# Formulation of Seismic Passive Resistance of Non-Vertical Retaining Wall Backfilled with c-Φ Soil

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**Abstract**

The seismic passive earth pressure is really the most important parameter in some special cases like key analysis, anchor analysis, foundation analysis etc. The simultaneous action of weight, surcharge, cohesion and adhesion is also taken into consideration. A visual presentation is made by plotting graphs with the wide range of variation Parameters like angle of internal friction ($\Phi$), angle of wall friction ($\delta$), wall inclination angle ($\alpha$), cohesion ($c$), adhesion ($c_a$), seismic accelerations ($k_h$, $k_v$), surcharge loading ($q$), unit weight ($\gamma$), height ($H$) to provide the variation of seismic passive earth pressure coefficient.

**Keywords:** Pseudo-static, seismic passive resistance, single wedge, c-Φ backfill, rigid retaining wall, wall inclination.

## 1. Introduction

Computation of passive resistance is extremely important and the level of importance of the passive earth pressure increases many fold under earthquake conditions due to the devastating effects of earthquake. Hence to analyze the retaining wall under passive condition for both under the static and seismic conditions, the basic theory is very complex and the several researchers have discussed on this topic. Initially Okabe (1926) and Mononobe and Matsuo (1929) had proposed the theory to compute the pseudo-static lateral earth pressure on the wall, which is commonly known as the Mononobe-Okabe method (see Kramer (1996)). Based on the classical limit equilibrium theory, this method is a direct modification of the Coulomb wedge method where the earthquake effects are replaced by quasi-static inertia forces, whose magnitude is computed with seismic coefficient concept. Again, by using the approximate method based on modified shear beam model Wu and Finn (1999) developed charts for seismic thrusts against rigid walls. Psarropoulos et al. (2005) have developed a general finite element solution for analyzing the distribution of dynamic earth pressures on rigid and flexible walls. Davies et al. (1986), Morrison and Ebeling (1995), Soubra (2000) and Kumar (2001) to name a few had analyzed the seismic passive earth pressure problems. All the analyses as mentioned above are for Φ backfill. Subba Rao and Choudhury (2005) had given a solution for seismic passive earth pressure supporting c-Φ backfill in such a way that they are getting separate critical wedge surfaces and separate coefficients for unit weight, surcharge and cohesion. But from practical point of view, this fact is not true, as for the simultaneous action of unit weight, surcharge and cohesion, we will get single failure surface. Keeping this fact in mind, here an attempt is made to develop a formulation for the seismic passive resistance on the back of a non-vertical retaining wall supporting c-Φ backfill in such a way that a single failure wedge is developed. A planar rupture surface is considered in that analysis to extend the Mononobe-Okabe concept for c-Φ backfill.

## 2. Method of Analysis for Seismic Passive Resistance

A schematic diagram of seismic passive earth pressure is shown in the fig.1. Here a rigid retaining wall of height H supporting $c$-$\Phi$ backfill of unit weight $\gamma$, unit cohesion $c$, unit adhesion $c_a$, angle of wall friction $\delta$, angle of soil friction $\Phi$, retaining wall inclination angle $\alpha$ is shown. On the top of the backfill a surcharge load of intensity $q$ per unit length is acting. At any stage of earthquake (having seismic acceleration coefficients $k_h$ and $k_v$) during passive state of equilibrium, if the planer wedge surface $BD$ generates an angle $\theta$ with the vertical, then the forces acting on

the wedge system as shown in Fig.1, $P_p$ and $R$ being the force on the retaining wall and reaction offered by the retained earth on the sliding wedge $ABD$ at the face $BD$ respectively.

Applying the force equilibrium conditions, $\sum H = 0$ and $\sum V = 0$,

$$P_p \cos(\delta - \alpha) + (W + Q)k_h - cH \tan\theta + c_a H \tan\alpha - R\cos(\phi - \theta) = 0 \tag{1}$$

$$P_p \sin(\delta - \alpha) + (W + Q)(1 \pm k_v) + c_a H + cH + R\sin(\phi - \theta) = 0 \tag{2}$$

