Chemical and Process Engineering Research
ISSN 2224-7467 (Paper) ISSN 2225-0913 (Online)
Vol.35, 2015

Soret, Dufour and Chemical Reaction Effects on Convective Heat and Mass Transfer over a Stretching Sheet with Heat Generating Sources: A Lie Group Analysis

C.Sulochana* H.Tayappa
Dept. of Mathematics, Gulbarga University, Gulbarga-585106, Karnataka, INDIA.

Abstract
In this paper we analysed the effects of Soret, Dufour and Chemical reaction on convective heat and mass transfer of an incompressible, electrically conducting fluid over a stretching sheet in the presence of heat generating sources. The similarity solutions are obtained by using scaling transformations. Furthermore, these similarity equations are solved numerically by using shooting technique with fourth-order Runge-Kutta integration scheme. A comparison with previously published work is performed and the results are found to be in good agreement. Numerical results of the local skin-friction coefficient, the local Nusselt number and the local Sherwood number as well as the velocity, the temperature and the concentration profiles are presented for different physical parameters. The result indicates that (i) for fluids with medium molecular weight (H_2, air). Dufour and Soret effects should not be neglected (ii)the thickness of concentration boundary layer enhances in the degenerating chemical reaction case and depreciates in the generating chemical reaction case.
Keywords: Soret and Dufour effects, Chemical reaction, Heat sources, Stretching sheet, Magnetic field, Scaling transformations, Similarity function and Shooting technique.

Nomenclature

a	parameters of the group
C_1, C_2, C_3, C_4	Constants
C	concentration of the fluid (kgm^{-2})
C_p	specific heat ($J\ kg^{-1}\ K^{-1}$)
D_m	coefficient of mass diffusivity ($m^2\ s^{-2}$)
Du	Dufour number
f	similarity function
g	acceleration due to gravity (ms^{-2})
Gc	local modified Grashof number
Gr	local Grashof number
k	chemical reaction parameter
M	magnetic parameter
m	index parameter
Nu	Nusselt number
Pr	Prandtl number
Re_x	local Reynolds number
Sc	Schmidt number
Sh	Sherwood number
Sr	Soret number
T	temperature of the fluid (K)
T_m	mean fluid temperature
$U(x)$	stretching speed of the plate (ms^{-1})
u,v	the x-and y-component of the velocity field (ms^{-1})
x,y	cartesian coordinates (m)
Greek symbols	
Ψ	stream function
η	similarity variable
θ	dimensionless temperature
φ	dimensionless concentration
ν	kinematic viscosity ($m^2\ s^{-1}$)
ρ	density ($kg\ m^{-3}$)
τ	skin-friction
α	heat source parameter
α_l	thermal diffusity ($m^2\ s^{-1}$)

β_T coefficient of thermal expansion (K^{-1})
βc coefficient of thermal expansion with concentration (kg^{-1} m^3)

Subscripts
w wall condition
∞ free stream condition
o constant condition

Superscript
$()'$ differentiation with respect to η

1. Introduction

The study of hydrodynamic flow and heat transfer over a stretching sheet has gained considerable attention due to its application in industry and important bearings on several technological processes. In particular, many metallurgical processes involve the cooling of continuous strips or filaments by drawing them though a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. In the case of annealing and thinning of copper wires, the properties of the final product depend to a great extent on the rate of cooling. By drawing such strips in an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final products of desired characteristics might be achieved and this has been studied by the authors Chakrabarti et al (1979). And also, in several engineering processes, materials manufactured by extrusion processes and heat treated materials travelling between a feed roll and a wind up roll on convey belts possess the characteristics of a moving continuous surface. The steady flow on a moving continuous flat surface was first considered by Sakiadis (1961) who developed a numerical solution using a similarity transformation. The researchers Ramana Reddy et al. (2014), sulochana et al. (2015), Sugunamma et al. (2014) and Veera Suneela Rani et al. (2012) discussed the radiation effects on convective flows through different channels . The two-dimensional flow caused solely by a linearly stretching sheet in an otherwise quiescent incompressible fluid which has a very simple closed from exponential solution was established by Crane (1970). Gupta and Gupta (1977) studied the heat and mass transfer in a stretching surface with suction or injection. Chen and Char (1998) studied the effects of variable surface temperature and variable surface heat flux on the heat transfer characteristics of a linearly stretching sheet. Gorla et.al (1998) studied the MHD effect on a vertical stretching surface with suction and blowing. Seddeek (2007) studied the effects of heat generation or absorption on heat and mass transfer of a visco-elastic fluid with a magnetic field over a stretching sheet. In all of the above mentioned studies the thermal-diffusion and the diffusion-thermo are negligible. The effects of the thermal-diffusion and the diffusion-thermo on the transport of heat and mass has been developed from the kinetic theory of gases by Chapman and Cowling (1952) and Hirshfelder et al (1954) explained the phenomena and derived the necessary formulae to calculate the thermal-diffusion coefficient and thermal-diffusion factor for monoatomic gases or for polyatomic gas mixtures. Kafoussias and Williams (1995) studied the effects of thermal-diffusion and diffusion thermo on steady mixed free-forced convective and mass transfer over a vertical flat plate, when the viscosity of the fluid varies with temperature. Alam et al (2005) studied the effects of Dufour and Soret numbers on unsteady free convection and mass transfer flow past an impulsively started infinite vertical porous flat plate, of a viscous incompressible and electrically conducting fluid, in the presence of an uniform transverse magnetic field. Alam et al (2006) studied the effects of Dufour and Soret numbers on unsteady MHD free convection and mass transfer flow past an infinite vertical porous plate embedded in a porous medium. Raju et al. (2015), Sandeep et al. (2012,2013 &2014) discussed the heat transfer characteristics of MHD flows through different channels. The method has been applied intensively by Pakdemirli (1994), Mukhopadhyay et al (2005) and Layek et al (2007). Mukhopadhyay et al (2012) have discussed lie group analysis of MHD boundary layer slip flow past a heated stretching sheet in presence of heat source/sink .Subhas Abel et al (2013) have investigated MHD flow and transfer of mixed hydrodynamic/thermal slip over a linear vertically stretching sheet. Dulal pal et al (2011) have studied the effect of Soret, Dufour, chemical reaction and thermal radiation on MHD, non-darcy, unsteady mixed convective heat and mass transfer over stretching sheet. Dulal pal et al (2012) have investigated the effects on MHD non-darcian mixed convective heat and mass transfer over a stretching sheet with non-uniform heat source/sink. Lakshminarayana et al (2010) have investigated Soret and Dufour effects on free convection along vertical wavy surface in a fluid saturated darcy porous medium. Ching-Yang-Cheng (2011) have investigated Soret and Dufour effects on free convective boundary layer over inclined wavy surface in a porous medium.

