Damage detection on bridge structures based on static deflection measurements of a single point

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Abstract. The principle of structural health monitoring of the bridge is the assessment of the structure performance or safety level comparing with a reference system. The most used technique is the dynamic methods which are employed to determine the structural dynamic characteristics and thereafter to locate the damages or changes in some zones of the structure. While static methods are not widely used although they are simpler than dynamic methods and also they do not require sophisticated equipment. In the last decade, some recent researches develop the interesting deterministic or probabilistic methods to evaluate the flexural rigidity or stiffness on a beam, a structure or a bridge and thus detect any damage.
The idea is to analyze the static deflections of one selected point or cross-section of a beam or a bridge with a variable position loading. The developed numerical approach uses an inverse method to solve the static equilibrium equations of a variable positions loading in the structure using the finite element method. A Matlab code is developed to solve this static inverse problem. By knowing the deflections amplitude of a selected point in the structure corresponding to several positions of a load, then the stiffness reduction factor in the bridge can be estimated.
Some examples for a beam are treated to test this new method for assessing its rigidity.

1 INTRODUCTION

Structure health monitoring of structures like buildings, bridges and dams is important for the civil engineers in the safety, security and the resistance evaluation. Another goal of this evaluation is to search, detecting and quantifying the eventual damages, cracks or changes in the structure comparing with the designed one. The non-destructive techniques are more common, economic and reliable to detect the global or local damages in structures.
The damages in structure produce changes to its stiffness. These changes make variation in their static and dynamic responses. In the dynamic case, it is observed relative changes in frequencies and modal shapes measured by an accelerometers system. In the static case, displacements, deformations or stresses variations are measured by strain gauges, fiber optic gauges or laser displacement sensor. The static load testing is been the first technique used essentially for the bridges. In the last decades, the vibration modal identification is so much used for detecting damages and then capacity assessment of the structure.

Many research papers treat the techniques for damage identification based on static approach. Sheena et al. (1982) [1] presented an analytical method to assess the stiffness matrix by minimizing the difference between the real and the analytical stiffness matrix subjected to the measured displacement constraints. Banan et al. (1993) [2,3] proposed the mathematical formulation of two least-squares parameter estimators that element constitutive parameters of a finite element model that corresponds to a real structural system from measured static response to a given set of loads. Stöhr et al. (2006) [4] could identify the existence and locations of stiffness changes in a beam by the difference analyze between the influence lines of inclination measured under original and under modified structural conditions. Eun et al. (2007) [5] proposed an analytical method to predict the damage location based on the moment diagram calculated by both the constraint forces at measured points and the known external forces. Wang et al. (2009) [6] developed a quasi-static approach to analyze the measured deflection influence line at certain points of the beam type structure due to loading vehicle slowly passing the structure. Cao et al. (2011) [7] investigated the sensitivity analyses of fundamental mode shape, deflection under tip-concentrated loading and deflection under uniformly distributed loading in cantilever beams using analytical models in conjunction with a three-dimensional finite element method. This approach permitted to detect damages or cracks.
The approach developed in this work is to indentify changes or damages in a beam by the only one beam point measurement of vertical displacement for a variable load position. The Bezier p-version finite element method is used to define the Euler-Bernoulli beam deflection. The used mathematical technique is an inverse method for estimating the flexural rigidity reduction factor along the beam. Two simple examples are treated in this paper: a beam with one change section and a beam with two change sections. It is also seen in the numerical examples that the damages cause perturbations in the rest of the beam.

2 MATHEMATICAL FORMULATION

2.1 The Bezier p-version finite element method

In the beam finite element, the vertical displacement *w* can be expressed at position *x*. The displacement function $w(x)$ is taken as summation of m Bernstein polynomials as follows [8]:

$$w(x)=\sum_{i=1}^{m} B_{m,i}(x).w_i=[B_{m,i}(x)].\{w_i\}=[N].\{\delta\} \quad (1)$$

where w_i are the displacement-control points to be determined and $B_{m,i}(x)$ are the Bernstein polynomials corresponding to the beam displacement.
The Bernstein polynomials are the blending functions defined as $x\in [0, L]$, with L is the beam length (Fig.1):

