Application of Basic Excel Programming to Linear Muskingum Model for Open Channel Routing

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Abstract
Flood routing as the process of determining the reservoir stage, storage volume of the outflow hydrograph corresponding to a known hydrograph of inflow. It is viable technique for determining the flood hydrograph at a section of a river by utilizing the flow data at one or more upstream sections. It can be hydraulic and hydrologic. Some hydrological routing techniques include Muskingum method, Muskingum-Cunge method, Lag method and Kalinin-Milyukov method while many sophisticated computer programs like Matlab had been deployed for river routing.Muskingum Method for stream routing was considered by using spreadsheet for. Coefficients were determine using various hydrologic data and formula for the Muskingum method. A popular data with other three data sets were considered in a linear model. The value of k and x was calculated using the basics of Microsoft Excel cell programming. Analysis of variance (One- way) was performed to detect any significant difference in the methods compared with other study without basics of excel.The result shows no significant difference with the values computed in this present study, limitations of Muskingum method were highlighted and further research the subject is recommended.
Keywords: Flood routing, hydrograph, Muskingum method, hydraulic, hydrologic,

1.0 INTRODUCTION

1.1 The Origin of Muskingum Method

The Muskingum method for flood routing was developed for the Muskingum Conservancy district (Ohio) flood control study in the 1930s and is one of the most popular methods of hydrological routing for drainage channels with all types of rivers and streams (Elbashir, 2011). The design of flood protection schemes in the Muskingum River Basin, Ohio, USA brought about this method of flow routing. Inflow and outflow is complex in a natural channel therefore, wedge and prism storage occurs in natural channels. Chin (2000) explained prism storage as the volume of a constant cross-section that corresponds to uniform flow in a prismatic channel. With the movement of flow, wedge storage is generated.

1.2 Open Flow in Rivers

Flow in open river channels especially natural channels like can be majorly unsteady, non-uniform and turbulent. These could also be sub-critical, critical and super-critical according to the Froude's number. Therefore the Froude's number is important parameter for analyzing open channel flow.

1.3 Equations Governing Open Channel Flow

The two most widely used formulas for solving open channel flow was given an Irish engineer Robert Manning and his engineering French colleague, Anthonie Chezy. The formulated what is known Mannings and Chezy's formula.

Manning's Formula

$$V = \frac{\left[R^{\frac{2}{3}}S^{\frac{1}{2}}\right]}{n} \tag{1.1}$$

$$Q = AV = \frac{A\left[R^{\frac{2}{3}}S^{\frac{1}{2}}\right]}{n} \tag{1.2}$$

$$R = \frac{A}{P} \tag{1.3}$$

The roughness coefficient (n) for the Manning equation indicates the resistance of the channel bottom to flowing water (Elbashir, 2011).

The wetted perimeter (P) is described as the distance along the channel bottom below the water surface (Boyd and Yoo, 1994).

Chezy's Formula

This formula is given as:

$$V = c\sqrt{RS} \tag{1.4}$$

$$Q = cA^{\frac{1}{2}}S^{\frac{1}{2}} \tag{1.5}$$

$$Q = \frac{\left[cA^{\frac{1}{2}}\right]}{\left[P^{\frac{1}{2}}\right]} \tag{1.6}$$

Where,

V = mean velocity (m/s)
R = hydraulic radius
S = slope of channel
Q = discharge in open channel flow (m^3 /s),
A = cross-sectional area of flow in (m^2)
P = wetted perimeter in m.
n = Manning's Coefficient
C = Chezy's resistant coefficient

The two equations above deals with velocity and discharge of open channel. However, stream routing is either hydraulic or hydrological models.

1.4 Hydraulic Routing

This is a type of flow routing model that put many equations together to compute the reach of a river. It combines momentum and continuity equations in both conservative and non-conservative form. Continuity equation together with the equation of motion of unsteady flow makes the technique a rather complex one. Its prediction is based on the fact that it allows flow computation to be varied in both time and space (Mays and Tung, 2002).

Momentum Equation (Non-Conservative form)

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0 \tag{1.7}$$

Momentum Equation (Conservative form)

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0 \tag{1.8}$$

Where,

$\frac{1}{A}\frac{\partial Q}{\partial t}$ = Local Acceleration

$\frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)$ = Conservative Acceleration

$g\frac{\partial y}{\partial x}$ = Pressure Fore

$g(S_o - S_f)$ = Friction Force

g = Gravity

S_o = Bed Slope

S_f = Friction Slope

Continuity Equation (Conservative one-dimensional form)

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{1.9}$$

Continuity Equation (Non-Conservative form)

$$\frac{\partial (Vy)}{\partial x} + \frac{\partial y}{\partial t} = 0 \qquad (1.10)$$

$$V\frac{\partial y}{\partial x} + y\frac{\partial V}{\partial x} + \frac{\partial y}{\partial t} = 0 \qquad (1.11)$$

Where,

V = the velocity of flow at any section in (m/s),
S_0 = the channel bed slope (Bed slope)
S_f = the slope of the energy line in (Friction Slope in m/m)

1.5 Hydrologic Routing

The hydrological methods for river routing combine the principle of continuity equation with some relationship between storage, outflow, and inflow. These relationships are usually assumed, empirical, or analytical in nature. Guo (2006) also stated that this method require a storage-stage-discharge-relation to determine the outflow for each time step
The change in storage (ΔS) equals the difference between inflow I(t) and outflow Q(t). The established equation (1.12).

$$\frac{dS}{dt} = I(t) - Q(t) \qquad (1.12)$$

Where,

S = storage between the upstream and downstream sections (m^3)
t = time in (s)
I (t) = inflow at upstream section at time t (m^3/s)
Q (t) = outflow at downstream section at time t (m^3/s)

Therefore, storage is a function of inflow and outflow i.e. S= f(I(t) and Q(t))

$$S = f(I, \frac{dI}{dt}, \cdots, Q, \frac{dQ}{dt}, \cdots) \qquad (1.13)$$

For open channel flow, the continuity equation is:

(1.14)

Equation (2.20) is a modified form of equation (2.27)

Where,

A = the cross-sectional area,
Q = channel flow, and
q = lateral inflow

Over the finite interval of time between t and t+ Δt or suppose, there are gauges both upstream (station 1) and downstream (station 2). Both have floodplains that store water. We could write a water balance equation with averages.

$$\frac{(I_1 + I_2)}{2} - \frac{Q_1 + Q_2}{2} = \frac{S_2 + S_1}{\Delta S} \qquad (1.15)$$

Average inflow - Average outflow = average change in storage

Where,

Subscripts 1 and 2 = variables at times t and t+ Δt respectively.
$I_1 = Q_1$ is assumed initially for flood routing models.

Muskingum method where storage is linear functions of inflow and outflow is a typical of hydrologic routing model which can be either nonlinear or linear.