Solving Eqn 1 and 2 and putting $W = \{\gamma H^2 (\tan\theta + \tan\alpha)\}/2$, $Q = qH(\tan\theta + \tan\alpha)$, $C = cH\sec\theta$, $C_a = c_a H\sec\alpha$, $\psi = \tan^{-1}(k_h/(1 \pm k_v))$ we get,

$$P_p \sin(\theta - \phi - \delta + \alpha) = \left(\gamma + \frac{2q}{H}\right)\frac{H^2}{2}(1 \pm k_v)\frac{(\tan\theta + \tan\alpha)}{\cos\psi}\cos(\theta - \phi + \psi) + cH\sec\theta\cos\phi$$
$$+ c_a H \cos(\theta - \phi + \alpha)\sec\alpha \tag{3}$$

Replacing $(\gamma + 2q/H)$ by $\gamma_e$, Eqn 3 can be written as

$$P_p = \frac{\gamma_e H^2}{2}(1 \pm k_v)\left[\begin{array}{l}\dfrac{(\tan\alpha + \tan\theta)\cos(\theta - \phi + \psi)}{\cos\psi\sin(\theta - \phi - \delta + \alpha)} + \dfrac{2c}{\gamma_e H(1 \pm k_v)}\dfrac{\cos\phi}{\cos\theta\sin(\theta - \phi - \delta + \alpha)} \\ + \dfrac{2c_a}{\gamma_e H(1 \pm k_v)}\dfrac{\sec\alpha\cos(\theta - \phi + \alpha)}{\sin(\theta - \phi - \delta + \alpha)}\end{array}\right] \tag{4}$$

Substituting $\dfrac{2c}{\gamma_e H(1 \pm k_v)} = n_c$ and $\dfrac{2c_a}{\gamma_e H(1 \pm k_v)} = m_c$, the above Equation reduces to

$$P_p = \frac{\gamma_e H^2}{2}(1 \pm k_v)\left[\frac{\sin(\alpha + \theta)\cos(\theta - \phi + \psi) + n_c \cos\phi\cos\psi\cos\alpha + m_c\cos\theta\cos(\theta - \phi + \alpha)\cos\psi}{\cos\theta\cos\alpha\cos\psi\sin(\theta - \phi - \delta + \alpha)}\right] \tag{5}$$

$$P_p = \frac{\gamma_e H^2}{2}(1 \pm k_v)k_p \tag{6}$$

Where $k_p = \left[\dfrac{\sin(\alpha + \theta)\cos(\theta - \phi + \psi) + n_c\cos\phi\cos\psi\cos\alpha + m_c\cos\theta\cos(\theta - \phi + \alpha)\cos\psi}{\cos\theta\cos\alpha\cos\psi\sin(\theta - \phi - \delta + \alpha)}\right]$

$$\tag{7}$$

In Eqn.7, all the terms are constant except $\theta$. The optimum value of $k_p$ is given by the condition $dk_p/d\theta = 0$, Applying this condition on Eqn.7, we get critical wedge angle $\theta_c$ as given by the following Eqn,

$$\theta_c = \cos^{-1}\sqrt{\frac{(p + q)q + r^2 - r\sqrt{q^2 + r^2 - p^2}}{2(q^2 + r^2)}} \tag{8}$$

Where

$$p = \sin(\psi + \delta) + m_c\cos\psi\sin\delta \tag{9}$$

$$q = \sin(\psi - \phi)\cos(2\alpha - \phi - \delta) - \sin(\phi + \delta)\cos(\psi - \phi) - m_c \cos\psi\cos\delta$$
$$- 2n_c \cos\alpha\cos\phi\cos\psi\cos(\alpha - \phi - \delta) \tag{10}$$

$$r = 2\cos\alpha\sin(\alpha - \phi - \delta)\{\sin(\phi - \psi) - n_c \cos\phi\cos\psi\} \tag{11}$$

Putting this value of $\theta_c$ in Eqn 7, we get the optimum value of passive earth pressure coefficient, which is represented here as $k_{pe}$. So, seismic passive resistance is given by,

$$P_{pe} = \frac{\gamma_e H^2}{2}(1 \pm k_v)k_{pe} \tag{12}$$

## 3. Parametric Study

From the Eqn 8 and its related Eqns 9, 10 and 11, it is seen that the coefficient for seismic passive resistance depends on $\Phi$, $\delta$, $\psi$, $\alpha$, $m_c$ and $n_c$. Coefficient $\psi$ depends on seismic acceleration coefficient $k_h$ and $k_v$. Cohesion factor $n_c$ is taking care of the effect of cohesion and adhesion factor $m_c$ is taking care of the effect of adhesion. Both $n_c$ and $m_c$ are also depend on $\gamma$, q, H and $k_v$. All the factors $\Phi$, $\delta$, $\alpha$, $k_h$ and $k_v$, $\gamma$, q, H, c and $c_a$ affects the magnitude of seismic passive earth pressure coefficient. In the following sub sections, the affects of all these parameters on the variation of seismic passive resistance coefficient are studied.

