

Study of the impact of dielectric constant perturbation on electromagnetic wave propagation through material medium: MathCAD solution

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ABSTRACT

We present the study of dielectric constant and its influence on wave propagation through an inhomogeneous material medium via mathcard approach. Mathcard was used to solve numerically the wave equation involving a perturbation in term of dielectric constant $\varepsilon_p(r) = \Delta\varepsilon_p(r) + \varepsilon_{ref}$ imposed on scalar wave equation. $\psi''(r) + \omega^2\mu_0\ \varepsilon_0\ \varepsilon_p(r) = 0$; The equation was reduced to a form suitable for numerical solution using mathcard on which we applied various values of dielectric constant perturbation for three regions or electromagnetic spectrum VIS UV, optical near infrared region. The correlation between the field profile and the propagation was analyzed.

Keyword: dielectric constant, perturbation, numerical solution, mathcard, scalar wave equation, electromagnetic spectrum, correlation, propagation distant, material medium.

INTRODUCTION

Analytic study of optical field propagating through an inhomogeneous material media have been presented only for a few specific geometries with a number of schemes for further study of the optical effect of layered. Inhomogeneous medium has been attempted (Wait, 1970; Brekhovakikh, 1980). Such a scheme as Abeles 2 x 2 propagation matrix, Jones 2 x 2 and Berreman 4 x 4 propagation matrices has been utilized. (Abele, 1950; Jones 1941; Azzum, 1972) for analytical study of the geometrical optical approximation. Also, generalized geometrical optic approximation, phase integral and perturbation theory have been applied (Ong, 1993, Budden, 1966, Ugwu, 2010).

However, one concept is paramount for its application. This is the concept of definition of generalized field vector in propagation matrix and perturbation term that depends on the dielectric parameters and other optical and solid state parameters of the material medium. The propagation distance of the medium depicting the thickness of the material ingestion and the propagation vector of the incident field is taken into consideration unless where approximation value is necessary (Ugwu, 2007: 2010).

In all, beam propagation method has remained a very good technique for analytical study of wave propagation through various material medium since it provides a unified treatment of various guided and radiation field problems subject to paraxiality and spatial frequency content of the index profile (Thylen, 1986).

It can also be used to analyze non linear directional coupler operation for various combinations of non linear materials and initially mismatched guide. (Steyeman, 1985; Jensen, 1982). This is because the technique involves propagating the input beam over small distance through homogeneous space in the material medium and then analyzing the influence of variation of the solid state properties of the material such as refractive index and dielectric constant on the propagating wave (Fleck *et al.,* 1976; Feit, 1979; Ugwu, 2011, Yeh, 1979). A number of derivations of the method adopted amenable to specific problems have been made possible using beam propagation method (Roay, 1981, Ugwu *et al.,* 2011, Yevick *et al.,* 1985, Kim *et al.,* 1990, Brykhovetskii *et al.,* 1985, Tatarskii, 1979, Rytov *et al.,* 1987).

THEORETICAL FRAMEWORK

We begin by defining scalar wave equation

Given

$$\psi''(r) + \omega^2 \mu_0 \varepsilon_0 \varepsilon_p(r) \psi(r) = 0$$

Which is a second order differential equation represent a scalar wave equation with a perturbation due to its propagation through a medium with a dielectric function,

$$\varepsilon_p(r) = \Delta\varepsilon_p(r) + \varepsilon_{ref} .$$

$$\therefore \psi''(r) + \omega^2 \mu_0 \varepsilon_0 \left(\Delta\varepsilon_p(r) + \varepsilon_{ref}\right)\psi(r) = 0$$
$$\Rightarrow \psi''(r) + \omega^2 \mu_0 \varepsilon_0 \Delta\varepsilon_p(r)\psi(r) = -\omega^2 \mu_0 \varepsilon_0 \varepsilon_{ref} \psi(r)$$

Assuming $\varepsilon_{ref} = 0$ and $-V(r) = k_0^2 \Delta\varepsilon_p(r)$, $k_0^2 = \omega^2 \mu_0 \varepsilon_0 = \left(\frac{2\pi}{\lambda}\right)^2$

$$\psi''(r) - V(r)\psi(r) = 0$$

Hence $V = k_0^2 \Delta\varepsilon_p(r)$ with the perturbed term contained in V, we transform the equation in the form that makes suitable for Mathcard solution.

