

Game Theory Modeling in River Valley Projects-Benin Owena River Basin

Dr. Eme Luke Chika, (B.Sc, PGD, PGDE, M.Eng, Ph.D),
Civil Engineering Department, Chukwuemeka Odumegwu Ojukwu University
(formerly Anambra State University), Uli – Nigeria.
Leeworks2002@yahoo.com

Ohaji Evans, (B.Eng, M.Eng)
Rural Infrastructural Engineer, Implementation Field Office,
Niger Delta Support Programme Component, Imo State, Nigeria.
evansohaji@gmail.com

Abstract

This study applied game theory based model to analyze and solve sharing conflicts on funds allocation to the multi-purpose and the multi-objectives in Benin-Owena River Basin. The model provides strategic decisions geared toward resolving the problem of apportioning ₦100 billion Naira development fund each to the two players, multi-objective [economic efficiency, regional economic distribution, state economic distribution, youth employment and environmental control] and the multi-purpose.[irrigation, hydropower, water supply, recreation, and erosion control]. The game simulation comprised five players on both the multi-purpose and multi-objective axis and the game theory converted to a linear programming problem and was analyzed using Simplex method. The analysis and presentation of results in this paper were based on Game Theory Simulation Model. However, Contingency and Association, Chi-square and Pearson Product Moment Correlation were carried out as Interaction, reliability and Validity tests. The result indicates the following proportional funds allocated in percentages to the multi-objectives: economic efficiency, regional economy distribution, state economic distribution, youth employment and environmental control are 23, 72, 0.00, 0.00, and 5%, respectively. And funds apportioned to the multipurpose are in the following order: Irrigation, hydroelectric power, water supply, and recreation and erosion control are: 0.0, 0.0, 0.26, 0.16, and 58%, respectively. This study gave the indication that funds were available for water supply, recreation and erosion control for the multipurpose, which gave rise to solving economic efficiency and regional Economic Distribution for the multi-objective. In additional, to avoid conflict, the results suggest a need to design a mechanism to reduce the risk of losses of those players by a side payment, which provides them with economic incentives to cooperate. Game theory application in River basin management is invaluable; it gives optimal solution on government investment and wellbeing of people within the region for both multi-purpose and multi-objectives simultaneously.

Keywords: Optimal, Logical Model, Investment Fund, Game Theory, River Valley.

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1.0 Introduction

This study applied game theory based models to analyze and solve seem sharing conflicts concerning funds allocation to the multi-purpose and the multi-objectives in Benin-Owena River Basin. This study covers the dynamics between five river basin purposes and five river Objectives and how the relation of the duo can be optimized using Game theory model for the benefit of the inhabitant of the basin. The horizon for the study was designed to cover a period of 5 years (2013 – 2017).

The Ondo State Government in 1976, commissioned the design of the Owena River Dam with the objective of supplying raw water from the resulting reservoir for the existing water scheme, but taken over by the Federal Government of Nigeria (through Benin-Owena River Basin Development Authority) and converted it to a multipurpose use in line with the functions of the River Basin Development Authorities. The design was reviewed to include in addition to provision of potable water, usage for irrigation of 3,000 hectares of farmland, fisheries, as well as generation of hydro-electric power. The dam sited on the Owena River and was designed to create an impoundment of 36.25 million cm³ gross capacity, covering an area of approximately 7.38 km² at the normal water level. Thus, this study examined the Owena multipurpose/multi-objective River basin, as a key activity in managing the water source.

Statement of Problem: According to Eme, L. (2013) stated that “A nation with Economic and great water resources potentials as Nigeria cannot prosper without the benefits of the resources development and utilization”. And to benefit from the resources development and utilization the required decisions will need to be made by concern stakeholders in the government and River Basin Development Authorities (RBDA)”. Therefore for the RBDA to thrive it must not be starve of funds. However, funds provided or generated within the system must be adequately distributed and/ or allocated. The problem now is, how can it be distributed equitably free from bias? The foregoing question conceived this research work. Furthermore this paper will make an attempt to solve the problem by applying game theory modeling in the equitably allocation of scares resources within Owena RBDA system.

Study Area: The study area is the Benin-Owena River Basin Area that lies between the west bank of the Niger River in the east and the Oni River in the west, and occupies the territory covered by Ondo, Ekiti, Edo and Delta states. The basin has a total area of 59 787 km², with a human population of over 13 million. The settlement pattern shows compactness in the towns and cities, with the major occupation of the rural population being agriculture; there are a few growing industries scattered across basin. The main north–south flowing rivers/streams of the basin are from west to east namely; Oni, Siluko, Benin, Escravos, Forcados, Ase, Niger and many other streams. Topographically, the basin can be divided into two belts; namely the northern highlands and southern plains or lowlands. It has an undulating land surface that descends gradually from an altitude of over 731.52m in Ekiti state to the lowlands. The Ishan plateau of the basin rises steeply from the Niger valley and has such striking characteristics as level topography, easily worked sandy soil and paucity of surface drainage which calls for extensive hydrological study in terms of groundwater occurrences.