In the risk assessment of nuclear power plants, the possibility and the consequences of a meltdown of the reactor core are usually considered. During the course of such an accident molten fuel and coolant may interact. Violent thermal reaction can dispose the molten fuel into fine particles. These small particles quickly solidify in the coolant and settle on internal structures of the reactor pressure vessel forming a saturated porous bed. The question arises under what conditions the nuclear decay heat can be removed from the particle bed to the ambient coolant by natural convection. Thus the problem of natural convection in saturated porous layers. This analysis of heat transfer in a viscous heat generating fluid also important in engineering processes pertain to flow in which a fluid supports an exothermal chemical or nuclear reaction or problems concerned with dissociating fluids and this has been studied by Kafoussias et al (1998). Angirasa et al (1997) have assumed the volumetric heat generation as constant. For example a hypothetical core-disruptive accident in a liquid metal fast breeder reactor (I MFBR) could result in the setting of fragmented fuel debris as horizontal surfaces below the core. The porous debris could be saturated sodium coolant and heat generation will result from the radioactive decay of the fuel particulate and this was studied by Christofer Philips (1990). The heat losses from the geothermal system in some cases can be treated as if the heat comes from the heat generating sources and this was analysed by Gebhart et al (1971). Keeping this in view porous medium with internal heat source have been discussed by several authors like Naga Radhika et al (2010). Recently Afify (2009) has studied free convective heat and mass transfer of an incompressible electrically conducting fluid past a stretching sheet in the presence of suction and injection with thermal diffusion (Soret) and diffusion-thermo (Dufour) effects. Ibrahim et al (2005) have discussed Lie group analysis of radiative and magnetic field effects on free convection and mass transfer flow past a semi- infinite vertical flat plate. Kalpakides et al (2004) have studied Symmetry groups and similarity solutions for a free convective boundary layer problem. Sivasankaran et al (2006) have discussed Lie group analysis of natural convective heat and mass transfer in an inclined surface.

In this paper we investigate the Lie-group analysis of Soret, Dufour and chemical reaction effects on convective heat and mass transfer flow over a stretching sheet with heat generating sources. The similarity solutions are obtained using scaling transformations. The similarity equations are solved numerically by using fourth order Range-Kutta integration scheme with shooting method. Numerical results of local Nusselt number and local Sherwood number as well, the velocity, temperature and the concentration profiles are presented for different values of the governing parameters. From this analysis we find that the velocity component enhances in the generating and degeneration of chemical reaction cases for $|k|\leq 0.5$ and depreciates for higher $|k|\geq 2.5$. The thickness of the thermal boundary layer decreases in the degenerating chemical reaction case and enhances in the generating chemical reaction case,while the thickness of concentration boundary layer enhances in the degenerating chemical reaction case and decreases in the generating chemical reaction case.

2. Governing equations

Consider the steady free convective heat and mass transfer flow of a viscous, incompressible and electrically conducting fluid past a stretching surface coinciding with the plane y=0. Keeping the origin fixed two equal and opposite forces are applied along the x-axis which results in stretching of the sheet and hence, the flow is generated. The non-uniform transverse magnetic field B(x) is imposed along they-axis. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible. The temperature and the concentration of the ambient fluid are $T_\infty$and C_∞ and those at the stretching surface are $T_w(x)$ and $C_w(x)$ respectively. It is also assumed that the pressure gradient, viscous and electrical dissipation are neglected. The fluid properties are assumed to be constant except the density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq's approximation. Under the above assumptions, the boundary layer form of the governing equations can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)u}{\rho}+g\beta_T(T-T_\infty)+g\beta c(C-C_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_1\frac{\partial^2 T}{\partial y^2}+\frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2}+Q(T_\infty-T) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2}-k_1(C-C_\infty) \tag{4}$$

The boundary conditions for Eqs. (1)–(4) are expressed as

$$u(x,0)=U(x)=C_1x^m,\ v(x,0)=V_w(x)=C_2x^n,\ T=T_w(x)=T_\infty+C_3x^r,\ C=C_w(x)=C_\infty+C_4x^r$$
$$u(x,\infty)=0,\ T(x,\infty)=T_\infty,\ C(x,\infty)=C_\infty \tag{5}$$

