Civil and Environmental Research www.iiste.org
ISSN 2224-5790 (Paper) ISSN 2225-0514 (Online)
Vol.11, No.9, 2019

Simple to Use Microsoft Excel Template for Estimating the Parameters of Some Selected Probability Distribution Model by Method of L-Moment

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Abstract
The focus of this research was to design a simple to use Microsoft excel algorithm that will aid in the estimation of the parameters of generalized extreme value probability distribution (GEV), generalized logistics probability distribution (GLO) and generalized pareto probability distribution (GPA), calculate the predicted rainfall/discharge based on L-moment procedures and compute the quantile estimates at various return periods.The algorithm was design based on the underlying mathematics of L-moment and has the capacity to handle forty (40) year's annual maximum series of either rainfall or discharge data which must first be ranked in ascending order of magnitude. Basic descriptive statistics such as the sample mean, variance, standard deviation, skewness, kurtosis, coefficient of variation have been built into the algorithm. Other exciting features include; the computation of Probability weighted moment parameters (b0, b1, b2 and b3), L-Moment values (ƛ1, ƛ2, ƛ3 and ƛ4)**,** L-Moment ratio values (Ʈ2, Ʈ3 and Ʈ4), and goodness of fit statistics (RRMSE, RMSE, MAE, MADI and PPCC). Others include; the shape (k), scale (α) and location (ξ) parameters of GEV, GPA and GLO probability distributions. To test the performance of the algorithm, forty (40) year's annual maximum rainfall data from Benin City was used. Basic time series analysis such as test of normality, test of homogeneity and outlier detection was conducted to ensure that the data used are adequate and suitable.Results obtained revealed that generalized logistics probability distribution GLO was the best fit distribution model for analyzing the annual maximum rainfall series at the study site. The predicted rainfall quantile magnitude (Qt) based on the GLO model ranges from 425.877mm at 2years return period to 762.759mm at 200years return period. The coefficient of determination (r^2) for the observed versus predicted rainfall based on the best fit model was observed to be 0.9793. It was thereafter concluded that L-moments and L –moment ratios are useful summary statistics for analyzing rainfall data.
Keywords: L-moments, probability distribution, normality test, goodness of fit statistics, coefficient of variation.
DOI: 10.7176/CER/11-9-05
Publication date:October 31st 2019

1.0 Introduction

Estimation of extreme flood discharge of known return period is paramount in the design of hydraulic structures such as culverts, dams, bridges and drainage systems. Owing to the stochastic nature of the hydrologic phenomena that governs extreme flood discharge, it is fundamental that we investigate most hydrologic processes such as rainfall and droughts by simply analyzing their records of observations (Ehiorobo & Izinyon, 2013). Effective analysis and determination of extreme flood discharge requires the use of statistical frequency analysis or fitting of probability distribution to the series of recorded annual maximum discharge (AMD) (Vivekanandan, 2015; Sharma & Singh, 2010). One of the widely used statistical frequency analysis methods is univariate frequency analysis technique. Univariate frequency analysis is widely used for analyzing hydrologic data, including rainfall characteristics, peak discharge series and low flow record of observations. Univariate frequency analyses are primarily employed in the estimation of exceedance probabilities and variable magnitudes. The basic assumption is that the data to be used must be satisfactorily homogeneous otherwise; the estimated probabilities or variable magnitude will be inaccurate (ECOST, 2012). The versatility of statistical frequency analysis makes it the most commonly used procedure for the analysis of flood data. Due to its wide application in the analysis of flood data, univariate statistical frequency analysis is sometime designated flood frequency analysis (FFA). A number of probability distributions such as Generalized Extreme Value (GEV), Generalized Pareto (GPA), Generalized Logistics (GLO), Gumbel distribution and Normal distribution are used in flood frequency analysis (Hosking & Wallis 1997). To employ any type of probability distribution for flood frequency analysis, the parameters of the distribution must first be estimated. Different types of probability distribution parameters estimation methods exist, namely; least square regression method (LSR), maximum likelihood estimation (MLE), method of moments (MOM) and method of L-moment. Of the four parameters estimation method, method of moments (MOM) and method of L-moment are widely used owing to their high level of sensitivity to rainfall and runoff data (Ahmad et al., 2011). Method of moments (MOM) has found a wide range of applicability in recent time based on it's used for the determination of parameters of different probability distributions. In some cases, it is always very difficult to assess the exact information about the shape of a distribution that is conveyed by its third and higher order moments (Landwehr, 1979). In addition, for small sample size, the numerical values of sample moments can be very different from those of the probability distribution from which the sample was drawn (Ehiorobo & Izinyon, 2013). On account of these limitations of MOM, alternative approach such as L-moments (LMO) was introduced to accurately estimate the parameters of probability distributions. L-Moment is a dramatic improvement over conventional product moment statistics for characterizing the shape of a probability distribution and estimating the distribution parameters, particularly for environmental data where sample sizes are commonly very small (Hosking, 1990; Izinyon & Ehiorobo, 2015).

Although, L-moment is more reliable compared to other conventional methods, but the underlying equations needed to obtain the L-moment parameters are usually very complex and requires an in-depth knowledge of mathematics. In addition, most of the tools (software) that can be employed in the determination of L-moment parameters such as L-RAP, DHI software are not only very expensive to acquire, they are also not user friendly. The implications are that so many researchers are not able to do much in this area of studies since they are either not able to purchase the needed software or are unable to solve the complex mathematical equations required to compute the L-moment parameters. The purpose of this research therefore, was to design an algorithm using Microsoft Excel that will help estimate the parameters of three probability distributions namely; Generalized Extreme Value (GEV), Generalized Logistics (GLO) and Generalized Pareto (GPA) using L-Moment procedure.

2.0 L-Moment Theory and Statistics

L-moments can be obtained by considering linear combinations of the observation in a sample of data that has been arranged in ascending order. Consider measurement of the shape of a distribution, given a small sample drawn from the distribution. Denote by $X_{1:1} \leq X_{2:n} \leq \ldots\ldots X_{n:n}$ (Hosking & Wallis, 1997; Eregno, 2014).

The basic steps in the determination of L-Moment statistics are described below;

Step One: Computation of probability weighted moments of distribution (pwms)

Probability weighted moments is needed for the calculation of L-moment. The data must first be ranked in ascending order of magnitude, thereafter; the following equations proposed by cunnane, 1989 can thus be applied

$$b_0 = \frac{1}{N}\sum_{j=1}^{n} X_{(j:n)} \quad (1)$$

$$b_1 = \frac{1}{N}\sum_{j=2}^{n} X_{(j:\mathrm{n})}\,[(j-1)/(n-1)] \quad (2)$$

$$b_2 = \frac{1}{N}\sum_{j=3}^{n} X_{(j:\mathrm{n})}\,[(\mathrm{j}-1)(\mathrm{j}-2)]/[(\mathrm{n}-1)(\mathrm{n}-2)] \quad (3)$$

$$b_3 = \frac{1}{N}\sum_{j=4}^{n} X_{(j:\mathrm{n})}\,[(\mathrm{j}-1)(\mathrm{j}-2)(\mathrm{j}-3)]/[(\mathrm{n}-1)(\mathrm{n}-2)(\mathrm{n}-3)] \quad (4)$$

Where;

$X_{(j)}$ represent the ranked annual maximum series in which $X_{(1)}$ is the smallest precipitation or stream flow data and $X_{(n)}$ is the largest. The parameters (b_0, b_1, b_2 and b_3) can easily be determined by using the developed Microsoft excel algorithm.