2.0 Data Collection and Analysis of Multi-Objectives Benefits

This is measured through benefits derived from water resources projects by various benefiting localities within a region as a result of location and size of project and with regards to various purposes involved. Such benefits vary with respect to decision variables (purposes).

However, the raw data were analyzed for interaction, reliability and validity through:

- Contingency coefficient & association
- Pearson moment correlation coefficient and
- Chi-square test.

Contingency and Reliability Test: Contingency and reliability in this paper is another alternative method of testing null hypothesis, the paper assesses the relationship and test the null hypothesis on: “Are there relationships between the Watershed Purposes and Objectives?”

Table 1: Observed Contingency Table

State of Nature	Course of Action	Economic Efficiency	Regional Economic Distribution	State Economic Distribution	Youth Employment	Environmental Control
Irrigation	4	4.75	25.5	18.3	1.95	54.5
Hydropower	29	5.3	19.2	17.9	4.9	76.3
Water supply	25	6.45	21.6	15.1	8.5	76.65
Recreation	16	8.3	14.5	13	12.9	64.7
Erosion Control	3	13.4	17.6	11.2	8.8	54
	77	38.2	98.4	75.5	37.05	326.15

Step I: Calculation of the expected contingency table using the formula:

$$Cell_{ij} = \frac{\text{ith Row Total} \times \text{jth Column Total}}{\text{Grand Total}}$$

Where I = is the ith and

J = is the jth column

$$A, B, C, \dots, Y = \frac{\text{ith Row Total} \times \text{jth Column Total}}{\text{Grand Total}} = \text{Cell}_{ij} \text{ Values in table Below:}$$

Table 2: Expected contingency Table

	A	B	C	D	E	Total
	12.86678	6.383259	16.44274	12.61613	6.191093	54.5
	F	G	H	I	J	
	18.01349	8.936563	23.01984	17.66258	8.66753	76.3
	K	L	M	N	O	
	18.09612	8.977556	23.12543	17.7436	8.70729	76.65
	P	Q	R	S	T	
	15.27487	7.577924	19.5201	14.97731	7.349793	64.7
	U	V	W	X	Y	
	12.74874	6.324697	16.29189	12.50038	6.134294	54
Total	77	38.2	98.4	75.5	37.05	326.15

Step II: Computation of Chi-square using the formula:

$$\chi^2 = \frac{\sum(O - E)^2}{E}$$

Table 3: Chi-square Table

O	E	0-E	(0-E) ²	(0-E) ² /E
4	12.86678	-8.866779089	78.61977	6.110292
4.75	6.383259	-1.633259237	2.667536	0.417896
25.5	16.44274	9.05725893	82.03394	4.989067
18.3	12.61613	5.683872451	32.30641	2.560723
1.95	6.191093	-4.241093055	17.98687	2.905282
29	18.01349	10.98650927	120.7034	6.700722
5.3	8.936563	-3.636562931	13.22459	1.47983
19.2	23.01984	-3.819837498	14.59116	0.633851
17.9	17.66258	0.237421432	0.056369	0.003191
4.9	8.66753	-3.767530277	14.19428	1.637639
25	18.09612	6.903878583	47.66354	2.633909
6.45	8.977556	-2.527556339	6.388541	0.711612
21.6	23.12543	-1.525433083	2.326946	0.100623
15.1	17.7436	-2.643599571	6.988619	0.393867
8.5	8.70729	-0.207289591	0.042969	0.004935
16	15.27487	0.725126476	0.525808	0.034423
8.3	7.577924	0.722075732	0.521393	0.068804
14.5	19.5201	-5.020098114	25.20139	1.291048
13	14.97731	-1.977311053	3.909759	0.261045
12.9	7.349793	5.55020696	30.8048	4.191247
3	12.74874	-9.748735245	95.03784	7.454688
13.4	6.324697	7.075302775	50.05991	7.914989
17.6	16.29189	1.308109765	1.711151	0.105031
11.2	12.50038	-1.300383259	1.690997	0.135276
8.8	6.134294	2.665705964	7.105988	1.158404
326.15	326.15	-2.75335E-14	7.58E-28	53.89839

Contingency coefficient, C is given by

$$C = \sqrt{\frac{X^2}{N + X^2}}$$

Where C = Contingency Coefficient

X^2 = Chi-square

N = Grand total of subjects or cases

$$X^2 = 53.89839$$

$$N = 326.15$$

$$C = \sqrt{\frac{53.89839}{326.15 + 53.89839}}$$

C = 0.376, the maximum Contingency coefficient can go is 0.8.

