Unsteady Variable Thermal Conductivity Gravity Flow of a Power-Law Fluid with Viscous Dissipation through a Porous Medium

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Abstract
In this work, the unsteady variable thermal conductivity gravity flow of a power-law fluid with viscous dissipation through a porous medium is examined. It is assumed that the fluid has variable temperature-dependent viscosity. The modified Darcy's law is considered together with the equation of energy transfer in such media. The governing partial differential equations were transformed into ordinary differential equations in terms of a suitable similarity variable. Criteria for existence and uniqueness of solution are formulated. Central finite difference technique and Galerkin weighted residual method were employed to solve the resulting non-linear equations. The effects of variable viscosity parameter, Reynolds number, Brinkman number, Prandtl number, Peclet number, and those of viscous dissipation parameter on the flow system were reported graphically.
Keywords: Non-Newtonian fluid , Weighted residual method, Power-law fluid and Viscous dissipation.

1.0: Introduction

The importance of studies involving gravity flows in porous media cannot be over-emphasized due to its variety of usefulness in various fields of human endeavours. Flows through porous media are very prevalent in nature, and has attracted the attention of many scientists in the recent times. This is due to its large area of applications in engineering practices, particularly in applied geophysics, geology, groundwater flow, food technology, geothermal reservoirs, enhanced oil recovery, oil reservoir engineering , oil recovery processes, to mention but just a few. Due to the increase in the production of heavy crude oils, and elsewhere where materials whose flow behaviour in shear cannot be characterized by Newtonian relationships; it has become necessary to have an adequate understanding of the rheological effects of non-Newtonian fluid flows and, as a result, a new stage in the evolution of fluid dynamic theory is in progress.

Kumar and Prasad [1] considered MHD pulsatile flow through a porous medium. Analytical solution was employed in solving the system of flow. Their result shows that an increase in the permeability parameter and Hartmann number leads to a decrease in the steady state velocity. Vajraveh et al [2] examined fluid flow and heat transfer over a permeable stretching cylinder. A numerical method involving second order finite difference scheme known as Keller Box method was employed to investigate the velocity and temperature distribution of the flow system. Their result shows that increasing values of the fluid viscosity parameter is to enhance the temperature. This is due to the fact that an increase in the fluid viscosity parameter results in an increase in the thermal boundary layer thickness. The effects of variable viscosity, viscous dissipation and chemical reaction on heat and mass transfer flow of MHD micropolar fluid along a permeable stretching sheet was examined by Salem [3]. A numerical method involving Runge-Kuta fourth order method and shooting technique were employed to investigate the velocity and temperature distribution of the flow system. The results show that as Prandtl number and viscosity parameter increases the velocity profile and the temperature profile decreases.

The effect of variable viscosity and thermal conductivity of micro polar fluid in a porous channel in the presence of magnetic field was studied by Gitima [4]. A numerical method involving Runge-Kuta fourth order method was employed to investigate the velocity and temperature distribution of the flow system. The results show that the velocity and temperature of the fluid increases as Darcy number, thermal conductivity variation parameter and magnetic field parameter increases.

Cortell [5] investigated on unsteady gravity flows of a power-law fluid through a porous medium. He analyzed the flow in two direction, one side both thinning and thickening of the fluids and on the other hand, two different types of solutions, for the case of a gravity flow generated by the injection of a power-law fluid at the well into an empty reservoir of an infinite extent. He employed shooting method to analyze the flow model. The result shows that as power-law index increases the velocity profile decreases. Ogunsola and Ayeni [6] considered the effects of temperature distribution of an Arrheniusly reacting unsteady flow through a porous medium with variable permeability. A numerical method involving shooting method was employed to investigate the velocity and temperature distribution of the flow system. Their result shows that as Frank-Kamenetskii parameter increases the fluid velocity and temperature increases.

Motivated by these facts, the present work has been undertaken in order to analyze the effects of unsteady variable thermal conductivity gravity flow of a power-law fluid with viscous dissipation through a porous medium on the flow system.

