Radiation and Chemical Reaction Effects on MHD Thermosolutal Nanofluid Flow over a Vertical Plate in Porous Medium

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Abstract
In this study we discussed the influence of radiation and chemical reaction on MHD thermosolutal nanofluid convective slip flow over a vertical plate in porous medium in presence of thermophoresis and Brownian motion effects. The governing boundary layer partial differential equations are transformed into system of ordinary differential equations by using similarity transformation and then solved numerically using bvp5c Matlab package. The effects of dimensionless governing parameters on the flow, heat and mass transfer was discussed and presented through graphs. Also, the skin friction coefficient and local Nusselt and Sherwood numbers are computed and discussed. Results indicate that an increase in chemical reaction parameter enhances the mass transfer rate.

Keywords: MHD, Radiation, Chemical Reaction, Nanofluid, Convection.

1. Introduction
Recently there has been an affordable amount of work carried out by scientists/researchers on radiative heat transfer in nanofluids due to their abundant applications. Particularly, the conception of radiative heat transfer is extremely employed in nanofluid solar collectors. Nanofluid transport in porous media has developed into a substantial area of research in recent years. The study of heat transfer in the presence of nanofluids is of great practical importance in engineering and sciences because many branches of science and engineering using cooling applications of nanofluids.

The mathematical modelling of radiative hydromagnetic thermal radiation processing of a nonmaterial fluid sheet extruded in porous media was discussed by Uddin et al. (2014). Radiative heat transfer of nanofluids over a nonlinearly stretching surface with thermal radiation in the presence of variable heat generation and viscous dissipation along with variable temperature is investigated by Anjali Devi and Mekala Selvaraj (2013). The study of mixed convection heat transfer over an inclined flat plate in a porous medium was discussed by Rasekh and Ganji (2013). The effect of local thermal non-equilibrium on transient MHD laminar boundary layer flow of viscous, incompressible nanofluid over a moving surface in a saturated porous media was discussed by Muthtamilselvan et al. (2014). A numerical and analytical study of the magneto-hydrodynamic mixed convection flow of nanofluid over a nonlinear stretching inclined transparent plate embedded in a porous medium under the solar radiation examined by Meisam Habibi Matin and Reza Hosseini (2014). The entropy generation of magnetohydrodynamic mixed convection flow of nanofluid over a nonlinear stretching inclined transparent plate embedded in a porous medium due to solar radiation is investigated numerically by Mohammad Dehsara et al. (2013). The study of mixed convection flow over a vertical cone embedded in a porous medium saturated with nanofluid in the presence of thermal radiation was analyzed by Ali J. Chamkha et al. (2012).

The effect of the complex interactions between the electrical conductivity of the conventional base fluid and the nano particles under the influence of magnetic field in a boundary layer flow with heat transfer over a convectively heated flat surface was stated by Oluwole Daniel and Winifred Nduku (2014). The study of heat and mass transfer in copper and silver nanofluid flow over stretching sheet placed in saturated porous medium with internal heat generation or absorption was discussed by Kameswaran et al. (2013). The magnetic field, radiation and soret effect of a nanofluid flow over a moving vertical plate in porous medium was analyzed by Raju et al. (2015). The effects of aligned magnetic field, radiation and rotation on unsteady hydromagnetic free convection flow of a viscous incompressible electrically conducting fluid past an impulsively moving vertical plate in a porous medium in presence of heat sources analyzed by Sandeep et al. (2014). The heat transfer characteristics of a two dimensional steady of hydromagnetic natural convection flow of a nanofluid over a nonlinear stretching sheet taking in to account the effects of radiation and convective boundary conditions has been investigated numerically by Rahman and Eltayed (2012).

The heat and mass transfer in thermophoretic radiative hydromagnetic nanofluid flow over an exponentially stretching porous sheet embedded in porous medium with internal heat generation/absorption, viscous dissipation and injection effects was illustrated by Sandeep and Sulochana (2015). The mixed convection flow of magnetohydrodynamic Jeffrey nanofluid over a radially stretching surface with radiative surface is studied and convective boundary conditions through heat and mass are employed by Bilal Ashraf et al. (2015). A numerical investigation of unsteady magnetohydrodynamic mixed convective boundary layer flow of a nanofluid over an exponentially stretching sheet in porous medium is presented by Anwar et al. (2013).
recently the researchers Raju et al. ((2015a), (2015b)) Mohankrishna et al. ((2013), (2014)), Sandeep et al. ((2012), (2013)), Ramana Reddy et al. (2014), Sugunamma et al. (2011) discussed the influence of radiation on MHD flows through different channels.