### 3.1. Effect of angle of internal friction of soil (Φ)

Fig.2 shows the variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for different values of $\Phi$ at $\delta = \Phi/2$, $k_v = k_h/2$, c = 10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 kN/m, $\gamma$ = 18 kN/m$^3$, H = 10 m. From the plot, it is seen that the magnitude of $k_{pe}$ appreciably increases with increase in $\Phi$. For example, for $k_h$ = 0.1, at $\Phi$ = 10°, 20°, 30° and 40°, the magnitude of $k_{pe}$ is 1.55, 2.21, 3.35 and 5.6 respectively. Due to increase in $\Phi$, the resistance capacity of the backfill increases which resembles for the fact to increase in $k_{pe}$.

### 3.2 Effect of angle of wall friction (δ)

Fig.3 shows the variation of $k_{pe}$ with $k_h$ for different value of $\delta$ at $\Phi$ =30°, $k_v = k_h/2$, c = 10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 kN/m, $\gamma$ = 18 kN/m$^3$, H = 10 m. From the plot, it is seen that the coefficient $k_{pe}$ increases due to increase in $\delta$. For example, at $k_h$ = 0.2, due to increase in $\delta$ from $-\Phi/2$ to $\Phi/2$, the coefficient $k_{pe}$ increases from 1.89 to 3.12.

### 3.3 Effect of $k_v/k_h$ ratio

Fig.4 shows the variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for ratio of $k_v/k_h$ from 0 to 1 at $\Phi$ =30°, $\delta = \Phi/2$, c = 10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 kN/m, $\gamma$ = 18 kN/m$^3$, H = 10 m. From the plot, it is seen that $k_{pe}$ decreases with increase $k_v/k_h$ ratio. Increase in $k_v/k_h$ ratio means increase in seismic disturbance of the backfill material and due to that the resistance capacity of the backfill material is going to be reduced which resembles for the fact of reduction of $k_{pe}$ due to increase in $k_v/k_h$ ratio. Here in Fig.3, upto $k_h$=0.2, the value of $k_{pe}$ is more or less same for $k_v/k_h$ ratio of 0, 1/2, 1.

### 3.4 Effect of cohesion (c)

From the earlier analyses [Saran and Gupta(2003); Ghosh and Saran (2007); Ghosh (2010); Ghosh and Sharma (2010)], it is seen that there is no effect of cohesion on the magnitude of seismic passive earth pressure ($k_{pe}$). But from the present analysis, it is seen that cohesion of the soil appreciably increases the magnitude of $k_{pe}$. Here, Fig.5 represents one such variation of $k_{pe}$ with $k_h$ for different value of c at $\Phi$ =30°, $\delta = \Phi/2$, $k_v = k_h/2$, $c_a$ = 0, q = 15 kN/m, $\gamma$ = 18 kN/m$^3$, H = 10 m. In this plot, it is seen that for $k_h$ = 0.3, $k_{pe}$ = 2.37 at c = 0 increases to 3.12 at c = 20 kN/m$^2$. Due to increase in c, intermolecular attraction of the soil particles increases which increases the resistance capacity of the backfill soil mass and thus increases $k_{pe}$.

### 3.5  Effect of adhesion ($c_a$)

Similar to the effect of cohesion, the adhesion also increases the magnitude of coefficient of seismic passive earth pressure $k_{pe}$. Fig.6 shows one such variation of $k_{pe}$ with $k_h$ for different value of $c_a$ at $\Phi = 30°$, $\delta = \Phi/2$,  $k_v = k_h/2$, c = 10kN/m$^2$,  q = 15 kN/m, $\gamma$ = 18 kN/m$^3$, H = 10 m. From this plot, it is seen that for $k_h$ = 0.2, $k_{pe}$ increases from 3.04 to 3.14 due to change in $c_a/c$ ratio from 0 to 1.

### 3.6  Effect of surcharge (q)

Fig.7 shows the variation of $k_{pe}$ with $k_h$ for different value of q at $\Phi = 30°$, $\delta = \Phi/2$,  $k_v = k_h/2$, c = 10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, $\gamma$ = 18 kN/m$^3$, H = 10 m. From the plot, it is seen that $k_{pe}$ decreases marginally due to increase in q. For example at $k_h$ =0.3, due to increase in q from 0 to 40 kN/m, the value of $k_{pe}$ decreases from 2.91 to 2.74.

