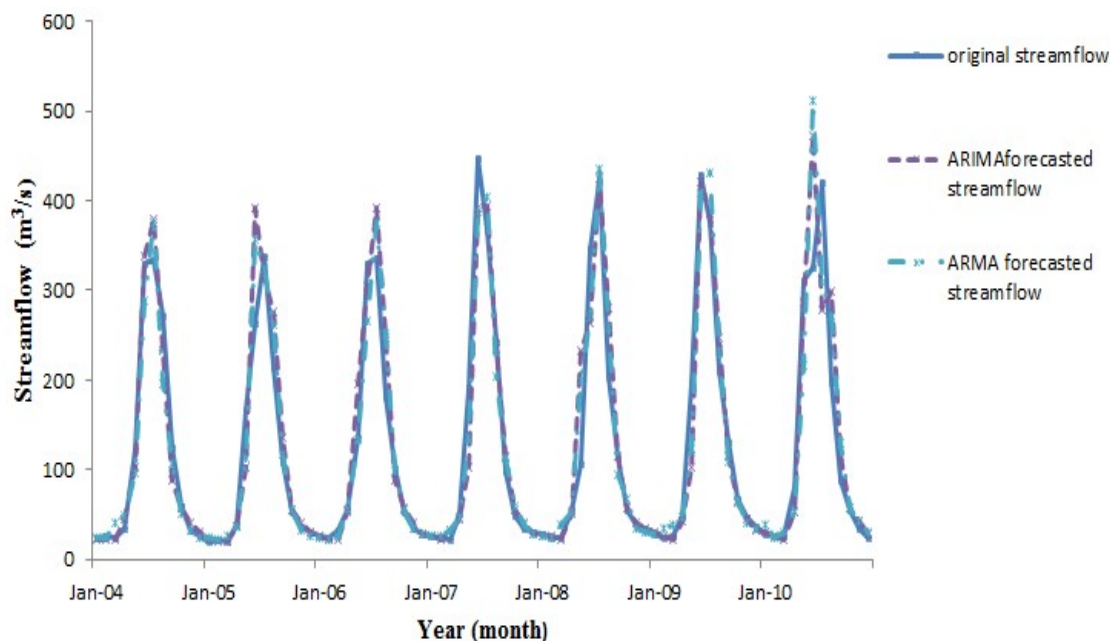


Figure 5. Observed and forecasted streamflow hydrograph using ARIMA and ARMA models for testing period

5. Conclusions

In this research, the stochastic nature of streamflow is analyzed with deseasonalized ARIMA and ARMA stochastic models. The ARIMA model has a better performance than ARMA because it makes time series stationary, in both training and forecasting. The values of mean absolute percentage error and root mean square error of ARIMA model was less than ARMA model. This indicated the superiority of ARIMA models to the ARMA models. it can be concluded that the ARIMA model could be used for forecasting one month ahead streamflow at this site.

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