2.0: MATHEMATICAL FORMULATION AND METHOD OF SOLUTION

Radial Flow in Porous media

The governing equations are conservation of mass, momentum and energy. In petroleum engineering we are often interested in fluid flowing towards a well, therefore it is more convenient to use cylindrical (radial) coordinates, rather than Cartesian coordinates. Considering a two dimensional flow in the $x-z$ plane where the free surface is a streamline at a point on the surface, we expressed the flow by a modified Darcy's law. There are three basic differential equations of fluid motion:

$$\Phi\frac{\partial h}{\partial t}+\frac{\partial hu}{\partial x}=0 \quad \text{(Mass conservation law-Continuity Equation)} \quad (2.1)$$

where $\Phi = K\rho g/2\mu m$, Φ being the porosity of the porous medium, which is assumed to be constant in both space and time.

It is a single phase flow where $\partial h/\partial s$ is the gradient in the flow direction and K is independent of the nature of the fluid but depends on the geometry of the medium.

$$\frac{\partial(hu)}{\partial r}=-\Phi\frac{\partial h}{\partial t} \quad \text{(Continuity Equation)} \quad (2.2)$$

Following Hupper (1982) and Hupper & Woods (1995) the local continuity condition is as follows :

$$\frac{1}{r}\frac{\partial(rhu)}{\partial r}=-\Phi\frac{\partial h}{\partial t} \quad \text{(Mass balance Equation-Continuity Equation)} \quad (2.3)$$

$$u=-\left(\frac{K\rho}{\mu_{ef}}\right)^{\frac{1}{n}}\frac{\partial h}{\partial s}\left|\frac{\partial h}{\partial s}\right|^{\frac{1-n}{n}} \quad \text{(Modified Darcy's law-Momentum Equation)} \quad (2.4)$$

as proposed by Cortell (2008) where s is measured along the streamline,

since $z=h$, on the free surface. The rheological parameter n is the dimensionless power-law exponent which represents Newtonian fluid when $n=1$, shear-thinning ($n<1$) and shear-thickening ($n>1$) fluids, K is the permeability. The Dupuit's approximation yields $\partial h/\partial s \cong \partial h/\partial x$ (2.5)

For small gradients which converts the problem into a one–dimensional problem. This approximation permits to assume a horizontal flow with $h=h(x,t)$ (t being the time).

We consider a semi-infinite reservoir underlain plane horizontal impermeable surface and bounded on the inlet side by a vertical plane perpendicular to the $x-$axis and passing through the point $r=0$.

The initial head of the fluid in the reservoir is assumed to be negligible, and the fluid is injected through a well with a negligibly small radius. So we let the initial head to be equal to zero, i.e. $h_1=0$. The head at the vertical boundary of the reservoir is assumed to vary according to the power-law, $h(0,t)=\sigma t^{\alpha}$, where $\sigma>0$ and α is a constant within the range $-1/2<\alpha<\infty$, we let $\sigma=h_1 t^{-\alpha}$. As the initial fluid level is uniform over the whole reservoir, the zero initial and infinity conditions are $h(r,0)=h_0, h(\infty,t)=h_0$, while the condition at the boundary $r=0$, is $h(0,t)=0$.

$$\rho c_p\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\mu\left(\frac{\partial u}{\partial r}\right)^2 \quad (2.6)$$

The appropriate initial and boundary conditions for this work are

$$T(r,0)=T_0$$

$$T(0,t)=T_1, T(\infty,t)=T_0 \qquad t>0 \quad (2.7)$$

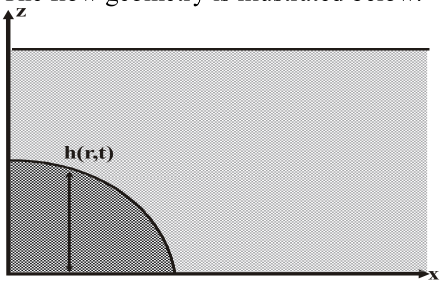
where

r is the distance of the given point of the reservoir from the axis of symmetry,

k -Thermal conductivity, $\rho - Density$, C_p -Specific heat at constant pressure, μ -Dynamic viscosity, $\mu(\partial u/\partial r)^2$ is the viscous heating term, u -Component of velocity in the radial direction, μ_{ef} -Effective viscosity, n - Dimensionless Power-law index, e^T -Thermal expansion, $k_0 -$ The thermal conductivity of the fluid, γ -Thermal expansion exponent, T_0 -Initial temperature and it is the reference temperature, T -Temperature within the boundary layer, T_1, T_2,....T_∞ - Temperature at the plate, $p=\rho gh$, $h-$is the height of the fluid, g is gravitational acceleration, η -Apparent viscosity, i.e. Similarity variable parameter, f -is a dimensionless stream function, m_0 or k_1 $-$Flow consistency index, θ -Dimensionless temperature.