1.6 Nonlinear Muskingum Equation

1.6.1 First form of Nonlinear Muskingum Model

Equation 1.16 was presented by Gill cited by Hamedi *et al.* (2014). This model is the most common nonlinear Muskingum model (Geem, Orouji *et al.*, Karahan *et al.*, all cited by Hamedi *et al.*, 2014). The equation improved on Tung's (1985) by better fitting. Barati (2011) also worked on this nonlinear model.

$$S_t = K[XI_t^p + (I - X)O_t^p] \tag{1.16}$$

1.6.2 Second form of Nonlinear Muskingum Model

This (Equation 1.17) nonlinear Muskingum model is first presented by Chow cited by Hamedi *et al.* (2014). Barati (2013) used excel solver to estimate the Muskingum parameter. The results present a viable way of nonlinear Muskingum parameter determination.

$$S_t = K[XI_t^p + (I - X)O_t^p]$$

$$S_t = K[XI_t + (I - X)O_t]^m \tag{1.17}$$

1.6.3 Third form of Nonlinear Muskingum Model

The third form (Equation 1.18) of nonlinear Muskingum model is presented by Gavilan and Houck cited by Hamedi *et al.* (2014). The result was compared with other nonlinear estimation by other authors and shows no significant difference. However, this book dwells majorly on linear Muskingum method.

$$S_t = K[XI_t^{p_1} + (I - X)O_t^{p_2}] \tag{1.18}$$

Where,

S_t = Storage at time, t
I_t = Inflow at time, t
O_t = outflow at time, t
K = storage coefficient
X = a dimensionless weighting factor

It should be noted that p_1, p_2, p, and m = exponent factors for considering the degree of nonlinearity of accumulated storage S and weighted flow [XI + (1 − X) O]. Hamedi *et al.* (2014) discussed more on nonlinear models. Vatankhah (2014) Solved nonlinear Muskingum model by Fourth-Order Runge-Kutta Method. Kim *et al.* (2001) used heuristic algorithm, called Harmony Search for the parameter estimation. The studies show no significant difference with Muskingum linear model. This book will compare the outflow with other work in linear and nonlinear Muskingum routing in natural channel.

1.7 Linear Muskingum Equation

For linear model, the equation is derived by the addition of prism storage and wedge storage.

$$S = K[XI + (I - X)O] \tag{1.19}$$

Where,

K = travel time of peak through the reach
X = weight on inflow versus outflow ($0 \le X \le 0.5$)
X = 0.0 - 0.3 for natural stream

Equation (1.19) is linear but the relationship between inflow, outflow and storage may not follow this pattern which is provided for in non-linearity.

2.0 METHODOLOGY

2.1 Muskingum Coefficient

The coefficients were determined for the data before the commencement of routing. By applying Equation 1.19 at any time increments, the storage S in the channel between inflow and outflow sections at (j+1) t can be written as:

$$S_{j+1} = K\{[XI_{j+1} + (1 - X)Q_{j+1}]\} \tag{2.1}$$

The change in storage over the time interval t is therefore given by:

$$S_{j+1} - S_j = K\{[XI_{j+1} + (1-X)Q_{j+1}] - [XI_j + (1-X)Q_j]\} \quad (2.2)$$

Recall continuity equation, Equation (2.10a) can be written as:

$$S_{j+1} - S_j = \frac{I_{j+1} + I_j}{2}\Delta t - \frac{Q_{j+1} + Q_j}{2}\Delta t \quad (2.3)$$

Combining the equations above gives:

$$Q_{j+1} = C_0 I_{j+1} + C_1 I_j + C_2 Q_j \quad (2.4)$$

$$C_0 = \frac{\Delta t - 2KX}{2K(1-X) + \Delta t} \quad (2.5)$$

$$C_1 = \frac{\Delta t + 2KX}{2K(1-X) + \Delta t} \quad (2.6)$$

Where,

$$C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t} \quad (2.7)$$

If I(t), K and X are known, Q(t) can be calculated using above equations. The routing time Δt should be kept smaller than 1/5 of the travel time of the flood peak through the reach. Equation 2.4 for the next outflow can be written as:

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 \quad (2.8)$$

Where

$$C_0 + C_1 + C_2 = 1 \quad (2.9)$$

Equation 2.8 can be repeated for Q_3, Q_4…..$Q_{n..}$ It should be note that K and Δt must have the same units, and for numerical accuracy; the equation (2.10) must be meet.

$$2Kx < \Delta t \leq K \quad (2.10)$$

Another suggestion by Chin (2000) is that, t, should be assigned any convenient value between K/3 and K. In addition, equation (2.10) must be unified because they are proportions. The routing procedure is accomplished successively, with Q_2 from Q_1 of the previous calculation.

2.2 Estimating for K and X

The Muskingum coefficient K is typically estimated from the travel time for a flood wave through the reach. The travel time is expected to change with the flow. Constant data needs to be gathered to know the time for the wave to travel. This is one of the shortcoming to Muskingum method which will be discussed later. However, if the two hydrographs are available for the stream, K and x can be better estimated. Storage, S is then plotted against the weighted discharge xI + (1-x) Q. Several values of X are tried in a trial and error basis. The value that gives the narrowest loop in the plotted relationship is taken as the correct X value and the slope of the plotted relationship is taken as the K value (Haan, Barfield and Hayes, 1994).

2.3 Analyzing data with Basics of Microsoft Excel

Routing Popular Data were routed using basics of Microsoft Excel spreadsheet. Ramirez Data, data reported by Wilson (Data Set 1 for the present study) cited by (Al-Humond and Esen, 2006) which are known to present a nonlinear relationship between weighted discharge and storage is also used. This data set has also been extensively studied by others (Gill, Tung, Yoon and Padmanabhan, Mohan, all cited by Al-Humoud and Esen, 2006). Karahan, (2009) had worked on the data. In addition to the data above, data sets by Viessman and Lewis, Wu *et al.* as cited by Al-Humoud and Esen (2006) was also routed as Data Set 2 and Data Set 3 respectively for the present study. Viessman and Lewis data is based on the inflow and outflow hydrographs exhibiting linear relationship. The methods used in previous studies include the Least Square Method (LSM) which Gill developed. The same author described approximate method to determine Muskingum parameters x and K. This approximate method gave rise to Method 1 and Method 2. The forth method which this present study employs is the use of spreadsheet for the Muskingum routing procedure. Inflow and outflow data should be placed in different columns in Excel while the coefficient formulas and equations above is entered into the first row cell. Analysis of variance (One - way) was carried out to detect any significant difference in the four methods.

3.0 RESULTS AND DISCUSSION

3.1 Muskingum Model Routing

Based on output from spreadsheet (Table 3.1), a value of x = 0.15 gave the straightest loop (Figure 3.1). The best fit to the corresponding points yields a value of k = 2.31 h. C_o, C_1, and C_2 was obtained using equation (2.5), (2.6) and (2.7) respectively (Table 3.2). Figure 3.2 is the hydrograph for the flood routing. For data set 1, table 3.3 gives the estimated value for x while figure 3.3 shows x= 0.555 is the straightest. Table 3.4 is routed data of computed outflow and Figure 3.4 compares outflow hydrographs of methods in data set 1.