In this study we discussed the influence of radiation and chemical reaction on MHD thermosolutal nanofluid convective slip flow over a vertical plate in porous medium in presence of thermophoresis and Brownian motion effects. The governing boundary layer partial differential equations are transformed into system of ordinary differential equations by using similarity transformation and then solved numerically using bvp5c Matlab package. The effects of dimensionless governing parameters on the flow, heat and mass transfer was discussed and presented through graphs. Also, the skin friction coefficient and local Nusselt and Sherwood numbers are computed and discussed.

2. Mathematical formulation
Consider a steady, two-dimensional, incompressible flow of a nanofluid over a vertical sheet. The \( \bar{x} \)-axis aligned horizontally and the \( \bar{y} \)-axis is normal to it. A transverse magnetic field of strength \( B_0 \) acts normal to the bounding surface. The magnetic Reynolds number is small so that the induced magnetic field is negligible when compared to the applied magnetic field. It is further assumed that the left of the plate is heated by the convection from the hot fluid of temperature \( T_f > T_w > T_\infty \) which provides a variable heat transfer coefficient \( h_f(\bar{x}) \). Consequently a thermal convective boundary condition arises. It is further assumed that the concentration in the left of the plate \( C_f \) is higher than that of the plate concentration \( C_w \) and free stream concentration \( C_\infty \) which provides a variable mass transfer coefficient \( h_m(\bar{x}) \). Chemical reaction effect along with the buoyancy forces are taken into account. The boundary layer equations as per the above assumptions are given by

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \quad (1)
\]

\[
\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g \beta T(\bar{T}_f - \bar{T}_w) + g \beta^* (C - C_w) - \frac{\sigma B_0^2 \bar{u}}{\rho \rho_f} - \frac{\nu}{\kappa_p}, \quad (2)
\]

\[
\bar{u} \frac{\partial C}{\partial \bar{x}} + \bar{v} \frac{\partial C}{\partial \bar{y}} = D_B \frac{\partial^2 C}{\partial \bar{y}^2} + \left(D_U \frac{\partial T}{\bar{T}_w} \right) \left( \frac{\partial T}{\partial \bar{y}} \right)^2 - \frac{1}{(\rho_c \rho_p)} \frac{\bar{q}_r}{\partial \bar{y}}, \quad (3)
\]

\[
\bar{u} \frac{\partial h_m}{\partial \bar{y}} = D_B \frac{\partial C}{\partial \bar{y}} + \left(D_U \frac{\partial T}{\bar{T}_w} \right) \frac{\partial^2 T}{\partial \bar{y}^2} - k_0 (C - C_w), \quad (4)
\]

The appropriate boundary conditions are

\[
\bar{u} = \bar{u}_b, \quad \bar{v} = 0, -k \frac{\partial T}{\partial \bar{y}} = h_f (T_f - T_w), -D_B \frac{\partial C}{\partial \bar{y}} = h_m (C_f - C_w), \quad at \quad \bar{y} = 0,
\]

\[
\bar{u} \rightarrow 0, \quad T \rightarrow T_w, \quad C \rightarrow C_w \quad as \quad \bar{y} \rightarrow \infty, \quad (5)
\]

where \( \alpha = k / (\rho c)_p \) : thermal diffusivity of the fluid, \( \tau = (\rho c)_p / (\rho c)_f \) : ratio of heat capacity of the nano particle and fluid, \( K_p \) : Permeability of the medium, \( (\bar{u}, \bar{v}) \) : velocity components along \( \bar{x} \) and \( \bar{y} \) axes, \( \bar{u}_b = U_r (\bar{x} / L) \) : velocity of the plate, \( L \) :characteristic length of the plate, \( \bar{u}_f = N_U (\partial \bar{u} / \partial \bar{y}) \) :linear slip velocity, \( \rho_f \) :density of the based fluid, \( \sigma \) :electric conductivity, \( \mu \) :dynamic velocity of the base fluid, \( \rho_p \) :density of the nano particles, \( (\rho c)_p \) :effective heat capacity of the fluid, \( (\rho c)_p \) :effective heat capacity of the nano particle material, \( \varepsilon \) :porosity, \( D_B \) :Brownian diffusion coefficient, \( D_U \) : thermophoretic diffusion coefficient, and \( q_r \) :radiative heat transfer in \( \bar{y} \)-direction. We consider fluid to be a gray, absorbing-emitting but non-scattering medium. We also assume that the boundary layer is optically thick and the Rosseland approximation or diffusion approximation for radiation is valid. Thus, the radiative heat flux for an optically thick boundary layer (with intensive absorption), as elaborated by Sparrow and Cess, is defined as
\[ q_r = -\left(4\sigma T^3 / k_i\right) (\partial T / \partial y) \], where \( \sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \) is the Stefan-Boltzmann constant and \( k_i \text{ (m}^{-1}) \) is the Rosseland mean absorption coefficient. Purely analytical solutions to the partial differential boundary value problem defined by (1)-(4) are not possible. Even a numerical solution is challenging. Here we aim to transform the problem to a system of ordinary differential equations. We define the following dimensionless transformation variables:

