Traffic Load Effect on a Highway Bridge

Z.Kaoutar[1] and K.Tahar[1]
[1] Abdelhamid Ibn Badis University, Materials and Construction Procedures Laboratory of Civil Engineering Department (MCPL), Mostaganem, Algeria

Abstract. Bridge plays an important role in a transportation system, and is directly linked to the country's development as well as people's daily living. However, it is subjected to the damaging effects of the daily traffic and freight trains. Experience shows that bridges with fundamental natural frequencies in the range f=2.0...4.0 Hz respond more strongly to the dynamic action of heavy commercial traffic than other bridges. To achieve better insight into the processes occurring during the passage of a vehicle over a highway bridge with a critical natural frequency dynamic load tests have been performed. This article is about the dynamic analysis of bridges under moving vehicles. In the first phase, dynamic equation of motion with the moving load for simply supported beam is solved by the analytical method of moments and then the results are compared with the time domain method. Secondly experimental analysis is presented in which the beam is divided into 20 stations and the deflection at mid-span is recorded while the mass moves at constant velocity through different stations with the help of oscilloscope. The result obtained is plotted in the form of graphs for different velocities of mass. Finally the dynamic amplification of displacements was extracted and compared with recommendations of current design codes.

1 Introduction

Bridges are lifeline structures. They act, as an important link in surface transportation network and they carry people and vehicles across natural or man-made obstacles. Bridges in service are subjected to the damaging effects due to a combination of various external loads resulting both from the live loads and exposure of the structures to the weather and environmental effects of nature [1], among which a very important load is the traffic load. The dynamic vehicle load information is very important for designing new bridges, assessing the condition of existing bridges, and maintaining old bridges when the applied loads cannot be measured directly, while the responses can be measured easily [2–5], especially when modern railway vehicles become lighter, run faster and carry heavier loads than ever before (Zhai, 2007).

Dynamic effects due to moving loads on bridges are of most concern at shorter spans. They are essentially transient effects. The magnitude of the forcing function will be changing with time and will have a definite beginning and end. Therefore, it is more convenient to analyze bridge dynamic response in the time domain by performing a 'time history' analysis rather than by using a spectral analysis approach in the frequency domain. Furthermore, it is preferable to use recorded wheel data rather to mathematically characterize it and regenerate it using a Monte Carlo simulation approach. Regeneration of continuous records from frequency domain spectral analysis data has been criticized because it 'tends to produce too many peaks' (Elnashai, 1995).

Various commercial finite element method (FEM) programs are available with the ability to perform time history calculations. It is not always easy to model multiple loads which are changing in space and time, and it is useful to consider more economical and simpler alternatives. These may also provide means of obtaining results for a variety of structures relatively quickly and economically. It is possible to analyze the structural response to a particular loading history independently in each of a number of independent modes of vibration, and use the principle of mode superposition to combine them. This would require prior analysis (using FEM or classical theory) to obtain the elastic properties which define each mode of vibration (mode shapes, frequencies, masses) [6].

The importance of investigating the moving loads on the bridge deck was first depicted in the 19th century as a reaction to the collapses of some railway bridges in Great Britain and further research on new techniques for the bridge design had been carried out (Cantieni 1983, 1992; Chan 1988, 1990). In order to evaluate the influence of a passing vehicle on a bridge deck, the dynamic problem is converted into a pseudo-static one with a dynamic amplification factor (DAF) in the design codes. However, the DAF may not always reveal the true dynamic behavior of the bridge. Lee and Park [7] analyzed the characteristics of the error in the force determination in structural dynamic systems, and they proposed a regularization procedure to reduce the force determination error. Tikhonov's regularization method has been used by Busby and Trujillo [8] in a modal based load identification problem. In a more recent work, Busby and Trujillo [9] used a first-order regularization, where the penalty is in terms of the derivative of the force rather than the force itself, and the regularization parameter is determined by the L-curved method [10] and the generalized cross-validation method [11]. Also a time domain method is presented [12] for estimating the discrete input forces acting on a structure based on system Markov parameters.

