Effect of Viscous Dissipation on Double Stratified MHD Free Convection in Micropolar Fluid Flow in Porous Media with Chemical Reaction, Heat Generation and Ohmic Heating

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Abstract
This present paper deals with the study of heat and mass transfer characteristics of the free convection on a vertical plate in porous media with variable wall temperature and concentration in a doubly stratified and viscous dissipating micropolar fluid in presence of chemical reaction, heat generation and Ohmic heating. A uniform magnetic field is applied normal to the plate. The governing non-linear partial differential equations are transformed into a system of non-linear ordinary differential equations using similarity transformations and then solved numerically using the Runge-Kutta-Fehlberg method with shooting technique. Effects of viscous dissipation on non-dimensional velocity, microrotation, temperature and concentration are presented graphically. An increase in Eckert number (which is measure of viscous dissipation) increases the velocity, temperature and microrotation, but decreases the concentration.
Keywords: Chemical reaction, Double stratification, Free convection, MHD, Micropolar fluids, Porous media, Heat generation, Ohmic heating.

MATHEMATICS SUBJECT CLASSIFICATION: 76A05; 76E06; 76S05; 80A20.

1. Introduction

The study of free convective flow, heat transfer in non-Newtonian fluid in porous media has been an active field of research as it plays an important role in diverse applications, for example, thermal insulation, extraction of crude oil and chemical catalytic reactors, the thermal designing of industrial equipment dealing with molten plastic, polymeric liquids, foodstuffs etc. Eringen [1-3] has introduced the theory of micropolar fluids that is capable to describe those fluids by taking into account the effect arising from local structure and micromotions of the fluid element. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory and in the theory of porous media are presented by Lukaszewicz [4]. J. Wright and W. Shyy [5] investigated the time-dependent development of convective intrusions in a thermohaline stratification using a composite grid method with local refinement. Srinivasacharya and RamReddy [6] analyzed the Natural convection heat and mass transfer along a vertical plate embedded in a doubly stratified micropolar fluid saturated by non-Darcy porous medium and found that an increase in the both thermal and solutal stratification parameters, the velocity, skin friction parameter and non-dimensional heat and mass transfer coefficients were decreasing but the wall couple stress was increasing.. Murthy et al. [7] studied the effect of magnetic field on free convection in a thermally stratified non-Darcy porous medium saturated with nanofluid with convective boundary condition. Upendar and Srinivasacharya [8] investigated the flow and heat and mass transfer characteristics of the natural convection on a vertical plate with variable wall temperature and concentration in a doubly stratified MHD micropolar fluid and founded that an increase coupling number reduce velocity but enhance the temperature and concentration distributions. Bataller [9] has proposed the effects of viscous dissipation, work due to deformation, internal heat generation (absorption) and thermal radiation. It was shown internal heat generation/absorption enhances or damps the heat transformation. Chien-Hsin Chen [10] has presented the problem of combined heat and mass transfer in buoyancy-induced MHD natural convection flow of an electrically conducting fluid along a vertical plate is investigated with Ohmic and viscous heating. Narayana and Sravanthi [11] studied the simultaneous effects of soret and Ohmic heating on MHD free convective heat and mass transfer flow for a micro polar fluid bounded by a vertical infinite surface. Kumar [12] analyzed the problem of MHD mixed convective flow of a micropolar fluid with the effect of Ohmic heating, radiation and viscous dissipation over a chemically reacting porous plate with constant heat flux. Rashad [13] has proposed the effect of thermal radiation with a regular three-parameter perturbation analysis in some free convection flows of Newtonian fluid in saturated porous medium. E1-Hakiem [14] has presented an analysis for the effect of thermal dispersion, viscous and Joule heating on the flow of an electrically conducting and viscous incompressible micropolar fluid past a semi-infinite plate whose temperature varies linearly with the distance from the leading edge in the presence of uniform transverse magnetic field. The skin friction factor and the rate of heat transfer decrease with the magnetic parameter and the micropolar parameter and they increase with the thermal dispersion parameter increase.