Therefore C = 0.47/0.8

$$C = 0.5875$$

$$C = 0.6$$

Correlation of attributes r, is given as:

$$r = \sqrt{\frac{X^2}{N(K-1)}}$$

$$r = \sqrt{\frac{53.89839}{326.15(5-1)}}$$

$$r = 0.2$$

2.1 Summary of Results

The Contingency of the raw data is = 0.6

The correlation of attributes of the raw data = 0.2

The X^2 value 53.89839 is interpreted from the X^2 table of probability values at 0.10 level of significance. The degree of freedom necessary to intercept X^2 values are always determined from the frequency table by the number of rows minus one times the number of columns minus one (r-1)(c-1) i.e. (5-1)(5-1) = 16

-Since the obtained X^2 value of 53.89839 is greater than the critical value of 32.000, therefore the null hypothesis is accepted. i.e.: $X^2(53.89839) > X^2_{0.10}(32.000)$. Therefore the Null hypotheses is rejected, a clear indication that there is a relationship between the watershed purposes and the Objectives/Benefits.

-Therefore there is relationship between the state of the system (Dam Purposes) and the Dam Objectives.

-The Chi Square was not based on a fictitious data, in the case of Bayesian Decision Modeling in Water Shade Management.

-In the test of how well the linear estimator, $y = a + bx$ fits the raw data, the correlation coefficient $r = 0.2$ resulting in a good fit or relationship for the raw data

Pearson Moment Correlation Coefficient

Using column 1 and 2 of table 4.10 the Pearson Moment Correlation Coefficient (r) = 0.7193 is represented in the graph below:

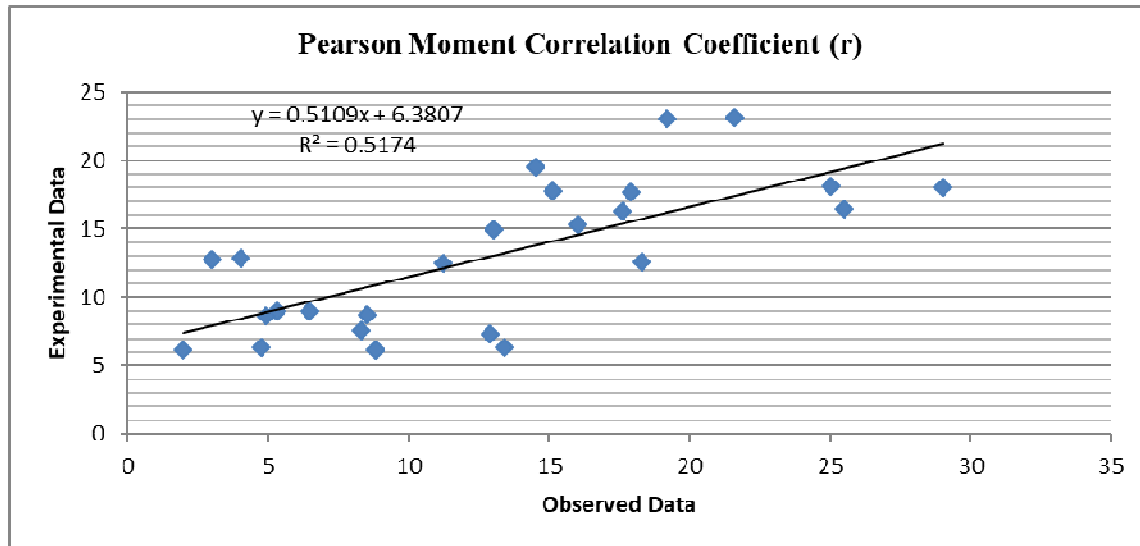


Figure 1: Pearson Moment Correlation Coefficient Graph

From the graph, $R^2 = 0.517$

Therefore $R = r = 0.719$

$Y = 0.517x + 6.380$

3.0 Methodology

There are six data categories that structure the multipurpose benefits framework. These categories are referred to herein as “uses”, and they represent a culmination of operations and services made possible due to existence of a reservoir. These uses are broadly classified to identify categories associated with a reservoir project, and serve as a foundation for assessing collective and inter-dependent relationships (Marisol Bonnet et al, 2015):

-Hydropower: Operation and use of generating facilities and/or equipment for producing power by the sole source of water.