Research work presented in this article aims to perform a theoretical study of the traffic load effect on a bridge with uncertainties, to develop new methods on dynamic analysis of bridge-vehicle system and to fill the gap of lacking the moving force identification technique. In this paper, the simple moving mass problem is represented with a simple supported beam over which the vehicle load is moving which was described as a combination of whole basis functions. The dynamic analysis of the vibrating beam is done by neglecting the disconnection of the moving mass from the beam during the motion and result is given by considering the mass moving at constant speed and in one direction. It is solved analytically by two different methods: the Method of Moments and the Time Domain Method where the results are compared and then experimentally analyzed, some conclusions finally made.

2 Formulation of the problem using a simply supported beam

The most fundamental problem that should be considered in the study of vehicle-induced vibrations on bridges is the dynamic response of a simply-supported beam subjected to a single moving load Fryba [13], the vehicle can be modeled as two-axle vehicle model, moving masses or moving forces [14]. Two effects are associated with the motion of a vehicle over a bridge, i.e., the gravitational effect and the inertial effect, both related to the mass of the vehicle. For the cases where the mass of the vehicle is small compared with that of the bridge, the vehicle can be represented as a concentrated load, with the inertial effect neglected. This is the so-called moving load model, the simplest case that can be conceived of a moving vehicle [15], in our work the vehicle is modeled as moving forces f(t). The bridge model is shown in figure 1 and the equation of motion is as follows:

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{2}{\rho L}P_n(t) \quad (n = 1,2,\ldots,\infty) \quad (1)$$

Where

$$\omega_n = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho}}, \xi_n = \frac{C}{2\rho\omega_n}, P_n(t) = f(t).\sin\frac{n\pi ct}{L} \quad (2)$$

ω_n : the nth modal frequency

ξ_n : the modal damping ratio

$P_n(t)$: modal load

ρ : the constant mass per unit length

L : span length of bridge

C : the proportional damping

A method based on force identification provides an effective way to solve the above problem. The main idea of this method is using the measured bridge responses to identify the parameters of a bridge-vehicle system, and subsequently to identify the contact forces *f*(*t*).

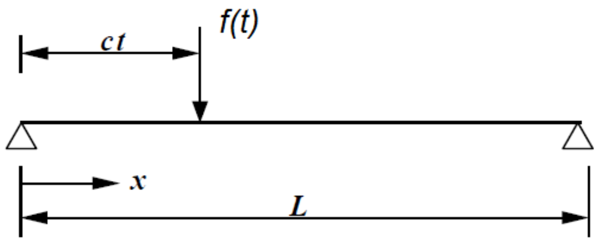

Fig. 1. Moving Load on an Euler's Beam

3 Moving force identification

Various methods were applied on different bridge-vehicle systems were developed to identify the interaction force between bridge and vehicle based on vibration theory and system identification technique and they can mainly be divided into two categories:
- Methods based on finite element method (FEM).
- Methods based on modal superposition technique with a continuous bridge model.

In the last kind, the modal superposition technique is firstly employed to decouple the equation of motion of the bridge and force model to a set of ordinary differential equations. Then the relationship between the moving forces and bridge responses in each mode can be formulated. Finally, the inverse problem can be solved by least-squares estimation with regularization or other optimization methods among which we use in this work are the Method of Moments and the Time Domain Method for a comparative study.

3.1 Method of Moments based algorithm (MOM)

This method was proposed by Yu et al. (2008a, 2008b) in which the moving vehicle loads were described as a combination of whole basis functions, such as the orthogonal Legendre or Fourier series, and the force identification can be transformed into a parameter identification problem.