The first two terms on the right hand side of Equation (2.6) represent the heat conduction and viscous dissipation term .

The flow geometry is illustrated below.



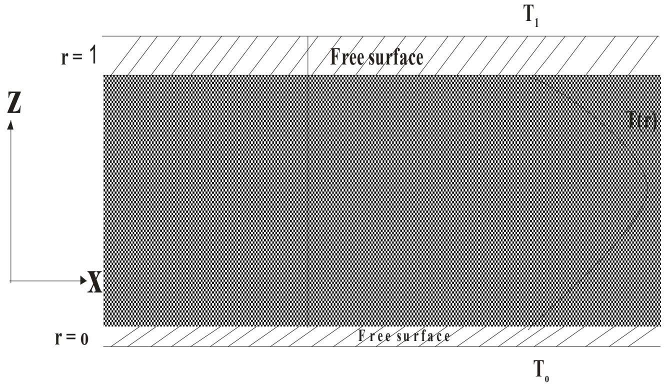


Figure 2.1: Shows the physical model

From Equation (2.6),we consider the case when the coefficient of viscous dissipation term is negligible $(i.e.k_1 << 1)$. The equation becomes

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk(T)\frac{\partial T}{\partial r}\right) \tag{2.8}$$

Following Gitima (2012), we consider a variable thermal conductivity of the form $k(T)=k_0 e^{-\gamma T}$ so that Eq.(2.8) becomes

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk_0 e^{-\gamma T}\frac{\partial T}{\partial r}\right) \tag{2.9}$$

We introduce the following non-dimensional variables

$$t' = \frac{t}{t_0}, \theta(r,t) = \frac{T-T_0}{T_1-T_0}, r' = \frac{r}{R}, h' = \frac{h}{h_0},\ t = t't_0, T = \theta(r,t)[(T_1 - T_0) + T_0], r = r'R \tag{2.10}$$

substituting the dimensionless variable (2.10) into Eq. (2.9) we obtain

$$\frac{\partial \theta}{\partial t'} = -\frac{Br}{\mathrm{Re}\,\mathrm{Pr}} k_0 \gamma e^{-\gamma_1 \theta}\left(\frac{\partial \theta}{\partial r'}\right)^2 + \frac{1}{\mathrm{Pr}\, r'_0} e^{-\gamma_1 \theta}\frac{\partial \theta}{\partial r'} + \frac{1}{\mathrm{Pr}} e^{-\gamma_1 \theta}\frac{\partial^2 \theta}{\partial (r')^2} \tag{2.11}$$

where

$$Br = \frac{u_0^2 \mu R_1}{k_0 (T_1 - T_0)}, \mathrm{Pr} = \frac{\mu \rho c_p}{R_2 k_0}, \mathrm{Re} = \frac{l_0 u_0 \rho}{\mu}, R_1 = \frac{e^{-\gamma_1 T_0} k_0 t_0 \rho (T_1 - T_0)^2}{l_0 R^2 \mu u_0},$$

$$R_2 = e^{-\gamma_1 T_0}\frac{\mu t_0}{R^2}, Peclet = \mathrm{Re} * \mathrm{Pr} \tag{2.12}$$

Dropping the primes, we have

$$\frac{\partial \theta}{\partial t} = -\frac{Br}{Pe}\gamma e^{-\gamma_1 \theta}\left(\frac{\partial \theta}{\partial r}\right)^2 + \frac{1}{\mathrm{Pr}\, r} e^{-\gamma_1 \theta}\frac{\partial \theta}{\partial r} + \frac{1}{\mathrm{Pr}} e^{-\gamma_1 \theta}\frac{\partial^2 \theta}{\partial r^2} \tag{2.13}$$

We seek similarity solution of the form

$$h_1 = h_3 t^{\alpha}, h(r,t) = h_3 t^{\alpha} f(\eta); \eta = rt^{\beta}, \theta(r,t) = t^{\lambda} g(\eta) \tag{2.14}$$

where $\alpha, \beta \in \Re$ are positive constants with dimension time
and f *is a function of* η *only*.
Substituting Equation (2.14) into (2.2) and (2.13) we obtain

$$\frac{d}{d\eta}\left(\eta f f' \left|f'\right|^{\frac{1-n}{n}}\right) = a^2\eta\left(\alpha f - \frac{n+\alpha}{n+1}\eta\frac{df}{d\eta}\right) \quad (2.15)$$

$$\beta\eta^2 g' = -e^{-\gamma_1 g}\left[\frac{Br}{Pe}\gamma\eta(g')^2 - \frac{1}{\Pr}g' - \frac{1}{\Pr}\eta g''\right] \quad (2.16)$$