\[ \eta = \frac{y}{\sqrt{K_p f}}, \quad \psi = U_r \frac{x}{L} \sqrt{K_p f} \eta, \quad \theta(\eta) = \frac{T - T_w}{T_r - T_w}, \quad \phi(\eta) = \frac{C - C_w}{C_f - C_w}, \quad (6) \]

where \( L \) is the characteristic length. From (6), we have \( T = T_w \{1 + (T_r - 1)\theta\} \), where \( T_r = T_r / T_w \) (the wall temperature excess ratio parameter) and hence \( T^4 = T_w^4 \{1 + (T_r - 1)\theta\}^4 \). Substitution of (6) into (2)-(4) generates the following similarity equations:

\[ f'''' + \text{Re} Da (f'' - f') + Gr \theta + Gc \phi - f' = 0, \quad (7) \]

\[ \theta' + \text{Pr} Da f \theta' + \text{Pr} \left[Nb \theta' \phi + Nt \theta^2\right] + \frac{4}{3R} \left[1 + (T_r - 1)\theta\right]^3 \theta' = 0, \quad (8) \]

\[ \phi' + Le \text{Re} Da f \phi' + \frac{Nt}{Nb} \theta' - Kr \phi = 0. \quad (9) \]

The relevant boundary conditions are

\[ f(0) = 0, \quad f'(0) = 1 + a f''(0), \quad \theta(0) = -Nc \left[1 - \theta(0)\right], \quad \phi(0) = -Nd \left[1 - \phi(0)\right], \quad \]

\[ f'(\infty) = \theta(\infty) = \phi(\infty) = 0, \quad (10) \]

where primes denotes differentiation with respect to \( \eta \). The thermophysical dimensionless parameters arising in (5)-(8) are defined as follows: \( \text{Re} = U_r L / \nu \) is the Reynolds number, \( Da = K_p / L^2 \) is the Darcy number, \( M = \sigma B_0^2 L / U_r \rho f \) is the magnetic parameter, \( Gr = g \beta (T_r - T_w) L^2 / U_r^2 \tau, Gc = g \beta^2 (C_f - C_w) L^2 / U_r^2 \tau \) are the thermal and mass Grashof numbers, \( Pr = \nu / \alpha \) is the Prandtl number, \( Nt = \tau D_f (T_r - T_w) / \nu T_w \) is the thermophoresis parameter, \( Nb = \tau D_f (C_f - C_w) / \nu \) is the Brownian motion parameter, \( Le = \nu / D_b \) is the Lewis number, \( a = Nc, \nu / \sqrt{K_p} \) is the hydrodynamic (momentum) slip parameter, \( Nd = h_m, \sqrt{K_p} / D_b \) is the convection-diffusion parameter, \( Kr = k_v L / U_r \) is the chemical reaction parameter and \( Nc = h_j \sqrt{K_p} / k \) is the convection-conduction parameter.

Quantities of physical interest are the local friction factor, \( C_f \), the local Nusselt number, \( Nu_\tau \), and the local Sherwood number, \( Sh_\tau \). Physically, \( C_f \) represents the wall shear stress, \( Nu_\tau \) defines the heat transfer rates, and \( Sh_\tau \) defines the mass transfer rates:

\[ C_f = 2 f''(0), \quad (11) \]

\[ Nu_\tau = -\left[1 + \frac{4}{3R} \left[1 + (T_r - 1)\theta(0)\right]^3\right] \theta(0), \quad (12) \]

\[ Sh_\tau = -\phi(0), \quad (13) \]

where \( Da_\tau = K_p / \bar{x}^2 \) the local Darcy is number for Darcian porous media and \( \text{Re}_\tau = \bar{u}_w \bar{x} / \nu \) is the local Reynolds number.

3. Results and Discussion

The coupled ordinary differential equations (7) to (9) subject to the boundary conditions (10) are solved
numerically using bvp5c Matlab Package. The results obtained shows the influences of the non dimensional governing parameters, namely magnetic field parameter \( M \), thermal Grashof number \( Gr \), radiation parameter \( R \), chemical reaction parameter \( Kr \), Darcy number \( Da \), Reynolds number \( Re \) on velocity, temperature and concentration profiles of the flow. Also, friction factor, local Nusselt and Sherwood numbers are discussed and presented through tables. For numerical results, we considered \( Nb = Nt = Kr = 0.1, a = 0.01, \)
\[ M = Re = Nc = Nd = R = Gr = Gc = 1, Da = 0.5, T_e = Le = 2, Pr = 6.8. \]
These values kept as common in entire study except the varied values as shown in figures.