The dynamic vehicle load *f*(*t*) can be expressed as follows in terms of a series of basis function $\psi_0(t)$, $\psi_1(t)$, $\psi_2(t)$, …, $\psi_n(t)$ (Harrington , 1968).

$$f(t) = \sum_k \alpha_k \psi_k(t) \quad (3)$$

Or in matrix form:

$$f(t) = \psi.\alpha \quad (4)$$

Where $\psi k(t)=Pk(t)$ or $\psi k(t)=sin(k\pi ct/L)$ (Jorgensen, 2004).

The bending moment of a beam is expressed after the use of a test function ω_j as:

$$m(x,t_j) = \sum_{k=0}^{m} \alpha_k . l_k \qquad (j = 0,1,..., N) \tag{5}$$

$$l_{jk} = \sum_{n=1}^{\infty} \frac{2EI\pi^2}{\rho L^3} \frac{n^2}{\omega_n'} \sin\frac{n\pi x}{L} \times \int_0^{tj} e^{-\xi_n \omega_n (t_j - \tau)} \sin\omega_n'(t_j - \tau).$$

$$\sin\frac{n\pi c\tau}{L}\psi_k(\tau)d\tau \tag{6}$$

Equations (5) and (6) can be rewritten in discrete terms and rearranged into a set of equations

$$\underset{(N-1)\times 1}{M} = \underset{(N-1)\times(m+1)}{L} \cdot \underset{(m+1)\times 1}{\alpha} \tag{7}$$

$$\underset{(N-1)\times(m+1)}{L} = \underset{(N-1)\times(N_B-1)}{B} \cdot \underset{(N_B-1)\times(m+1)}{\psi} \tag{8}$$

Where:
Ψ: the matrix of basic functions
M: the time-series vector of the measured bending moment responses
α: the coefficient vector
- If N-1=m+1, the coefficient α can be obtained directly by solving Equation (7).
- If N-1>m+1 *or* N-1<m+1, the least-squares method can be used to find the coefficient α.

Substituting α into Equation (4), the time history of the moving loads can be obtained finally.

3.1 Time Domain Method (TDM)

This method was firstly proposed by Law et al. (1997) in which the relationship of moving axle force and modal response is formulated by convolution integral. The discrete form of equation of motion of the system for each vibration mode can be obtained by assuming the time series of moving forces to be step functions in small time intervals. The time varying forces on a simply supported beam can be identified by solving the resulting discrete equations. The application of this method on identifying the moving forces on a multi-span continuous bridge was investigated by Zhu and Law (2000, 2001a, 2002b). The research was also extended to study the possibility of identifying axle loads when applied to real bridge-vehicle system with road surface roughness and incomplete vehicle speed. Experimental tests showed that the method can identify individual axle loads travelling at non-uniform speed with small error (Zhu and Law 2003c). The effect of bearing stiffness on the bridge support was also included in this MFI procedure by Zhu and Law (2006).

Solving the equation of motion of the bridge Eq. (1) and the dynamic deflection of the beam at point and time Eq. (6) in time domain can be obtained by deriving the same procedure of the MOM using a system of equation, and then be solved by many regularization methods as the least-squares method in time domain and Tikhonov regularization.

4 Numerical simulations

In order to confirm the accuracy of the developed numerical model, a simply supported beam at two opposite edges and subjected to two moving vehicle loads is simulated and illustrated.

4.1 Bridge and moving force model

The information below gives details of the material properties and the moving force:
Time-varying loads:
$f1(t) = 58\,800\times[1+0.1\sin(10\pi t)+0.05\sin(40\pi t)]$ N
$f2(t) = 137\,200\times[1-0.1\sin(10\pi t)+0.05\sin(50\pi t)]$ N
ls = 8 m
EI =1.27914×1011N·m2
ρ =12 000 kg/m
L=40m
f1 =3.2 Hz, f2 =12.8 Hz, f3 =28.8 Hz
c=40m/s
Only the three first modes of the beam are included in the calculation because the analysis frequency is in the range 0 to 40 Hz

Random noise is added to the calculated responses to simulate the polluted measurements as one in Ref (Yu 2002). The Fourier basis functions are only adopted for the MOMA in the following simulation. The MOMA is used to identify both the two axle constant and time-varying loads from bending moment and/or acceleration responses at 1/4, 1/2, and 3/4 spans in twelve combination cases.