### 3.7  Effect of unit weight of backfill material ($\gamma$)

Similar to the effect of q, the coefficient $k_{pe}$ also decreases due to increase in $\gamma$. Fig.8 shows one such variation of $k_{pe}$ with $k_h$ for different value of $\gamma$ at $\Phi = 30°$, $\delta = \Phi/2$,  $k_v = k_h/2$, c = 10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 kN/m, H = 10 m. From this plot, it is seen that at $k_h$ = 0.3, $k_{pe}$ decreases from 3.11 to 2.66 due to increase in $\gamma$ from 10 kN/m$^3$ to 30 kN/m$^3$.

### 3.8  Effect of height of retaining wall (H)

Height of retaining wall also appreciably reduces the coefficient of seismic passive earth pressure. Fig.9 shows the variation of $k_{pe}$ with $k_h$ for different heights of retaining wall at $\Phi = 30°$, $\delta = \Phi/2$,  $k_v = k_h/2$, c = 10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 kN/m, $\gamma$ = 18 kN/m$^3$. In this plot, for example, at $k_h$ = 0.2, due to the reduction of height from 10 m to 2 m, the coefficient $k_{pe}$ increases from 3.12 to 4.03.

### 3.9. Effect of wall inclination ($\alpha$)

Fig.10 shows the variation of seismic passive earth pressure coefficient ($k_a$) with $k_h$ for for $\Phi = 30°$, $k_v=k_h/2$, $\delta=\Phi/2$, $q$= 15 kN/m, $c$ = 10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, $\gamma$=18 kN/m$^3$, H=10 m for different wall inclination angle ($\alpha$). From the plot, it is seen that the effect of wall inclination angle is very a prominent factor for the determination of seismic passive earth pressure coefficient ($k_{pe}$). For example at $k_h$=0.3 for the value of $\alpha$=20° to -20° $k_a$ increases from 2.83 to 10.11.

### 3.10.    Collapse Mechanism

Critical wedge surface is the wedge surface, at which we get the optimum value of seismic passive earth pressure coefficient. Here in this analysis, it is represented by $\theta_c$ (measured with the vertical) and given by Eqn.8. Fig.11 shows the variation of $\theta_c$ with $k_h$ for different value of $\Phi$ at $\delta=\Phi/2$, $k_v = k_h/2$, c = 10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 kN/m, $\gamma$ = 18 kN/m$^3$, H = 10 m. From the plot, it is seen that due to increase in $\Phi$, the magnitude of the inclination of the critical wedge surface also increases.

## 4. Comparison of results

Very few studies are made for the determination of seismic passive resistance supporting c-$\Phi$ backfill. Mononobe Okabe, Subba Rao and Choudhury (2005) had given a solution for the seismic passive resistance supporting c- $\Phi$ backfill and a comparison of the present study is made with Mononobe Okabe, Subba Rao and Choudhury (2005) in Table 1.

 Table 1 shows that present study provides the value of seismic passive resistance in little higher side in comparison to Subba Rao and Choudhury (2005). The concept of present study is the extension of Mononobe-Okabe solution for c-$\Phi$ backfill. So, the findings of present study exactly matches Mononobe-Okabe (1929) for c = 0, $c_a$ = 0 and q= 0.

## 5. Conclusion

Using the limit equilibrium method of analysis with pseudo-static approach, the seismic passive resistance formulation on the back of a retaining wall has been developed. The basic theme of the analysis is to generate a single failure wedge surface for the simultaneous action of unit weight, surcharge, cohesion and adhesion. A wide range of variation of parameters like cohesion, adhesion, angle of wall friction, angle of soil friction, wall inclination are used to note down the variation of coefficient of seismic passive earth pressure. The basis of the analyses available at present is that the coefficient of seismic passive earth pressure does not depend on height of retaining wall, cohesion, adhesion, surcharge loading, unit weight of the backfill material. But present analysis represents that the coefficient for seismic passive earth pressure increases with increase in cohesion, adhesion but decreases due to increase in unit weight of backfill material, surcharge loading and height of retaining wall. Matching with the other available analysis, present analysis represents increase in coefficient for seismic passive earth pressure due to increase in angle of internal friction and angle of wall friction of the backfill material and decreases due to increase in seismic acceleration coefficients.