$$B_{m,i}(x)=C_{m-1}^{i-1}.\left(\frac{x}{L}\right)^{i-1}.\left(1-\left(\frac{x}{l}\right)\right)^{m-i} \quad (2)$$

where C_{m-1}^{i-1} is the binomial coefficient :

$$C_{m-1}^{i-1}=\frac{(m-1)!}{(i-1)!.(m-i)!} \quad (3)$$

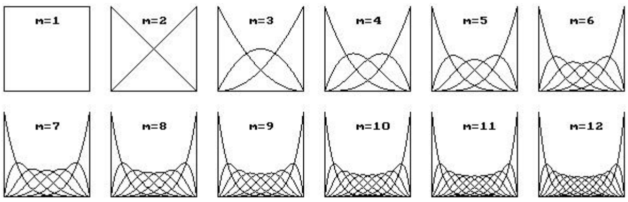


Fig.1. The typical Bernstein polynomials (m=1 to 12 order).

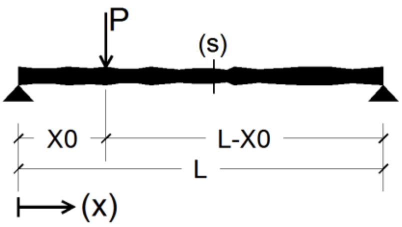


Fig.2. The representation of the beam.

In this study, the simply supported beam subjected to a variable concentrated load with a small variation in their cross-section is treated (Fig.2).

In the linear analysis of the beam, the static problem is defined by the typical matrix equation. It is shown as follows:

$$[K].\{\delta\}=\{F\} \quad (4)$$

where the beam stiffness matrix is defined as:

$$[K]=E.\int_0^L\left(I_y(x).\frac{\partial^2 B_{m,i}(x)}{\partial x^2}.\frac{\partial^2 B_{m,j}(x)}{\partial x^2}\right).dx \quad (5)$$

In the equation (5), E and $I_y(x)$ are the Young modulus of the bean material and the variable cross-section moment of inertia about the horizontal local axis respectively.

The beam support type (simply supported or clamped) can be defined by a simple elimination of the one extreme or two extremes of Bernstein polynomials respectively [8].
The force vector for the concentrated (P) load is defined as:

$$\{F\}=P.\{B_{m,i}(x=x_0)\}_{i=1,m} \quad (6)$$

After solving the equilibrium equation (eq.4), the vertical displacement in the chosen beam section is:

$$w(x_s)=\lfloor B_{m,j}(x=x_s)\rfloor\{\delta\} \quad (7)$$

After developing the eq.7 using eq.4 and eq.6, it is written:

$$w(x_s)=[B_{m,j}(x=x_s)][K]^{-1}.P.\{B_{m,i}(x=x_0)\} \quad (8)$$

2.2 The inverse method algorithm

For each position ($x_{0,k}$) of the load (P), the vertical displacement ($w_{s,k}$) of the only chosen section (s) is measured (Fig.2).
Then for (n) lectures of the displacement (w_s), the equation (8) becomes:

$$w_{s,k}=w(x_s)=[(B_{m,j}(x_s))][K]^{-1}.P.\{B_{m,i}(x_{0,k})\}$$
$$\text{for} \quad k=1,n \quad (9)$$

The damage or change of the beam is modeled through the reduction of its bending rigidity ($E.I_y$) by a factor(α). The beam can be decomposed into (p) equal distance intervals. Therefore, the reduction factor is defined (α_r) for each interval [x_{r-1}, x_r], $r=1,p$. Then the damaged or changed beam stiffness matrix can be written as:

$$[K]=[K_0]+\sum_{r=1}^{p}[K_r] \quad (10)$$

where [K_0] is the undamaged beam stiffness matrix and [K_r] is the reduced stiffness matrix localized in the interval (r). The matrix [K_r] can be defined also as a perturbed localized stiffness matrix in the interval [x_{r-1}, x_r]. The matrices [K_0] and [K_r] are expressed respectively as:

$$[K_0]=E.I_y.\int_0^L\left(\frac{\partial^2 B_{m,i}(x)}{\partial x^2}.\frac{\partial^2 B_{m,j}(x)}{\partial x^2}\right).dx \quad (11)$$

$$[K_r]=\alpha_r.\left(E.I_y.\int_{x_{r-1}}^{x_r}\left(\frac{\partial^2 B_{m,i}(x)}{\partial x^2}.\frac{\partial^2 B_{m,j}(x)}{\partial x^2}\right).dx\right) \quad (12)$$

where (I_y) is the constant cross-section moment of inertia of the undamaged beam.