-Flood Control: Dams that facilitate the prevention and/or lessen the severity of flood damage to valuable resources within a flood basin.

-Water Transport & Navigation: The operation and control of locks to facilitate the transportation of goods via inland waterways.

-Recreation: The use of water bodies (reservoirs or rivers) for physical and recreational activities (boating, fishing, swimming, etc.).

-Water Supply: Public and private withdrawals of water used for consumption, municipal, and industrial needs.

-Irrigation: The withdrawal and use of water from reservoirs to meet the needs and requirement for crop and plant irrigation to sustain growth and production.

Based on the availability of both public and proprietary data, the following represent the methodologies used to compute the economic benefit of each multipurpose use.

3.1 Conversion of a Game Theory into a Linear Programming Problem [Simplex Method]

Let us consider the 3 x 3 matrix

$$\begin{matrix}
 & \begin{matrix} B1 & B2 & B3 \end{matrix} \\
 \begin{matrix} A1 \\ A2 \\ A3 \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
 \end{matrix}$$

As per the assumptions, A always attempts to choose the set of strategies with the non-zero

Probabilities say p_1, p_2, p_3 where $p_1 + p_2 + p_3 = 1$ that maximizes his minimum expected gain. Similarly B would choose the set of strategies with the non-zero probabilities say $q_1, q_2, q_3=1$ that minimizes his maximum expected loss. However the LPP can be solved through the following steps

Step 1

Find the minimax and maximin value from the given matrix

Step 2

The objective of A is to maximize the value, which is equivalent to minimizing the value $1/V$. The LPP is written as

$$\text{Min } 1/V = p_1/V + p_2/V + p_3/V$$

and constraints ≥ 1

It is written as

$$\text{Min } 1/V = x_1 + x_2 + x_3$$

and constraints ≥ 1

Similarly for B, we get the LPP as the dual of the above LPP

$$\text{Max } 1/V = Y_1 + Y_2 + Y_3$$

and constraints ≤ 1
 Where $Y_1 = q_1/V, Y_2 = q_2/V, Y_3 = q_3/V$

Step 3

Solve the LPP by using simplex table and obtain the best strategy for the players

Ultimately step 3 was handled in this research work by applying Linear Solver 1.11.1.0[Lips IDE Application] developed by Michael Melnic [2009-2013].

4.0 Model Simulation

Let’s consider Federal Government Allocation to Benin-Owena River Basin as **N100 Billion** to be spent on a multi-purpose/multi-objective water resources development project. The purposes of interest are irrigation, hydro-electric power generation, water supply, recreational and erosion control. The objectives to be simultaneously achieved at optimum level are economic efficiency, regional economic redistribution, State economic distribution, youth employment and environment.

The problem then becomes how to apportion the **N100 Billion** development fund among the various purposes so as to optimize the objective even under the worst situation of conflict.

Suppose a benefit study of the five purposes under each of the five objectives was carried out. The results being the values as shown in table 2. What we have by the table is basically a game situation. The entries (benefits) are the pay-offs. The purposes are the strategies of one player (the maximizer) and the objectives are the strategies of the player.

Table 4: Benefit to N100 Billion under various objectives [N X 10⁹]

Purposes	Objectives				
	Economic efficiency allow	Regional economy	State economic distribution	Youth employment	Environment
Irrigation	4	4.75	25.5	18.3	1.95
Hydropower	29	5.3	19.2	17.9	4.9
Water supply	25	6.45	21.6	15.1	8.5
Recreation	16	8.3	14.5	13	12.9
Erosion Control	3	13.4	17.6	11.2	8.8

The above game problem can be solved from the maximize point of view with the understanding that all purposes should be undertaken at positive level even under the worst circumstances or condition.

- Let probability X1 represent Irrigation
- Let probability X2 represents Hydro-electric power
- Let probability X3 represents Water Supply
- Let probability X4 represents Recreation
- And Let probability X5 represents Erosion Control

Then the game problem can be stated as follows:

Inputs for Computer Simulation

Objective Function Max: $X1 + X2 + X3 + X4 + X5$;

Constrains/Subjects:

- Row1: $4 * X1 + 4.75 * X2 + 25.5 * X3 + 18.3 * X4 + 1.95 * X5 \leq 1$;
- Row2: $29 * X1 + 5.3 * X2 + 19.2 * X3 + 17.9 * X4 + 4.9 * X5 \leq 1$;
- Row3: $25 * X1 + 6.45 * X2 + 21.6 * X3 + 15.1 * X4 + 8.5 * X5 \leq 1$;
- Row4: $16 * X1 + 8.3 * X2 + 14.5 * X3 + 13 * X4 + 21.9 * X5 \leq 1$;
- Row5: $3 * X1 + 13.4 * X2 + 17.5 * X3 + 11.2 * X4 + 8.8 * X5 \leq 1$;
- Row6: $X1 \geq 0$;
- Row7: $X2 \geq 0$;
- Row8: $X3 \geq 0$;
- Row9: $X4 \geq 0$;
- Row10: $X5 \geq 0$;

Solving the above by simplex method [Lips IDE Application] developed by Michael Melnic [2009-2013]. We now have the following results [Section 6.3]:

Result Output of Computer Simulation

The table 3 and 4 below shows results output of computer simulation

- >> Optimal solution FOUND
- >> Maximum = 617/6654

Computer Simulation Results

Table 5: Output of Computer Simulation * Results - Variables *****

Variable	Value	Obj. Cost	Reduced Cost
X1	0.0211159	1	0
X2	0.0666264	1	0
X3	0	1	0.68345
X4	0	1	0.159126
X5	0.00498387	1	0

Table 6: Output of Computer Simulation * Results - Constraints *****

Constraint	Value	RHS	Dual Price
Row1	0.410658	1	0
Row2	0.989901	1	0
Row3	1	1	0.0241826
Row4	1	1	0.0146003
Row5	1	1	0.0539432

$X1 = 0.23$; $X2 = 0.72$; $X3 = 0.00$; $X4 = 0.00$; $X5 = 0.05$ and $V = 10.78$
 $Y1 = 0.00$; $Y2 = 0.00$; $Y3 = 0.26$; $Y4 = 0.16$; $Y5 = 0.58$

The above result means that for the five objectives to be simultaneously optimized even under the worst possible condition, the development should be apportioned as follows:

- Let probability $Y1$ represent Irrigation
- Let probability $Y2$ represents Hydro-electric power
- Let probability $Y3$ represents Water Supply
- Let probability $Y4$ represents Recreation
- Let probability $Y5$ represents Erosion Control
- Let probability $X1$ represent Economic Efficiency
- Let probability $X2$ represents Regional Economy
- Let probability $X3$ represents State Economy
- Let probability $X4$ represents Youth employment
- Let probability $X5$ represents Environment

Value $[V] = 10.78$

Table 7: Percentage Allocation to the Benefits [Objectives]

Strategy For The Objective	Percentage Apportion
Economic Efficiency	0.23
Regional Economy	0.72
State Economy	0.00
Youth employment	0.00
Environment	0.05

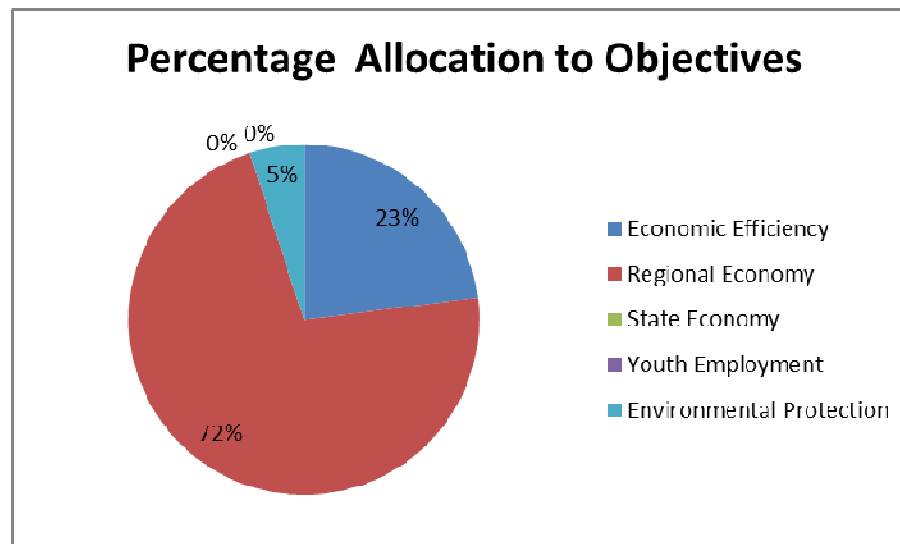


Figure 2: Percentage allocation to Objectives

Table 8: Funds Allocation to Objectives [N100Billion]

Strategy For The Objective	Apportion
Economic Efficiency[X1]	N23.0 Million
Regional Economy[X2]	N72.0 Million
State Economy[X3]	N0.00 Million
Youth employment[X4]	N0.00 Million
Environment[X5]	N5.00 Million