Since the initial temperature may not necessarily be zero, the above flow is characterized by the existence of a moving boundary conditions which in terms of $f(\eta)$ *leads to*

$$f(0)=0, f(\infty)=1, f'(\infty)=1, g(0)=1, g(\infty)=0 \quad (2.17)$$

Case 1.1

From Equation (3.12) divide through by a^2 and let $a^{-2} << 1$ we obtain

$$f(\eta) = \eta^{\frac{\alpha(n+1)}{\alpha+n}} \quad (2.18)$$

The result is presented in Figure 2.1

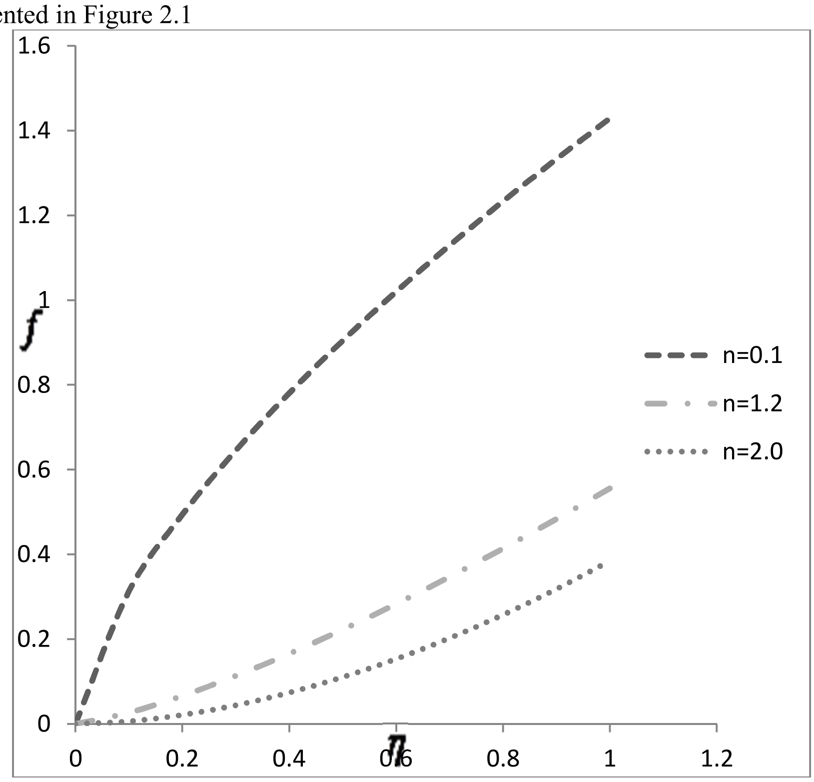


Figure 2.1: Graph of the velocity function f for various values of power-law index n when $\alpha = 0.6, 1.0$.

For the original problem we now proceed to obtain a numerical solution for the velocity profile. We solve Equations (2.16) subject to (2.17) using Galerkin-Weighted Residual method as follows:

$$\text{let } f = \sum_{i=0}^{2} A_i e^{-i/4\,\eta}, g = \sum_{i=0}^{2} B_i e^{-i/4\,\eta} \quad (2.18)$$

A maple pseudo code was used to solve problem (2.16)
The results are presented in Figures 2.2-2.3