Where u, v are the velocity components in the x- and y-directions respectively, ν is the kinematic viscosity, σ is the electrical conductivity, ρ is the density of the fluid, β_T is the coefficient of thermal expansion, β_C is the coefficient of thermal expansion with concentration. T_m is the mean fluid temperature. T and T_∞ are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream, respectively, while C and C_∞ are the corresponding concentrations. Also D_m is the coefficient of mass diffusivity, C_1,C_2,C_3,C_4 are the constants, $U(x)=C_1x^m$ is the stretching speed of the plate, $V_w(x)=C_2x^n$ is the transverse velocity at the surface, $B(x)=B_0x^s$ is the applied magnetic field, C_p is the specific heat at constant pressure, α_l is the thermal diffusivity, K_T is the thermal-diffusion ratio, C_s and is the concentration susceptibility. Q is the co-efficient of temperature dependent heat source, $k_1$is the chemical reaction co-efficient. The stream function Ψ (x,y) is defined by $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ such that continuity equation (1) is satisfied automatically. We introduced the following non-dimensional temperature $\theta=\frac{T-T_\infty}{T_w-T_\infty}$ and non-dimensional concentration $\varphi=\frac{C-C_\infty}{C_w-C_\infty}$ thenEqs. (2)–(4) and boundary condition (5) become

$$\psi_y\psi_{xy}-\psi_x\psi_{yy}-v\psi_{yyy}+\frac{\sigma B_0^2x^{2s}\psi_y}{\rho}-g\beta_T\theta(T_w-T_\infty)-g\beta_C\varphi(C_w-C_\infty)=0 \tag{6}$$

$$\psi_y((T_w-T_\infty)\theta)_x-\psi_x((T_w-T_\infty)\theta)_y-\alpha((T_w-T_\infty)\theta)_{yy}-\frac{D_mK_T}{C_sC_p}((C_w-C_\infty)\varphi)_{yy}+Q\theta(T_w-T_\infty)=0 \tag{7}$$

$$\psi_y((C_w-C_\infty)\varphi)_x-\psi_x((C_w-C_\infty)\varphi)_y-D_m((C_w-C_\infty)\varphi)_{yy}-\frac{D_mK_T}{T_m}((T_w-T_\infty)\theta)_{yy}+k_1\varphi(C_w-C_\infty)=0 \tag{8}$$

$$\psi_y(x,0)=C_1x^m,\ \psi_y(x,0)=-C_2x^n,\ \theta(x,0)=1,\ \varphi(x,0)=1 \quad \text{and}$$
$$\psi_y(x,\infty)=0\ \theta(x,\infty)=0,\ \varphi(x,\infty)=0 \tag{9}$$

3. Group theory analysis and similarity equations

Application of one-parameter transformations group:

The first step of the analysis is to introduce the one-parameter transformation group

$$\bar{x}=a^px,\ \bar{y}=a^qy,\ \bar{\psi}=a^d\psi,\ \bar{\theta}=\theta,\ \bar{\varphi}=\varphi,\ \bar{T}-\bar{T}_\infty=a^e(T-T_\infty),\ \bar{C}-\bar{C}_\infty=a^z(C-C_\infty) \tag{10}$$

Where'a' is the parameter of the group, p,q,d,e and z are real constants to be determined, then substituting group transformation (10) into Eqs. (6)–(8) and boundary conditions (9), we get.

$$\bar{\psi}_{\bar{y}}\bar{\psi}_{\bar{x}\bar{y}}-\bar{\psi}_{\bar{y}}\bar{\psi}_{\bar{y}\bar{y}}+\frac{\sigma B_0^2\bar{x}^{2s}\bar{\psi}_{\bar{y}}}{\rho}-g\beta_T(\bar{T}_w-\bar{T}_\infty)\bar{\theta}-g\beta_c(\bar{C}_w-\bar{C}_\infty)\bar{\varphi}$$
$$=a^{2d-p-2q}\psi_y\psi_{xy}-a^{2d-p-2q}\psi_x\psi_{yy}-a^{d-3q}v\psi_{yyy}-a^{d+2sp-q}\frac{\sigma B_0^2x^{2s}\psi_y}{\rho}-a^eg\beta_T(T_w-T_\infty)\theta-a^zg\beta_c(C_w-C_\infty)\varphi \tag{11}$$

$$\bar{\psi}_{\bar{y}}((\bar{T}_w-\bar{T}_\infty)\bar{\theta})_{\bar{x}}-\bar{\psi}_{\bar{x}}((\bar{T}_w-\bar{T}_\infty)\bar{\theta})_{\bar{y}}-\alpha((\bar{T}_w-\bar{T}_\infty)\bar{\theta})_{\bar{y}\bar{y}}-\frac{D_mK_T}{C_sC_p}((\bar{C}_w-\bar{C}_\infty)\bar{\varphi})_{\bar{y}\bar{y}}+Q(\bar{T}-\bar{T}_\infty)\bar{\theta}$$
$$=a^{d+e-p-q}\psi_y((T_w-T_\infty)\theta)_x-a^{d+e-p-q}\psi_x((T_w-T_\infty)\theta)_y-a^{e-2q}\alpha((T_w-T_\infty)\theta)_{yy}-a^{z-2q}\frac{D_mK_T}{C_sC_p}((C_w-C_\infty)\varphi)_{yy} \tag{12}$$
$$+Qa^e(T_w-T_\infty)$$

$$\bar{\psi}_{\bar{y}}((\bar{C}_w-\bar{C}_\infty)\bar{\varphi})_{\bar{x}}-\bar{\psi}_{\bar{x}}((\bar{C}_w-\bar{C}_\infty)\bar{\varphi})_{\bar{y}}-D_m((\bar{C}_w-\bar{C}_\infty)\bar{\varphi})_{\bar{y}\bar{y}}-\frac{D_mK_T}{T_m}((\bar{T}_w-\bar{T}_\infty)\bar{\theta})_{\bar{y}\bar{y}}+k_1(\bar{C}_w-\bar{C}_\infty)\bar{\varphi}$$
$$=a^{d+z-p-q}\psi_y((C_w-C_\infty)\varphi)_x-a^{d+z-p-q}\psi_x((C_w-C_\infty)\varphi)_{yy}-a^{z-2q}D_m((C_w-C_\infty)\varphi)_{yy}-a^{e-2q}\frac{D_mK_T}{T_m}((T_w-T_\infty)\theta)_{yy} \tag{13}$$
$$+k_1a^z(C_w-C_\infty)\varphi$$