So, extending the Mononobe-Okabe concept for the determination of seismic passive response on the back of a non-vertical retaining wall supporting c-Φ backfill, a formulation is developed for the simultaneous action of weight, surcharge, cohesion and adhesion which is reasonable and easy to use.

**6. References**

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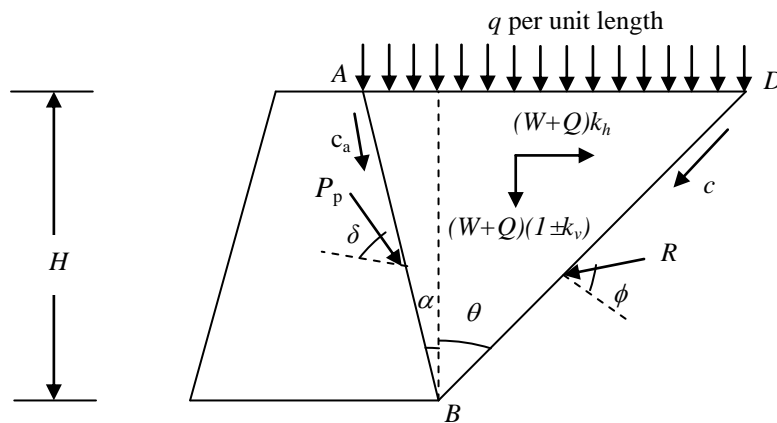
Fig.1. Forces acting on retaining wall – soil wedge system during passive state of equilibrium.
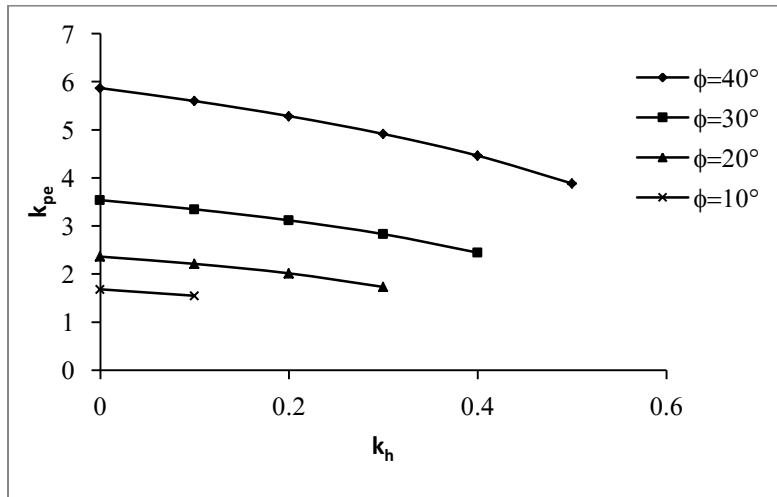
Fig.2.Variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for $k_v=k_h/2$, $\delta=\Phi/2$, $\gamma = 18$ kN/m$^3$, c=10 kN/m$^2$, $c_a = 8$ kN/m$^2$, q = 15 kN/m, H=10 m, $\alpha=20°$
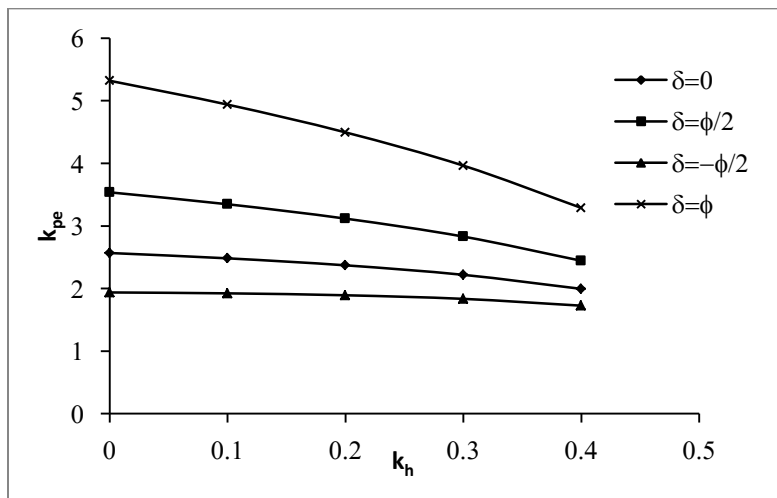