Let $x_1 = \psi \Rightarrow x_1'' - Vx_1 = 0,\ x_1' = \frac{dx_1}{dr}$.

Let $x_1' = x_2$, the differential equation becomes;

$$\begin{cases} x_1' = x_2 \\ x_2' = Vx_1 \end{cases} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 0 & x_2 \\ Vx_1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The form of this equation is suitable for numerical solution by MathCAD.
In the solution, we considered three different values associated with the value of dielectric perturbation $\Delta\varepsilon_p$:

$\Delta\varepsilon_p = 0.5; 3.5$ and 10.50 which represents negligible, limited and strong absorbing medium. In the mathcard numerical solution as used here we considered the ultraviolet, optical and near infrared region of electromagnetic wave spectrum.

MathCAD Solution

$r_0 = 0$ $r_1 = 150$ Solution interval endpoints (5)

$ic = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ Initial condition vector (6)

$N = 1500$ Number of solution values on (t_0, t_1) (7)

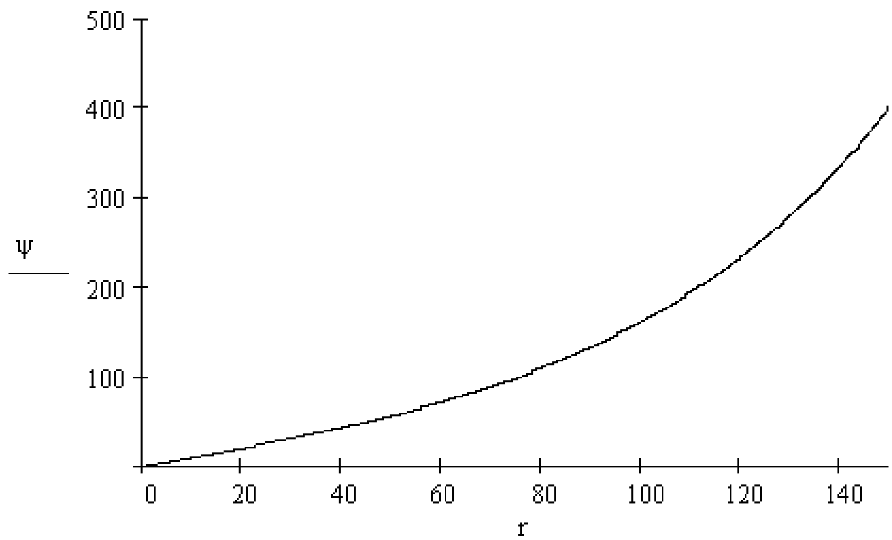
$\mathrm{D}(r, X) = \begin{bmatrix} X_1 \\ 4.\dfrac{\pi^2}{\lambda^2}.\varepsilon.X_0 \end{bmatrix}$ Derivative function (8)

Solution matrix:

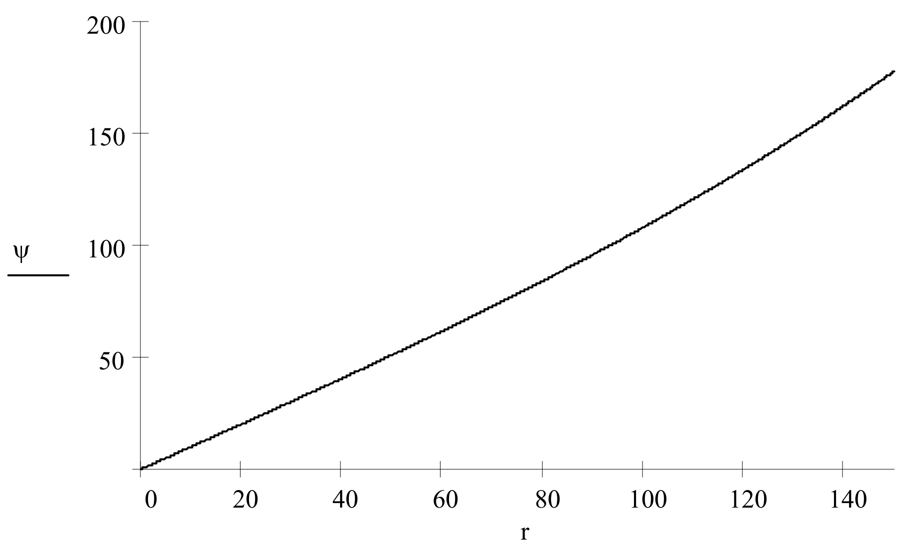
S = rkfixed (ie, r0, N D)