The reduction stiffness matrix [K_r] can be rewritten as:

$$[K_r]=\alpha_r.[K_{rd}] \qquad r=1,p \quad (13)$$

Since the coefficients (α_r) are unknowns, the inverse of the matrix [K], in eq.10, is so difficult or impossible with the use of software packages like Mathematica, Maple or Matlab for example.

The idea developed in this work is to apply the Neumann series method to inverse the stiffness matrix [9]:

$$[K]^{-1} = \left([K_0] + \sum_{r=1}^{p}[K_r]\right)^{-1} = \left(\sum_{q=0}^{T}\left(-[K_0]^{-1}.\left(\sum_{r=1}^{p}[K_r]\right)\right)^{q}\right).[K_0]^{-1} \tag{14}$$

This Neumann series converge if the norm $\left\| [K_0]^{-1}.\left(\sum_{r=1}^{p}[K_r]\right)\right\| < 1$. It is indicating that the rigidity reduction caused by damages must have been small. The parameter (T) is the order or the maximum of the Newmann series.

In the equation 14, the inverse matrix $[K]^{-1}$ become an algebraic matrix which their components are polynomials of the coefficients (α_r).

After substituting equation 14 in equation 9, we have the followed relation:

$$w_{s,k} = [(B_{m,j}(x_s))].\left(\sum_{q=0}^{T}\left(-[K_0]^{-1}.\left(\sum_{r=1}^{p}[\alpha_r.K_{rd}]\right)\right)^{q}\right).[K_0]^{-1}.P.\{B_{m,i}(x_{0,k})\} \tag{15}$$

This relation becomes a nonlinear function noted $f_i(\alpha_r)$, specific for one lecture of flexural displacement (k) for a position (k) of the pointed load. It can be rewritten as:

$$f_i(\alpha_r) = f_i(\alpha_1, \alpha_2, \cdots, \alpha_p) = w_{s,k} - [(B_{m,j}(x_s))].\left(\sum_{q=0}^{T}\left(-[K_0]^{-1}.\left(\sum_{r=1}^{p}\alpha_r.[K_{rd}]\right)\right)^{q}\right).[K_0]^{-1}.P.\{B_{m,i}(x_{0,k})\} \tag{16}$$

Collecting the (n) lectures of each position ($x_{0,k}$) of the pointed load, the nonlinear system is built:

$$f(\alpha_1, \alpha_2, \cdots, \alpha_p) = f(\alpha_r) = \begin{Bmatrix} f_1(\alpha_r) \\ f_2(\alpha_r) \\ \vdots \\ f_n(\alpha_r) \end{Bmatrix} = \{w_{s,k}\}^T - [(B_{m,j}(x_s))].\left(\sum_{q=0}^{T}\left(-[K_0]^{-1}.\left(\sum_{r=1}^{p}[K_r]\right)\right)^{q}\right).[K_0]^{-1}.P.[\{B_{m,i}(x_{0,k})\}] \tag{17}$$

The unknowns of this nonlinear system are the interval rigidity reduction coefficients (α_r), $r=1,p$.

This mathematical formulation has been programmed in a Matlab code [10]. Solving the nonlinear system, defined in equation 17, consist of an optimization problem. The Levenberg-Marquardt algorithm is the most adapted for solving this problem and then to determine the interval rigidity reduction coefficients (α_r). The used termination tolerance on the function value is 1.10^{-9}.

It is observed in several numerical examples, that the Neumann series order (T) can be limited to 4 with a significant accuracy.

It is noted that the number (p) of interval rigidity reduction coefficients (α_r) is lower or equal of the displacement lectures number (n).

3 NUMERICAL SIMULATIONS

Here two examples are treated for proving the last algorithm to identify damages or changes in the beam. The first and the second examples concern the same simply supported beam with one and two section changes respectively. These examples are presented in the figure 3.

