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On Steady Free Convective Reacting Flows on Porous Plate

A.J. Omowaye
Department Of Mathematical Sciences, Federal University Of Technology, Akure

Abstract
This work investigate the existence of solution for steady free convective reacting flows on porous plate. Our approach to the problem is by the method of upper and lower solutions.
Keywords: Upper and Lower solutions, reacting, convection, porous, steady.

1. Introduction

The phenomenon of free convection arises in the fluid when temperature changesncause density variation leading to buoyancy forces acting on the fluid element.This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences.

Jha and Ajibade (2009) discussed free convective flow of heat generating/absorbing fluid beeetween vertical parallel porous plates due to peridic heating of the plate.The momentum and energy equations which arises from the definition of velocity and temperature were written in dimensionless form seperating the temperature and velocity field into steady and periodic parts,the resulting second order differential equations were solved to obtain the expressions for velocity,temperature,skin friction and the rate of heat transfer.It was shown that temperature and velocity reduces as Prandtl number increases. In much earlier work Soundalgekar (1977) studied the free convection effects on the stokes problem for an infinite vertical plate using a convection current that was due to temperature difference.

Recently,Omowaye(2011) studied steady free convective hdromagnetic flow of a reactive viscous fluid in a bounded domain.He proved the relevant theorems of existence and uniqueness of solution of momentum and energy equations.He showed that velocity increases as Grashof number and Prandtl number increases and temperature increases as Frank-Kamenetskii parameter increases.The aim of this work is to investigate free convective reacting flows on porous plate and to provide relevant theorems on the work.

2. Mathematical Model

The non-dimensional reacting convective flows are

$$x^2\frac{d^2u}{dx^2}+(v_0+1)x\frac{du}{dx} + G\theta= 0 \qquad (2.1)$$

$$\frac{x^2}{P}\frac{d^2\theta}{dx^2}+\frac{(v_0P+1)}{P}x\frac{d\theta}{dx} + \delta\, e^{\frac{\theta}{1+\in\theta}} = 0 \qquad (2.2)$$

$$u(0) = 0 \qquad u(1) = 1 \qquad (2.3)$$

$$\theta(0) = 0 \qquad \theta(1) = 1 \qquad (2.4)$$

where u is fluid velocity, v_0 -velocity of suction/injection,G-Grashof number,
θ -dimensionless temperature ,P-Prandtl number,$\in$ -activation energy parameter and
δ - Frank- Kameneskii parameter.

2.1 Definition

2.1.1 A smooth function w is said to be an upper solution of the problem

$$LW = f(y,W)$$

where $L \equiv \frac{d^2}{dy^2} + a(y)\frac{d}{dy} + b(y)$

if w satisfies

$$Lw \leq -f(y,w)$$

$$w(y_1) \geq f(y) \qquad w(y_2) \geq f(y)$$

Similarly,

2.1.2 A smooth function s is said to be a lower solution of the problem

$$Ls = f(y, s)$$

where $L \equiv \frac{d^2}{dy^2} + a(y)\frac{d}{dy} + b(y)$

if s satisfies

$$Ls \geq -f(y,s)$$

$$s(y_1) \leq f(y) \qquad s(y_2) \leq f(y)$$

3. Main Results

THEOREM 1:

If $1-2v_0 p < 0, \in \geq 0, 0 < \delta < \delta^*$. Then problem (2.2) satisfying (2.4) has a solution.

Proof:

It suffices to provide upper and lower solutions $\bar{\theta}$ and $\underline{\theta}$. Clearly, $\underline{\theta} \equiv 0$ is a lower solution for $L\underline{\theta} = 0$ and $L\underline{\theta} > -\delta$. Also, $\bar{\theta} = \delta p e^{\in} x^{\frac{1}{2}-\frac{1}{2}}$ is an upper solution for $L\bar{\theta} = -\delta p e^{\in} x^{\frac{1}{2}-\frac{1}{2}}\alpha$ where $1-2v_0 p = -\alpha$. So, $L\bar{\theta} = -\delta e^{\frac{\theta}{1+\in\theta}}$ since $0 < x < 1$, problem (2.2) has solution. This completes the proof.

THEOREM 2:

Let $\frac{x^2}{P}\frac{d^2\theta}{dx^2} + \frac{(v_0 P+1)}{P} x \frac{d\theta}{dx} + \delta e^{\frac{\theta}{1+\in\theta}} = 0$

$$\theta(0) = 0 \qquad \theta(1) = 1$$

If $v_0 = -\frac{1}{P}$. Then $\theta'(x) > 0$.

Proof:

Let $v_0 = -\frac{1}{P}$. Then $\frac{d^2\theta}{dx^2} = -\frac{\delta p}{x^2} e^{\frac{\theta}{1+\in\theta}} = 0$. Using Ayeni (1978), we obtain

$$\theta(x) = \delta p \int_0^1 k(x,t) \frac{1}{t^2} e^{\frac{\theta(t)}{1+\in\theta(t)}} dt$$

where $k(x,t) = \begin{cases} x, & 0 \leq x \leq t \\ t, & 1 \leq x \leq 1 \end{cases}$

So,

$$\theta'(x) = \delta p \frac{1}{x^2} e^{\frac{\theta(x)}{1+\in\theta(x)}} + \delta p \int_x^1 \frac{1}{t^2} e^{\frac{\theta(t)}{1+\in\theta(t)}} dt - \delta p \frac{1}{x^2} e^{\frac{\theta(x)}{1+\in\theta(x)}}$$

$$= \delta p \int_x^1 \frac{1}{t^2} e^{\frac{\theta(t)}{1+\in\theta(t)}} dt$$

Hence, $\theta(x)$ is strictly monotonically increasing for $x \in (0,1)$. This completes the proof.