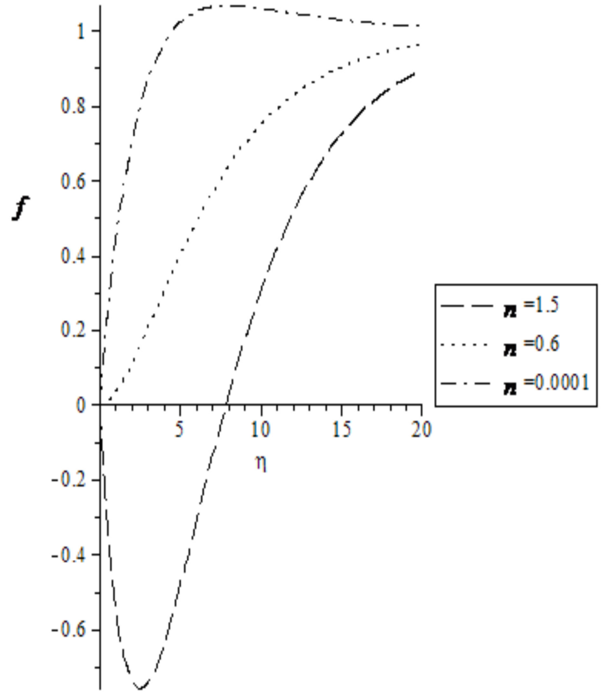


Figure 2.2: Graph of the velocity function f for various value of porous radial flow with constant viscosity when $\alpha = 0.5$.

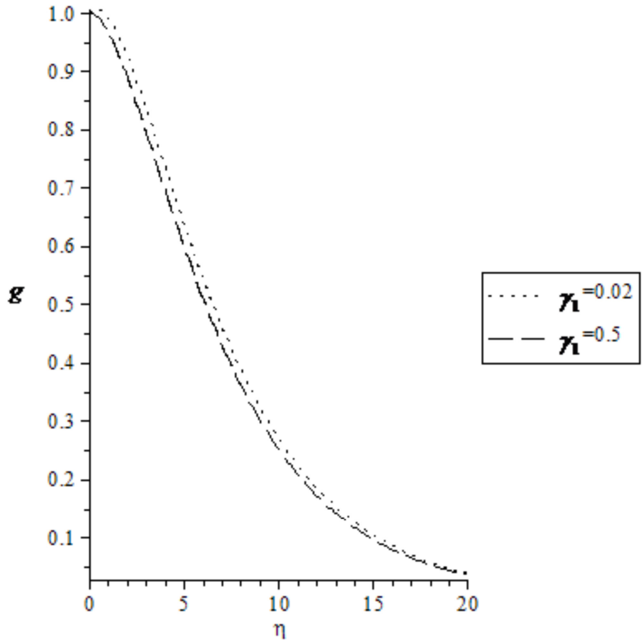


Fig.2.3: Graph of the temperature function g for various value of g when $\beta = 1.5, n > 0, \Pr = 0.25, \mathrm{Re} = 1.0, Br = 0.75$.

In this section, we consider the energy equation through a porous medium with viscous dissipation as follows:

$$\frac{1}{r}\frac{\partial(rhu)}{\partial r} = -\Phi \frac{\partial h}{\partial t} \tag{2.19}$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(k_0 e^{-\gamma T} r \frac{\partial T}{\partial r}\right) + \mu\left(\frac{\partial u}{\partial r}\right)^2 \quad (2.20)$$

The appropriate initial and boundary conditions are

$$h(r,) = h_0, h(0,t) = h_1, h(\infty,t) = 0$$

$$T(r,0) = T_0,\ T(0,t) = T_1, T(\infty,t) = T_0 \qquad t > 0 \quad (2.21)$$

substituting the dimensionless variable Eq. (2.21) into (2.20) we obtain

$$\frac{\partial\theta}{\partial t} = -\gamma k_0 e^{-s\theta}\frac{Br}{Pe}\left(\frac{\partial\theta}{\partial r}\right)^2 + \frac{1}{\Pr}e^{-s\theta}\frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{1}{\Pr}k_0 e^{-s\theta}\frac{\partial^2\theta}{\partial(r)^2} + p_0\left[\frac{\partial}{\partial r}\left(\frac{\partial h}{\partial r}\right)^{\frac{1}{n}}\right]^2 \quad (2.22)$$

where $s = \gamma(T_1 - T_0), p_0 = \frac{\mu h_0^{2/n}}{\rho c_p (T_1 - T_0)}\left(\frac{K\rho}{R\mu_{ef}}\right)^{2/n}, R_3 = \frac{t_0}{R^2}e^{\gamma_1 T_0}, Peclet = \text{Re} * \Pr,$

$$Br = \frac{u_0^2 \mu R_4}{k_0(T_1 - T_0)}, \Pr = \frac{\mu\rho c_p}{R_3 k_0}, \text{Re} = \frac{l_0 u_0 \rho}{\mu}, R_4 = \frac{e^{-\gamma_1 T_0} k_0 t_0 \rho (T_1 - T_0)^2}{l_0 R^2 \mu u_0} \quad (2.23)$$