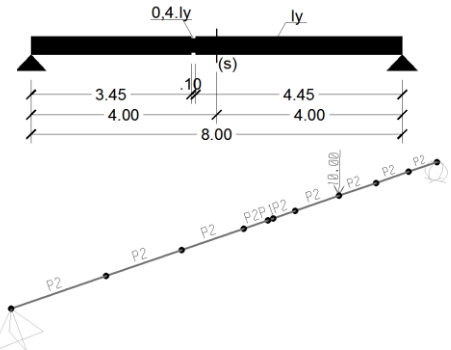


Fig.3. Finite element model of the 1st example of the modified beam with one section change.

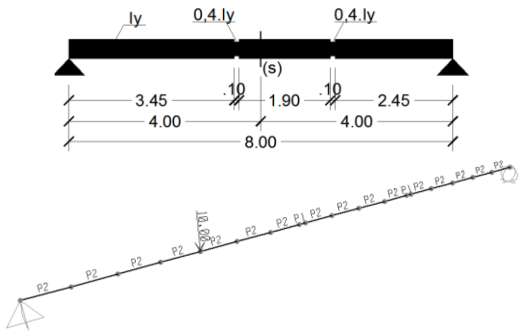


Fig.4. Finite element model of the 2nd example of the modified beam with two sections change.

The initial beam is considered with a constant flexural rigidity ($E.I_y$=cst). The section change is defined about 0.10m length with -60% of moment inertia reduction. For these simulations, a finite element model is used to measure flexural displacements using Sap2000 code [11]. The vertical displacements are measured at mid-span.

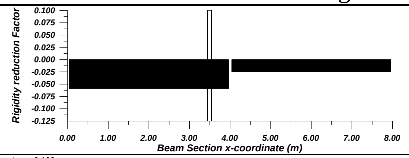
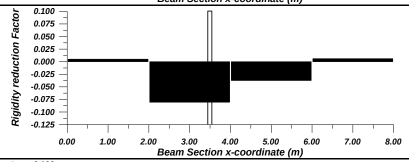
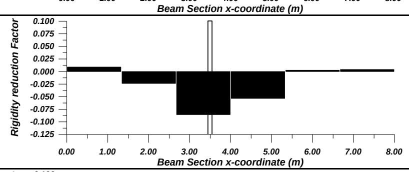
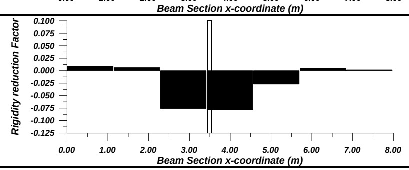
The initial beam is modeled in 8 frame elements. The beams with one and two section changes are modeled in

10 frame elements and 18 frame elements respectively. A pointed load (*P*) of 10kN is used for a number of variable positions ($x_{0,k}$).
The initial beam is analyzed by the developed algorithm. In the tables 1 or 2, it is observed that the flexural rigidity is reduced of -2.3%. These results will be the reference of comparison for change identification and evaluation.

3.1. Example with one section change

For this example, seven (7) lectures of vertical displacement at mid-span are used for 7 different positions of the pointed load *P* with an equal distance. The algorithm is used for different number of intervals (*p*=2, 4, 6 and 7). The reduction of the flexural rigidity is observed in Table 1. In comparison with the initial beam, the flexural rigidity reduction is identified correctly in the region the section change. The maximum reduction reached to -8% for a beam subdivision with 7 intervals.

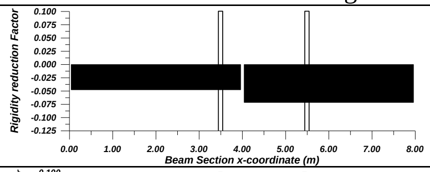
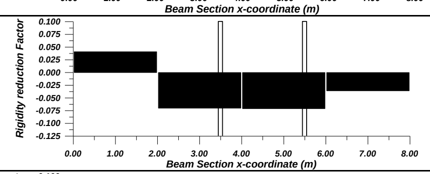
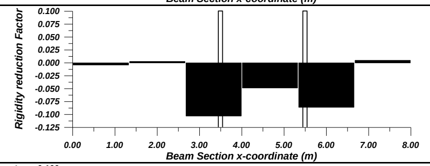
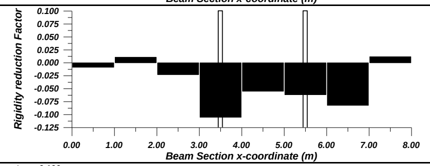
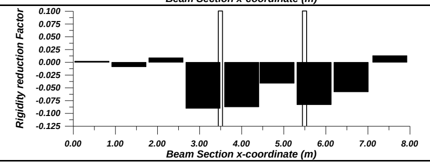
Table 1. The variation of flexural rigidity reduction along the beam - The beam with one section change.