$r = S^{<0>}$ Independent variable values

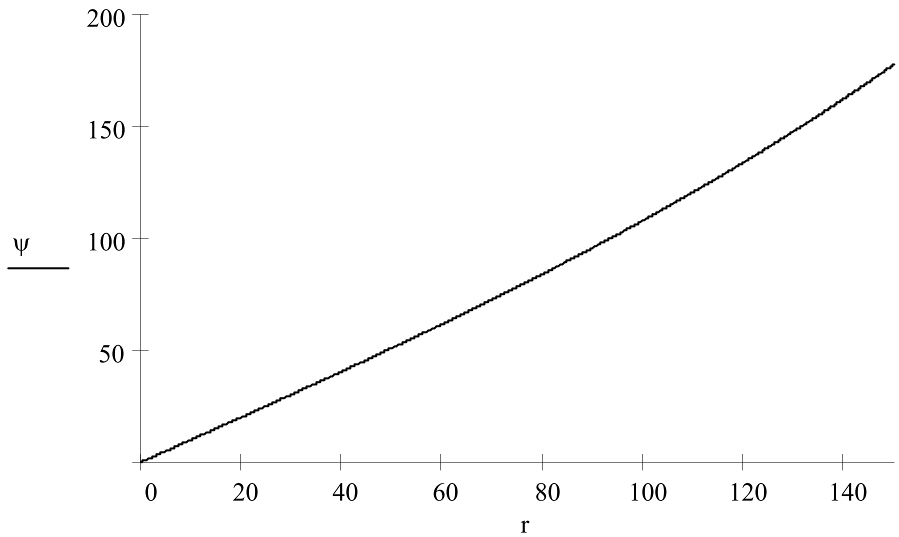
$\psi = S^{<1>}$ Solution function values



Fig;1 Field,Ψ VS Propagation distant for $\Delta\varepsilon_p$= 0.5 within UV region



Fig;2. Field,Ψ VS Propagation distant for $\Delta\varepsilon_p$= 0.5 within optical region



Fig;3. Field,Ψ VS Propagation distant for $\Delta\varepsilon_p$= 0.5 within near infrared region