$$\theta(r,0) = 0, \theta(0,t) = 1, \theta(\infty,t) = 0 \quad (2.24)$$

We seek similarity solution of the form

$$h_1 = h_3 t^{\alpha}, h(r,t) = h_3 t^{\alpha} f(\eta), \theta(r,t) = t^{\lambda}\phi(\eta), \eta = rt^{\beta} \quad (2.25)$$

substituting Equation (2.25) into (2.22) we obtain

$$\beta\eta\phi' e^{s\phi} = \left[-\frac{Br}{Pe}\gamma(\phi')^2 + \frac{1}{\Pr}(\eta^{-1}\phi' + \phi'')\right] + p_0 e^{s\phi}\left[\frac{d}{d\eta}(f')^{\frac{1}{n}}\right]^2 \quad (2.26)$$

Since the initial temperature may not necessarily be zero we have

$$\phi(0) = 1, \phi(\infty) = 0, \eta \in (0,\infty) \quad (2.27)$$

EXISTENCE AND UNIQUENESS

We again examine the existence and uniqueness of problem (2.26) subject to (2.27) and we have the following theorem:

let $\beta > 1$

Theorem 2.1:

Let

$$e^{-s\phi}\left[-Br\eta(\phi')^2 + \text{Re}(\eta^{-1}\phi' + \phi'')\right] + \text{Re}\Pr p_0\left[\frac{d}{d\eta}\left(\frac{df}{d\eta}\right)^{1/n}\right]^2 - \text{Re}\Pr\beta\eta\phi' e^{s\phi} = 0 \quad (2.28)$$

Which satisfies

$$\phi(0) = 1, \phi(\infty) = 0 \quad (2.29)$$

Problem (2.28) subject to (2.29) has a unique solution

Proof:

Let

$$x_1 = \eta, x_2 = \phi, x_3 = \phi' \quad (2.30)$$

Then

$$\phi'' = Br(\phi')^2 - \eta^{-1}\phi' - \frac{\text{Re}\Pr p_0}{e^{-s\phi}}\left[\left(\frac{2\alpha}{\alpha+1}\right)\left(\frac{\alpha-1}{\alpha+1}\right)\right]^2 \eta^{-4/\alpha+1} - \text{Re}\Pr\beta\eta\phi' e^{s\phi} \quad (2.31)$$

The system of equation (2.31) can be written in vector form using

$$x_1' = 1 \qquad = f_1(x_1, x_2, x_3) \quad (2.32)$$

$$x_2' = x_3 \qquad = f_2(x_1, x_2, x_3) \quad (2.33)$$

$$x_3' = \frac{\mathrm{Re}\,\mathrm{Pr}\,\beta x_1 x_3}{e^{-sx_2}} + \frac{Br(x_3)^2}{1} - \frac{x_3}{x_1} - \frac{\mathrm{Re}\,\mathrm{Pr}\,p_0 e^{sx_2}}{1}\left[\left(\frac{2\alpha}{\alpha+1}\right)\left(\frac{\alpha-1}{\alpha+1}\right)\right]^2 x_1^{-4/\alpha+1} = f_3(x_1,x_2,x_3) \quad (2.34)$$

Satisfying

$$1 \le x_1 \le \infty$$
$$-k_2 \le x_2 \le k_2$$
$$-\alpha_1 \le x_3 \le \alpha_1, i.e -1 \le \alpha_1 \le 1 \quad (2.35)$$
$$\phi(0)=1, \phi(\infty)=0 \quad (2.36)$$

Then

$$\left|\frac{\partial f_3}{\partial x_1}\right| \le \left|0 - \frac{\mathrm{Re}\,\mathrm{Pr}(\beta x_3)}{e^{-sx_2}}\right| \le \left|0 + \frac{\alpha_1\,\mathrm{Re}\,\mathrm{Pr}\,\beta}{e^{-sk_2}}\right| \quad (2.36)$$

$$\left|\frac{\partial f_3}{\partial x_2}\right| \le \left|\frac{\alpha_3\,\mathrm{Re}\,\mathrm{Pr}\,\beta x_3 x_1}{e^{-sx_2}} - \frac{se^{sx_2}\,\mathrm{Re}\,\mathrm{Pr}\,p_0\left[\left(\frac{2\alpha}{\alpha+1}\right)\frac{\alpha-1}{\alpha+1}\right]^2}{1} x_1^{-4/\alpha+1}\right| \quad (2.37)$$

