Radiation and Magnetic field effects on Unsteady Natural Convection Flow of a Nanofluid Past an Infinite Vertical Plate with Heat Source

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Abstract
In this study, we analyze the effects of thermal radiation and magnetic field on unsteady natural convective flow of a nanofluid past an impulsively started infinite vertical plate in presence of heat source. The nanofluid contains Copper, Nickel, Zinc Oxide nanoparticles with water as base fluid. The partial differential equations governing the flow are solved numerically by Laplace Transform Technique. The effects of various parameters on velocity and temperature profiles examined and presented graphically. It is found that the increase in magnetic field causes the decrease in fluid velocity, fluid velocity and temperature profiles are more influenced by Radiation, Heat source and Volume fraction of the nanoparticles. Shape of nanoparticles does not show effect on the velocity of the fluid.

Key Words: Heat transfer, Nanofluids, Thermal radiation, MHD, Heat source, Volume fraction.

Introduction
Unsteady natural convection flow of a nanofluid past an impulsively infinite vertical plate in presence of radiation and magnetic field have received a lot of attention in the field of several industrial, scientific, and engineering applications in recent years. Nanofluids have many applications in the industries since materials of nanometer size have unique chemical and physical properties. With regard to the sundry applications of nanofluids, the cooling applications of nanofluids include silicon mirror cooling, electronics cooling, vehicle cooling, transformer cooling, etc. This study is more important in industries such as hot rolling, melt spinning, extrusion, glass fiber production, wire drawing, and manufacture of plastic and rubber sheets, polymer sheet and filaments, etc. among the tasks facing by the engineer is the development of ultrahigh-performance cooling in many industrial technologies. This is where nanotechnology takes important part for further development of high performance, compact, cost-effective liquid cooling systems.


Takhar et al. (1996) studied the radiation effects on the MHD free convection flow of a gas past a semi-infinite vertical plate. Ghaly (2002) considered the thermal radiation effect on a steady flow, whereas Rapits and Massalas (1998) and El-Aziz (2009) analyzed the unsteady case. Sattar and Alam (1994) presented unsteady free convection and mass transfer flow of a viscous, incompressible, and electrically conducting fluid past a moving infinite vertical porous plate with thermal diffusion effect. Radiation convection interaction problems are found in consideration of the cooling of high temperature components, convection cells and their effect on radiation from stars, furnace design where heat transfer from surfaces occurs by parallel radiation and convection, the interaction of incident solar radiation with the earth’s surface to produce complex free convection patterns and thus to complicate the art of weather forecasting and marine environment studies for predicting free convection patterns in oceans and lakes this was discussed by Siegel and Howell (1981). Hady (2012) analysed radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet.

Yao et al. (2011) have recently investigated the heat transfer of a viscous fluid flow over a permeable stretching/shrinking sheet with a convective boundary condition. Magyari and Weidman (2006) analyzed the heat transfer characteristics on a semi-infinite flat plate due to a uniform shear flow, both for the prescribed surface temperature and prescribed surface heat flux. It is worth pointing out that a uniform shear flow is driven by a viscous outer flow of rotational velocity whereas the classical Blasius flow is driven over the plate by an in viscid outer flow of irrotational velocity. Sandeep et. al (2013) discussed Radiation effects on an Unsteady Natural Convective Flow of a EG-Nimonic 80a Nanofluid Past an Infinite Vertical Plate and they concluded that nanofluid velocity influenced by radiation parameter. Sandeep and Sugunamma (2014) studied Radiation and
Inclined Magnetic Field Effects on Unsteady Hydromagnetic Free Convection Flow past an Impulsively Moving Vertical Plate in a Porous Medium.

To the author’s knowledge, no studies have been communicated so far with regard to radiation and magnetic field effects on an unsteady natural convective flow of a nanofluid past an infinite vertical plate. The objective of this paper is to analyze the effects of radiation and magnetic field on transient natural convective flow of nanofluid past an infinite vertical plate in presence of heat source. The unsteadiness is caused by the impulsive motion of the vertical plate. The present study is of immediate application to all those processes which are highly affected with heat enhancement concept.