Results for data set 2 is tabulated with observed outflow in Al-Humoud and Esen (Table 3.5 and 3.6). Figure 3.5 gives the narrowest loop at x = 0.25 and final outflow hydrograph is represented in Figure 3.6.

Data Set 3 observed and computed values of outflow was routed using values of C_0, C_1 and C_2 (Table 3.8) from value of x = 0.08 (Table 3.7 and Figure 3.7). Figure 3.7 is the plot of storage versus [xI + (1-x) O]. The hydrograph for the flow data is presented in Figure 3.8.

3.2 Statistical Analysis

The routing data for data set 1, 2 and 3 were put side by side with the other estimated values from other authors of the data sets analyzed. Statistical analysis Analysis of Variance (ANOVA) was carried out to compare the means of the four different approaches. The Least Square Method, Method 1, Method 2 and the Present Study were put into statistical perspective. The basic purpose of the analysis of variance is to test the homogeneity of several data; ANOVA is a technique that enables evaluation of several populations means simultaneously (Gupta, 2008).

Null hypotheses: There is no significant difference in the methods

$$H_o: \mu_1 = \mu_2 = \mu_3 = \mu_4$$

Alternative hypotheses: There is a significant difference in the methods

$$H_1: \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$$

μ_1 = LSM
μ_2 = Method 1
μ_3 = Method 2
μ_4 = Present Study
Alpha value: 0.01

For data set 1, Variance ratio (F) < Critical value at alpha =0.01. Since the calculated value of the F = 0.104 is less than critical value 4.024 (Table 3.10 and 3.11), it is not significant, hence the Null hypotheses is not rejected at 1 percent level of significance. Therefore there is no significant difference between the LSM, Method 1, Method 2 and the values routed with Muskingum using Microsoft excel in the Present study and conclude with 99 percent confidence level that the methods does not differ significantly. There was also no significant difference with Vatankhah's (2014) study of "Solving nonlinear Muskingum model by Fourth-Order Runge-Kutta Method" and Chu (2009) using Fuzzy Inference System.

For data set 2, Variance ratio < Critical value at alpha =0.01. Also, the calculated value of the F= 0.015 is less than critical value 4.002, the Null hypotheses is accepted at 1 percent level of significance. Therefore there was no significant difference between the LSM, Method 1, Method 2 and the values routed with Muskingum using Microsoft Excel in the Present study (Table 3.12 and 3.13) and conclude that with 99 percent confidence that the methods has no significant difference.

Data set 3 at alpha =0.01, Variance ratio 0.296 is less than critical value 4.018 (Table 3.14 and 3.15), with 99 percent confidence level, it is not significant. For this reason, the Null hypothesis is accepted at 1 percent level of significance and the alternative hypothesis rejected. No significant difference exists between the LSM, Method 1, Method 2 and the outflow values in the Present study. It is therefore concluded with 99 percent certainty that the methods does not differ. Moreover, data set 1, 2, and 3 shows that the model parameter (Table 3.9) results are in good agreement with the observation values and gave more flexible results.

3.3 Limitations to Muskingum Method

Despite the flexibility, simplicity and advantage of this flood routing technique, better knowledge of its limitations can help scientists and hydrologists improved on the generality of models. Some of the shortcomings include:

i. Muskingum method assumes a single stage-discharge relationship. This assumption may not be possible in natural open channel. For instance, the friction slope drawn on the rising limb of the flood hydrograph for a given flow, may be rather dissimilar than for the falling limb of the hydrograph for the same given flow.

   Some flood wave may have all the three propagation in the same flow.
   - Entire reach flooded I =Q
   - Advancing Flood Wave I > Q
   - Receding Flood Wave Q > I

ii. Moreover, the method has the drawback of producing a negative initial outflow which is commonly referred to as 'reduced flow' at the beginning of the routed hydrograph. This view has been supported in the work of Perumal as cited by Elbashir (2008) and Luo and Xuewei (1987).

iii. This limitation above makes Muskingum not suitable for very steep channel. Thus, it is not applicable to steeply rising hydrographs such as dam breaks.

iv. In many flow cases, K is generally assumes constant for easy computation which may be incorrect at all point of the stream.
v. The method also pays little and sometimes no attention to variable backwater effects such as downstream dam, bridges barrier, wave, human and geological influences.

4.0 CONCLUSION AND RECOMMENDATION

Flood routing using Microsoft excel (spreadsheet) was implemented in this work using Muskingum method for three different popular data sets. In spite of the simplicity of linear method, nonlinear also yielded outflow with no significant difference. Though the method has limitations, it produces similar routing effects according to the values of the available data. It is recommended that future work should dwell on comparing output of other hydraulic and hydrological routing methods in channel routing to Muskingum models so as to improve on routing techniques. More studies on estimating flow routing parameters simultaneously by using Excel Solver should also be conducted.

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Table 3.1 Values for X computed for Ramirez Data Set

A	B	C	D	E	F x=0.40	G x=0.35	H x=0.30	I x=0.25	J x=0.20	K x=0.15
Inflow, I (m^3/s)	Outflow, O (m^3/s)	Ave. Inflow (m^3/s)	Ave. Outflow (m^3/s)	Storage (m^3)	Weighted Average Flux - xI + (1-x) O (m^3/s)					
93	85			715000	88.2	87.8	87.4	87.0	86.6	86.2
137	91	115.0	88.0	812200	109.4	107.1	104.8	102.5	100.2	97.9
208	114	172.5	102.5	1064200	151.6	146.9	142.2	137.5	132.8	128.1
320	159	264.0	136.5	1523200	223.4	215.4	207.3	199.3	191.2	183.2
442	233	381.0	196.0	2189200	316.6	306.2	295.7	285.3	274.8	264.4
546	324	494.0	278.5	2965000	412.8	401.7	390.6	379.5	368.4	357.3
630	420	588.0	372.0	3742600	504.0	493.5	483.0	472.5	462.0	451.5
678	509	654.0	464.5	4424800	576.6	568.2	559.7	551.3	542.8	534.4
691	578	684.5	543.5	4932400	623.2	617.6	611.9	606.3	600.6	595.0
675	623	683.0	600.5	5229400	643.8	641.2	638.6	636.0	633.4	630.8
634	642	654.5	632.5	5308600	638.8	639.2	639.6	640.0	640.4	640.8
571	635	602.5	638.5	5179000	609.4	612.6	615.8	619.0	622.2	625.4
477	603	524.0	619.0	4837000	552.6	558.9	565.2	571.5	577.8	584.1
390	546	433.5	574.5	4329400	483.6	491.4	499.2	507.0	514.8	522.6
329	479	359.5	512.5	3778600	419.0	426.5	434.0	441.5	449.0	456.5
247	413	288.0	446.0	3209800	346.6	354.9	363.2	371.5	379.8	388.1
184	341	215.5	377.0	2628400	278.2	286.1	293.9	301.8	309.6	317.5
134	274	159.0	307.5	2093800	218.0	225.0	232.0	239.0	246.0	253.0
108	215	121.0	244.5	1649200	172.2	177.6	182.9	188.3	193.6	199.0
90	170	99.0	192.5	1312600	138.0	142.0	146.0	150.0	154.0	158.0