Fig.3.Variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for $k_v=k_h/2$, $\phi=30°$, $\gamma = 18$ kN/m$^3$, c=10 kN/m$^2$, $c_a = 8$ kN/m$^2$, q = 15 kN/m, H=10 m, $\alpha=20°$
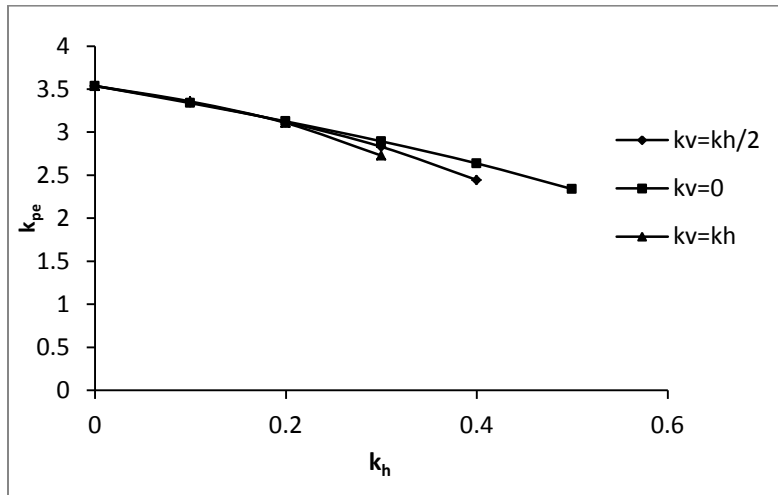
Fig.4.Variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for $\phi$=30°, $\delta$=$\Phi$/2, $\gamma$ = 18 kN/m$^3$, c=10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 kN/m, H=10 m, $\alpha$=20°
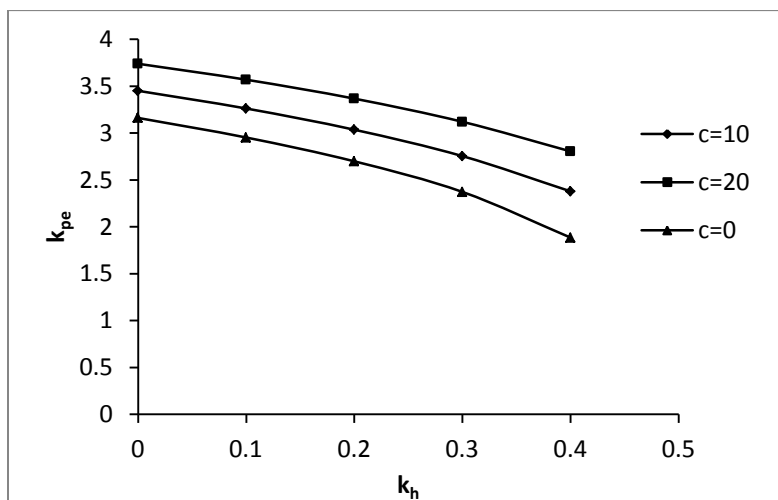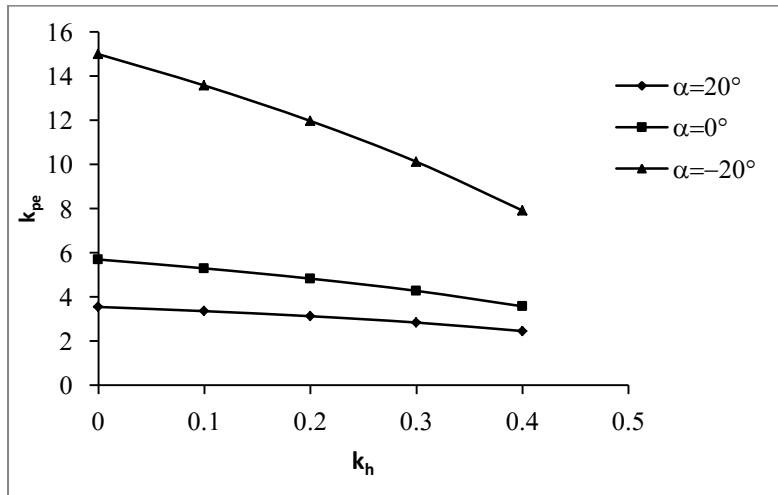