Number of intervals	The beam with one section change case
2	
4	
6	
7	

3.2. Example with two section changes

For this example, nine (9) lectures of vertical displacement at mid-span are used for 9 different positions of the pointed load *P* with an equal distance. The algorithm is used for different number of intervals (*p*=2, 4, 6, 7 and 9). With a 9 equal intervals subdivision, the reduction of the flexural rigidity reached to -8.5% in the region of the first section change and -7.4% in the region of the second section change (Table 2). Between these two section changes, the rigidity is perturbed and reduced at -3.8%.

Table 2. The variation of flexural rigidity reduction along the beam - The beam with two section changes.

Number of intervals	The beam with two section changes case
2	
4	
6	
7	
9	

4 CONCLUSION

This paper presents a new approach for damage identification in beams utilizing lectures of flexural displacement of only one section with a variable position for an applied static force. The method concerns the solving of an inverse problem of a static equilibrium equation using Neumann series method. The solution of the developed computing code is the assessment of the flexural rigidity reduction coefficients defined by intervals. The numerical results of the two examples show that this method can locate damage and quantify the reduction of rigidity along the beam. For the next step, this approach requires to be proved by the experimental tests on beams and bridges.

REFERENCES

1. Sheena Z., Unger A. and Zalmanovish A., “Theoretical stiffness matrix correction by static test

results", Israel Journal of Technology, Vol. 20, pp. 245-253, 1982.

2. Banan M.R., Banan M.R. and Hjelmstad K.D., "Parameter estimation of structures from static response. I : Computational aspects", Journal of Structural Engineering, Vol. 120 (11), pp. 3243-3258, 1993.
3. Banan M.R., Banan M.R. and Hjelmstad K.D., "Parameter estimation of structures from static response. II : Numerical simulation studies", Journal of Structural Engineering, Vol. 120 (11), pp. 3259-3283, 1993.
4. Stöhr S., Link M., Rohrmann R. and Rüker W., "Damage detection based on static measurements on bridge structures", IMAC XXIV Conference and Exposition on Structural Dynamics, 30 Jan. - 2 Feb., St Louis, Missori, USA, 2006.
5. Eun H.C. and Lee E.T., "Damage detection of flexural member by estimated moment diagram", The Open Construction and Building Technology Journal, 1, 1-7, 2007.
6. Wang C.Y., Huang C.K., Zeng Y.T., Chen C.S. and Chen M.H., "Damage Assessment of Beam by a Quasi-Static Moving Vehicular Load", Proceeding of the 10th International Conference on Structural safety and Reliability, Osaka, Japan, pp. 803-809, 2009.
7. Maosen Cao, Lin Ye, Limin Zhou, Zhongqing Su and Runbo Bai, "Sensitivity of fundamental mode shape and static deflection for damage identification in cantilever beams", Mechanicals Systems and Signal processing, 25, pp. 630-643, 2011.
8. Boumechra N. and Kerdal D.E., "The p-version finite element method using Bezier-Bernstein functions for frames, shells and solids". Journal of Applied Sciences, ISSN 1812-5654, Vol. 11 (6), pp. 2334-2357, 2006.
9. Meyer C.D., "Matrix analysis and applied linear algebra", Society of Industrial and Applied Mathematics (SIAM), 2001.
10. MATLAB, version 7.3 (2006), The MathWorks Inc.: Natick, MA (USA).
11. Computers and Structures Inc. (2007), SAP 2000 - Structural Analysis Program: Non Linear Version 11.0.0, Berkeley, California.