The boundary conditions

$$y=0:\bar{\psi}_y-C_1\bar{x}^m=a^{d-q}\psi_y-a^{mp}C_1x^m=0,\ \bar{\psi}_x-C_2\bar{x}^n=a^{d-p}\psi_x+C_2a^{np}x^n=0$$

$$(\bar{T}_w-\bar{T}_\infty)-C_3\bar{x}=a^e(T_w-T_\infty)-a^{rp}C_3x^r=0,\ (\bar{C}_w-\bar{C}_\infty)-C_4\bar{x}^r=a^z(C_w-C_\infty)-a^{rp}C_4x^r=0$$

$$y\to\infty:\bar{\psi}_y=a^{d-q}\psi_y=0,\ (\bar{T}-\bar{T}_\infty)=a^e(T-T_\infty)=0,\ (\bar{C}-\bar{C}_\infty)=a^z(C-C_\infty)=0 \quad (14)$$

The condition of conformal invariance implies

$$2d-p-2q=d-3q=d+2sp-q=e=z$$
$$d+e-p-q=e-2q=z-2q=e \quad (15)$$
$$d+z-p-q=z-2q=e-2q=z$$
$$d-q=mp,\ d-p=np,\ e=rp,\ z=rp$$

By solving the previous conditions together, we obtain

$$p=\frac{2q}{1-m},\ d=\left(\frac{1+m}{1-m}\right)q,\ s=\frac{m-1}{2},\ n=\frac{m-1}{2},\ e=z=\frac{2(2m-1)}{(1-m)}q,\ r=2m-1 \quad (16)$$

Under the condition in invariant transformation, the group transformation (10) becomes

$$\bar{x}=a^{\left(\frac{2}{1-m}\right)q}x,\ \bar{y}=a^qy,\ \bar{\psi}=a^{\left(\frac{1+m}{1-m}\right)q}\psi,\ \bar{T}_w-\bar{T}_\infty=a^{\left(\frac{2(2m-1)}{1-m}\right)q}(T_w-T_\infty),\ \bar{C}_w-\bar{C}_\infty=a^{\left(\frac{2(2m-1)}{1-m}\right)q}(C_w-C_\infty),$$

$$\bar{\theta}=\theta,\ \bar{\varphi}=\varphi \quad (17)$$

Absolute invariants:

First we find an absolute invariant which is a function of the dependent variable, namely ζ and $\zeta=yx^N$. For this purpose we write.

$$\bar{x}=Ax,\ A=a^{\left(\frac{2}{1-m}\right)},\ \bar{y}=A^{\left(\frac{1-m}{2}\right)}y,\ \bar{\psi}=A^{\left(\frac{1+m}{2}\right)}\psi,\ \bar{T}_w-\bar{T}_\infty=A^{2m-1)}(T_w-T_\infty),\bar{C}_w-\bar{C}_\infty=A^{2m-1)}(C_w-C_\infty) \quad (18)$$

To establish $\overline{yx}^N=yx^N$, we have $\overline{yx}^N=yx^NA^{\left(\frac{1-m}{2}\right)}+\text{N}$. Putting $\left(\frac{1-m}{2}\right)+N=0$, we get $\overline{yx}^N=yx^N$.

Since $N=\frac{m-1}{2}$ and $\zeta=yx^{\left(\frac{m-1}{2}\right)}$ is an absolute invariant.

We now calculate a second absolute invariant $f(\zeta)$, which involves the dependent variable ψ. Let us assume that $f(\zeta)=\bar{x}^{-L}\bar{\psi}$. Now $\bar{x}^{-L}\bar{\psi}=A^{\left(\frac{1+m}{2}\right)+L}x^L\psi$

Putting $\left(\frac{1+m}{2}\right)+L=0$ we get, $L=-\left(\frac{1+m}{2}\right)$. Thus we get the second absolute invariant $f(\zeta)$ as $f(\zeta)=x^{-\left(\frac{1+m}{2}\right)}\psi$, finally $\psi=x^{\left(\frac{1+m}{2}\right)}f(\zeta)$.Similarly, we obtained $T_w-T_\infty=C_3x^{(2m-1)}$, $C_w-C_\infty=C_4x^{2m-1}$. We have also $\theta=\theta(\zeta)$, $\varphi=\varphi(\zeta)$

It can be seen from Eq. (16), that when the sheet is stretched with a speed $U(x)=C_1x^m$ there exists as similarity solution to this problem provided

$$B(x)=B_0x^{\frac{(m-1)}{2}},\ V_w=C_2x^{\frac{(m-1)}{2}} \quad (19)$$

from which the similarity variable and the dependent variables turn out to be of the form

$$\zeta=yx^{\left(\frac{m-1}{2}\right)},\ \psi=x^{\left(\frac{m-1}{2}\right)}f(\zeta),\ \theta=\theta(\zeta),\varphi=\varphi(\zeta),T_w-T_\infty=C_3x^{(2m-1)},\ C_w-C_\infty=C_4x^{(2m-1)}. \quad (20)$$