Step Two: Computation of L-Moment Values

L-moment values are easily calculated in terms of probability weighted moment (PWMs). In particular, the first four L-moment values are given as follows (Hosking & Wallis, 1997).

$$\lambda_1 = L_1 = b_0 \quad (5)$$

$$\lambda_2 = L_2 = (2b_1 - b_0) \quad (6)$$

$$\lambda_3 = L_3 = (6b_2 - 6b_1 + b_0) \quad (7)$$

$$\lambda_4 = L_4 = (20b_3 - 30b_2 + 12b_1 - b_0) \quad (8)$$

The parameters ($\lambda_1 = L_1$; $\lambda_2 = L_2$; $\lambda_3 = L_3$; $\lambda_4 = L_4$) can easily be determined by using the developed Microsoft excel algorithm that requires forty year's annual maximum monthly rainfall or discharge data.

Step Three: Computation of L-Moment Ratio

L-Moment ratio used for expressing the parameter estimates are as follows (Hosking and Wallis, 1997).

$$L-CV\ (\text{Coefficient of variability}) = (\tau_2) \quad (9)$$

$$L-\text{Skewness} = (\tau_3) \quad (10)$$

$$L\text{ - Kurtosis} = (\tau_4) \tag{11}$$

L-Cv is a dimensionless measure of variability. For a distribution or sample data that only has positive values, L-Cv is normally in the range of 0 < |L-Cv| < 1. Negative values of L-Cv are only possible if the at-site mean has a negative value (Sanjib, 2016; Herlina, 2015). The descriptions of the relative magnitude of variability is presented in Table 1

Table 1: Magnitude of L-Cv

Range of L-CV	Descriptions of Relative Magnitude of L-Cv
0.000 < | L-CV| < .025	minimal variability
.025 < | L-Cv | < .075	minor variability
.075 < | L-Cv | < .150	moderate variability
.150 < | L-Cv | < .400	large variability
.400 < | L-Cv |	very large variability

L-Skewness is a dimensionless measure of asymmetry, which may take on positive or negative values. For a distribution or sample data, L-skewness is in the range 0 < |L-Skewness| < 1 (CEH, 2001). The descriptions of the relative magnitude of asymmetry is presented in Table 2

Table 2: Relative magnitude of Asymmetry

Magnitude of L-skewness	Descriptions of Relative Magnitude of L-Skewness
L-skewness = 0.0	symmetrical distribution
0.000<|L – skewness | ≤0.050	minor skewness
0.050 <|L – Skewness | ≤0 .150	moderate skewness
0.150 < |L – Skewness| ≤0 .300	large skewness
0.300 < |L – Skewness|	very large skewness

L-kurtosis refers to any measure of the "peakedness" of the probability distribution of a real-valued random variable. The parameters (τ_2 τ_3 and τ_4) are computed using the formula below (Hosking & Wallis, 1997; Gubareva & Gartsman, 2010).

$$\tau_2 = \frac{\lambda_2}{\lambda_1} = \frac{L_2}{L_1} \tag{12}$$

$$\tau_3 = \frac{\lambda_3}{\lambda_2} = \frac{L_3}{L_2} \tag{13}$$

$$\tau_4 = \frac{\lambda_4}{\lambda_3} = \frac{L_4}{L_3} \tag{14}$$

The parameters (τ_2 τ_3 and τ_4) can easily be determined by using the developed Microsoft excel algorithm that requires forty year's annual maximum monthly rainfall or discharge data.

2.1 Advantages of L-Moment

The main advantage of L-moment over conventional moments is that L-moments, being linear functions of the data, suffer less from the effects of sampling variability and are more robust compared to conventional moments in handling outliers. In addition (Hosking & Wallis, 1997). Some of the underlying simplicity of L-Moments are;

i. L-moment is based on linear combination of data that have been arranged in ascending order of magnitude. It provides an advantage as it is easier to work with, and more reliable since it is less sensitive to outliers.
ii. The method of L-moment calculates more accurate parameter than method of moment (MOM) technique especially for smaller sample size.
iii. MOM techniques only apply to limited range of parameters, whereas L-moment can be more widely used, and are also nearly unbiased

iv. L- Moment allow for the generation of ratio diagrams which are helpful in identifying the distribution properties of highly skewed data

v. L-Moment is a dramatic improvement over conventional product moment statistics for characterizing the shape of a probability distribution and estimating the distribution parameters, particularly for environmental data where sample sizes are commonly very small

vi. In practice, an in-depth knowledge and accurate estimation of the L-moment ratios of L-Cv (τ_2), L-Skewness (τ_3) and L-kurtosis (τ_4) is a key determinant in quantifying the success of the regional frequency analysis in computing quantile estimates for selected stations (Hosking & Wallis, 1997).

3.0: Development and Execution of the Algorithm

The Microsoft Excel algorithm was design to capture three probability distributions, namely; Generalized Extreme Value (GEV), Generalized Logistics (GLO) and Generalized Pareto (GPA). The GEV distribution is a flexible distribution which has been found to fit flood and rainfall extremes in a variety of environments. The underlying equations including the quantile function (x_p) corresponding to the non-exceedance probability (p) and the return period (T) corresponding to the non-exceedance probability (p) have been incorporated into the algorithm. The probability density functions and or cumulative distribution functions, range and moments for the selected distributions captured by the algorithm are presented in Table 3 and 4 respectively while the goodness of fit statistics captured by the algorithm is presented in Table 5

Table 3: Probability distribution model captured by the algorithm

Distribution	Parameters/Range	Probability density function f(x)	Cumulative distribution function F(x)	Quantile function (x_p)
Generalized Extreme Value (GEV)	**Parameters:** ξ (location), α (scale), k(shape) **Range:** α > 0, ξ + α/k ≤ x < ∞ for k < 0, -∞ ≤ x ≤ ξ + α/k for k > 0	$f(x)=\frac{1}{\alpha}\left[1-\frac{k(x-\xi)}{\alpha}\right]^{1/k-1}\exp\left\langle-\left[1-\frac{k(x-\xi)}{\alpha}\right]^{1/k}\right\rangle$	$F(x)=\exp\left(-\left[1-\frac{k(x-\xi)}{\alpha}\right]^{1/k}\right)$	$x_p=\xi+\frac{\alpha}{k}(1-[-\ln(F)]^k)$
Generalized Pareto (GPA)	**Parameters:** ξ (location), α (scale), k(shape) **Range:** α > 0, ξ ≤ x < ∞ for k < 0, ξ ≤ x ≤ ξ + α/k for k > 0	$f(x)=\frac{1}{\alpha}\left[1-k\frac{x-\xi}{\alpha}\right]^{1/k-1}$	$F(x)=1-\left[1-k\frac{x-\xi}{\alpha}\right]^{1/k}$	$x_p=\xi+\frac{\alpha}{k}\left[1-(1-F)^k\right]$
Generalized Logistics (GLO)	**Parameters:** ξ (location), α (scale), k(shape) **Range:** α > 0, ξ + α/k ≤ x < ∞ for k < 0, -∞ ≤ x ≤ ξ + α/k for k > 0	$\gamma=[1-\frac{K(x-\xi)}{\alpha}]^{\frac{1}{k}}$ $f_x=(\frac{1}{\alpha})[\frac{\gamma^{(1-x)}}{(1+\gamma)}]^2$ $for\ k\neq 0$	$F_x(x)=\frac{1}{1+\gamma}$	$x_p=\xi+\frac{\alpha}{k}\left[1-\left(\frac{1-F}{F}\right)^k\right]$