$$\le \left|\frac{\alpha_1\alpha_3 Pe\beta}{e^{-sk_2}} + \frac{\alpha_3 Pep_0}{e^{-sx_2}}\left[\left(\frac{2\alpha}{\alpha+1}\right)\left(\frac{\alpha+1}{\alpha+1}\right)\right]^2 x_1^{-5/\alpha+1}\right| \le \left|\frac{\alpha_1\alpha_3 Pe\beta}{e^{-sk_2}}\right|$$

$$\left|\frac{\partial f_3}{\partial x_3}\right| \le \left|\frac{Pe\beta}{e^{-sx_2}} + \frac{2Brx_3}{1} - \frac{1}{x_1}\right| \le \left|\frac{Pe\beta}{e^{-sk_2}} + \frac{2Br\alpha_1}{1}\right| \quad (2.38)$$

The upper bound of K , i.e.

$$K \ (\max) = \left|\frac{\partial f_3}{\partial x_3}\right| \le \left|\frac{Pe\beta}{e^{-sk_2}} + \frac{2Br\alpha_3}{1}\right| \quad (2.39)$$

$$\therefore \ K = \left|\frac{\mathrm{Re}\,\mathrm{Pr}\,\beta}{e^{-sk_2}} + \frac{2Br\alpha_3}{1}\right|. \quad (2.40)$$

The partial derivatives $\left|\frac{\partial f_i}{\partial x_j}\right|, i,j = 1,2,\ldots n$ are bounded since there exists Lipschitz constant k. Hence $\left|\frac{\partial f_i}{\partial x_j}\right|, i,j=1,2,3$ are Lipchitz continuous and are bounded in D for every bounded x_1 *and* x_2.Therefore; problem (2.28) subject to (2.29) has a unique solution. This established the proof.

We now solve Equation (2.28) subject to (2.29) using Central finite difference approximations

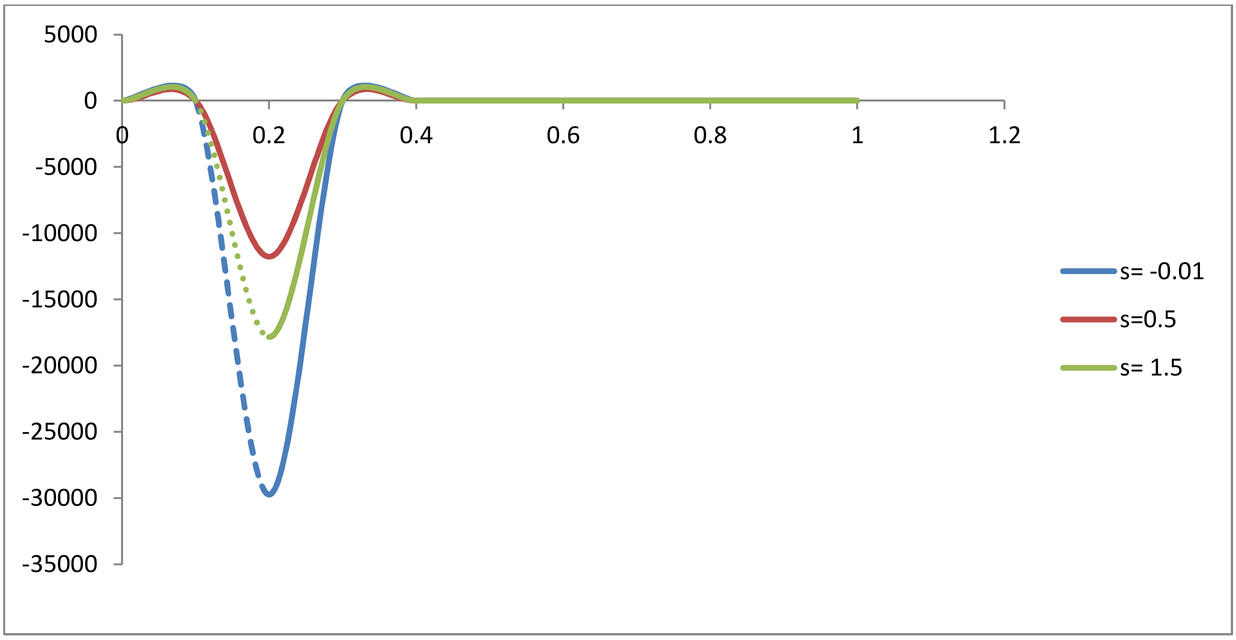


Figure 2.4: Graph of the temperature function ϕ for various values of viscous dissipation parameter when $\beta = -0.001$, $\alpha = 0.001$, $p_0 = 0.001$, $k_0 = \Pr = n = \mathrm{Re} = 1.0$, $a^2 << 1$, $Br = 0$