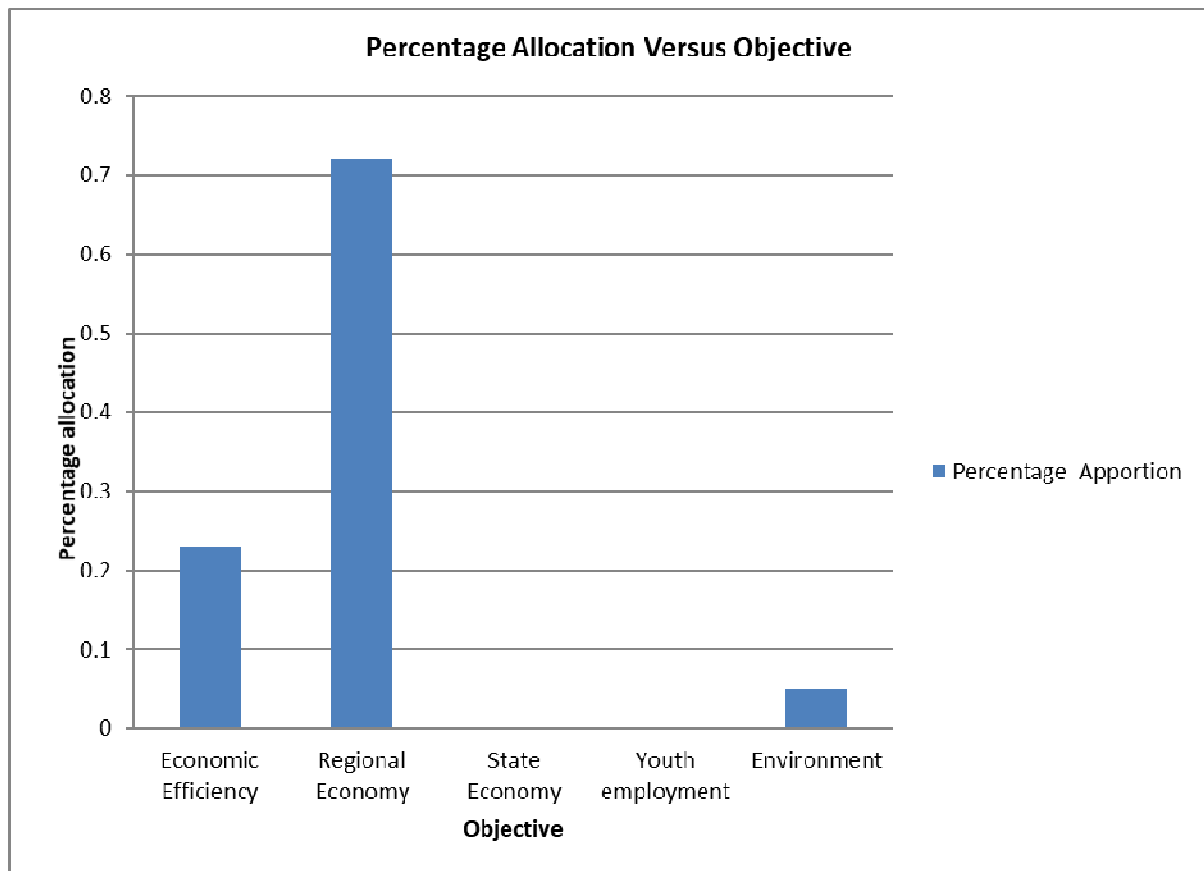


Figure 3: Funds Allocation to Objectives

Table 9: Percentage Allocation to the Purposes

Strategy For The Purposes	Percentage Allocation to the Purposes
Irrigation[Y1]	0
Hydro-electric Power[Y2]	0
Water Supply[Y3]	0.26
Recreation[X4]	0.16
Erosion Control[Y5]	0.58