Table 4: L-Moment parameter estimates for probability distributions captured by the algorithm

Distribution	Quantile function (x_p)	L-Moment Parameter Estimates Equations
Generalized Extreme Value (GEV)	$x_p = \xi + \frac{\alpha}{k}(1-[-\ln(F)]^k)$	$\alpha = \frac{l_2 K}{\Gamma(1+K)\Gamma(1-2^{-K})}$ $\xi = l_1 + \frac{\alpha(\Gamma(1+K)-1)}{K}$ $K = 7.8590C + 2.9554C^2$ $C = \frac{2}{3+\tau_3} = \frac{\ln 2}{\ln 3}$
Generalized Pareto (GPA)	$x_p = \xi + \frac{\alpha}{k}\left[1-(1-F)^k\right]$	$\alpha = l_2[(K+l)(K+2)]$ $\xi = l_1 = l_2(K+2)$ $K = \frac{(1-3\tau_3)}{(1+\tau_3)}$
Generalized Logistics (GLO)	$x_p = \xi + \frac{\alpha}{k}\left[1-\left(\frac{1-F}{F}\right)^k\right]$	$\alpha = \frac{l_2}{\Gamma(1+K)\Gamma(1-K)}$ $\xi = l_1 + \frac{(l_2-\alpha)}{K}$ $K = -\tau_3$

Table 5: Goodness of fit statistics captured by the algorithm

Statistics	Equation
Root Mean Square Error (RMSE)	$RMSE = \left(\frac{\sum(x_i - y_i)^2}{n-m}\right)^{1/2}$
Relative Root Means Square Error (RRMSE)	$RRMSE = \left(\frac{\sum\left(\frac{x_i - y_i}{x_i}\right)^2}{n-m}\right)^{1/2}$
Mean Absolute Deviation Index	$MADI = \frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{x_i - y_i}{x_i}\right\rvert$
Maximum Absolute Error	MAE = max $(\lvert x_i - y_i\rvert)$
Probability Plot Correlation Coefficient	PPCC = $\frac{\sum[(x_i-\bar{x})(y_i-\bar{y})]}{\left[\sum(x_i-\bar{x})^2\sum(y_i-\bar{y})^2\right]^{1/2}}$

Root mean square error (RMSE), relative root means square error (RRMSE) and maximum absolute deviation index (MADI) were selected since they can adequately assess the fitted distribution at a site. They possess an added advantage of being able to summarize the deviation between observed precipitation and predicted precipitation. In addition, RRMSE can also provide a better picture of the overall fit of a distribution as it calculates each error in proportion to the size of observation thus helping to eliminate or reduce the effects of bias commonly associated with hydrological data (Tao *et al*, 2008).

To develop the excel algorithm, the underlying mathematics of L-moment were employed. The complex equations were first digested and rewritten in a more simplified format that is usable by Microsoft Excel. Two algorithms were developed, namely; **algorithm one** and **algorithm two**. Algorithm one uses the ranked discharge or precipitation data (Xi) (40 years annual maximum data) as the input data to calculates the following parameters:

i. Variance
ii. Standard deviation
iii. Skewness
iv. Kurtosis
v. Coefficient of variation (CV)
vi. Probability weighed moment's parameters (b0, b1, b2 and b3)
vii. L-Moment values (ƛ1, ƛ2, ƛ3 and ƛ4)
viii. L-Moment ratio values (Ʈ2, Ʈ3 and Ʈ4)
ix. Shape parameter (k) of GEV, GPA and GLO probability distribution
x. Scale parameter (α) of GEV, GPA and GLO probability distribution
xi. Location parameter (ξ) of GEV, GPA and GLO probability distribution

Algorithm two uses the 40 years precipitation or discharge data including the calculated shape parameter (k), scale parameter (α) and location parameter (ξ) from algorithm one as input data to calculates the following parameters:

i. The predicted rainfall or discharge values (yi)
ii. The goodness of fit statistics (RMSE, RRMSE, MADI, MAE and PPCC)
iii. Quantile estimates at various return periods
iv. Exceedance and non-Exceedance probabilities

3.1: Computation of L-moment parameters Using Algorithm One

The following steps are involved in using algorithm one to compute the parameters of GEV, GLO and GPA probability distributions.

i. The algorithm is available on request. Table 6a and 6b shows a section of the algorithm
ii. The algorithm requires 40 years annual maximum data (precipitation or discharge) which must be ranked in ascending order of magnitude. The data must be prepared in excel, copy and paste on cell B2 – B41 of Algorithm one
iii. Descriptive statistics of the data presented to the algorithm will be gotten immediately from cell D4 – D10
iv. The probability weighted moment statistics (b_0, b_1, b_2 and b_3) will be gotten immediately from cell I10 - I13
v. The L-moment values ($\lambda_1 = L_1$; $\lambda_2 = L_2$; $\lambda_3 = L_3$; $\lambda_4 = L_4$) will be gotten immediately from cell K10 - K13
vi. The L-moment ratio values (τ_2 τ_3 and τ_4) will be gotten immediately from cell M10 – M12
vii. The parameters of the generalized extreme value probability distribution (GEV) which include; Shape parameter (k), Scale parameter (α) and Location parameter (ξ) will immediately be computed and presented in cell O3, O5 and O6
viii. The parameters of the generalized logistics probability distribution (GLO) which include; Shape parameter (k), Scale parameter (α) and Location parameter (ξ) will immediately be computed and presented in cell O16, O18 and O17
ix. The parameters of the generalized pareto probability distribution (GPA) which include; Shape parameter (k), Scale parameter (α) and Location parameter (ξ) will immediately be computed and presented in cell O26, O28 and O27

3.2: Predicted values and Quantile Estimates based on selected Return Period Using Algorithm Two

The following steps are involved in using algorithm two to determine the quantile estimates for selected return periods based on the probability distribution.

i. The algorithm is available on request. Table 7a and 7b shows a section of the algorithm
ii. The same 40 years annual maximum data (precipitation or discharge) used in Algorithm one is also needed to run Algorithm two. Go to cell B2 – B41 of Algorithm one where you have your 40 years annual maximum data (precipitation or discharge) copy and paste on cell E2 – E41 of GEV, GLO and GPA in Algorithm two
iii. Copy the computed parameters (Shape parameter (k), Scale parameter (α) and Location parameter (ξ)) of each distribution from algorithm one. Replicate them to 40 years data and paste in cell F2 – F41, G2 – G41 and H2 –H41 of GEV, GLO and GPA in Algorithm two
iv. The predicted discharge or precipitation values of generalized extreme value (GEV) will be gotten from

cell N2 – N41 of Algorithm two; that of generalized logistics (GLO) will be gotten from cell O2 – O41 of Algorithm two and that of generalized pareto (GPA) will be gotten from cell N2 – N41 of algorithm two.

v. The goodness of fit statistics for selecting the probability distribution that best fit your data, namely; root mean square error (RMSE), relative root mean square error (RRMSE) and maximum absolute deviation index (MADI)) will be gotten from cell R3, V3 and X3 of algorithm two for generalized extreme value distribution (GEV), cell S3, W3 and Y3 of algorithm two for generalized logistics distribution (GLO) and cell R3, W3 and Y3 of algorithm two for generalized pareto distribution (GPA)

vi. Cell AE2 to AE11 of Algorithm two gives the computed quantile estimates for the generalized extreme value distribution (GEV), generalized logistics distribution (GLO) and generalized Pareto distribution (GPA).

vii. The cumulative probability of non-exceedance is also computed in addition to the graphical visualization of your data with respect to GEV, GLO and GPA probability distribution.