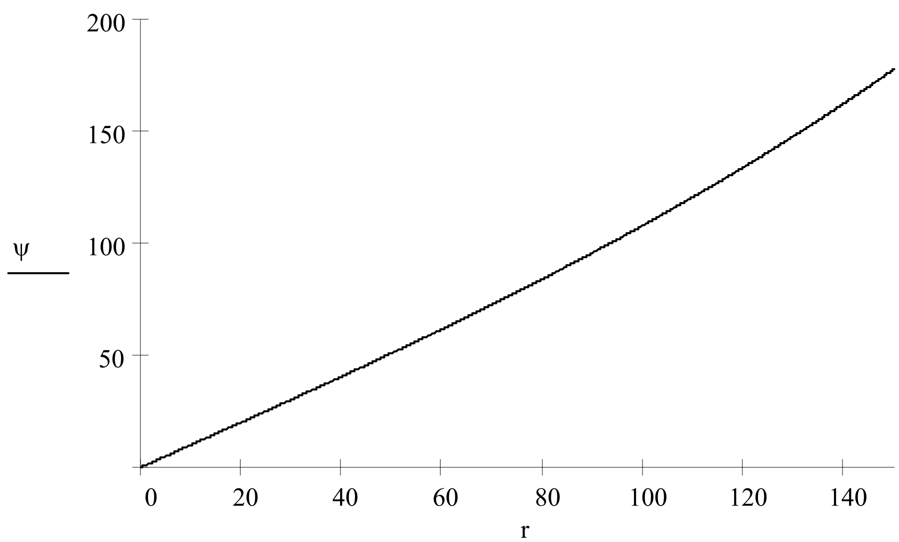
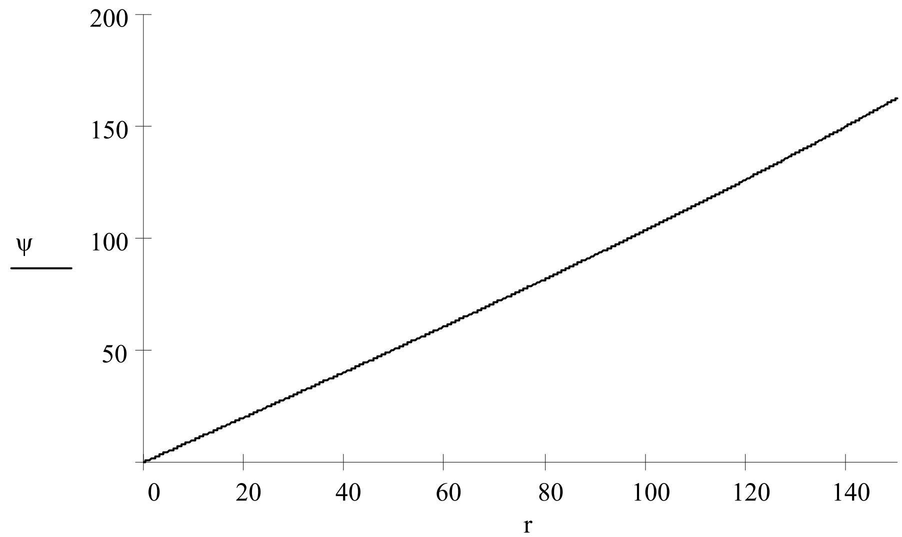


Fig.4 Field,Ψ VS Propagation distant for $\Delta\varepsilon_p$= 3.5 within UV region