4.2 Simulation results

Table 1 shows the comparison on the RQPE values of two axle constant loads identified by both the TDM and MOMA under the 5% noise level as well as including the effect of two different solutions, i.e. the SVD and regularization solutions. Selecting four out of twelve combination cases, Table 2 gives the comparison on the RQPE values of two axle time-varying loads identified by TDM and MOMA when the SVD solution is adopted only. In addition, the effect of different noise levels on the RQPE values is also considered.

Table 1. Comparison of RQPE of two axle constant loads under 5% Noise using the regularization and the SVD solutions for different sensor locations.

TDM				MOM			
Axle 1		Axle 2		Axle 1		Axle 2	
SVD	Reg	SVD	Reg	SVD	Reg	SVD	Reg
*	36,5	*	28,5	1,06	0,76	0,25	0,05
*	34,4	*	27,6	0,79	0,39	0,37	0,04
55,8	14,1	25,8	10,9	0,18	0,18	0,24	0,24
2,58	2,58	1,40	1,40	0,10	0,10	0,21	0,21
*	35,0	*	24,6	0,26	0,26	0,15	0,15
*	25,2	*	23,2	0,13	0,13	0,11	0,11
55,0	16,6	25,9	10,8	0,04	0,04	0,18	0,18
*	28,2	*	23,5	0,17	0,17	0,20	0,20
62,8	14,6	28,2	11,9	0,25	0,25	0,20	0,20
*	38,9	*	25,5	0,41	0,41	0,18	0,18
*	29,8	*	22,2	0,23	0,23	0,13	0,13
53,2	16,6	24,9	10,2	0,14	0,14	0,22	0,22

** refers to the errors exceeding 100%*

Table 2. Comparison of RQPE of two axle time varying loads identified via SVD for different sensor locations.

	Noise					
	1%		5%		10%	
	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2
TDM	97,8	55,4	*	*	*	*
MOM	7,35	1,81	36,7	9,03	73,5	18,1
TDM	*	29,6	*	*	*	*
MOM	4,45	1,50	22,3	7,50	44,5	15,0
TDM	31,5	22,1	*	*	*	*
MOM	1,31	0,76	6,54	3,81	13,1	7,62
TDM	0,93	0,63	4,66	3,13	9,30	6,25
MOM	0,86	0,31	4,29	1,56	8,58	3,11

** refers to the errors exceeding 100%*

4.3 Results discussion

We can observe from the obtained results of both tables that the MOMA results are obviously better than the TDM ones whether for two constant loads or for two time-varying loads.

For the cases of two axle constant load identification, the RQPE values by the MOMA are very low and less than 1.06% for all twelve cases in Table 1. They are dramatically lower than the RQPE values by the TDM. It shows that the MOMA is a very good identification method, which is especially suitable for two axle constant load identification.

Compared the SVD results with the regularization results, it can be found from Table1 that the RQPE values for all cases, except for the case of 1/4a&1/2a&3/4a, are significantly reduced if the regularization solution are adopted instead of the SVD solution for the TDM. For the MOMA, the RQPE values are also significantly improved when the bending moment responses are only used to identify the two moving loads. However, when only the acceleration responses, or the combination of acceleration and bending moment responses are used to identify the two moving loads, the RQPE values are close to each other whether the SVD or the regularization solution is adopted.

For case comparison, Table 1 also shows that, the more the measurement station is, or the more the number of measured acceleration involved is, the better the identified results are. It shows that adopting more responses for two moving load identification is beneficial to both the TDM and the MOMA. From Table 2, it can be seen that the more the number of bending moment responses replaced with acceleration responses is, the better both the TDM and the MOMA results are. The best sensor arrangement is when all three sensors are accelerometers, i.e. 1/4a&1/2a&3/4a, for both the two methods.