Mathematical Formulation

Consider an unsteady, incompressible, two dimensional flow of a nanofluid past an impulsively started infinite vertical plate. The flow is considered along x-axis, which is taken along the plate in the vertically upward direction and the y-axis is measured normal to the surface of the plate. At time \( t^* \leq 0 \), the plate and the fluid are at the same temperature. At time \( t^* > 0 \), the plate is given an impulsive motion in the vertically upward direction with the constant velocity \( u_0 \). The surface of the plate is maintained at a constant temperature \( T_w \) higher than the temperature \( T_0 \) of the ambient nanofluid. The fluid is water based nanofluid containing the nano particles, Zinc Oxide (Zno), Nickel (Ni) and Copper (Cu). In this study, nanofluids are assumed to behave as single phase fluids with local thermal equilibrium between the base fluid and the nano particles suspended in them, so that no slip occurs between them. A schematic representation of physical model and coordinate system is depicted in Fig. 1. The thermophysical properties of the nanofluids are given in Table 1. The basic unsteady momentum and thermal energy equations according to the model for nanofluids in the presence of radiation and magnetic field along with heat source are as follows:

\[
\frac{\partial u}{\partial t} = \frac{1}{\rho_n^f} \left[ \mu_n^f \frac{\partial^2 u}{\partial y^2} + (\rho \beta)^f \rho g (T - T_w) - \sigma B_0^2 u \right]
\]

(1)

\[
\frac{\partial T}{\partial t} = \left( \frac{\rho c_p}{\rho_n^f} \right) \left[ k_n^f \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q(T - T_w) \right]
\]

(2)

The boundary conditions for the problem are

\( t^* \leq 0, \quad u = 0, T = T_w \text{ for all } y, \)

\( t^* > 0, \quad u = u_0, T = T_w \text{ for } y = 0, \)

\( u \rightarrow 0, T \rightarrow T_w \text{ as } y \rightarrow \infty. \)

(3)

Where \( u \) is the velocity along x-axis, \( \rho_n^f \) is the effective density of the nanofluid, \( \mu_n^f \) is the effective dynamic viscosity of the nanofluid, \( \beta_n^f \) is the thermal expansion of the nanofluid, \( g \) is the acceleration due to gravity, \( k_n^f \) is the thermal conductivity of the nanofluid, \( q_r \) is the radiative heat flux, \( \sigma \) is surface tension and \( B_0 \) is applied magnetic field. For nanofluids the expressions of density \( \rho_n^f \), thermal expansion coefficient \( (\rho \beta)^f \) and heat capacitance \( (\rho c_p) \) are given by
\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s \\
\mu_{nf} = \mu_f (1 - \phi)^{2.5} \\
(r \beta)_{nf} = (1 - \phi) (r \beta)_f + \phi (r \beta)_s \\
(r c_p)_{nf} = (1 - \phi) (r c_p)_f + \phi (r c_p)_s 
\]

The effective thermal conductivity of the nanofluid according to Hamilton and Crosser model is given by
\[
k_{nf} = k_s + (n-1)k_f - (n-1)\phi(k_f - k_s)
\]
where \(n\) is the empirical shape factor for nanoparticle. In particular \(n=3\) for spherical shaped nanoparticles and \(n=3/2\) for cylindrical shaped nanoparticles. \(\phi\) is the solid volume fraction of the nanoparticles, \(k\) is thermal conductivity. Here the subscripts \(nf\), \(f\) and \(s\) represents the thermo physical properties of nanofluid, base fluid and solid nanoparticles respectively.

By using Rosseland approximation (26) the radiative heat flux leads to
\[
q_r = -\frac{4\sigma T^4}{3k} \frac{\partial T}{\partial y} 
\]

Where \(\sigma\) and \(k^*\) are Stefan-Boltzmann constant and the mean absorption coefficient respectively.

\[
\frac{\partial q_r}{\partial y} = -4a^* \sigma^* \left( T_w^4 - T^4 \right) 
\]

If the temperature differences are within the flow are sufficiently small such that \(T^4\) may be expressed as a linear function of temperature, then expanding \(T^4\) in Taylors series about \(T_\infty\) and neglecting higher order terms we get
\[
T^4 \equiv 4T_\infty^4T - 3T_\infty^4 
\]

In view of equations (6)-(8) and introducing the following non dimensional variables in equations (1)-(3)
\[
U = \frac{u}{u_0}, \quad Y = \frac{y}{u_0}, \quad t = \frac{t}{u_0^2}, \\
\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad R = \frac{16a^* \sigma^* \nu_f^2 \nu_f T_w^2}{k_f u_0}, \quad M = \frac{\sigma B_0^2 \nu_f}{u_0^2}, \\
Gr = \frac{g \beta (T_w - T_\infty) \nu_f}{u_0^3}, \quad Pr = \frac{\nu_f}{\nu_f}, \quad H = \frac{Q \nu_f^2}{k_f u_0^2} 
\]