Table 3.2 Observed and Computed Values of Outflow for Ramirez Data Set

Time (h)	Inflow (m3/s)	C_o x I_{i+1} 0.063	C_1 x I_i 0.344	C_2 x O_i 0.593	Computed Outflow (m3/s)	Observed Outflow (m^3/s)
1	93				93	85
2	137	8.6310	31.9920	55.1490	96	91
3	208	13.1040	47.1280	56.7928	117	114
4	320	20.1600	71.5520	69.3957	161	159
5	442	27.8460	110.0800	95.5369	233	233
6	546	34.3980	152.0480	138.4435	325	324
7	630	39.6900	187.8240	192.6595	420	420
8	678	42.7140	216.7200	249.1629	509	509
9	691	43.5330	233.2320	301.5979	578	578
10	675	42.5250	237.7040	342.9692	623	623
11	634	39.9420	232.2000	369.5565	642	642

12	571	35.9730	218.0960	380.5272	635	635
13	477	30.0510	196.4240	376.3156	603	603
14	390	24.5700	164.0880	357.4548	546	546
15	329	20.7270	134.1600	323.8449	479	479
16	247	15.5610	113.1760	283.8880	413	413
17	184	11.5920	84.9680	244.6866	341	341
18	134	8.4420	63.2960	202.3593	274	274
19	108	6.8040	46.0960	162.5397	215	215
20	90	5.6700	37.1520	127.7557	171	170