To avoid the fluid properties appearing explicitly in the coefficients of the equations, we have the following similarity transformations;

$$\eta=y\sqrt{\frac{m+1}{2}\frac{U(x)}{\nu x}},\ \psi=\sqrt{\frac{2}{m+1}\nu xU(x)}f(\eta) \quad (21)$$

$$\theta=\theta(\eta) \quad (22)$$

$$\varphi=\varphi(\eta) \quad (23)$$

$$T_w - T_\infty = C_3 x^{(2m-1)},\quad C_w - C_\infty = C_4 x^{2m-1}. \tag{24}$$

Substituting Eqs. (21)–(24) into the governing Eqs. (6)–(8) and the boundary condition (9), we finally obtain a system of non-linear ordinary differential equations with appropriate boundary conditions :

$$f''' + ff'' - \left(\frac{2m}{m+1}\right)f'^2 - \left(\frac{2m}{m+1}\right)Mf' + \left(\frac{2m}{m+1}\right)Gr\theta + \left(\frac{2m}{m+1}\right)Gc\varphi = 0 \tag{25}$$

$$\left(\frac{1}{\Pr}\right)\theta'' - \frac{2(2m-1)}{m+1}\theta f' + f\theta' + Du\varphi'' - \alpha\theta = 0 \tag{26}$$

$$\left(\frac{1}{Sc}\right)\varphi'' - \frac{2(2m-1)}{m+1}f'\varphi + f\varphi' + Sr\theta'' - k\varphi = 0 \tag{27}$$

The boundary conditions are

$$f(0) = f_w,\ f'(0) = 1,\ \theta(0) = 1,\ \varphi(0) = 1$$
$$f'(\infty) = 0,\ \theta(\infty) = 0,\ \varphi(\infty) = 0 \tag{28}$$

Where primes denote differentiation with respect to η , $M = \dfrac{\sigma\beta_o^2}{\rho c_1}$ is the magnetic parameter, $Gc = \dfrac{g\beta_c(C_w - C_\infty)}{c_1^2 x^{2m-1}}$ is the local modified Grashof number, $Gr = \dfrac{g\beta_T(T_w - T_\infty)}{C_1^2 x^{2m-1}}$ is the local Grashof number, $\Pr = \dfrac{\nu}{\alpha}$ is the Prandtl number, $Du = \dfrac{DmK_T(C_w - C_\infty)}{C_pC_s(T_w - T_\infty)\nu}$ is the Dufour number, $\alpha = \dfrac{2Q}{c_1(m+1)x^{m-1}}$ is the heat source parameter, $Sc = \dfrac{\nu}{D_m}$ is the Schmidt number, $Sr = \dfrac{D_m(T_w - T_\infty)}{T_m\nu(C_w - C_\infty)}$ is the Soret number, $k = \dfrac{k_1}{C_1 x^{m-1}}$ is the chemical reaction parameter, $\mathrm{Re}_x = \dfrac{xU(x)}{\nu}$ is the local Reynolds number, $f_w = \left(-C_2\sqrt{\dfrac{2}{(m+1)\nu C_1}}\right)$ is the suction or injection parameter which is kept constant.

The quantities of physical interest in this problem are the local skin friction coefficient, the local Nusselt number and the local Sherwood numbers which are defined by $Cf = \dfrac{\tau_w}{\left(\dfrac{\rho U^2(x)}{2}\right)} = 2\sqrt{\dfrac{m+1}{2}\dfrac{1}{\mathrm{Re}_x}}f''(0)$,

$$Nu = \frac{xq_w}{k^*(T_w - T_\infty)} = -\sqrt{\frac{m+1}{2}\mathrm{Re}_x}\,\theta'(0) \quad \text{and} \quad Sh = \frac{xm_w}{D_m(C_w - C_\infty)} = -\sqrt{\frac{m+1}{2}\mathrm{Re}_x}\,\varphi'(0)$$
(29)

Where

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)(x,0),\ q_w = -k^*\left(\frac{\partial T}{\partial y}\right)(x,0) \text{ and } m_w = -D_m\left(\frac{\partial C}{\partial y}\right)(x,0) \tag{30}$$

4. Numerical method for solution

The Eqs. (25)–(27) constitute a highly non-linear coupled boundary value problem of third and second-order. So we develop most effective numerical shooting technique with fourth-order Runge-Kutta integration algorithm. To select η_∞ we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $f''(0)$, $\theta'(0)$ and $\varphi'(0)$. The solution process is repeated with another larger value of η_∞ until two successive values of $f''(0)$, $\theta'(0)$ and $\varphi'(0)$ differ only after desired digit signifying the limit of the boundary along η. The last value of η_∞ is chosen as appropriate value for that particular set of parameters. Eqs (25)–(27) of third-order in f and second-order in θ and φhave been reduced to a system of seven simultaneous equations of first-order for seven unknowns following the method of superposition Which was given by Hasen (1979). To solve this system we require seven initial conditions whilst we have only two initial conditions $f'(0)$ and $f(0)$ on f, two initial conditions on each on θ and φ. Still there are three initial conditions $f''(0)$, $\theta'(0)$ and $\varphi'(0)$ which are not prescribed. Now, we employ numerical shooting technique where these two ending boundary conditions are utilized to produce two known initial conditions at $\eta=0$. In this calculation, the step size $\Delta\eta=0.001$ is used while obtaining the numerical solution with $\eta_{max}=11$ and five-decimal accuracy as the criterion for convergence.

5. Results and Discussion

By employing one-parameter group theory to analyse the governing equations and the boundary conditions, the two independent variables are reduced by one, consequently the governing equations reduce to a system of non-linear ordinary differential equations with the appropriate boundary conditions. Finally the system of similarity equations (25)-(27) with boundary conditions (28) is solved numerically by using fourth order Range-Kutta integration with shooting method. In order to get a clear insight of the physical problem, the velocity f', the temperature θ and the concentration φ have been discussed by assigning numerical values to the parameters encountered in the problem.