Fig;5. Field,Ψ VS Propagation distant for Δεp= 3.5 within near UV region

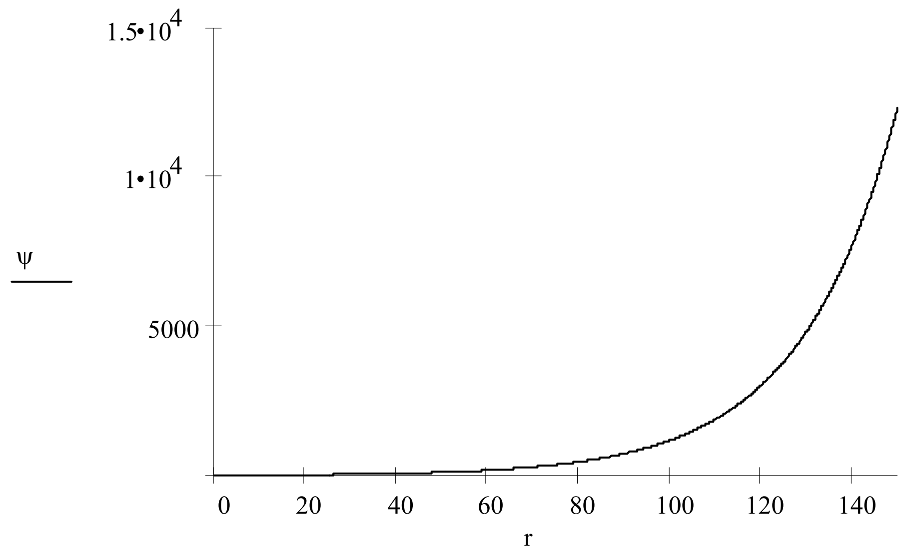


Fig.6 Field,Ψ VS Propagation distant for Δεp= 3.5 within near infrared region

$\lambda := 650 \qquad \varepsilon := 3.5$

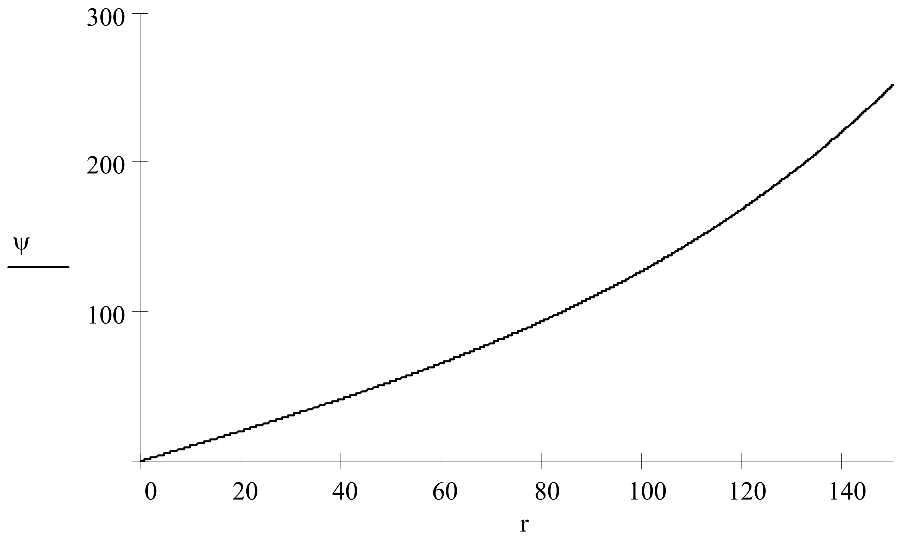


Fig.7 Field,Ψ VS Propagation distant for $\Delta\varepsilon_p$= 10.5 within UV region

$\lambda := 950 \qquad \varepsilon := 3.5$

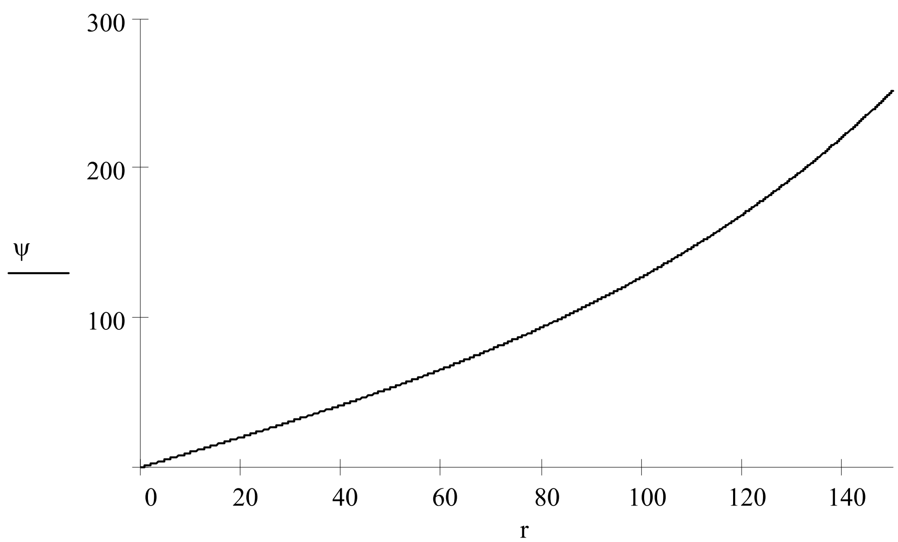
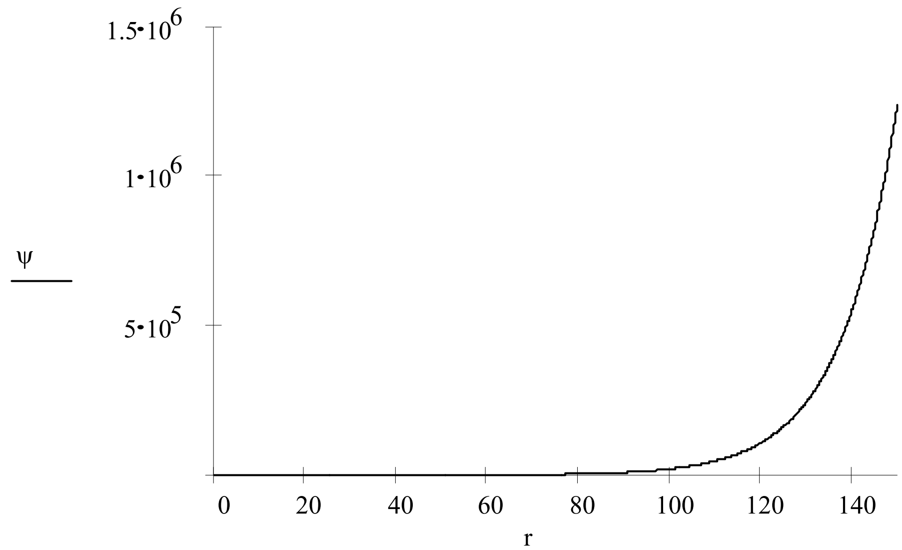