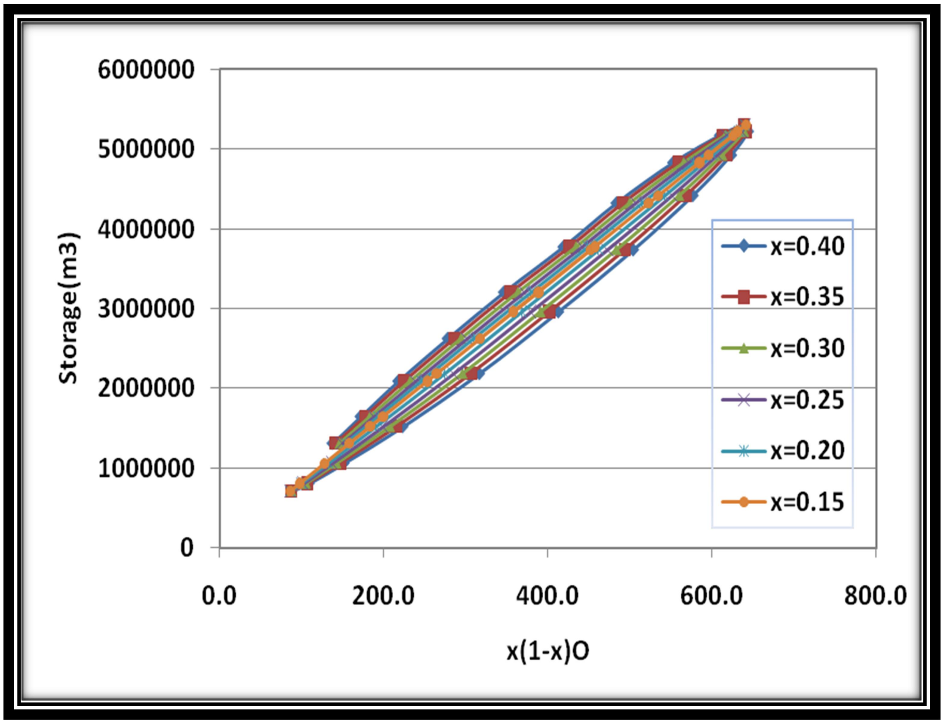

Figure 3.1 Graph of Storage versus [xI + (1-x) O] at different value of x

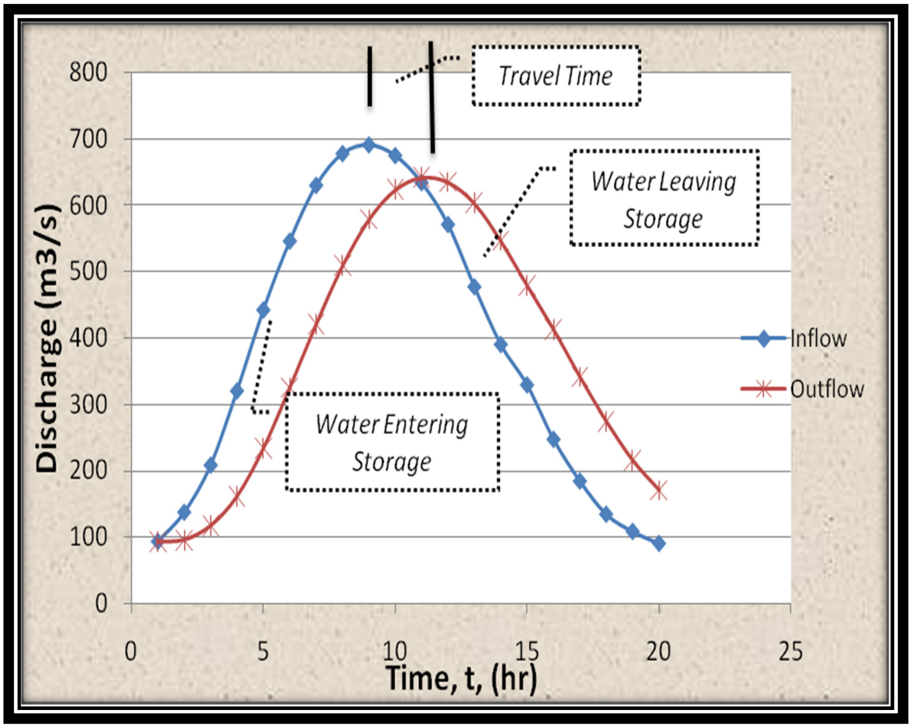

Figure 3.2 Storage versus the storage discharge

Data Set 1

Table 1.4 Estimated value of x for Data set 1

A	B	C	D	E	F x=0.40	G x=0.35	H x=0.30	I x=0.20	J x=0.26	K x=0.25
Inflow, (m^3/s)	Outflow (m^3/s)	Ave. Inflow (m^3/s)	Ave. Outflow (m^3/s)	Storage (m^3)	Weighted Average Flux - xI + (1-x) O (m^3/s)					
22	22			21600	22.0	22.0	22.0	22.0	22.0	22.0
23	21	22.5	21.5	43200	21.8	21.7	21.6	21.4	21.5	21.5
35	21	29.0	21.0	216000	26.6	25.9	25.2	23.8	24.6	24.5
71	26	53.0	23.5	853200	44.0	41.8	39.5	35.0	37.7	37.3
103	34	87.0	30.0	2084400	61.6	58.2	54.7	47.8	51.9	51.3
111	44	107.0	39.0	3553200	70.8	67.5	64.1	57.4	61.4	60.8
109	55	110.0	49.5	4860000	76.6	73.9	71.2	65.8	69.0	68.5
100	66	104.5	60.5	5810400	79.6	77.9	76.2	72.8	74.8	74.5
86	75	93.0	70.5	6296400	79.4	78.9	78.3	77.2	77.9	77.8
71	82	78.5	78.5	6296400	77.6	78.2	78.7	79.8	79.1	79.3
59	85	65.0	83.5	5896800	74.6	75.9	77.2	79.8	78.2	78.5
47	84	53.0	84.5	5216400	69.2	71.1	72.9	76.6	74.4	74.8
39	80	43.0	82.0	4374000	63.6	65.7	67.7	71.8	69.3	69.8
32	73	35.5	76.5	3488400	56.6	58.7	60.7	64.8	62.3	62.8
28	64	30.0	68.5	2656800	49.6	51.4	53.2	56.8	54.6	55.0
24	54	26.0	59.0	1944000	42.0	43.5	45.0	48.0	46.2	46.5
22	44	23.0	49.0	1382400	35.2	36.3	37.4	39.6	38.3	38.5
21	36	21.5	40.0	982800	30.0	30.8	31.5	33.0	32.1	32.3
20	30	20.5	33.0	712800	26.0	26.5	27.0	28.0	27.4	27.5
19	25	19.5	27.5	540000	22.6	22.9	23.2	23.8	23.4	23.5
19	22	19.0	23.5	442800	20.8	21.0	21.1	21.4	21.2	21.3
18	19	18.5	20.5	399600	18.6	18.7	18.7	18.8	18.7	18.8

Table 3.4 Observed and Computed Values of Outflow for Data Set 1

Time (h)	Inflow (m3/s)	Observed Outflow (m3/s)	Computed O (m3/s)				C_o x I_{i+1} (m3/s)	C_1 x I_i (m3/s)	C_2x O_i (m3/s)
			LSM	Method 1	Method 2	Present Study	(-) 0.179	0.434	0.745
0	22.0	22.0	22.0	22.0	22.0	22.0			
6	23.0	21.0	21.8	21.8	21.7	21.8	4.117	9.548	16.390
12	35.0	21.0	20.0	19.5	18.9	20.0	6.265	9.982	16.257
18	71.0	26.0	17.5	16.4	12.0	17.4	12.709	15.190	14.880
24	103.0	34.0	24.9	27.1	12.5	25.3	18.437	30.814	12.934
30	111.0	44.0	42.4	50.2	23.8	43.7	19.869	44.702	18.857
36	109.0	55.0	59.3	70.6	37.1	61.2	19.511	48.174	32.549
42	100.0	66.0	72.9	85.3	49.9	75.0	17.900	47.306	45.603
48	86.0	75.0	81.8	93.3	60.9	83.9	15.394	43.400	55.882
54	71.0	82.0	85.4	94.3	68.4	87.1	12.709	37.324	62.496
60	59.0	85.0	84.0	89.3	71.8	85.2	10.561	30.814	64.898
66	47.0	84.0	80.0	82.1	73.0	80.6	8.413	25.606	63.437
72	39.0	80.0	73.4	72.4	71.2	73.5	6.981	20.398	60.070
78	32.0	73.0	66.3	63.0	68.3	65.9	5.728	16.926	54.748
84	28.0	64.0	58.7	53.7	64.0	58.0	5.012	13.888	49.129
90	24.0	54.0	52.0	46.2	59.7	51.1	4.296	12.152	43.214
96	22.0	44.0	45.6	39.3	55.0	44.5	3.938	10.416	38.047
102	21.0	36.0	40.0	33.9	50.4	39.0	3.759	9.548	33.171
108	20.0	30.0	35.6	29.9	46.3	34.6	3.580	9.114	29.025
114	19.0	25.0	33.0	26.9	42.7	31.0	3.401	8.680	25.747
120	19.0	22.0	28.9	24.3	39.2	28.0	3.401	8.246	23.114
126	18.0	19.0	26.6	22.8	36.5	25.9	3.222	8.246	20.830