To be realistic, the values of the Schmidt number are chosen for hydrogen (Sc=0.22), water vapour (Sc=0.6) and ammonia (0.78) at temperature 25^0c and one atmospheric pressure. Prandtl number takes values 0.7, 1, 2 (especially for air Pr=0.71 at temperature 20^0c and one atmospheric pressure). Due to free convention problem local Grashof number for heat transfer takes value 2 and local modified Grashof number for mass transfer takes value 10. Index parameter (m) takes values 0, 1, 2, magnetic parameter takes values 0, 0.5, 1. The values of Dufour and Soret numbers are chosen in such a way that their product is constant provided that the mean temperature T_m is kept constant as well. Dufour number takes values 0.03, 0.15, 0.60 and Soret number takes values 2, 0.4, 0.1. The heat source parameter (α) takes values 2, 4, 6 and the chemical reaction parameter (k) takes values ±0.5, ±1.5, ±2.5.

The variation of f', θ & φ with heat source parameter α is shown in figs 1-3. It is found from Fig.1 that increase in α enhances $|f'|$ at $\eta=0$. From Fig.2 we find that an increase in α enhances the actual temperature at $\eta=0$. From Fig.3 we find that the actual concentration depreciates with increase in α at $\eta=0$. Figs 4-6 represent the variation of f', θ & φ with increase in Dufour parameter Du (or decrease in Soret parameter Sr). It is observed that the velocity component and temperature enhance while the concentration reduces with increase in Dufour parameter (or decreases in Soret parameter). The thickness of temperature boundary layer enhances and the thickness of concentration boundary layer reduces with increase in Dufour parameter (or decrease in Soret parameter).

Figs. 7-9 represent the variation of f', θ & φ with index parameter m. It is observed that the velocity component enhances with increase in m. From figs. 7 & 8 we notice that the thickness of the temperature and concentration boundary layers decrease in the flow region. Figs. 10-12 represent the variation of f', θ & φ with chemical reaction parameter k. It is observed from fig.10 that $|f'|$ reduces in the degenerating chemical reaction case and enhances in the generating chemical reaction case with increase in k. From figs.11 & 12 we observe that the temperature and concentration boundary layers decrease in the degenerating chemical reaction case and enhance in the generating chemical reaction case.

Figs. 13-15 represent the variation of f', θ & φ for different values of suction/injection f_w. It is observed from the fig. 13 that the velocity component reduces with increase in suction parameter f_w while it enhances with increase in injection parameter $|f_w|$. From fig. 14 we find that the thickness of the temperature boundary layer depreciates with f_w while increase in suction parameter $|f_w|$ leads to an enhancement in the thickness of the temperature boundary layer. From fig. 15 we notice that the actual concentration enhances with increase in $|f_w|$.