The aim of present work is to investigate the effects of thermal and solutal stratification and Eckert number on the MHD free convection heat and mass transfer from a vertical plate embedded in micropolar fluid in porous medium with viscous dissipation, heat generation and Ohmic heating. The equations thus obtained have been solved numerically using Runge–Kutta–Fehlberg method with shooting technique. The effects of different parameters on velocity, microrotation, temperature and concentration are presented graphically.

2. Mathematical formulation

The graphical model of the problem has been given along with flow configuration and coordinate system in figure 1. The system deals with a steady, laminar, incompressible, two-dimensional free convective heat and mass transfer along a semi-infinite vertical plate in porous media embedded in a doubly stratified, electrically conducting micropolar fluid. The heat generation as well as viscous dissipation and Ohmic heating terms have been retained in the energy equation. Choose the coordinate system such that *x* axis is along the vertical plate and *y* axis normal to the plate. The plate is maintained at temperature $T_w(x)$ and concentration $C_w(x)$. The temperature and the mass concentration of the ambient fluid are assumed to be linearly stratified in the form $T_\infty(x) = T_{\infty,0} + A_1 x$ and $C_\infty(x) = C_{\infty,0} + B_1 x$ respectively, where A_1 and B_1 are constants and varied to alter the intensity of stratification in the medium and $T_{\infty,0}$ and $C_{\infty,0}$ are the beginning ambient temperature and concentration at $x = 0$, respectively. A uniform magnetic field of magnitude B_0 is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field.

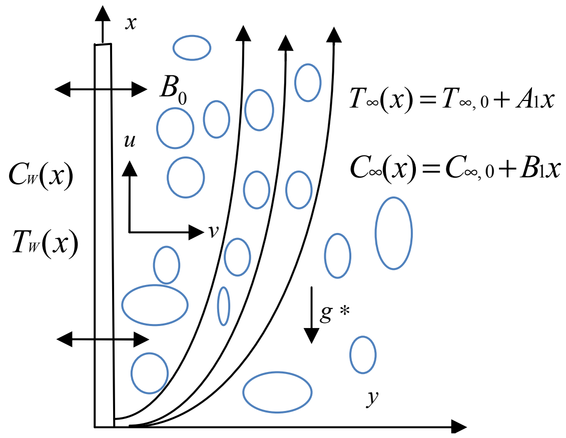

Figure 1 Physical model and coordinate system.

Using the Boussinesq and boundary layer approximations, the governing equations for the micropolar fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial \omega}{\partial y} + g^*\left(B_T(T - T_\infty) + B_C(C - C_\infty)\right) - \frac{\sigma B_0^2}{\rho}u + \frac{\mu\varepsilon}{\rho k_1}u \tag{2}$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 \omega}{\partial y^2} - \frac{\kappa}{\rho j}\left(2\omega + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{(\mu+\kappa)}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{\rho c_p}u^2 + \frac{\theta_0}{\rho c_p}(T - T_\infty) \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - R_r(C - C_\infty) \tag{5}$$

where u and v are the component of velocity along x and y directions, respectively, ω is the component of microrotation whose direction of rotation lies in the xy-plane, g* is the gravitational acceleration, ρ is fluid density, T is the temperature, C is the concentration, B_T is the coefficient of thermal expansions, B_C is the coefficient of solutal expansions, B_0 is the coefficient of the magnetic field, ε is porosity of porous media, k_1 is permeability of porous media, μ is the dynamic coefficient of viscosity of the fluid, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, σ is the magnetic permeability of the fluid, υ is the kinematic viscosity, α is the thermal diffusivity, k is the thermal conductivity of the fluid and D is the molecular diffusivity, c_p is the specific heat, θ_0 is the internal heating, R_r is the chemical reaction rate constant.