It can also be found from Table 2 that the RQPE values are almost proportional to the noise levels. Obviously, the MOMA identification accuracy is higher than the TDM accuracy for each case. It shows that the MOMA immunity to the noise is higher than the TDM immunity when 1%, 5% and 10% noise were added into the responses. In other words, the proposed MOMA method is more suitable for identification of moving loads from the measured response signals contaminated by measurement noise.

5 Effects of Different Solutions on MOM

Figure 2 illustrates a comparison on the identified moving forces due to the two solutions for MOMA. Basically, the regularization results are in agreement with the SVD results except for the moment at the beginning and the end of time histories of moving forces as well as the moment at the accessing and exiting of vehicle. It shows that the fluctuation of identified moving forces can be effectively bounded at the moment mentioned above if the Regularization solution is adopted to solve the system equation for MOMA. The identified results by the Regularization solution are obviously improved. They are clearly better than the results by the SVD solution and more reasonable in practice.

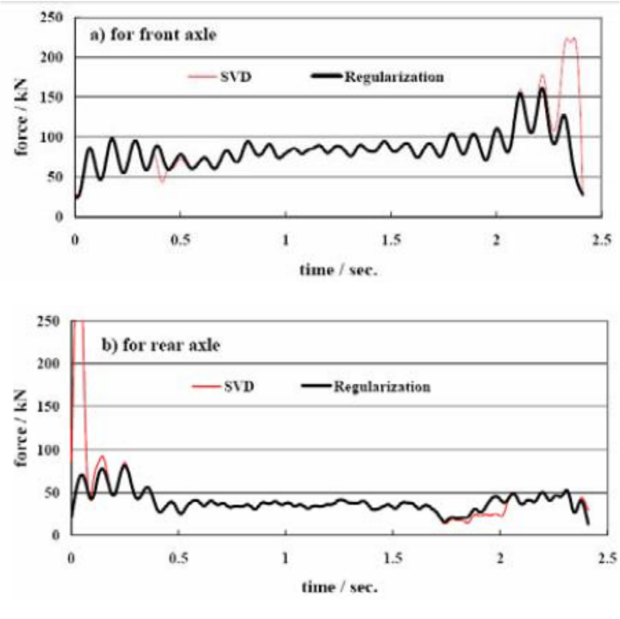

Fig. 2. Effect of two solutions on moving forces for MOMA

6 Experimental analysis of the vehicle load effect on bridge structure

The objective of this test is to show experimentally the effects of moving force on the modal parameters of the bridge. The dynamic response characteristic of a simple beam bridge that is likely to be of most concern is that in bending.

As is usual, acceleration responses were employed because of the simplicity of instrumentation. A referencing and digital oscilloscope technique was adopted in the present experiments for a simply supported beam shown in figure 3.

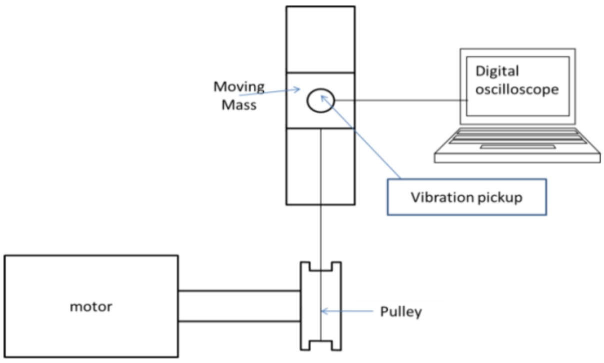

Fig. 3. Experimental setup showing different equipment's.