Governing equations reduces to
\[
\frac{\partial U}{\partial t} = \alpha_1 \frac{\partial^2 U}{\partial Y^2} + \alpha_2 \theta - \alpha_3 \theta \\
\frac{\partial \theta}{\partial t} = \alpha_1 \frac{\partial^2 \theta}{\partial Y^2} - \alpha_2 \theta 
\]

The corresponding dimensionless boundary conditions are
\[
t \leq 0, \quad U = 0, \quad \theta = 0 \quad \text{for all} \ Y, \\
t > 0, \quad U = 0, \quad \theta = 1 \quad \text{for} \ Y = 0, \\
U \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as} \ Y \rightarrow \infty.
\]
Where

\[ \alpha = \frac{1}{1 - \phi + \phi \left( \frac{x_p}{x_f} \right)} \left( \frac{k_f}{k_j} \right) \text{Pr}, \]

\[ \alpha_2 = \frac{R}{1 - \phi + \phi \left( \frac{x_p}{x_f} \right)}, \]

\[ \alpha_5 = \frac{1}{(1 - \phi)^{25}} \left( \frac{1 - \phi + \phi \left( \frac{\rho_\beta}{\rho_f} \right)}{1 - \phi + \phi \left( \frac{\rho_\beta}{\rho_f} \right)} \right), \]

\[ \alpha_4 = \left[ \frac{1 - \phi + \phi \left( \frac{\rho_\beta}{\rho_f} \right)}{1 - \phi + \phi \left( \frac{\rho_\beta}{\rho_f} \right)} \right] \text{Gr}, \]

\[ \alpha_6 = \frac{1}{(1 - \phi)^{25}} \left( \frac{1 - \phi + \phi \left( \frac{\rho c_p}{\rho_f} \right)}{1 - \phi + \phi \left( \frac{\rho c_p}{\rho_f} \right)} \right), \]

\[ \alpha_7 = \alpha_2 - \alpha_6 \]

Equations (10) and (11) subjected to boundary conditions in equation (12) are solved using Laplace transform technique and the solutions are given by

\[ \Theta (y, t) = \frac{1}{2} \left[ \exp \left( 2 \eta \sqrt{\frac{\alpha \gamma}{\alpha_1}} t \right) \text{erfc} \left( \frac{\eta}{\sqrt{\alpha_1}} + \sqrt{\alpha \gamma t} \right) \right] \]

\[ + \exp \left( -2 \eta \sqrt{\frac{\alpha \gamma}{\alpha_1}} t \right) \text{erfc} \left( \frac{\eta}{\sqrt{\alpha_1}} - \sqrt{\alpha \gamma t} \right) \]

(14)
\begin{equation}
U(y,t) = \frac{1}{2} \left[ \exp \left(2\eta \sqrt{\frac{\alpha_3 t}{\alpha_3}} \right) \text{erfc} \left( \frac{\eta}{\sqrt{\alpha_3}} + \sqrt{\frac{\alpha_3 t}{\alpha_3}} \right) + \exp \left(-2\eta \sqrt{\frac{\alpha_3 t}{\alpha_3}} \right) \text{erfc} \left( \frac{\eta}{\sqrt{\alpha_3}} - \sqrt{\frac{\alpha_3 t}{\alpha_3}} \right) \right]
\end{equation}

\begin{equation}
= \frac{1}{2} \left[ \exp \left(2\eta \sqrt{\frac{\alpha_4 t}{\alpha_4}} \right) \text{erfc} \left( \frac{\eta}{\sqrt{\alpha_4}} + \sqrt{\frac{\alpha_4 t}{\alpha_4}} \right) + \exp \left(-2\eta \sqrt{\frac{\alpha_4 t}{\alpha_4}} \right) \text{erfc} \left( \frac{\eta}{\sqrt{\alpha_4}} - \sqrt{\frac{\alpha_4 t}{\alpha_4}} \right) \right]
\end{equation}

Where \( z = \frac{\alpha_4 - \alpha_3}{\alpha_4} \), \( \eta = \frac{Y}{2\sqrt{t}} \)