The boundary conditions are:

$$u = 0, \quad v = 0, \quad \omega = 0, \quad T = T_w(x), \quad C = C_w(x), \quad at\ y = 0$$

$$u \to 0, \quad \omega \to 0, \quad T \to T_\infty(x), \quad C \to C_\infty(x) \quad as\ y \to \infty \tag{6}$$

where the subscripts w and ∞ indicate the conditions at wall and at the outer edge of the boundary layer, respectively. The continuity equation (1) is satisfied by introducing the stream function ψ such that

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x} \tag{7}$$

Equations 2, 3 and 4 can be transformed into a set of nonlinear ordinary differential equations by using the following similarity variables:

$$\psi = A\,x\,f(\eta), \quad \eta = B\,y, \quad \omega = E\,x\,g(\eta), \quad \theta(\eta) = \frac{T - T_{\infty,0}}{\Delta T} - \frac{A_1 x}{\Delta T}, \quad \Delta T = T_w(x) - T_{\infty,0} = M_1(x)$$

$$\emptyset(\eta) = \frac{C - C_{\infty,0}}{\Delta C} - \frac{B_1 x}{\Delta C}, \qquad \Delta C = C_w(x) - C_{\infty,0} = N_1(x) \tag{8}$$

where the constants A, B, E, M_l and N_l have respectively, the dimension of velocity, reciprocal of length, the reciprocal of the product of length and time, the ratio of temperature and length and the ratio of concentration and length. The transformed ordinary differential equations are:

$$\left(\frac{1}{1-N}\right) f''' + f f'' + \left(\frac{N}{1-N}\right) g' - (f')^2 + \theta + L\emptyset - Mf' + \Lambda f' = 0 \tag{9}$$

$$\lambda g'' - \left(\frac{N}{1-N}\right) £\,(2g + f'') + f g' - f' g = 0 \tag{10}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta + H\theta + MEc f'^2 + \left(\frac{1}{1-N}\right) Ec f''^2 - \varepsilon_1 f' = 0 \tag{11}$$

$$\frac{1}{Sc}\emptyset'' + f\emptyset' - f'\emptyset - \Omega\emptyset - \varepsilon_2\emptyset = 0 \tag{12}$$

where primes denotes differentiation with respect to similarity variable η, Pr = υ/α is Prandtl number, Sc = υ/D is the Schmidt number, $£ = 1/(jb^2)$ is the micro-inertia density, $N = \kappa / (\mu + \kappa)$, $(0 \leq N < 1)$ is the Coupling number, $\lambda = \gamma / j\rho\upsilon$ is the spin – gradient viscosity, $L = \frac{B_C}{B_T}\frac{\Delta C}{\Delta B}$ is the buoyancy parameter, $M = \frac{\sigma B_0^2}{\mu B^2}$ is the magnetic field parameter, $\Lambda = \frac{\varepsilon}{k_1 B^2}$ is the porous parameter, $H = \frac{\theta_0}{\mu C_p B^2}$ is the heat generation parameter, $Ec = \frac{(\upsilon x B^2)^2}{C_p \Delta T}$ is the Eckert number, $R = \frac{R_r}{\upsilon B^2}$ is the chemical reaction parameter, $\varepsilon_1 = \frac{x}{\Delta T}\frac{d}{dx}[T_\infty(x)]$ is the thermal stratification parameter and $\varepsilon_2 = \frac{x}{\Delta C}\frac{d}{dx}[C_\infty(x)]$ is the solutal stratification parameter.