Table 6a: Sectional view of algorithm one

1	Rank (j)	Discharge/Precipitation (Xj)		b0 = Mean	b1	b2	b3	PWMs Parameters	PWMs Statistics
2	1	222.3			j = 2 [(j-1)(Xj)]	j = 3	j = 4		
3	2	235.2			=(A3-1)*B3	(j-1)(j-2)(Xj)			
4	3	305.2	n	=A41	=(A4-1)*B4	=(A4-1)*(A4-2)*(B4)	(j-1)(j-2)(j-3)(Xj)		
5	4	338.4	Mean	=AVERAGE(B2:B41)	=(A5-1)*B5	=(A5-1)*(A5-2)*(B5)	=(A5-1)*(A5-2)*(A5-3)*(B5)	b1-(SUM)	=SUM(E3:E41)
6	5	342.9	Variance	=VARA(B2:B41)	=(A6-1)*B6	=(A6-1)*(A6-2)*(B6)	=(A6-1)*(A6-2)*(A6-3)*(B6)	b2-(SUM)	=SUM(F4:F41)
7	6	351.2	Standard Deviation	=STDEV.S(B2:B41)	=(A7-1)*B7	=(A7-1)*(A7-2)*(B7)	=(A7-1)*(A7-2)*(A7-3)*(B7)	b3-(SUM)	=SUM(G5:G41)
8	7	357.1	Skewness	=SKEW(B2:B41)	=(A8-1)*B8	=(A8-1)*(A8-2)*(B8)	=(A8-1)*(A8-2)*(A8-3)*(B8)		
9	8	391	Kurtosis	=KURT(B2:B41)	=(A9-1)*B9	=(A9-1)*(A9-2)*(B9)	=(A9-1)*(A9-2)*(A9-3)*(B9)		
10	9	393.3	CV = [(mean)/(S.d)]	=(D7/D5)	=(A10-1)*B10	=(A10-1)*(A10-2)*(B10)	=(A10-1)*(A10-2)*(A10-3)*(B10)	b0	=(AVERAGE(B2:B41))
11	10	394.8			=(A11-1)*B11	=(A11-1)*(A11-2)*(B11)	=(A11-1)*(A11-2)*(A11-3)*(B11)	b1	=((I5)/(D4*(D4-1)))
12	11	403			=(A12-1)*B12	=(A12-1)*(A12-2)*(B12)	=(A12-1)*(A12-2)*(A12-3)*(B12)	b2	=((I6)/((D4*(D4-1)*(D4-2)))
13	12	413.6			=(A13-1)*B13	=(A13-1)*(A13-2)*(B13)	=(A13-1)*(A13-2)*(A13-3)*(B13)	b3	=((I7)/((D4)*(D4-1)*(D4-2)*(D4-3)
14	13	425.2			=(A14-1)*B14	=(A14-1)*(A14-2)*(B14)	=(A14-1)*(A14-2)*(A14-3)*(B14)		
15	14	427.8			=(A15-1)*B15	=(A15-1)*(A15-2)*(B15)	=(A15-1)*(A15-2)*(A15-3)*(B15)		
16	15	433.9			=(A16-1)*B16	=(A16-1)*(A16-2)*(B16)	=(A16-1)*(A16-2)*(A16-3)*(B16)		
17	16	445.4			=(A17-1)*B17	=(A17-1)*(A17-2)*(B17)	=(A17-1)*(A17-2)*(A17-3)*(B17)		
18	17	453.1			=(A18-1)*B18	=(A18-1)*(A18-2)*(B18)	=(A18-1)*(A18-2)*(A18-3)*(B18)		
19	18	454			=(A19-1)*B19	=(A19-1)*(A19-2)*(B19)	=(A19-1)*(A19-2)*(A19-3)*(B19)		
20	19	454.5			=(A20-1)*B20	=(A20-1)*(A20-2)*(B20)	=(A20-1)*(A20-2)*(A20-3)*(B20)		
21	20	455.4			=(A21-1)*B21	=(A21-1)*(A21-2)*(B21)	=(A21-1)*(A21-2)*(A21-3)*(B21)		
22	21	458.8			=(A22-1)*B22	=(A22-1)*(A22-2)*(B22)	=(A22-1)*(A22-2)*(A22-3)*(B22)		
23	22	461.4			=(A23-1)*B23	=(A23-1)*(A23-2)*(B23)	=(A23-1)*(A23-2)*(A23-3)*(B23)		
24	23	462.4			=(A24-1)*B24	=(A24-1)*(A24-2)*(B24)	=(A24-1)*(A24-2)*(A24-3)*(B24)		
25	24	462.5			=(A25-1)*B25	=(A25-1)*(A25-2)*(B25)	=(A25-1)*(A25-2)*(A25-3)*(B25)		
26	25	467.8			=(A26-1)*B26	=(A26-1)*(A26-2)*(B26)	=(A26-1)*(A26-2)*(A26-3)*(B26)		
27	26	472.5			=(A27-1)*B27	=(A27-1)*(A27-2)*(B27)	=(A27-1)*(A27-2)*(A27-3)*(B27)		
28	27	476			=(A28-1)*B28	=(A28-1)*(A28-2)*(B28)	=(A28-1)*(A28-2)*(A28-3)*(B28)		

3.3

Table 6b: Sectional view of algorithm one

L-Moment Equation	L - Moment Values	L-Moment Ratios	L-Moment Ratios Value	PARAMETERS ESTIMATION	
				(1): GENERALIZED EXTREME VALUE (GEV)	
				K = 7.8590C + 2.9554C^2 (parameter 1)	=(7.859*O4)+(2.9554*O4^2)
				C = [(2/(3+τ3)) - (Ln2/Ln3)]	=((2/O9)-(O7))
				α = [(λ2K)/((1 - 2^{-K}) Γ(1 + K))] (parameter 2)	=((O12)/((O11)*(O10)*(O8)))
				ξ = λ1 -[α(1 - Γ(1 + K)]/K] (parameter 3)	=((K10)-((O5/O3)*(1-((O10)*(O8)))))
				(Ln2/Ln3)	=(LN(2)/LN(3))
				(1 + K)	=(1+O3)
				(3 + τ3)	=(3+M11)
λ1 = L1 = b0	=(I10)	L-CV (τ2)= (λ2/λ1)	=(K11/K10)	Γ	1.011218862
λ2 = L2 = 2b1 - bo	=((2*I11)-(I10))	L-Skewness (τ3) = (λ3/λ2)	=(K12/K11)	(1 - 2^{-K})	=(1-2^-O3)
λ3 = L3 = 6b2 - 6b1 + b0	=((6*I12)-(6*I11)+(I10))	L- Kurtosis (τ4) = (λ4/λ2)	=(K13/K11)	(λ2*K)	=(K11*O3)
λ4 = L4 = 20b3 - 30b2 + 12b1 - b0	=((20*I13)-(30*I12)+(12*I11)-(I10))				
				(2): GENERALIZED LOGISTICS (GLO)	
				K = -τ3 (parameter 1)	=(-1*M11)
				ξ = L1 + [(L2 - α)/K] (parameter 2)	=((K10)+(O22/O16))
				α = [(L2)/Γ(1 + K)*Γ(1 - K)] (parameter 3)	=((K11)/(O19*O20))
				Γ(1 + K)	=(O21*(1+O16))
				Γ(1 - K)	=(O21*(1-O16))
				Γ	1.011218862
				(L2 - α)	=(K11-O18)
				(3): GENERALIZED PARETO (GPA)	
				K = [(1 - 3τ3)/(1 + τ3)] (parameter 1)	=((1-O30)/(O29))
				ξ = L1 - (L2(K + 2)) (parameter 2)	=(K10-((K11*O32)))
				α = L2 [(K + 1)(K+2)] (parameter 3)	=((K11)*(O31*O32))