Fig.7 Field,Ψ VS Propagation distant for $\Delta\varepsilon_p$= 10.5 within UV region



Fig;9 Field,Ψ VS Propagation distant for $\Delta\varepsilon_p$= 10.5 within near infrared region

RESULT OF DISCUSSION

Figure 1 to figure 9 depict the behaviour of propagated wave profile through the medium against the propagation distance presented in accordance with the values of dielectric perturbation.

In fig 1, the field profile when $\Delta\varepsilon_p = 0.5$ within UV region appear parabolic in nature but tends to a straight-line with the optical and near infrared region as in fig 2 and 3.

When the dielectric perturbation increases to 3.5, the field behaviour within the UV region remained constant up to 49nm of the propagation distant after which it rise up sharply to maximum of 1.3 x 10^4 within 140nm of the propagation distance as in fig. 4. In fig 5 and 6 depict the field profile within optical and near infrared region respectively, the correlation between the field and the propagation distant tends to increase for $\Delta\varepsilon_p = 10.5$ which can note the strong absorption value, the field behaviour within the UV increases sharply after 129nm of the propagation distant and begins to experience a slight increment on correlation between the field profile and propagation distant for both optical and near infrared region. comparing the work with the one obtained using theoretical solution of the scalar wave equation, one observed that the wave profile exhibited a defined behaviour in relation, the propagation distance unlike the results from the theoretical solution technology and W. K. B approximation which only depicted the oscillatory pattern of the wave as it propagates through the medium (Ugwu *et al.,* 2011, Ugwu and Maliki, 2011).

Another interest phenomenon is the correlation between the field profile within the UV region for $\Delta\varepsilon_p, = 0.5, 3.5$ and 10.5. One observes that the correlation increases with the increase in the value of perturbed dielectric constant. Unlike the field profile behaviour in the case of the theoretical result whereby the oscillatory wave pattern is dampened as $\Delta\varepsilon_p$ increase especially within the near infrared region (Ugwu and Maliki 2011).

CONCLUSION

The study of this work, we have presented the propagation behaviour of wave through a medium with variation of dielectric perturbation using numerical solution by Mathcard. Though other method such as theoretical solution of scalar wave equation and W. K B approximation teeth: we has been used, all exhibited different wave profile uniquely in relation to the propagation distant for various values of $\Delta\varepsilon_p$, However, numerical solution by mathcard exhibited greater correlation between the wave profile and propagation distant than the theoretical solution of scalar wave equation and W. K. B approximation technique. This formalism allowed the understanding of how the dielectric constant perturbation influences the correlation between the filed profile and propagation distance which signifies the thickness of the medium through which the wave propagate.

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