Figure 4 shows the beam model which is of 1m length 5cm breadth, 0.5cm width, E=200GPa and the mass per unit length is 3kg/m is divided into 20 stations and a vibration pickup is attached at mid-span. The moving mass is 0.9kg and 1.8kg, the velocity of mass is 1, 2.5, 5, and 7 m/s. The vibration pickup is connected to the digital oscilloscope which shows the wave pattern generated on the screen. Amplitude of vibration or deflection at mid span of the beam can be recorded from the oscilloscope.

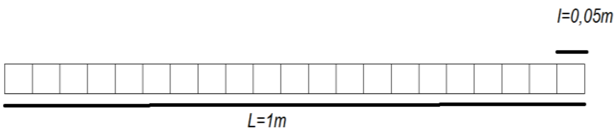

Fig. 4. Simple supported beam model.

Figures 5 and 6 show the beam deflection at mid span for moving mass traversing through different stations, results are arranged in graphs for different mass and speed values.

The results show that the maximum deflection of beam increase with the velocity and mass increasing, we can observe also that the position of maximum deflection deviates from mid-span of the beam.

Practical bridge design codes usually provide load models which will provide 'nominal' load effects which have some pre-determined probability of exceedence. If the load model has been derived separately for static and dynamic effects, there remains the problem of combining the two analysis results into a single design model, which is related in some pre-determined manner to the statistically determined extreme of the joint effects of static and dynamic loading. It does appear that, for most practical structures, dynamic magnification or reduction of static load effects is caused mainly by the effects of uneven road profile. To a first approximation, therefore, the DAF is a unique (although uncertain) property of each bridge (or, at least, of the transit of each individual type of vehicle). Thus, the extreme static load effect will be a function of the lifetime exposure of the bridge to traffic, but the extreme dynamic load effect will be a property of the bridge. When the Highways Agency's (1997) assessment rules were developed, it had to be assumed that there were generally no site specific strain records, and the uncertainty in DAF was treated as a structural property. After much consideration, the rules were finally based on reviewing variations in static load effects derived from a large number of continuous wheel load measurements from a set of vehicles which was broadly representative of the types of vehicle in common use in the UK.

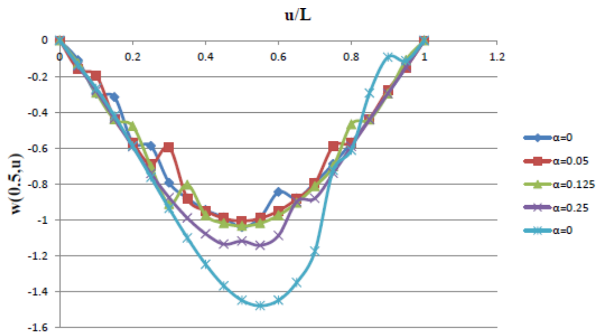

Fig. 5. Mid-span deflection of a simple supported beam traversed by a moving mass (0.9kg) at different values of α.

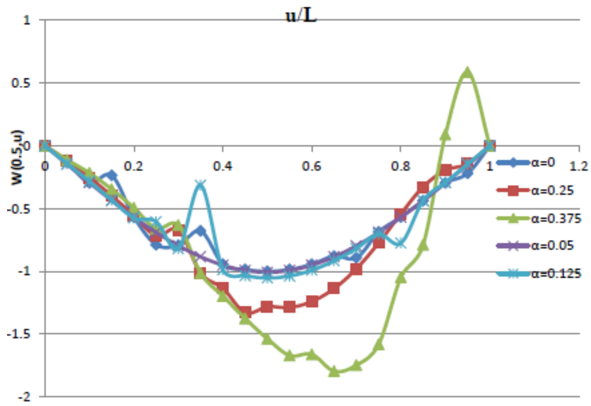

Fig. 6. Mid-span deflection of a simple supported beam traversed by a moving mass (1.8kg) at different values of α.