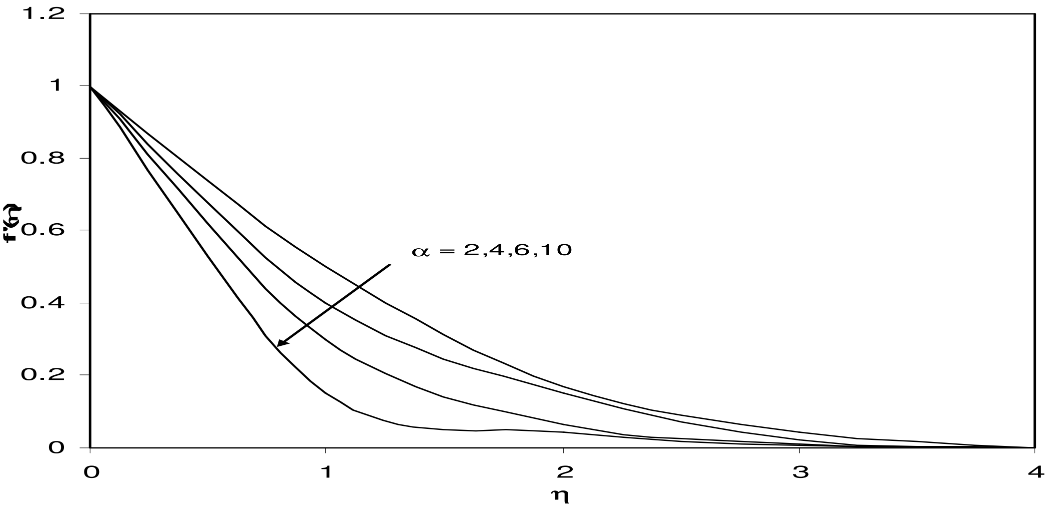

Fig. 1 Variation of $f'(\eta)$ with α

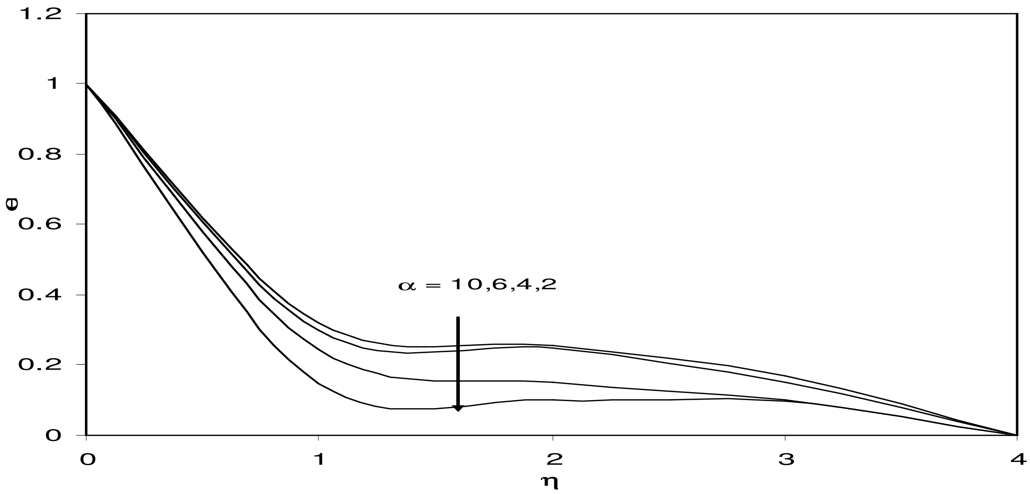

Fig. 2 Variation of θ with α

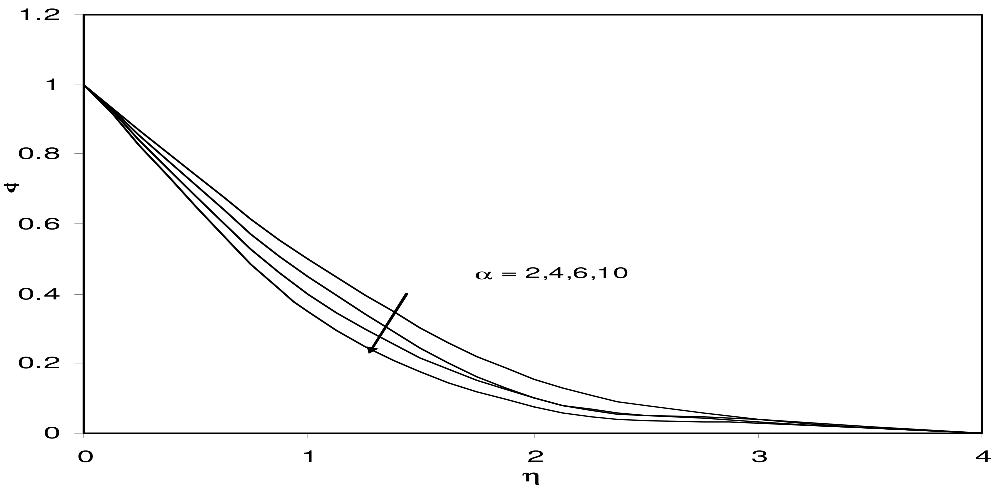

Fig. 3 Variation of ϕ with α

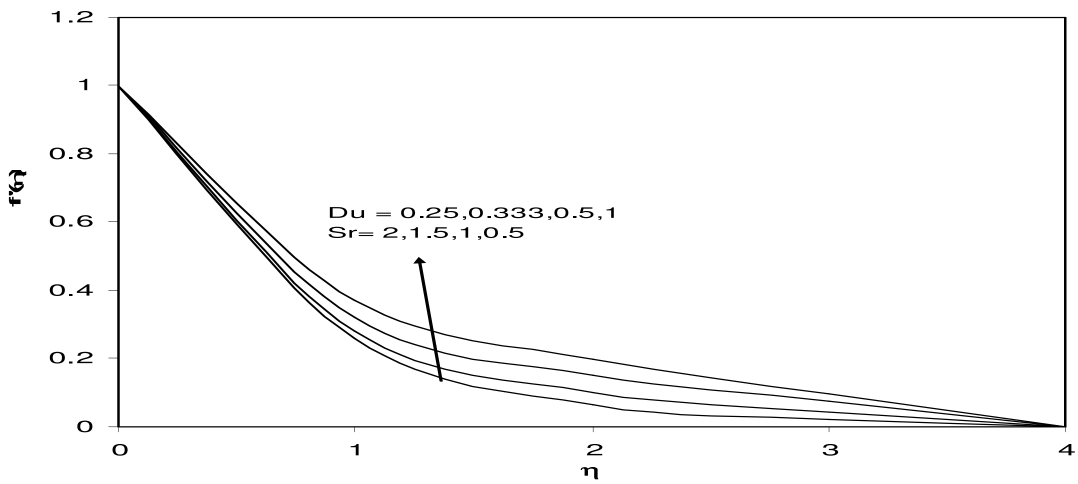

Fig. 4 Variation of f ′(η) with Du and Sr

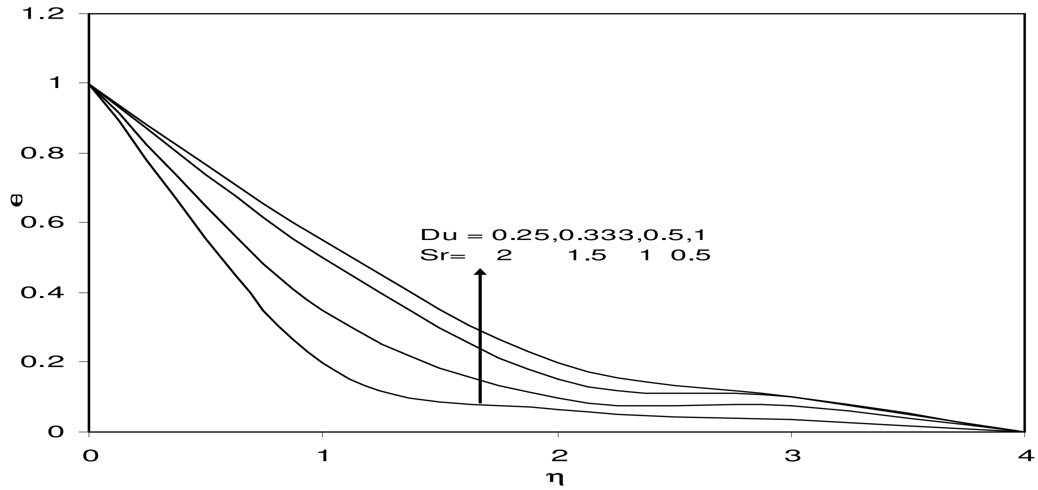