Table 7a: Sectional view of algorithm two

1	m	n	(n+1)	F = m/(n+1)	Observed (Xi)	K	ξ	α	[1 - F]	[(1 - F)^k]	[1 - (1 - F)^k]	[(α/k)]
2	1	40	41	0.024390244	222.3	**0.865993717**	**291.8408521**	**316.7041936**	0.975609756	0.978843362	0.021156638	365.7118839
3	2	40	41	0.048780488	235.2	**0.865993717**	**291.8408521**	**316.7041936**	0.951219512	0.957615719	0.042384281	365.7118839
4	3	40	41	0.073170732	305.2	**0.865993717**	**291.8408521**	**316.7041936**	0.926829268	0.936315002	0.063684998	365.7118839
5	4	40	41	0.097560976	338.4	**0.865993717**	**291.8408521**	**316.7041936**	0.902439024	0.914939028	0.085060972	365.7118839
6	5	40	41	0.12195122	342.9	**0.865993717**	**291.8408521**	**316.7041936**	0.87804878	0.893485483	0.106514517	365.7118839
7	6	40	41	0.146341463	351.2	**0.865993717**	**291.8408521**	**316.7041936**	0.853658537	0.87195192	0.12804808	365.7118839
8	7	40	41	0.170731707	357.1	**0.865993717**	**291.8408521**	**316.7041936**	0.829268293	0.85033574	0.14966426	365.7118839
9	8	40	41	0.195121951	391	**0.865993717**	**291.8408521**	**316.7041936**	0.804878049	0.828634182	0.171365818	365.7118839
10	9	40	41	0.219512195	393.3	**0.865993717**	**291.8408521**	**316.7041936**	0.780487805	0.806844304	0.193155696	365.7118839
11	10	40	41	0.243902439	394.8	**0.865993717**	**291.8408521**	**316.7041936**	0.756097561	0.78496297	0.21503703	365.7118839
12	11	40	41	0.268292683	403	**0.865993717**	**291.8408521**	**316.7041936**	0.731707317	0.762986825	0.237013175	365.7118839
13	12	40	41	0.292682927	413.6	**0.865993717**	**291.8408521**	**316.7041936**	0.707317073	0.740912277	0.259087723	365.7118839
14	13	40	41	0.317073171	425.2	**0.865993717**	**291.8408521**	**316.7041936**	0.682926829	0.718735466	0.281264534	365.7118839
15	14	40	41	0.341463415	427.8	**0.865993717**	**291.8408521**	**316.7041936**	0.658536585	0.69645224	0.30354776	365.7118839
16	15	40	41	0.365853659	433.9	**0.865993717**	**291.8408521**	**316.7041936**	0.634146341	0.674058116	0.325941884	365.7118839
17	16	40	41	0.390243902	445.4	**0.865993717**	**291.8408521**	**316.7041936**	0.609756098	0.651548242	0.348451758	365.7118839
18	17	40	41	0.414634146	453.1	**0.865993717**	**291.8408521**	**316.7041936**	0.585365854	0.628917351	0.371082649	365.7118839
19	18	40	41	0.43902439	454	**0.865993717**	**291.8408521**	**316.7041936**	0.56097561	0.606159706	0.393840294	365.7118839
20	19	40	41	0.463414634	454.5	**0.865993717**	**291.8408521**	**316.7041936**	0.536585366	0.583269034	0.416730966	365.7118839
21	20	40	41	0.487804878	455.4	**0.865993717**	**291.8408521**	**316.7041936**	0.512195122	0.560238454	0.439761546	365.7118839
22	21	40	41	0.512195122	458.8	**0.865993717**	**291.8408521**	**316.7041936**	0.487804878	0.53706038	0.46293962	365.7118839
23	22	40	41	0.536585366	461.4	**0.865993717**	**291.8408521**	**316.7041936**	0.463414634	0.513726415	0.486273585	365.7118839
24	23	40	41	0.56097561	462.4	**0.865993717**	**291.8408521**	**316.7041936**	0.43902439	0.490227213	0.509772787	365.7118839
25	24	40	41	0.585365854	462.5	**0.865993717**	**291.8408521**	**316.7041936**	0.414634146	0.466552314	0.533447686	365.7118839
26	25	40	41	0.609756098	467.8	**0.865993717**	**291.8408521**	**316.7041936**	0.390243902	0.44268994	0.55731006	365.7118839
27	26	40	41	0.634146341	472.5	**0.865993717**	**291.8408521**	**316.7041936**	0.365853659	0.418626729	0.581373271	365.7118839
28	27	40	41	0.658536585	476	**0.865993717**	**291.8408521**	**316.7041936**	0.341463415	0.394347408	0.605652592	365.7118839

Table 7a: Sectional view of algorithm two

R = α/k* [1 - (1 - F)^k]	(yi) Predicted = [ξ + R]	[n - m]	[(Xi - yi)^2]	Sum[(xi - yi)^2]	RMSE	[(xi - yi)]	[(xi - yi)/xi]	[(xi - yi)/xi]^2
7.737233992	299.5780861	39	5971.902587	36486.00901	986.1083516	-77.27808607	-0.347629717	0.12084642
15.50043529	307.3412874	38	5204.365344	n = 40	31.4023622	-72.14128738	-0.306723161	0.094079097
23.29036042	315.1312125	37	98.62898175	m = 3 (number of parameters estimated)		-9.931212501	-0.032540015	0.001058853
31.10780846	322.9486605	36	238.743891	n - m = 37		15.45133946	0.045659987	0.002084834
38.95362469	330.7944768	35	146.5436925	37		12.10552322	0.035303363	0.001246327
46.82870462	338.6695567	34	157.0120093			12.5304433	0.035678939	0.001272987
54.73399845	346.5748505	33	110.7787713			10.52514947	0.029473955	0.000868714
62.6705162	354.5113683	32	1331.420245			36.48863172	0.093321309	0.008708867
70.63933334	362.4801854	31	949.8609704			30.81981457	0.078362102	0.006140619
78.64159727	370.4824493	30	591.3432696			24.31755065	0.061594607	0.003793896
86.67853455	378.5193866	29	599.3004308			24.48061337	0.060745939	0.003690069
94.75145927	386.5923113	28	729.4152463			27.00768865	0.065299054	0.004263966
102.8617825	394.7026345	27	930.0892998			30.49736546	0.071724754	0.00514444
111.011023	402.8518751	26	622.4089344			24.94812487	0.058317262	0.003400903
119.2008204	411.0416725	25	522.5031381			22.85832754	0.052681096	0.002775298
127.4329488	419.2738008	24	682.5782821			26.12619915	0.058657834	0.003440741
135.7093345	427.5501866	23	652.7929634			25.54981337	0.056388906	0.003179709
144.0320759	435.872928	22	328.5907392			18.127072	0.039927471	0.001594203
152.4034665	444.2443186	21	105.1790006			10.25568138	0.022564756	0.000509168
160.8260234	452.6668755	20	7.469969612			2.733124515	0.006001591	3.60191E-05
169.3025204	461.1433725	19	5.491394627			-2.34337249	-0.005107612	2.60877E-05
177.8360288	469.6768808	18	68.50675632			-8.276880833	-0.017938623	0.000321794
186.4299662	478.2708183	17	251.8828744			-15.87081833	-0.034322704	0.001178048
195.088158	486.9290101	16	596.776534			-24.42901009	-0.052819481	0.002789898
203.814912	495.6557641	15	775.9435921			-27.85576407	-0.059546311	0.003545763
212.6151142	504.4559663	14	1021.18378			-31.95596627	-0.067631675	0.004574043
221.4943502	513.3352023	13	1393.917333			-37.33520233	-0.078435299	0.006152096

