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ABSTRACT
From the conjugacy classes in the Symmetric inverse transformation semigroup, we obtained its nilpotent conjugacy classes. A general expression was obtained for the number of nilpotent conjugacy classes in the Symmetric inverse transformation semigroup.

Keywords: Nilpotent conjugacy classes, Symmetric inverse transformation semigroup

1. PRELIMINARIES

Let $X_n = \{1, 2, \ldots, n\}$. Then a (partial) transformation $\alpha: Domain\,\alpha \subseteq X_n \rightarrow Im\alpha$ is said to be full or total if $Dom\ \alpha = X_n$, otherwise it is called strictly partial.

The set of all partial transformation on n-object forms a semigroup under the usual composition of functions. It is denoted by P_n, when it is strictly partial, T_n when it is full or total and I_n when it is partial 1-1(or the symmetric inverse).

An element $\alpha \in S$ is nilpotent$(\alpha^n = 0)$ for some $n > 0$. A property of nilpotent element among others is $x\alpha \neq x, \forall x \in X_n$, where α is nilpotent.

Let $\alpha, \beta \in I_n$, then chart α is conjugate to chart β if and only if α and β have the same path structure

2. NILPOTENT CONJUGACY CLASSES IN THE SYMMETRIC INVERSE TRANSFORMATION SEMIGROUP

The following nilpotent conjugacy classes are arranged according to the number of their images in any number of I_n.

I_n	Number of images	Conjugacy classes
When n = 1	No image	(1]

Total nilpotent conjugacy classes = 1

I_n	Number of Image	Conjugacy classes
When n = 2	No Image	(1](2]
	1 Image	(12]

Total nilpotent conjugacy classes = 2

I_n	Number of Image	Conjugacy classes
When n = 3	No Image	(1](2](3]
	1 Image	(12](3]
	2 Images	(123]

Total nilpotent conjugacy classes = 3

I_n	Number of Image	Conjugacy classes
When n = 4	No Image	(1](2](3](4]
	1 Image	(12](3](4]
	2 Images	(12](34], (123](4]
	3 Images	(1234]

Total nilpotent conjugacy classes = 5

I_n	Number of Image	Conjugacy classes
When n = 5	No Image	(1](2](3](4](5]
	1 Image	(12](3](4](5]
	2 Images	(12](34](5], (123](4](5]
	3 Images	(123](45],(1234](5]
	4 Images	(12345]

Total nilpotent conjugacy classes = 7

I_n	Number of Image	Conjugacy classes
When n = 6	No Image	(1](2](3](4](5](6]
	1 Images	(12](3](4](5](6]
	2 Images	(12](34](5](6], (123](4](5](6]
	3 Images	(12](34](56], (123](45](6], (1234](5](6]
	4 Images	(123](456],(1234](56], (12345](6]
	5 Images	(123456]

Total nilpotent conjugacy classes = 11

I_n	Number of Image	Conjugacy classes
When n = 7	No Image	(1](2](3](4](5](6](7]
	1 Image	(12](3](4](5](6](7]
	2 Images	(12](34](5](6](7], (123](4](5](6](7]
	3 Images	(12](34](56](7], (123](45](6](7], (1234](5](6](7]
	4 Images	(123](45](67], (123](456](7], (1234](56](7], (12345](6](7]
	5 Images	(1234](567], (12345](67], (123456](7]
	6 Images	(1234567]

Total nilpotent conjugacy classes = 15

3. RESULTS

From the enumeration above, a summary of the sequence of the number of nilpotent conjugacy classes of I_n is listed below

1, 2, 3, 5, 7, 11, 15, … where n = 1, 2, …
Let a(n) be the number of nilpotent conjugacy classes in I_n

a(n) is the number of partitions of n(the partition numbers)which is generally given as

$a(n) = \frac{1}{n}\sum_{k=0}^{n-1}[S(n-k)a(k)]$, where

$a(0) = 1$ and $S(k)$ is the sum of divisors of k.

For sum of divisors of n, for example,
$S(8) = 1 + 2 + 4 + 8 = 15$.

For higher values of k, we use the formula

$S(k) = \prod_{i=1}^{m} \frac{p_i^{r_i+1}}{p_i - 1} : k = p_1^{r_1} p_2^{r_2}, \ldots p_m^{r_m}$

where the ps are distinct primes.

$$a(4) = \frac{1}{4}(7 \times 1 + 4 \times 1 + 3 \times 2 + 1 \times 3) = \frac{1}{4}(20) = 5$$

4. CONCLUSION

It has been shown that the number of nilpotent conjugacy classes in I_n for $n \geq 1$, can be calculated using the formula:

$a(n) = \frac{1}{n}\sum_{k=0}^{n-1}[S(n-k)a(k)]$, where

$a(0) = 1$ and $S(k)$ is the sum of divisors of k.

For higher values of k, we use the formula

$S(k) = \prod_{i=1}^{m} \frac{p_i^{r_i+1}}{p_i - 1} : k = p_1^{r_1} p_2^{r_2}, \ldots p_m^{r_m}$

where the ps are distinct primes.

REFERENCES

[1] Cadogan, C. C. (1971), On partly ordered partitions of a positive integer, Fibonacci Quart. 9, no 3, pp.329-336

[2] Charles, V. E. (2001), Elementary number theory, 2nd Edition, Mcgraw-Hill, New York, pp.81

[3] Gupta et al, H. (2002), Tables of partition, Royal Society Mathematical tables, Vol. 4, Cambridge University press, pp. 90

[4] Howie, J. M. (1976), An introduction to semigroup theory, Academic press London, pp.7-10

[5] Lipscomb, S. (1995), Symmetric inverse semigroup, Mathematical surveys and monographs, Volume 46, pp.1-15

[6] Sloane, N. J. A. (1973), A handbook of integer sequences, Academic press, London.

[7] Sloane, N. J. A. (1995), The encyclopaedia of integer sequence*s*, Academic press, London.

[8] www.Integer Sequences.com

[9] Umar, A. (2000), Some combinatorial problems in the theory of transformation semigroup, American Mathematical Society

[10] Wikipaedia, the free encyclopaedia online

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