The boundary conditions (6) in terms of f, g, θ and $\emptyset$ becomes

$$f(0) = 0, \quad f'(0) = 0, \quad g(0) = 0, \quad \theta(0) = 1 - \varepsilon_1, \quad \emptyset(0) = 1 - \varepsilon_2 \quad \text{at } \eta = 0$$
$$f'(\infty) \to 0, \quad g(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \emptyset(\infty) \to 0 \quad \text{as } \eta \to \infty. \tag{13}$$

The physical parameters of interest are the skin friction coefficient C_f, the Couple stress coefficient M_w, the local Nusselt number Nu and the Sherwood number Sh which are defined as

$$C_f = \frac{2\tau_w}{\rho A^2}, \quad M_w = \frac{B}{\rho A^2} m_w, \quad Nu_x = \frac{q_w}{B\,k\,(T_w - T_\infty)} \quad \text{and} \quad Sh = \frac{q_m}{D\,B\,[C_w - C_\infty]} \tag{14}$$

where τ_w, m_w, q_w and q_m are the wall shear stress, the wall couple stress, the wall heat flux and the wall mass flux, respectively, are given by

$$\tau_w = \left[(\mu + \kappa)\left(\frac{\partial u}{\partial y}\right) + \kappa\omega\right]_{y=0}, \quad m_w = \gamma\left(\frac{\partial\omega}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad \text{and} \quad q_m = -D\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{15}$$

Hence using (8), we get

$$C_f = \left(\frac{2}{1-N}\right) f''(0)\,\bar{x}, \quad M_w = \frac{\lambda}{£} g'(0)\,\bar{x}, \quad Nu = -\theta'(0) \quad \text{and} \quad Sh = -\emptyset'(0) \tag{16}$$

where $\bar{x} = B\,x$.

3. Method of Solution

The non-linear ordinary differential equations (9)-(12) subject to the boundary condition (13) have been solved using the Runge-Kutta-Fehlberg method along with shooting technique. This method is based on the discretization of the problem domain and the calculation of unknown boundary conditions. The domain of the problem is discretized and the boundary conditions for η_∞ are replaced by $f'(\eta_\infty) = 0$, $g(\eta_\infty) = 0$, $\theta(\eta_\infty) = 0$, and $\varnothing(\eta_\infty) = 0$, where η_∞ is sufficiently large value of η at which boundary conditions (13) for $f(\eta)$ are satisfied. To solve the problem the non-linear equations (9)-(12) have been converted into nine first orders linear ordinary differential equations. There are five conditions at boundary $\eta = 0$ and four conditions at boundary $\eta = \infty$. To find the solution of problem, one will need four more conditions $f''(0)$, $g'(0)$, $\theta'(0)$ and $\varnothing'(0)$ at $\eta = 0$. These conditions have been found by the shooting technique. Finally the problem has been solved by the Runge-Kutta-Fehlberg method along with calculated boundary conditions.

4. Results and discussion

To validate the results obtained, the authors compared results in the absence porous parameter Λ, heat generation parameter H, Eckert number Ec, chemical reaction parameter R with reported by D. Srinivasacharya and M. Upendar [3] and found that they are in good agreement, as shown in Table1.

In order to study effects of Eckert number Ec on the physical quantities of the flow, the other parameters are fixed as $N = 0.5$, $M=1$, $H = 0.2$, $\Lambda = 0.3$, $L = 1$, $Pr = 1$, $R=0.3$, $Sc = 0.2$, $\lambda = 1$ and £ = 0.1. The values of micropolar parameter λ and £ are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen [1]. Figures 2-5 depict the variation of Eckert number Ec on the non-dimensional velocity, microrotation, temperature and concentration with $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.2$. It is seen from figure 2 that the velocity increases with increasing value of Eckert number Ec. It is noticed from figure 3 that the values of microrotation changes sign from negative to positive within boundary layer. Also, it is observed that the magnitude of the microrotation increases with increasing value of Eckert number Ec. Figure 4 demonstrate that the non-dimensional temperature of the fluid increases with increasing value of Eckert number Ec. It is observed from figure 5 that the concentration of the fluid decreases with increasing value of Eckert number Ec.

Table 1 Comparison between $f''(0)$ and $-\theta'(0)$ calculated by the present method and Ref. [3] for $\Lambda = R = H = Ec = 0$, $Pr = 1$ and $Sc = 0.2$.