THEOREM 3:

Let $v_0 = -\frac{1}{P}$. If θ_1 and θ_2 are two distinct solution of the problem (2.2) satisfying (2.4) Such that $\theta_1'(0) \neq \theta_2'(0)$. Then either $\theta_1(x) < \theta_2(x)$ or $\theta_2(x) < \theta_1(x)$, $x \in (0,1)$.

Proof:

We following the lines of reasoning of Ayeni(1978). It is well known that if a function h is strictly monotonic continuous mapping on an interval I, then the inverse h^{-1} is strictly monotone and continuous on the interval h (I)

(Ayeni(1978)) .

Let $v(\theta(x))$ where v is the inverse function.

$$\text{Let} \quad f(x,\theta) = \frac{\delta p}{x^2} e^{\frac{\theta(x)}{1+\in\theta(x)}}$$

Suppose there exits $a\in(0,1)$ such that $\theta_1(a) = \theta_2(a)$. Let us assume that for $x\in(0,a)$

$$\theta_1(x) < \theta_2(x)$$

$$\text{Then} \quad \theta_1'(0) < \theta_2'(0)$$

But

$$(\theta_1')^2(a) - (\theta_2')^2(a) = 2\delta p[\int_0^{\theta_1(1)} f(v_1(v),v)dv - \int_0^{\theta_2(1)} f(v_2(v),v)dv] +$$

$$2\delta p[\int_0^{\theta_1(a)} f(v_1(v),v)dv - \int_0^{\theta_2(a)} f(v_2(v),v)dv]$$

Since $\theta_1(x) < \theta_2(x)$ then $v(\theta_1) > v(\theta_2)$

It follows that

$$\int_0^{\theta_1(a)} f(v_1(v),v)dv - \int_0^{\theta_2(a)} f(v_2(v),v)dv$$

is non-positive. $\theta_1'(0) < \theta_2'(0)$ implies that

$\int_0^{\theta_1(1)} f(v_1(v),v)dv - \int_0^{\theta_2(1)} f(v_2(v),v)dv$ is non-negative .Hence there exists no $a\in(0,1)$ such that $\theta_1(a) = \theta_2(a)$ .This completes the proof.

THEOREM 4: (Protter and Weinberger(1967))

$$\text{Let} \quad u'' + g(x)u' + h(x)u = f(x) \qquad (3.1)$$

where the functions f,g and h are given in an interval (a,b) with g and h bounded .Assume

$$u(a) = \gamma_1 \qquad u(b) = \gamma_2 \qquad (3.2)$$

suppose $u_1(x)$ and $u_2(x)$ are solutions of (3.1) which satisfy (3.2).If g(x) $\leq$ 0 in (a,b) then $u_1 \equiv u_2$.

Proof: see(Protter and Weinberger(1967))

Now, Buckmaster and Ludford (1982) have shown that

$$\frac{d^2\phi}{dx^2} + \delta e^{\phi} = 0 \qquad x\in(-1,1)$$

$$\phi(-1) = \phi(1) = 0$$

has two solutions. It therefore follows that the following proposition holds.

Proposition 1:

For each θ and $\in \to 0$ equation (2.1) has a unique solution which satisfies (2.3) for each $v_0 < 0$.

Proof:

Let u_1, u_2 be any two solutions then

$$x^2\frac{d^2\phi}{dx^2} + (v_0+1)x\frac{d\phi}{dx} = 0$$

$$\phi(0) = 0 \qquad \phi(1) = 1$$

Where $\phi = u_1 - u_2$.Then $\phi = \frac{cx^{\alpha}}{\alpha} + c_1$,where $\alpha = -v_0$.

Hence, $\phi \equiv 0$

Proposition 2:

Let $v_0 = -\frac{1}{P}$, $\in \to 0$ .Then problem (2.2) which satisfies (2.4) has two solutions for $0 < \delta p < \delta c$.:

Proof:

Butckmaster and Ludford [2] has shown that $\frac{d^2\theta}{dx^2} + \delta e^{\theta} = 0 \quad x \in (0,1)$

Has two solutions $\theta_1(x), \theta_2(x)$ for $0 < \delta < \delta c$ such that $\theta_1(x) < \theta_2(x)$ or $\theta_2(x) < \theta_1(x)$ $x \in (0,1)$ .So,it suffices to show that each solution of

$\frac{x^2}{P}\frac{d^2\theta}{dx^2} + \delta e^{\theta} = 0$ is an upper solution for each solution of $\frac{d^2\theta}{dx^2} + \delta e^{\theta} = 0$ .Now

$$\frac{d^2\theta}{dx^2} + \frac{p}{x^2}\delta e^{\theta} = 0$$

So, $\frac{d^2\theta}{dx^2} < \delta e^{\theta} = 0$ .The result follows from theorem 4 and the result of Butckmaster and Ludford [2] .

4. Conclusion

In this paper the problem of steady convective reacting flows on porous plate is considered. The existence of solution of the problem was established using method of lower and upper solution.

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