4.0: Application of the Algorithms for modeling Rainfall Data

4.1: Description of study area

Benin City, the capital of Edo State, Nigeria, the study area, is located in the rain forest zone, with geographical coordinates of latitudes 6º 17^1 N, 6º 26^1 N and longitudes 5º 35^1 E, with an annual mean temperature of 27.5ºC (Ikhuoria, 1987). It has two main seasons, wet and dry; from April to November and November to April respectively, with an annual mean rainfall of about 2095mm. In matters of hydrogeology, Benin City lies on the Benin Formation, with an aquifer of mean dept of 136m. Schools, hospitals, markets and cemeteries are among the social services provided in the City. Figure 1 shows the digitized map of the study area

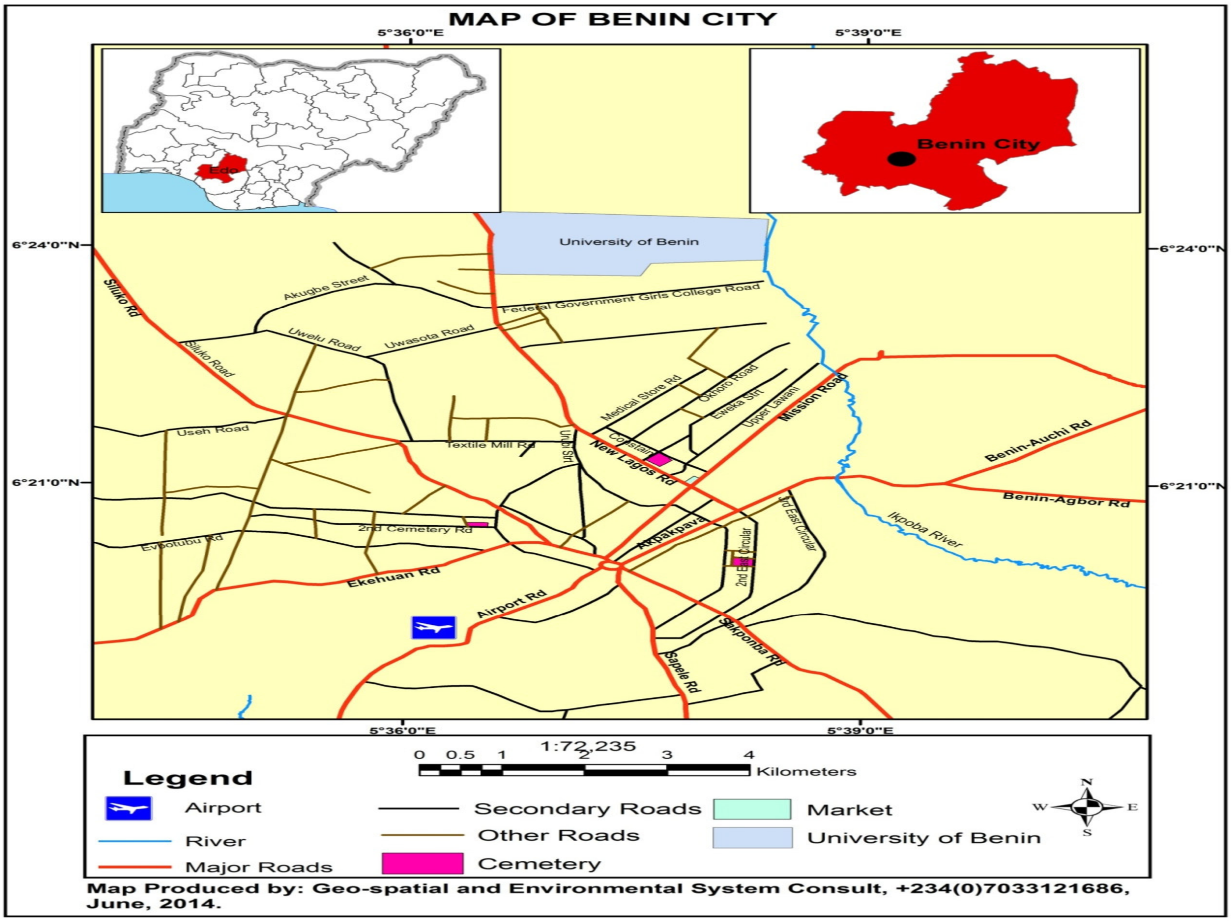

Figure 1: Map of Benin City

4.2: Data collection

The data used for this study was collected from the Nigerian Meteorological Agency, Oshodi; Lagos State, Nigeria. The data includes monthly precipitation data for 40 years spanning between; 1974 to 2013. The data were then sorted to obtain the annual maximum precipitation records.

5.0: Results and Discussion

The time series plot of the data presented in Figure 2 shows the presence of seasonal variability since rainfall depth varies within the period understudy as some years experienced extreme precipitation compared to others.

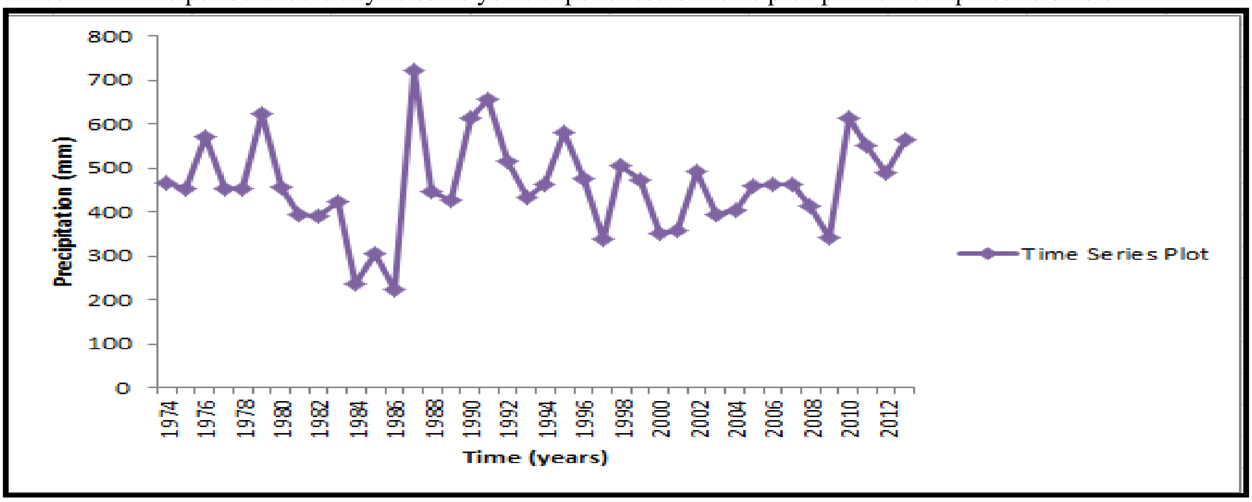

Figure 2: Time series plot

On whether the rainfall data used in this study are from the same population distribution, homogeneity test was performed using hydrological software (RAINBOW). Result of the test presented in Figure 3 shows that the data used is homogeneous.