6 Conclusions

A comparative study was presented in this work between two numerical methods: the Method of Moments and the Time Domain Method in order to solve the dynamic equation of a simple supported beam to identify moving loads on a bridge deck based on the measured responses. The bridge deck is modeled as a homogeneous beam and the loads are modeled as moving forces. Simulations and experimental studies give the following conclusions:

- The factors that need to be considered in analyzing the response of the VBI systems include the dynamic properties and driving frequencies of the moving vehicles, and the dynamic properties and surface roughness of the bridge
- Even though vehicle models of higher complexities, e.g., those consisting of dozens of DOFs, can be employed in studying the VBI problems nowadays, the use of simplified vehicle and bridge models is helpful, since it allows us to identify the key parameters dominating the dynamics of the VBI systems.
- The proposed MOMA is a successful method for the identification of moving loads from the responses induced by the moving vehicles on bridges.
- The MOMA is obviously better than the existed TDM from all the aspects, especially for the constant load identification cases.
- The MOMA can give satisfactory results with higher accuracy and computation efficiency when whether the SVD or regularization method is used.
- The basis function terms play an important role in the MOMA. The different patterns and the number of basis function can lead to different computation efficiency, therefore, they should be properly selected and appropriately determined in order to keep the MOMA more effective.
- The MOMA has higher computation efficiency and better flexibility than the TDM. When the Fourier series are adopted as the basis function of the MOMA.
- As a feasible and reasonable identification method, the MOMA should be firstly recommended as a practical method of moving force identification in situ.
- The experimental study shows that the position of maximum deflection of beam occurs far from mid span.
- The dynamic response of beam is more influenced by the mass speed changing.
- Identification using bending moment will give better result as compared with that using displacement in design codes.

References

1. W. Radomski, *Bridge Rehabilitation*, 11 (2002)
2. Yen, C.S , and Wu, On the Inverse Problem of Rectangular Plates Subjected to Elastic Impact, Part I: Method Development and Numerical Verification, ASME J. Appl. Mech, **62**(3) 692–698 (1995)
3. Yen, C.S and Wu, On the Inverse Problem of Rectangular Plates Subjected to Elastic Impact, Part II: Experimental Verification and Further Application, ASME J. Appl. Mech, **62**(3) 699–705 (1995)
4. Choi, K. Y., and Chang, X,Identification of Impact Force and Location Using Distributed Sensors, AIAA J., **34**(1)136–142 (1995)
5. Moller, P. W., Load Identification Through Structural Modification, ASME J. Appl. Mech., 236–241(1999).
6. A.J.Kappos, *Dynamic Loading and Design of Structures* (Spon Press, 2002)
7. Busby, H. R., and Trujillo, Solution of An Inverse Dynamics Problem Using an Eigenvalue Reduction Technique, Comput. Struct., **25**(1), 107–117 (1987)
8. Busby, H. R., and Trujillo, Optimal Regularization of Inverse Dynamics Problem. Comput. Struct., **63**(2), 243–248 (1997)
9. Hansen, P. C., Analysis of Discrete Ill-Posed Problems by Means of the L-Curve, SIAM Rev., **34**(4), 561–580 (1992)
10. Golub, G. H., Heath, M., and Wahaba, G., Generalized Cross-Validation as a Method for Choosing a Good Ridge Parameter, Technometrics, **21**(2), 215–223 (1979)
11. Kammer, D. C., Input Force Reconstruction Using A Time Domain Technique, ASME J. Vibr. Acoust., **120**(4), 868–874 (1998)
12. O'Connor, C., and Chan, T. H. T., , ''*Dynamic Wheel Loads*,(1988)
13. Fryba. L , *Vibration of Solids and Structures under Moving Loads*, (Noordhoff International Publishing, 1972).
14. B.Sc, M.Sc, *Bridge-Vehicle System and the Interaction Force Identification*, (Pao Yue-kong Library, 2010)
15. Y. B. Yang, J. D. Yau and Y. S. Wu, *Vehicle–Bridge Interaction Dynamics: With Applications to High-Speed Railways*, (World Scientific Publishing Co. Pte. Ltd, 2004)