Figs. 1-3 show the influence of magnetic filed parameter on velocity, temperature and concentration profiles of the flow. It is clear that an increase in the magnetic field parameter decreases the velocity and increases the temperature and concentration profiles of the flow. This is due to the Lorentz’s for acts opposite to the flow direction. Figs. 4 and 5 depicts the effect of thermal Grashof number on temperature and concentration profiles of the flow. It is evident that with the increase in thermal Grashof number we observed depreciation in the temperature and concentration profiles of the flow. This is due to the fact that an increase in thermal Grashof number develops the buoyancy forces; these forces reduce the thermal and concentration boundary layer thickness.

Figs. 6 and 7 illustrate the effect of radiation parameter on the velocity and temperature profiles of the flow. It is observed a hike in the velocity and temperature profiles for higher values of the radiation parameter. Generally, an increase in the radiation parameter releases the heat energy to the flow, these causes to develop the momentum and thermal boundary layer thickness. Figs. 8 and 9 show the influence of Darcy number on the velocity and temperature profiles of the flow. It is evident that an increase in the Darcy parameter depreciates the velocity and temperature profiles of the flow. Physically, increase in Darcy number implies a greater permeability in the porous medium. This corresponds to a decrease in presence of solid fibers and a reduction in thermal conduction heat transfer within the medium. Figs. 10 and 11 depict the effect of Reynolds umber on velocity and temperature profiles of the flow. With the increase in Reynolds number we noticed similar type of results as we observed in the Darcy case. Generally, increasing in the Reynolds number reduces the thermal and momentum boundary layers. Fig. 12 illustrates the effect of chemical reaction parameter on the concentration profiles of the flow. It is clear that an increase in the chemical reaction parameter declines the concentration profiles of the flow.

Table 1 displays the effects of non-dimensional governing parameters on the friction factor, local Nusselt and Sherwood numbers. It is evident from the table that with the increase in chemical reaction parameter, Darcy number and Reynolds number we noticed depreciation in friction factor and increase in heat and mass transfer rate. But we observed reverse results to above with the increase in radiation parameter. An increase in thermal Grashof number enhances the friction factor, local Nusselt and Sherwood numbers. But a raise in the magnetic field parameter showed reverse action to that of thermal Grashof number.

**Table 1 Variation in \( f^*(0), \theta^*(0) \) and \( \phi^*(0) \) for different non-dimensional parameters**

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<th>( Gr )</th>
<th>( R )</th>
<th>( Kr )</th>
<th>( Da )</th>
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Fig. 1 Velocity profiles for different values of η (M=1,3,5)

Fig. 2 Temperature profiles for different values of η (M=1,3,5)

Fig. 3 Concentration profiles for different values of η (M=1,2,3)
Fig. 4 Temperature profiles for different values of $\theta$ for $Gr=1, 2, 3$

Fig. 5 Concentration profiles for different values of $\theta$ for $Gr=1, 2, 3$

Fig. 6 Velocity profiles for different values of $\eta$ for $R=1, 1.5, 2$
Fig. 7 Temperature profiles for different values of $R=1, 1.5, 2$

Fig. 8 Velocity profiles for different values of $Da=0.5, 1, 1.5$

Fig. 9 Temperature profiles for different values of $Da=0.5, 1, 1.5$
Fig. 10 Velocity profiles for different values of

Fig. 11 Temperature profiles for different values of

Fig. 12 Concentration profiles for different values of
4. Conclusions
This study presents the effects of radiation and chemical reaction on MHD thermosolutal nanofluid convective slip flow over a vertical plate in porous medium in presence of thermophoresis and Brownian motion effects. The governing boundary layer partial differential equations are transformed into system of ordinary differential equations by using similarity transformation and then solved numerically using bvp5c Matlab package. The effects of dimensionless governing parameters on the flow, heat and mass transfer was discussed and presented through graphs. Also, the skin friction coefficient and local Nusselt and Sherwood numbers are computed and discussed. The conclusions of the present study are made as follows:

- An increase in magnetic filed parameter reduces the friction factor and heat and mass transfer rate.
- Darcy number have tendency to enhance the heat and mass transfer rate.
- A raise in the chemical reaction parameter reduces the friction factor and enhance the mass transfer rate.
- An increase in the radiation parameter enhances the momentum and thermal boundary layer thickness.

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