N	M	ε_1	ε_2	Ref. [8]		Present	
				$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.0	1.0	0.1	0.2	0.97648	0.62178	0.97649	0.62178
0.3	1.0	0.1	0.2	0.78120	0.58833	0.78120	0.58833
0.5	2.0	0.1	0.2	0.55448	0.51043	0.55449	0.51042
0.5	1.0	0.2	0.2	0.59187	0.52654	0.59187	0.52653
0.5	1.0	0.1	0.4	0.55160	0.52112	0.55161	0.52111

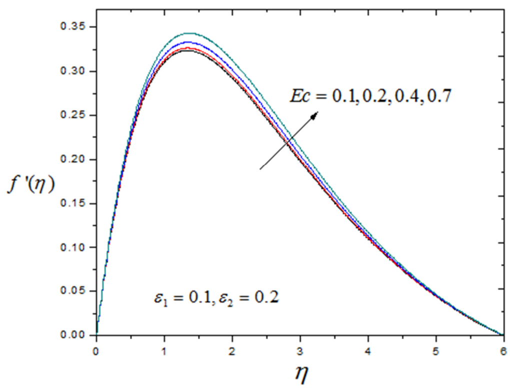

Figure 2 Velocity profile for various values of Eckert number Ec.

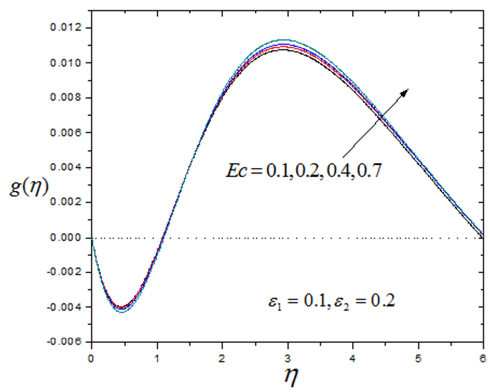

Figure 3 Microrotation profile for various values of Eckert number Ec.

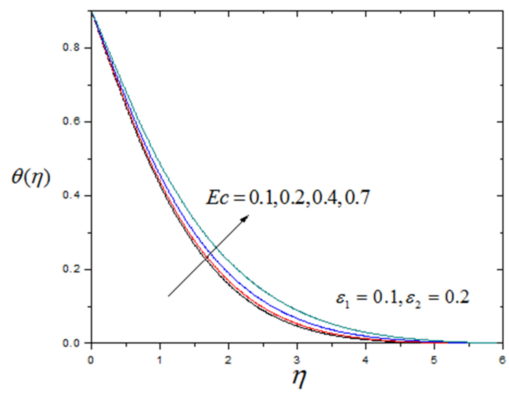

Figure 4 Temperature profile for various values of Eckert number Ec.

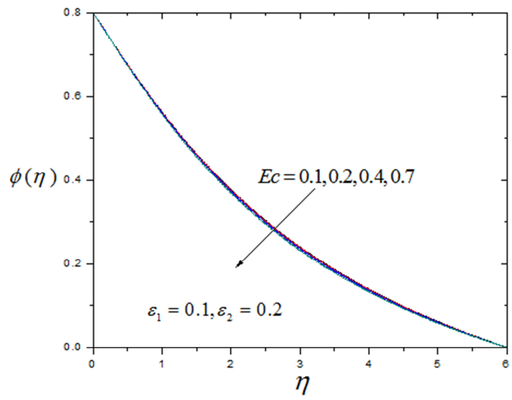

Figure 5 Concentration profile for various values of Eckert number Ec.

5. Conclusion

Free convection heat and mass transfer in an electronically conducting and chemical reacting micropolar fluid over a vertical plate with magnetic, thermal and solutal stratification and viscous dissipation effects are considered. The non-linear partial differential equations are transformed into a system of coupled non-linear ordinary differential equations by using similarity variables and then solved numerically using the Runge-Kutta-Fehlberg method along with the shooting technique. An increase in Eckert number (which is measure of viscous dissipation) increases the velocity, temperature and microrotation, but decreases the concentration.

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