We now proceed to solve the original Equation (2.28) subject to (2.29) numerically using Galerkin-Weighted Residual Method as follows:

let $f = \sum_{i=0}^{2} A_i e^{\left(-i/4\right)\eta}$ (2.41)

A maple pseudo code was used to solve problem (2.28)
The result is presented in Figure 2.5

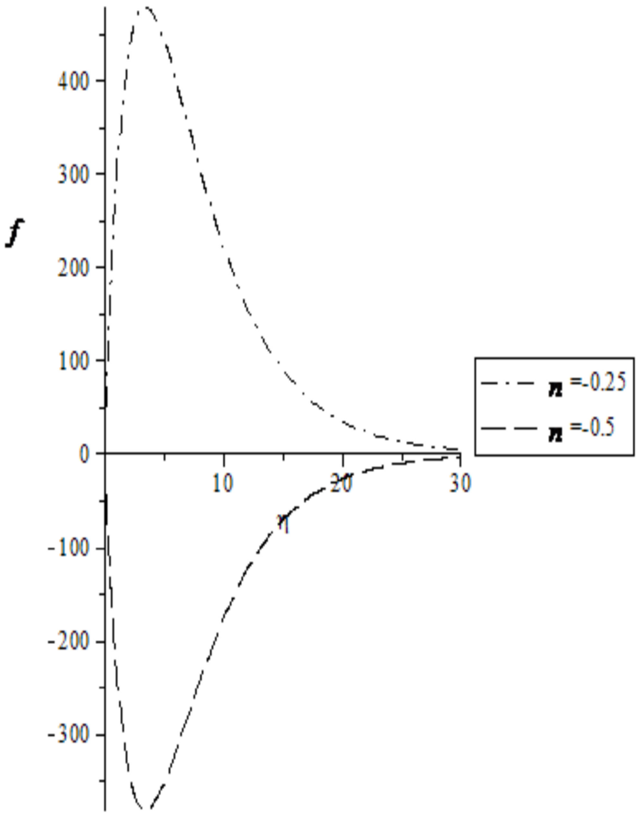


Figure 2.5: Graph of the velocity function f for various values of Viscous dissipation parameter when

$$\beta = 0.1\ , \Pr = \text{Re} = 0.001,\ k_0 = 0.5,\ s = 0.5$$

3.0: Discussion of Results and Conclusion

Discussion of Results

We have considered a suitable model of unsteady variable thermal conductivity gravity flow of a power-law fluid with viscous dissipation through a porous medium. The result from Figure 2.1shows that as power-law index increases the velocity express in terms of stream function f increases monotonically. The result from Figure 2.2 shows that as power-law index decreases the fluid velocity increases monotonically. From Figure 2.3 the results show that the temperature decreases monotonically with increase in each of Brinkman number, Prandtl number, Reynolds number, k_0, s *and* γ_1 thermal conductivity parameters. The result from Figure 2.4 shows that the temperature profile decreases as viscous dissipation parameter increases. From Figure 2.5 the result shows that the velocity profile decreases monotonically as power-law index decreases. Physically, increase in the Prandtl number is due to an increase in the viscosity of the fluid. On the other hand, from analytical and numerical calculations we also see that the parameter $n, s, \Pr, \text{Re}, p_0, \beta, \varepsilon$ *and* Br affects the flow characteristics significantly.

Conclusion

A set of non-linear coupled differential equations governing the fluid temperature is solved analytically and numerically for various parameters. We show that the problem has a solution and the solution is unique. It is noted that the influence of viscous dissipation parameter and thermal conductivity parameter on the flow system is to increase the fluid temperature.

It can be concluded that the increase physical parameters i.e. Reynolds number, Prandtl number, Brinkman number and Peclet number; thermal conductivity parameter and viscous dissipation parameter leads to a corresponding decrease in the viscosity of the fluid. This will be of great importance for the field engineers in various processes of oil recovery.

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