Results and Discussion

The partial differential equations (10) and (11) subject to the boundary conditions (12) were solved numerically by using Laplace Transform Technique. We consider \( Cu \), \( Ni \) and \( Zno \) nanoparticles with water as the base fluid. \textit{Table 1 shows the thermo physical Properties of water and nanoparticles.}

\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|}
\hline
 & \( \rho \) (Kg m\(^{-3}\)) & \( c_p \) (J Kg\(^{-1}\) K\(^{-1}\)) & \( k \) (Wm\(^{-1}\) K\(^{-1}\)) & \( \beta \times 10^5 \) (K\(^{-1}\)) \\
\hline
\textit{H}_2\textit{O} & 997.1 & 4179 & 0.613 & 21 \\
\hline
\textit{Cu} & 8933 & 385 & 401 & 1.67 \\
\hline
\textit{Ni} & 8890 & 456 & 702 & 1.33 \\
\hline
\textit{Zno} & 5600 & 514 & 147 & 3.02 \\
\hline
\end{tabular}
\end{table}
Fig. 1 Effect of Magnetic parameter $M$ on the velocity of Cu water nanofluid when $Gr=5$, $H=2$, $Pr=6.2$, $R=1$, $t=0.5$, $\phi=0.04$.

Fig. 2 Effect of Magnetic parameter $M$ on the velocity of Ni water nanofluid when $Gr=5$, $Pr=6.2$, $R=1$, $t=0.5$, $\phi=0.04$, $H=2$.

Fig. 3 Effects of Magnetic parameter $M$ on the velocity of Zno water nanofluid when $Gr=5$, $Pr=6.2$, $R=1$, $t=0.5$, $\phi=0.04$, $H=2$. 
Fig. 4 Comparison of velocity profiles of nanofluids in the presence of magnetic field.

Fig. 5. Effects of Heat source parameter $H$ on the velocity field of $Cu$ water nanofluid when $Gr = 5$, $Pr = 6.2$, $M = 1$, $R = 1$, $t = 0.5$, $\varphi = 0.04$.

Fig. 6. Effects of Heat source parameter $H$ on the velocity of $Ni$ water nanofluid when $Gr = 5$, $Pr = 6.2$, $M = 1$, $R = 1$, $t = 0.5$, $\varphi = 0.04$. 
Fig. 7 Effects of Heat source parameter $H$ on the velocity of $Zno$ water nanofluid when $Gr=5$, $Pr=6.2$, $M=1$, $R=1$, $t=0.5$, $\phi=0.04$.

Fig. 8 Comparison of velocity profiles of water and nanofluids in the presence of heat source (H=0.2)

Fig. 9 Effects of volume fraction $\phi$ on the velocity of $Cu$ water nanofluid when $Gr=2$, $Pr=6.2$, $M=1$, $R=2$, $t=0.6$, $H=3$. 

46
Fig. 11. Effects of volume fraction $\phi$ on the velocity of $\text{Ni}_{\text{water}}$ nano fluid when $Gr=2$, $Pr=6.2$, $M=1$, $R=2$, $t=0.6$, $H=3$.

Fig. 10. Effects of volume fraction $\phi$ on the velocity of $ZnO_{\text{water}}$ nanofluid when $Gr=2$, $Pr=6.2$, $M=1$, $R=2$, $t=0.6$, $H=3$.

Fig. 12 Comparison of velocity profiles of water and nanofluids in the presence of volume fraction ($\phi=0.2$)
Fig: 13. Effects of Radiation parameter $R$ on the temperature of Cu water nanofluid when $Pr = 6.2$, $t = 0.6$, $\phi = 0.04$, $H = 2$.

Fig: 14. Effects of Radiation parameter $R$ on the temperature of Ni water nanofluid when $Pr = 6.2$, $t = 0.6$, $\phi = 0.04$, $H = 2$.

Fig: 15. Effects of Radiation parameter $R$ on the temperature of Zno water nanofluid when $Pr = 6.2$, $t = 0.6$, $\phi = 0.04$, $H = 2$. 
Fig. 16. Comparison of temperature profiles of nanofluids in presence of radiation.

Fig. 17. Effects of Heat source parameter $H$ on the temperature of $Cu$ water nano fluid when $Pr = 6.2$, $R = 6$, $t = 0.1$, $\phi = 0.04$.