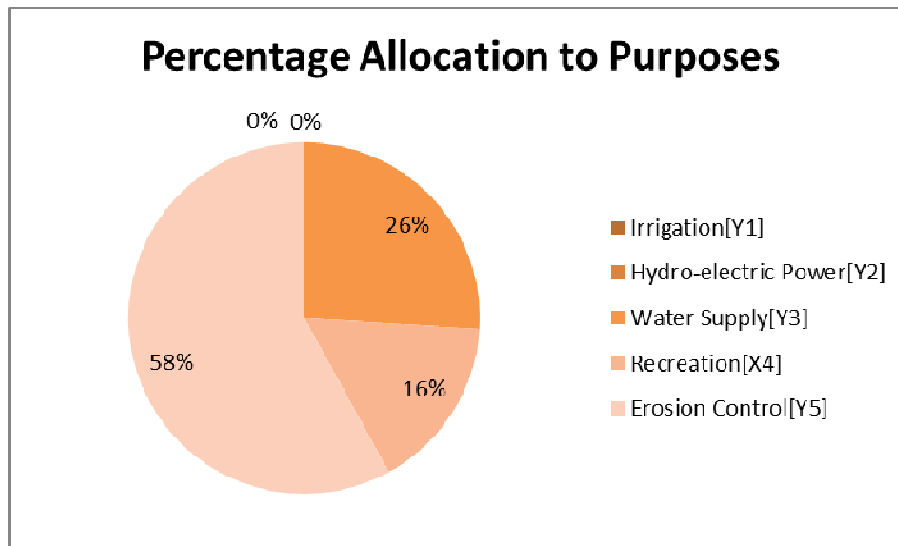


Figure 4: Percentage Allocation to Purposes

Table 10: Funds [N100Billion] Allocation to the Purpose

Strategy For The Purposes	Apportion
Irrigation[Y1]	N0.00 Million
Hydro-electric Power[Y2]	N0.00 Million
Water Supply[Y3]	N26.00 Million
Recreation[X4]	N16.0 Million
Erosion Control[Y5]	N58 Million

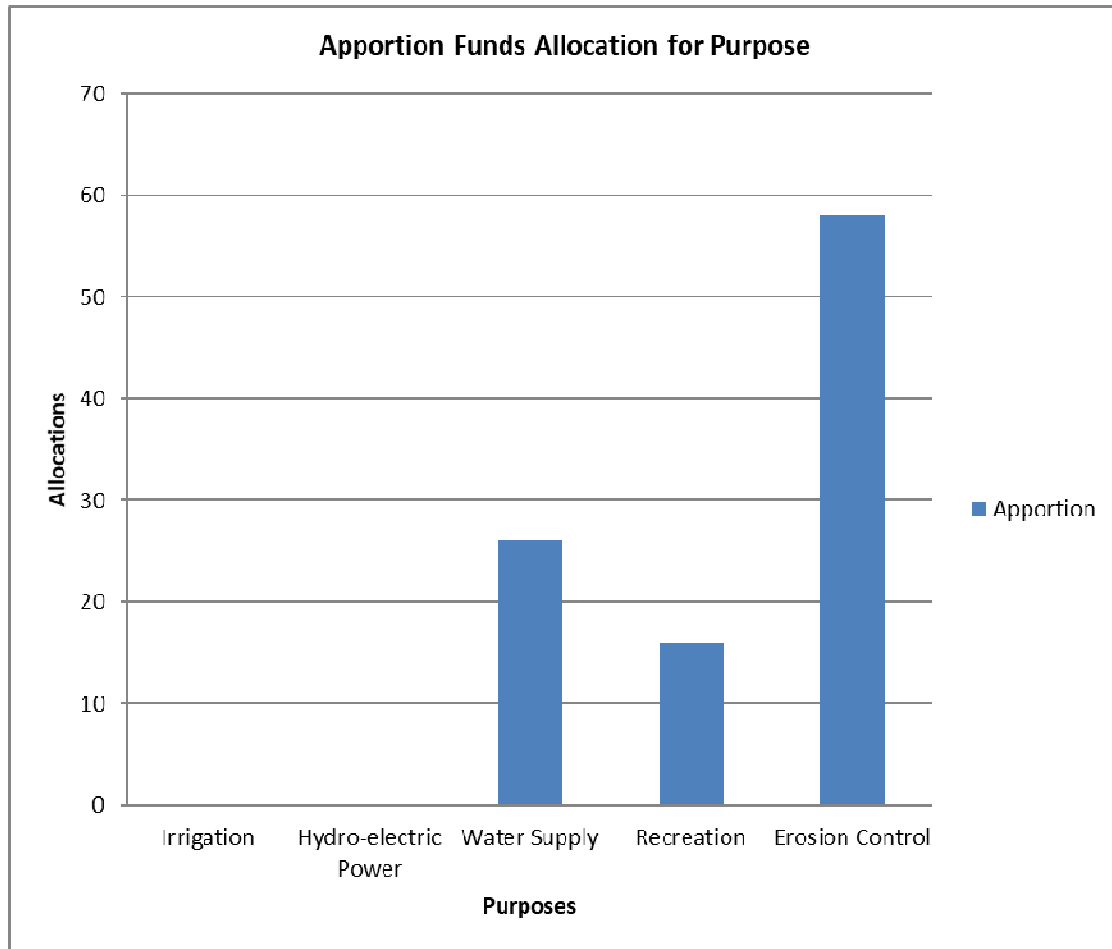


Figure 5: Funds Allocation Purposes

5.0 Discussion of Results

Apparently, from this research work, it is evident that game theory can be effectively applied in optimum policy decision making in multi-purpose/multi-objective water resources management. Such areas as exemplified in this paper are:

Integrated Water Resources Management (IWRM) using Game Theory Decision Modeling will achieve the expected or desired level of success in the Owena River Basin when applied. It serves as a useful tool for IWRM management. This kind of model can be used to allocate resources to both the purposes and the Objectives simultaneously. However, the analysis of the model generated the following result in allocating resources to the purposes and the objectives:

Funds allocated to Objectives in percentage are: Economic efficiency, Regional economy, State economy, Youth Employment and Environment are 23, 72, 0.00, 0.00, and 5% respectively.

Funds allocated to Purposes in percentages are: Irrigation, Hydroelectric Power, Water Supply, and Recreation and Erosion Control are: 0.0, 0.0, 0.26, 0.16, and 58%, respectively.

(2) Ultimately, one concludes that benefits allocation and cooperative game theory applications have been more common among water resource researchers than other applications and methods. This might be because bargaining and cost sharing in cooperative game theory is easily understandable by water engineers as the solutions are sometimes similar to solutions in optimization where the problem can be solved by having a single objective function, which tries to address the conflicting goals within the system, and a set of constraints. Nevertheless, the increasing number of game theory researches by water scholars in recent decades underscores the growing desire for application of this methodology in resolving water conflicts. However, there is still a lack of knowledge about the value of application of game theory in water resources management and many water scholars have not learned the basic concepts of game theory from the work published outside the water area.

5.1 Summary of Findings

The application of Game Decision theory on Benin-Owena River basin resulted to the following outputs:

-Funds allocated to Objectives in percentage are: Economic efficiency, Regional economy, State economy, Youth Employment and Environment are 23, 72, 0.00, 0.00, and 5% respectively. The allocations in monetary values are depicted in Table 6.

-Funds allocated to Purposes in percentages are: Irrigation, Hydroelectric Power, Water Supply, and Recreation and Erosion Control are: 0.0, 0.0, 0.26, 0.16, and 58%. Data Analysis:

-The Contingency of the raw data is = 0.6

-The attributes of the raw data = 0.2

-The X^2 value 53.89839 is interpreted from the X^2 table of probability values at 0.10 level of significance. The degree of freedom necessary to intercept X^2 values are always determined from the frequency table by the number of rows minus one times the number of columns minus one $(r-1)(c-1)$ i.e. $(5-1)(5-1) = 16$

-Since the obtained X^2 value of 53.89839 is greater than the critical value of 23.54, therefore reject null hypothesis i.e.: $X^2(53.89839) > X^2_{0.10}(23.54)$. Therefore the Null hypotheses is rejected, a clear indication that there is a relationship between the watershed purposes and the Objectives/Benefits. Therefore there is relationship between the state of the system (Dam Purposes) and the Dam Objectives.

-The Chi Square was not based on a fictitious data, in the case of Game Decision Modeling in River Valley Project.

-In the test of how well the linear estimator, $y=a + bx$ fits the raw data, the Pearson Moment Correlation Coefficient $(r) = 0.7193$ gives an indication of a good raw data. Good relationship of the river basin variables. Hence the equation is defined as $Y = 0.517x + 6.380$.

5.2 Conclusion

This study gave the indication that funds were only available for water supply, recreation and erosion control for the multipurpose, while funds are available for only economic efficiency and regional Economic Distribution for the multi-objective. In additional, to avoid conflict, the results suggest a need to design a mechanism to reduce the risk of losses of those players by a side payment, which provides them with economic incentives to cooperate. The application of Game Decision theory on Benin-Owena River basin resulted to the following outputs: -Funds allocated to Objectives in percentage are: Economic efficiency, Regional economy, State economy, Youth Employment and Environment are 23, 72, 0.00, 0.00, and 5% respectively. The allocations in monetary values are depicted in Table 6. -Funds allocated to Purposes in percentages are: Irrigation, Hydroelectric Power, Water Supply, and Recreation and Erosion Control are: 0.0, 0.0, 0.26, 0.16, and 58%.

5.3 Contribution to Knowledge

The study can provide an organized baseline for future work, mainly in obtaining superior estimates for institutional water use and planning conjunctive uses of water resources. However, the findings of the study can be vital input into the demand management process for long term sustainable water supply within Benin-Owena River Valley region and beyond.

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