Fig.5.Variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for $k_v$=$k_h$/2, $\phi$=30°, $\delta$=$\Phi$/2, $\gamma$ = 18 kN/m$^3$, $c_a$ = 0 kN/m$^2$, q = 15 kN/m, H=10 m, $\alpha$=20°
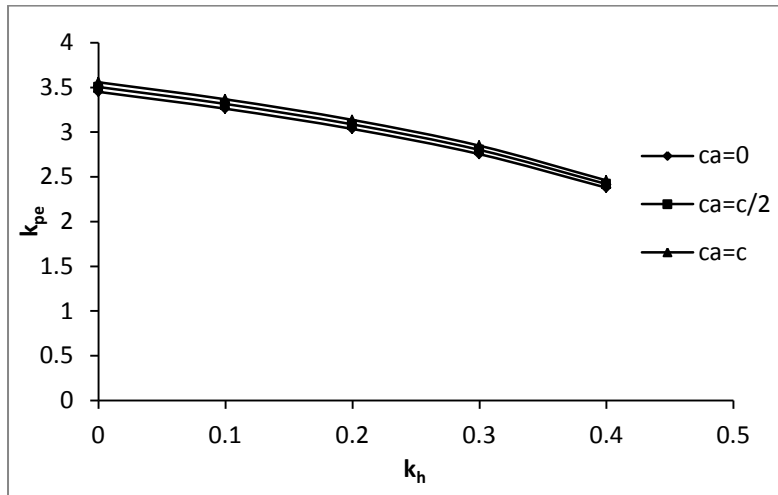
Fig.6.Variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for $k_v=k_h/2$, $\phi= 30°$, $\delta=\Phi/2$, $\gamma = 18$ kN/m$^3$, c=10 kN/m$^2$, q = 15 kN/m, H=10 m, $\alpha$=20°
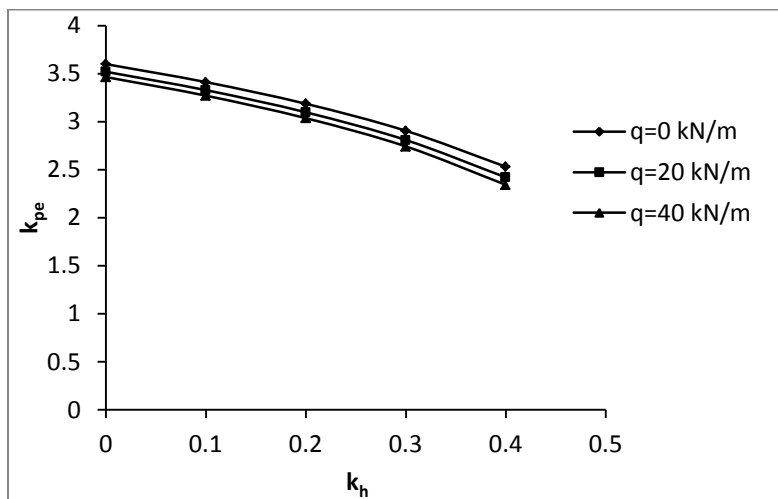


Fig.7.Variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for $k_v=k_h/2$, $\phi=30°$, $\delta=\Phi/2$, $\gamma = 18$ kN/m$^3$, c=10kN/m$^2$, ca = 8 kN/m$^2$, H=10 m, $\alpha$=20°
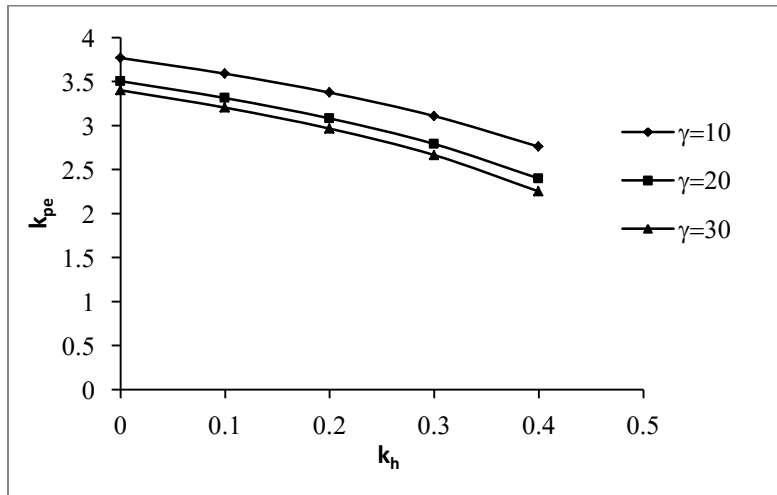
Fig.8.Variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for $k_v=k_h/2$, $\phi=30°$, $\delta=\Phi/2$, c=10 kN/m$^2$, ca = 8 kN/m$^2$, q=15 kN/m, H=10 m, $\alpha=20°$
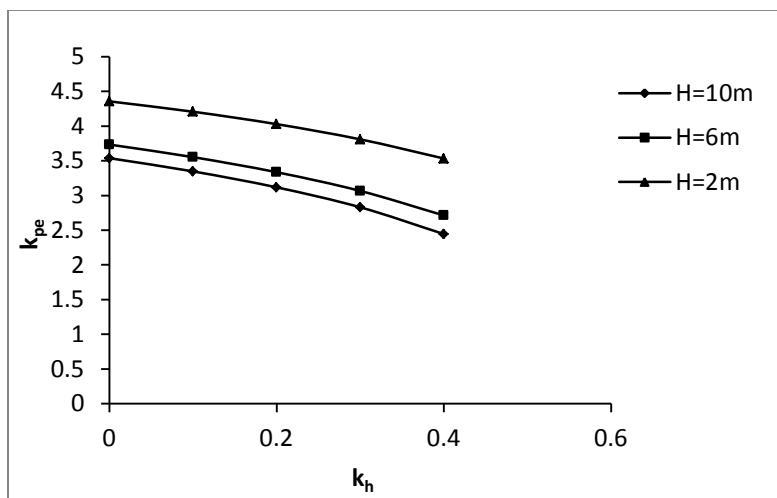