Fig. 18. Effects of Heat source parameter $H$ on the temperature of $Ni$ water nanofluid when $Pr = 6.2$, $R = 6$, $t = 0.1$, $\phi = 0.04$. 
Fig. 19. Effects of Heat source parameter $H$ on the temperature of $ZnO$ water nanofluid when $Pr=6.2$, $R=6$, $t=0.6$, $\phi=0.04$.

Fig. 20. Comparison of temperature profiles nanofluids in the presence of heat source

Fig. 21. Effects of volume fraction $\phi$ on the temperature of $Cu$ water nanofluid when $Pr=6.2$, $R=4$, $t=0.6$, $H=2$. 

50
Fig: 22. Effects of volume fraction $\phi$ on the temperature of Ni water nanofluid when $Pr=6.2$, $R=4$, $t=0.6$, $H=2$.

Fig: 23. Effects of volume fraction $\phi$ on the temperature field of Zno water nanofluid when $Pr=6.2$, $R=4$, $t=0.6$, $H=2$.

Figs. 1, 2 and 3 shows the effect of Magnetic field on Cu water, Ni water and Zno water nanofluids respectively. Here we observed that increase in magnetic field causes the decrease in fluid velocity. These may happen due to the magnetic field pull of the Lorentz force acting on the flow field. From Fig. 4 it is clear that magnetic field effect is more on Cu water nanofluid.

Figs. 5, 6 and 7 depicts the effect of Heat source parameter on Cu water, Ni water and Zno water nanofluids respectively. Here we observed that increase in Heat source parameter causes the increase in fluid velocity. Here heat is generated the buoyancy force, which induces the flow rate to increase giving rise to the increase in the velocity profiles. From fig. 8 we found that increase in heat source parameter shows much effect on increasing in fluid velocity in Cu water nanofluid.

Figures 9, 10 and 11 depict the effect of Volume fraction parameter on Cu water, Ni water and Zno water nanofluids respectively. Here we observed that decrease in Volume faction parameter causes the increase in fluid velocity. These may happen due to the number of surface atoms per unit of interior atoms of nanoparticles is very large when we reduce the volume fraction. From figure 12 it is clear that variation of volume fraction shows much effect to increase the fluid velocity of Zno water nanofluid.

Figs. 13, 14 and 15 shows the effect of Radiation parameter on Cu water, Ni water and Zno water nanofluids respectively. Here we observed that increase in Radiation parameter causes the decrease in fluid temperature, these may happen due to Rosseland radiation absorptiveness. From Fig. 16 it is clear that effect of Radiation parameter is more on Zno water nanofluid.
Figures 17, 18 and 19 shows the effect of Heat source parameter on Cu water, Ni water and Zno water nanofluids respectively. Here we observed that increase in Heat source parameter causes the increase in fluid temperature. From figure 20 it is clear that effect of Heat source parameter is more on Ni water nanofluid.

Figures 21, 22 and 23 shows the effect of Volume fraction parameter on Cu water, Ni water and Zno water nanofluids respectively. Here we observed that increase in Volume fraction initially decrease the fluid temperature after certain period it is reversed. But in case of Zno water nanofluids increase in volume fraction of nanoparticles causes the increase in fluid temperature. These may happen due to the reason that the decrease in volume fraction improves the heat transfer rate.

Conclusions
The effects of radiation and magnetic field on unsteady natural convective flow of nanofluids past an infinite vertical plate with heat source using Rosseland approximation for the radiative heat flux are analyzed. The governing partial differential equations are solved by Laplace Transform Technique. The effects of the nanoparticle volume fraction, thermal radiation, magnetic field and heat source on fluid velocity and temperature determined for Cu water, Ni water and Zno water nanofluids.

The conclusions are as follows:
1. Velocity of water nanofluid is more influenced by magnetic field parameter.
2. The effect of radiation on water nanofluid velocity is less compared to water, water nanofluids.
3. Heat sources parameter improves the velocity of water, water nanofluids.
4. Decrease in Volume fraction of all nanoparticles causes the increase in fluid velocity.
5. The increase in thermal radiation is causes the decrease in temperature of the fluid and it is more influenced the water nanofluid.
6. Increase in heat source parameter causes the increase in fluid temperature. Water nanofluid is much influenced by heat source parameter.
7. Decrease in volume fraction of the nanoparticles causes the increase in fluid temperature.

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