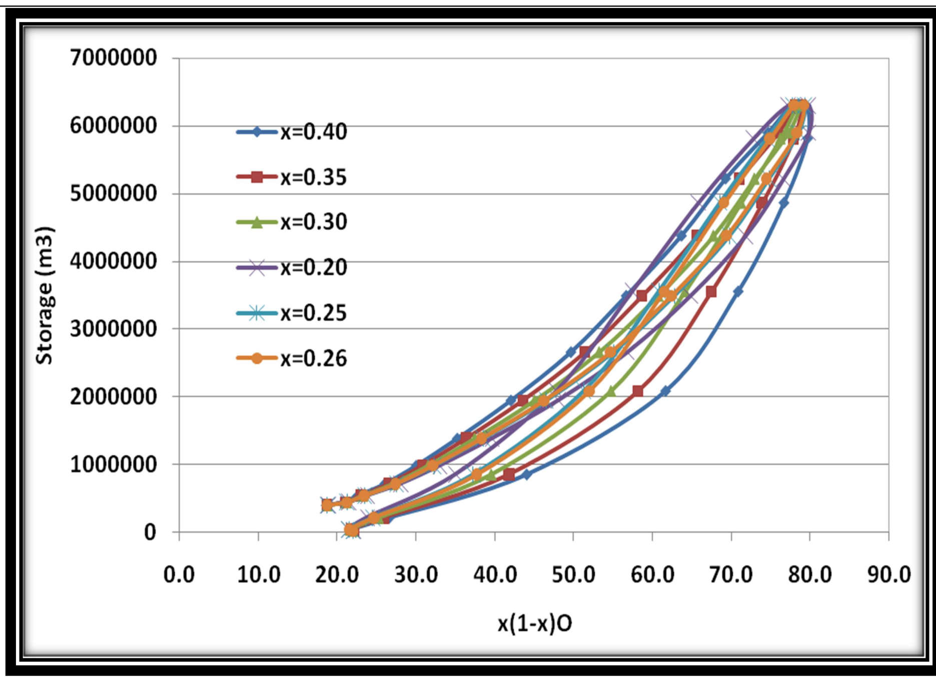

Figure 3.3 Graph of Storage versus [xI + (1-x) O] at different value of x

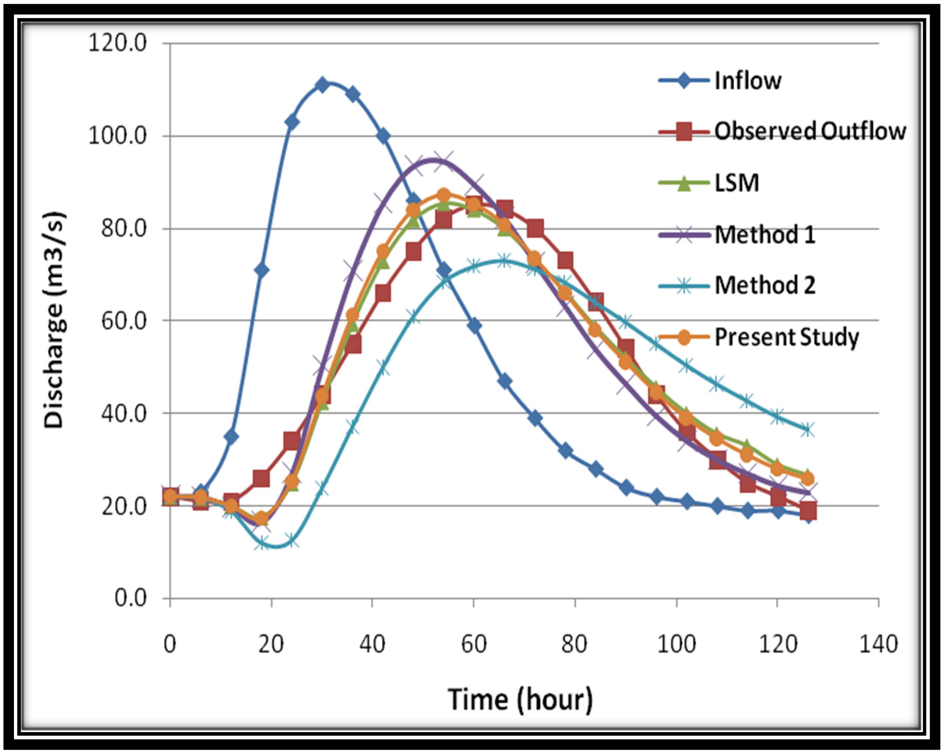

Figure 3.4 Outflow hydrographs for the data set 1 four methods

Data Set 2

Table 3.5 Estimated value of x for Data Set 2

A	B	C	D	E	F x=0.40	G x=0.35	H x=0.30	I x=0.25
Inflow, I (m^3/s)	Outflow, O (m^3/s)	Ave. Inflow (m^3/s)	Ave. Outflow (m^3/s)	Storage (m^3)	Weighted Average Flux - xI + (1-x) O			
166.2	118.4			4924800	137.5	135.1	132.7	130.4
263.6	197.4	214.9	157.9	9849600	223.9	220.6	217.3	214.0
365.3	214.1	314.5	205.8	19241280	274.6	267.0	259.5	251.9
580.5	402.1	472.9	308.1	33480000	473.5	464.5	455.6	446.7
594.7	518.2	587.6	460.2	44491680	548.8	545.0	541.2	537.3
662.6	523.9	628.7	521.1	53788320	579.4	572.4	565.5	558.6
920.3	603.1	791.5	563.5	73483200	730.0	714.1	698.3	682.4
1568.8	829.7	1244.6	716.4	119115360	1125.3	1088.4	1051.4	1014.5
1775.5	1124.2	1672.2	977.0	179180640	1384.7	1352.2	1319.6	1287.0
1489.5	1379.0	1632.5	1251.6	212090400	1423.2	1417.7	1412.2	1406.6
1223.3	1509.3	1356.4	1444.2	204508800	1394.9	1409.2	1423.5	1437.8
713.6	1379.0	968.5	1444.2	163408320	1112.8	1146.1	1179.4	1212.7
645.6	1050.6	679.6	1214.8	117167040	888.6	908.9	929.1	949.4
1166.7	1013.7	906.2	1032.2	106280640	1074.9	1067.3	1059.6	1052.0
1427.2	1013.7	1297.0	1013.7	130753440	1179.1	1158.4	1137.8	1117.1
1282.8	1013.7	1355.0	1013.7	160241760	1121.3	1107.9	1094.4	1081.0
1098.7	1209.1	1190.8	1111.4	167097600	1164.9	1170.5	1176.0	1181.5
764.6	1248.8	931.7	1229.0	141410880	1055.1	1079.3	1103.5	1127.8
458.7	1002.4	611.7	1125.6	97005600	784.9	812.1	839.3	866.5
351.1	713.6	404.9	858.0	57857760	568.6	586.7	604.9	623.0
288.8	464.4	320.0	589.0	34611840	394.2	402.9	411.7	420.5
228.8	325.6	258.8	395.0	22844160	286.9	291.7	296.6	301.4
170.2	265.6	199.5	295.6	14541120	227.4	232.2	237.0	241.8
143.0	222.6	156.6	244.1	6981120	190.8	194.7	198.7	202.7

Table 3.6 Observed and Computed Values of Outflow for Data Set 2

Time (days)	Inflow (m3/s)	Outflow (m3/s)				Outflow (m3/s)	C_o x I_{i+1} (m^3/s)	C_1 x I_i (m^3/s)	C_2 x O_i (m^3/s)
			LSM	Method 1	Method 2	Present Study			
							0.016	0.508	0.476
0	166.2	118.4	118.4	118.4	118.4	118.4			
1	263.6	197.4	146.1	138.7	131.6	146.6	5.845	84.430	56.358
2	365.3	214.1	209.6	206.3	170.0	264.7	9.288	185.572	69.797
3	580.5	402.1	296.4	284.1	226.0	430.4	9.515	294.894	125.977
4	594.7	518.2	442.2	466.3	331.9	517.6	10.602	302.108	204.864
5	662.6	523.9	522.4	539.3	409.8	597.7	14.725	336.601	246.365
6	920.3	603.1	602.7	590.9	482.5	777.1	25.101	467.512	284.501
7	1569	829.7	786.8	732.7	606.2	1195.3	28.408	796.950	369.906
8	1776	1124.2	1193.6	1230.5	891.9	1494.7	23.832	901.954	568.946
9	1490	1379.0	1481.7	1595.4	1159.4	1487.7	19.573	756.666	711.492
10	1223	1509.3	1476.8	1555.4	1261.2	1341.0	11.418	621.436	708.160
11	713.6	1379.0	1330.1	1398.6	1255.5	1011.2	10.330	362.509	638.323
12	645.6	1050.6	1012.5	981.0	1094.2	827.9	18.667	327.965	481.313
13	1167	1013.7	842.3	723.4	954.1	1009.6	22.835	592.684	394.102
14	1427	1013.7	1016.9	972.9	1014.8	1226.1	20.525	725.018	480.579
15	1283	1013.7	1221.9	1268.0	1139.8	1252.9	17.579	651.662	583.634
16	1099	1209.1	1246.9	1294.8	1184.6	1166.7	12.234	558.140	596.369
17	764.6	1248.8	1160.0	1205.4	1162.7	951.1	7.339	388.417	555.369
18	458.7	1002.4	947.5	961.8	1047.0	691.4	5.618	233.020	452.736
19	351.1	713.6	693.9	660.6	872.2	512.1	4.621	178.359	329.093
20	288.8	464.4	516.5	475.0	717.0	394.1	3.661	146.710	243.747
21	228.8	325.6	398.1	365.5	589.6	306.6	2.723	116.230	187.600
22	170.2	265.6	309.5	286.5	482.3	234.7	2.288	86.462	145.920
23	143.0	222.6	237.4	217.1	389.2	184.3	0.000	72.644	111.703