Fig.9.Variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for $k_v=k_h/2$, $\phi=30°$, $\delta=\Phi/2$, $\gamma$ = 18 kN/m$^3$, c=10 kN/m$^2$, ca = 8 kN/m$^2$, q=15 kN/m, $\alpha=20°$

Fig.10.Variation of seismic passive earth pressure coefficient ($k_{pe}$) with $k_h$ for $k_v=k_h/2$, $\phi=30°$, $\delta=\Phi/2$, $\gamma = 18$ kN/m$^3$, c=10 kN/m$^2$, ca = 8 kN/m$^2$, q=15 kN/m, H=10 m
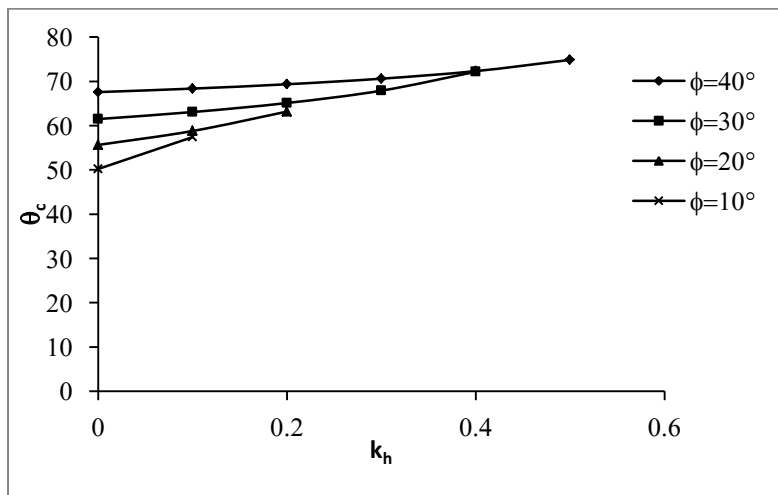


Fig.11.Variation critical wedge angle ($\theta_c$) with $k_h$ for $k_v=k_h/2$, $\delta=\Phi/2$, $\gamma = 18$ kN/m$^3$, c=10 kN/m$^2$, $c_a$ = 8 kN/m2, q = 15 kN/m, H=10 m, a=20 °

Table 1: Comparison of the results obtained from the present study with Mononobe Okabe, Subba Rao and Choudhury'2005 [$\Phi = 30°$, $\delta = \Phi/2$, c= 0 kN/m2, ca = 0 kN/m2, q = 0 kN/m, $\gamma$ = 18 kN/m3]

| $k_h$ | $k_v$ | Mononobe Okabe ($k_{pe}$ in kN/m ) | Subba Rao and Choudhury (2005) | Present study ($k_{pe}$ in kN/m ) |
|---|---|---|---|---|
| 0 | 0 | 4.976 | 4.458 | 4.976 |
| 0.1 | 0 | 4.562 | 4.24 | 4.562 |
| 0.2 | 0 | 4.129 | 3.86 | 4.129 |