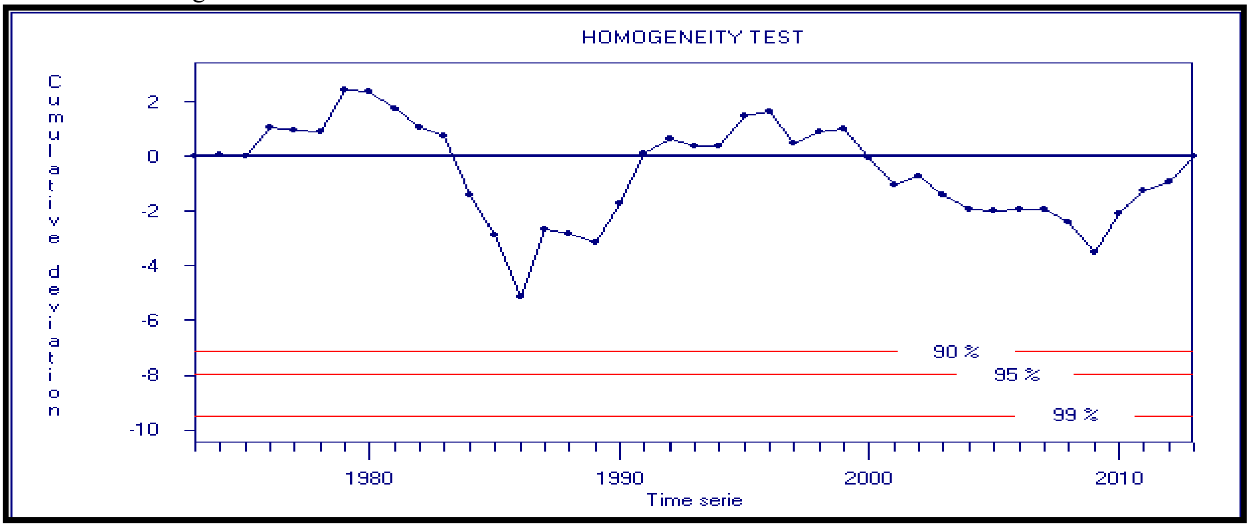

Figure 3: Homogeneity test of data

For a homogeneous record, the rainfall data points normally fluctuate around the zero-center point in the residual mass curve as observed in Figure 3. The descriptive statistics of the rainfall data and the computed probability weighted moment statistics (b0, b1, b2 and b3), the L-moment values (λ_1, λ_2, λ_3 and λ_4) and the L-moment ratio values (τ_2, τ_3 and τ_4) were obtained from **algorithm one** and presented in Table 8

Table 8: Descriptive and L-moment statistics obtained from algorithm one

Probability Weighted Moment Values	L-Moment Values	L-Moment Ratios	Basic Statistics
b_0 = 461.565 b_1 = 260.3925 b_2 = 183.807 b_3 = 143.190	λ_1 = 461.565 λ_2 = 59.220 λ_3 = 2.0527 λ_4 = 12.739	L-CV = 0.128 L-Skewness = 0.0347 L-Kurtosis = 0.2151	n = 40 Mean = 461.565 Variance = 11280.1 Standard Dev. = 106.208 C.V = 0.230

The L-moments λ_1 and λ_2, their ratio ($\tau=\frac{\lambda_2}{\lambda_1}$) termed L-CV, and L-moment ratios λ_3 and λ_4 are the most useful quantities for summarizing probability distribution (Maleki-Nezhad, 2006). The value of λ_1 (L- mean) is a measure of central tendency; λ_2 (L- standard deviation) is a measure of dispersion and L-CV (λ) is the coefficient of L-variation. L-Skewness (λ_3) measures whether the distribution is symmetric with respect to the dispersion from the mean and L-kurtosis (λ_4) refers to the weight of the tail of a distribution. The values of λ_3 and λ_4 are constrained to be between -1 and +1 and λ_4 is constrained by λ_3 to be no lower than -0.25 (Eslamian and Feizi, (2007). The parameters of location (ξ), scale (α) and shape (k) of the selected distributions estimated using **algorithm one** is presented in Table 9

Table 9: Probability parameter estimate based on L-moment using algorithm one

Probability Distribution Model	Shape Parameter (k)	Scale Parameter (α)	Location Parameter (ξ)
GEV	0.223348438	74.54719112	540.6930545
GLO	-0.03466283	57.98293611	425.8765149
GPA	0.865993717	316.7041936	291.8408521

The estimated parameters for the different distributions were applied to the relevant quantile function given in Table 3. The predicted annual maximum precipitation records based on L-moment using the three-probability distribution model, namely; GEV, GLO and GPA was obtained from **algorithm two** and presented in Table 10

Table 10: Observed and predicted annual maximum precipitation based on GEV, GLO and GPA using L-moment obtained from algorithm two

Rank	Observed annual maximum precipitation (xi)	Predicted annual maximum precipitation (yi) based on GEV	Predicted annual maximum precipitation (yi) based on GLO	Predicted annual maximum precipitation (yi) based on GPA
1	222.3	427.0489297	225.0946321	299.5780861
2	235.2	447.2249508	262.213348	307.3412874
3	305.2	460.7610137	284.9515664	315.1312125
4	338.4	471.3912158	301.7342325	322.9486605
5	342.9	480.3639873	315.2418773	330.7944768
6	351.2	488.26164	326.681309	338.6695567
7	357.1	495.4068129	336.7022738	346.5748505
8	391	501.9987395	345.6961962	354.5113683
9	393.3	508.1701477	353.9181094	362.4801854
10	394.8	514.0146504	361.5442227	370.4824493
11	403	519.6013347	368.7020615	378.5193866
12	413.6	524.9831494	375.4875291	386.5923113
13	425.2	530.2020282	381.9751792	394.7026345
14	427.8	535.29218	388.2247225	402.8518751
15	433.9	540.2822948	394.2853316	411.0416725
16	445.4	545.1970825	400.1986036	419.2738008
17	453.1	550.0583886	406.0006771	427.5501866
18	454	554.8860348	411.7238054	435.872928
19	454.5	559.6984815	417.3975769	444.2443186
20	455.4	564.5133759	423.0499088	452.6668755
21	458.8	569.3480325	428.7079054	461.1433725
22	461.4	574.2198818	434.3986499	469.6768808
23	462.4	579.1469178	440.1499871	478.2708183
24	462.5	584.1481753	445.9913571	486.9290101
25	467.8	589.2442708	451.9547409	495.6557641
26	472.5	594.4580483	458.0757995	504.4559663
27	476	599.8153864	464.3953135	513.3352023
28	490.4	605.3462472	470.9610814	522.2999154
29	491.8	611.0860883	477.8305154	531.3576148
30	506.1	617.0778262	485.0743203	540.5171568
31	515.9	623.3746618	492.7818904	549.7891329
32	550.8	630.0442946	501.0695375	559.1864205
33	564	637.1754714	510.0935822	568.7249902
34	572.7	644.8886628	520.0722686	578.4251385
35	580.8	653.3545277	531.3248014	588.313466
36	614.5	662.8283195	544.3465967	598.4262568
37	615.1	673.7206674	559.9703872	608.8157625
38	623.1	686.7649251	579.7661189	619.5633394
39	656.2	703.5073961	607.2890118	630.8123007
40	722.5	728.4376937	654.0454185	642.8811053