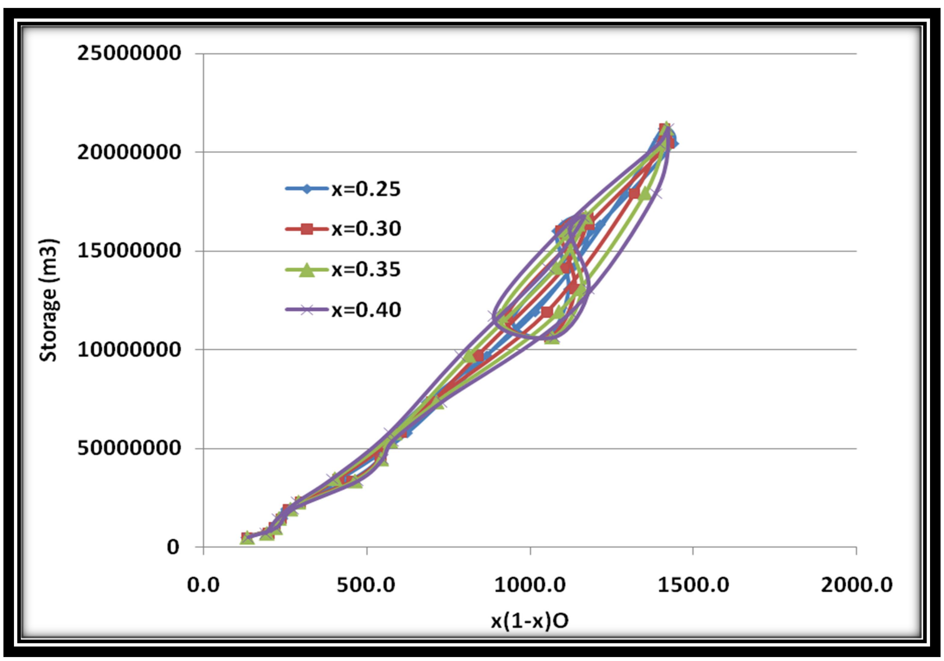

Figure 3.5 Graph of Storage versus [xI + (1-x) O] at different value of x

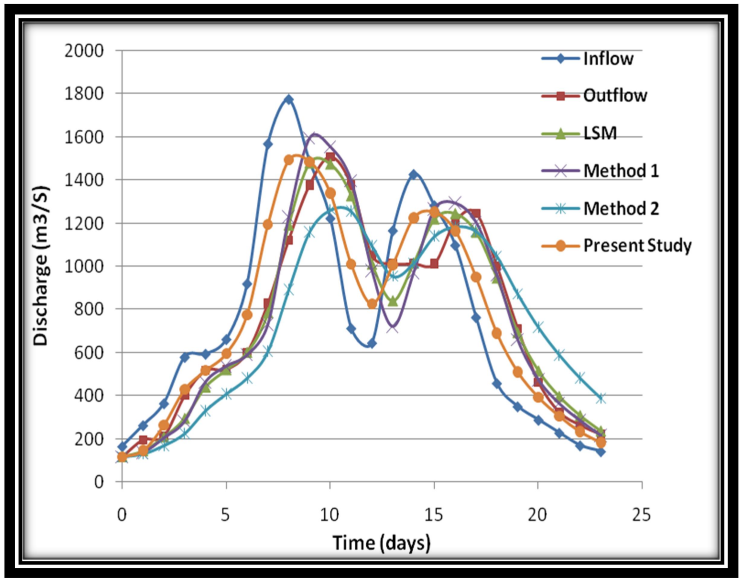

Figure 3.6 Outflow hydrographs for the data set 2 methods

Data Set 3

Table 3.7 Estimated value of x for Data Set 3

A	B	C	D	E	F x=0.20	G x=0.15	H x=0.10	I x=0.05	J x=0.08
Inflow, I (m^3/s)	Outflow, O (m^3/s)	Ave. Inflow (m^3/s)	Ave. Outflow (m^3/s)	Storage (m^3)	Weighted Average Flux - xI + (1-x) O (m^3/s)				
1.87	1.87			45360	1.9	1.9	1.9	1.9	1.9
7.08	2.89	4.5	2.4	90612	3.7	3.5	3.3	3.1	3.2
15.57	5.24	11.3	4.1	247428	7.3	6.8	6.3	5.8	6.1
18.85	7.50	17.2	6.4	481572	9.8	9.2	8.6	8.1	8.4
11.89	9.49	15.4	8.5	630072	10.0	9.9	9.7	9.6	9.7
8.35	10.48	10.1	10.0	632988	10.1	10.2	10.3	10.4	10.3
5.95	10.42	7.2	10.5	561708	9.5	9.7	10.0	10.2	10.1
4.16	8.78	5.1	9.6	463536	7.9	8.1	8.3	8.5	8.4
2.83	6.94	3.5	7.9	369252	6.1	6.3	6.5	6.7	6.6
2.10	5.66	2.5	6.3	286416	4.9	5.1	5.3	5.5	5.4
1.70	4.67	1.9	5.2	215892	4.1	4.2	4.4	4.5	4.4
1.44	3.74	1.6	4.2	158976	3.3	3.4	3.5	3.6	3.6
1.30	2.83	1.4	3.3	117612	2.5	2.6	2.7	2.8	2.7

Table 3.8 Observed and Computed Values of Outflow for Data Set 3

Time (hours)	Inflow (m3/s)	Observed Outflow (m3/s)	Computed Outflow (m3/s)				C_o x I_{i+1}	C_1 x I_i	C_2 x O_i
			LSM	Method 1	Method 2	Present Study	0.092	0.237	0.672
0	1.87	1.87	1.87	1.87	1.87	1.87			
6	7.08	2.89	2.31	1.39	1.09	2.35	0.651	0.443	1.257
12	15.57	5.24	4.32	2.89	0.79	4.69	1.432	1.678	1.580
18	18.85	7.50	7.61	7.69	2.71	8.58	1.734	3.690	3.152
24	11.89	9.49	10.02	12.82	6.40	11.32	1.094	4.467	5.763
30	8.35	10.48	10.22	12.78	7.83	11.20	0.768	2.818	7.610
36	5.95	10.42	9.52	11.22	8.28	10.05	0.547	1.979	7.524
42	4.16	8.78	8.41	9.27	8.17	8.55	0.383	1.410	6.754
48	2.83	6.94	7.16	7.34	7.71	6.99	0.260	0.986	5.743
54	2.10	5.66	5.94	5.59	7.02	5.56	0.193	0.671	4.697
60	1.70	4.67	4.88	4.22	6.28	4.39	0.156	0.498	3.737
66	1.44	3.74	4.00	3.23	5.57	3.49	0.132	0.403	2.951
72	1.30	2.83	3.31	2.52	4.91	2.80	0.120	0.341	2.343

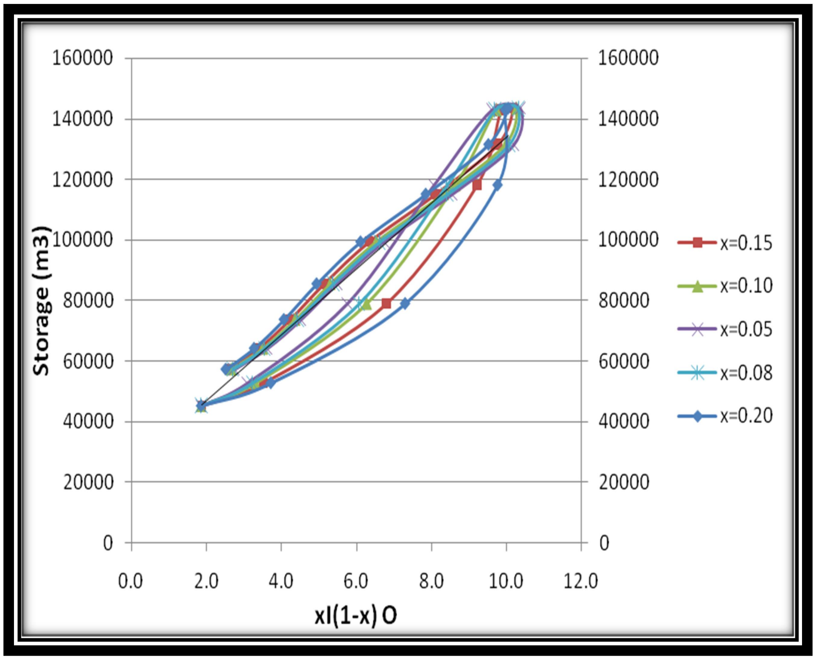

Figure 3.7 Graph of Storage versus [xI + (1-x) O] at different value of x

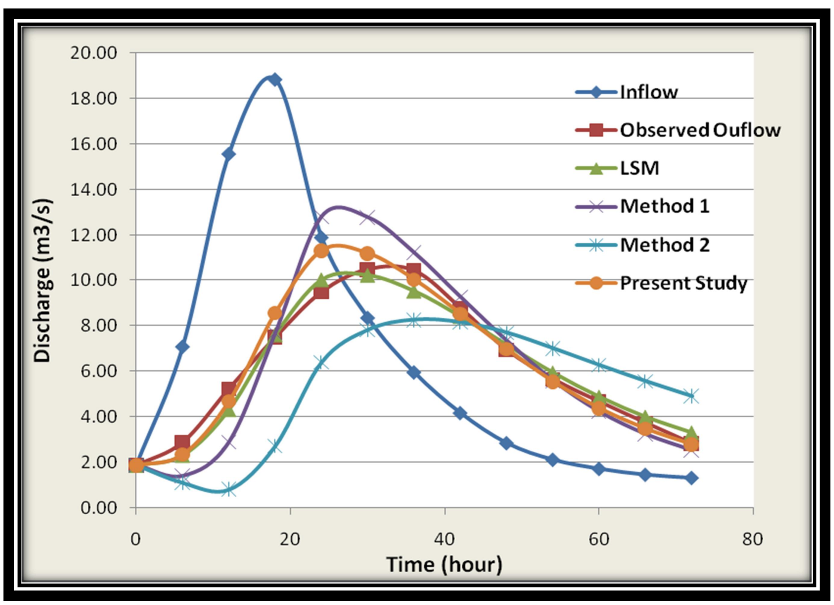

Figure 3.8 Outflow hydrographs for the data set 3 methods

Table 3.9 Comparison of Model Parameters for Different Data Sets

Data Set	X	K
Data by Ramirez	0.15	2.31
Present Study	0.15	2.32
Data Set 1, k in hours		
LSM	0.25	29.1
Method 1	0.32	22.4
Method 2	0.26	51.3
Present Study	0.26	27.8
Data Set 2, K in days		
LSM	0.23	1.87
Method 1	0.37	1.77
Method 2	0.16	3.38
Present Study	0.25	1.88
Data Set 3, K in hours		
LSM	0.005	20.5
Method 1	0.269	16.3
Method 2	0.202	42.2
Present Study	0.080	16.6

Table 3.10 Summary of Data Set 1

Groups	Count	Sum	Average	Variance
LSM	22	1072.10	48.73	553.36
Method 1	22	1084.30	49.29	745.00
Method 2	22	1005.30	45.70	417.84
Present Study	22	1074.55	48.84	585.98

Table 3.11 ANOVA (One- Way) for Data Set 1

Source of Variation	SS	df	MS	F	P-value	F Critical
Between Groups	178.96	3	59.65	0.104	0.958	4.024
Within Groups	48345.86	84	575.55			
Total	48524.82	87				

Table 3.12 Summary of Data Set 2

Groups	Count	Sum	Average	Variance
LSM	24	18210.20	758.76	195206.12
Method 1	24	18268.60	761.19	221700.09
Method 2	24	17691.90	737.16	154301.17
Present Study	24	18139.92	755.83	201239.51

Table 3.13 Anova (One- Way) for Data Set 2

Source of Variation	SS	Df	MS	F	P-value	F critical
Between Groups	8613.00	3	2871	0.015	0.998	4.002
Within Groups	17766278.47	92	193112			
Total	17774891.47	95				

Table 3.14 Summary of Data Set 3

Groups	Count	Sum	Average	Variance
LSM	13	79.57	6.12	8.49
Method 1	13	82.83	6.37	17.03
Method 2	13	68.63	5.28	7.60
Present Study	13	81.84	6.30	11.34

Table 3.15 ANOVA (One- Way) for Data Set 3

Source of Variation	SS	df	MS	F	P-value	F critical
Between Groups	9.86	3	3.29	0.296	0.828	4.218
Within Groups	533.63	48	11.12			
Total	543.48	51				

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