The best from among candidate distributions fitted to the observed data at the station was selected by subjecting their respective predicted precipitation values to five statistical goodness-of-fit tests. The computed goodness of fit statistics based on the three probability distributions was obtained from **algorithm two** and presented in Table 11

Table 11: Goodness of fit statistics based on L-moment obtained from algorithm two

GoF Statistics	RMSE	RRMSE	MADI	MAE	PPCC
GEV	115.6429728	0.326199623	0.269482909	0.45673	0.9675
GLO	37.79785402	0.079475417	0.001362853	0.00354	0.9793
GPA	30.20182487	0.089637382	0.006644185	0.0674	0.9171

The overall goodness of fit of each distribution was judged using a ranking scheme by comparing the three categories of test criteria based on the relative magnitude of the statistical test results. The distribution with the lowest RMSE, lowest RRMSE, lowest MADI, lowest MAE and highest PPCC was assigned a score of 3, the next was given the score 2, while the worst was given the score 1. The overall score of each distribution was obtained by summing the individual point scores and the distribution with the highest total point score was selected as the best fit distribution model. The scoring scheme and the overall ranking of the distributions models at the stations based on the goodness of fit tests is presented in Table 12

Table 12: Scoring and ranking scheme for selected probability distribution models

Test Criteria	Distribution Scoring		
	GEV	GLO	GPA
RMSE	1	2	3
RRMSE	1	3	2
MADI	1	3	2
MAE	1	3	2
PPCC	2	3	1
Total Score/ Rank	6 (3rd)	14 (1st)	10 (2nd)

Based on the result of Table 12, generalized logistics probability distribution (GLO) with the highest total score of 14 was selected as the best probability distribution model for analyzing annual maximum rainfall series in Benin City followed by GPA and then GEV. The quantile estimates (Qt) based on 2, 5, 10, 20, 50, 100, 200 and 500 years was also obtained based on L-moment procedure using algorithm two and results obtained is presented in Tables 13

Table 13: Computed quantile estimates based on selected return periods for GLO using algorithm two

Z	AA	AB	AC	AD	AE	AF
Return Periods (T)	(T - 1)	[(T - 1)^-K]	R = [1 -(T - 1)^-K]	Z = [(α/k)*(R)]	QT = (ξ + Z)	Exceedence Probability E = (1/T)
2	1	1	0	0	425.8765149	0.5
5	4	1.049226143	-0.049226143	82.34400664	508.2205215	0.2
10	9	1.079137404	-0.079137404	132.3786615	558.2551764	0.1
20	19	1.107452783	-0.107452783	179.7437724	605.6202873	0.05
50	49	1.144424059	-0.144424059	241.5882084	667.4647233	0.02
100	99	1.172666081	-0.172666081	288.8306102	714.7071251	0.01
200	199	1.201392049	-0.201392049	336.8825427	762.7590576	0.005
500	499	1.240291595	-0.240291595	401.952529	827.8290439	0.002
1000	999	1.27049637	-0.27049637	452.478166	878.3546809	0.001

The graphical visualization of the observed and predicted annual maximum precipitation for GLO was obtained from **algorithm two** and presented in Figure 4

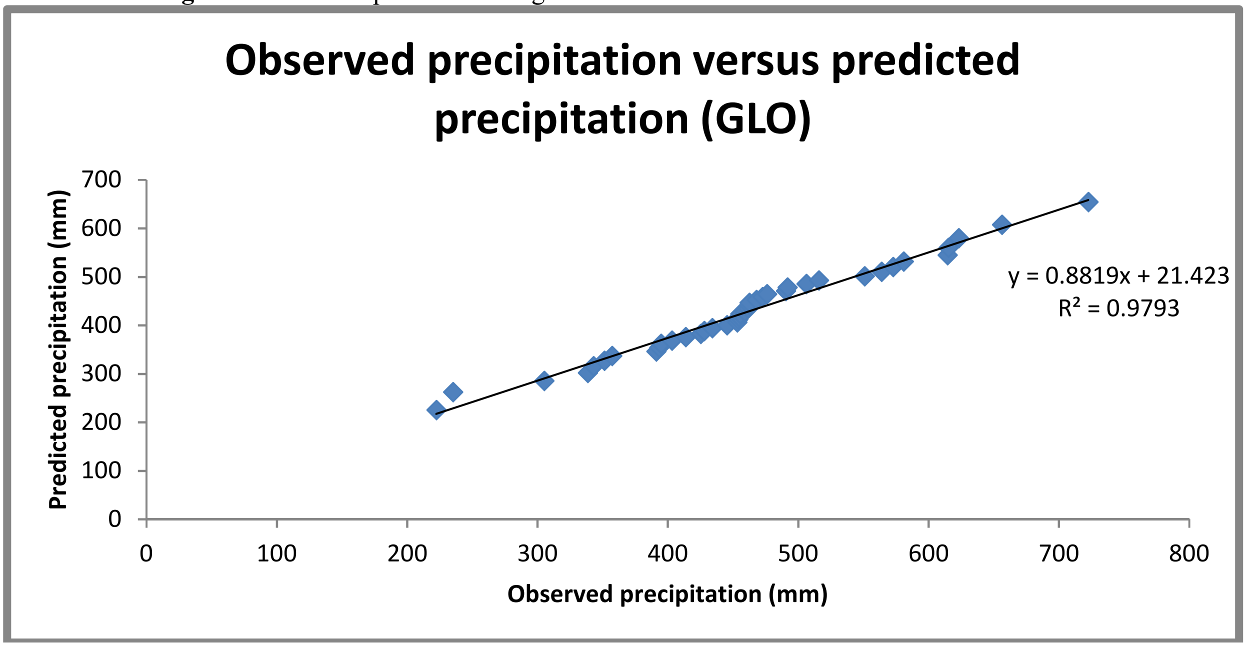

Figure 4: Observed versus predicted precipitation based on GLO distribution

The computed coefficient of determination (r^2) between the observed and predicted precipitation was observed to be 0.9793 for generalized logistics distribution (GLO). Based on the computed coefficient of determination, it was concluded that generalized logistics distribution had a better fit of the annual maximum precipitation data.

6.0 Conclusion

This paper gave a detail description of the current method of statistical parameter estimation of selected probability distribution model, namely; Generalized Extreme Value (GEV), Generalized Logistics (GLO) and Generalized Pareto (GPA) probability distributions – the L-moments method. A simple to use Microsoft Excel Algorithm have been developed for estimating basic descriptive statistics such as the sample mean, variance, standard deviation, skewness, kurtosis, and coefficient of variation. Other exciting features include; the computation of Probability weighted moment parameters (b0, b1, b2 and b3), L-Moment values ($\lambda 1$, $\lambda 2$, $\lambda 3$ and $\lambda 4$)**,** L-Moment ratio values ($\tau 2$, $\tau 3$ and $\tau 4$), and goodness of fit statistics (RRMSE, RMSE and MADI). The algorithm will not only find use in the practice of engineering hydrological computation, it will also help design engineers in estimating the magnitude of peak rainfall/discharge for various return periods.

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