Fig. 5 Variation of θ with Du & Sr

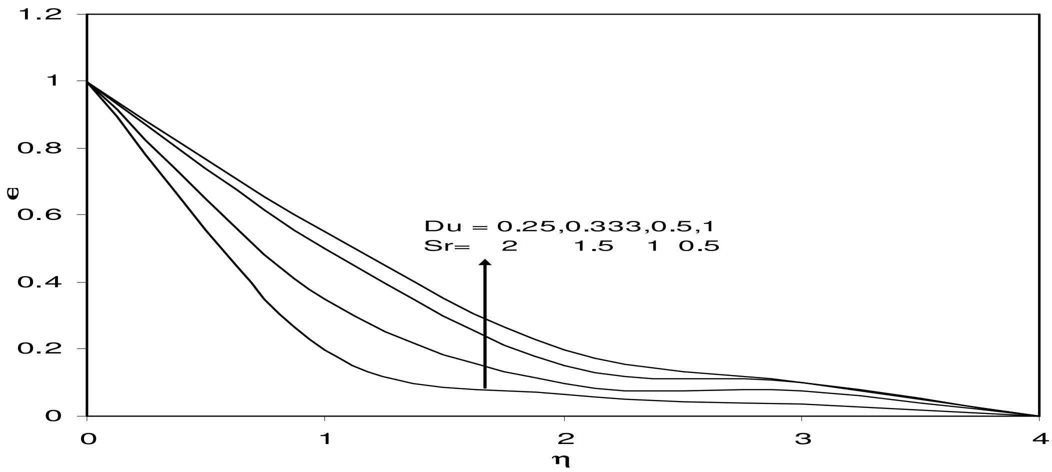

Fig. 6 Variation of ϕ with Du and Sr

Chemical and Process Engineering Research
ISSN 2224-7467 (Paper) ISSN 2225-0913 (Online)
Vol.35, 2015

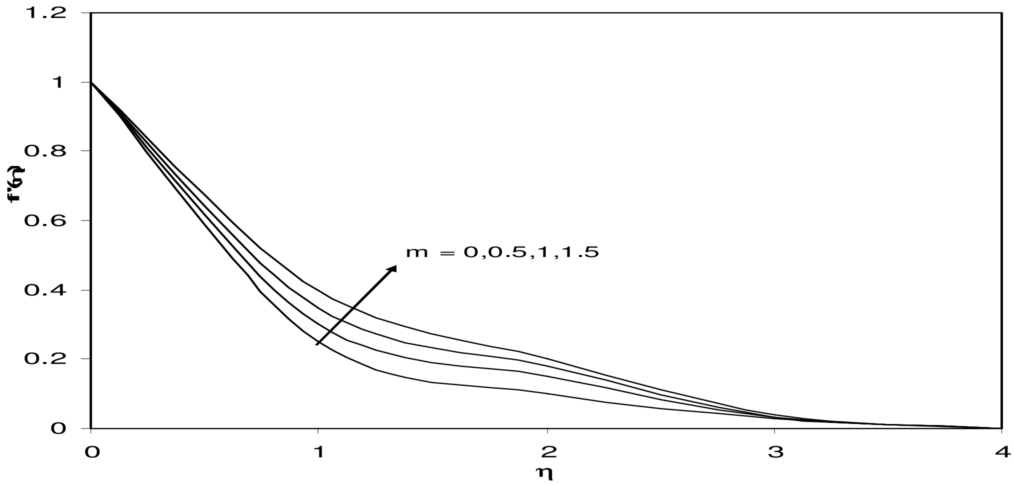

Fig. 7 Variation of $f'(\eta)$ with m

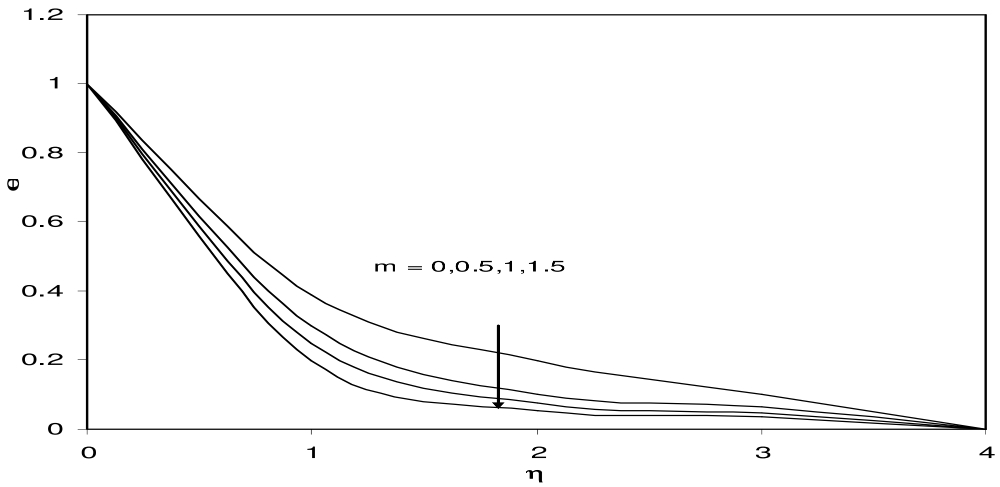

Fig.8 Variation of θ with m

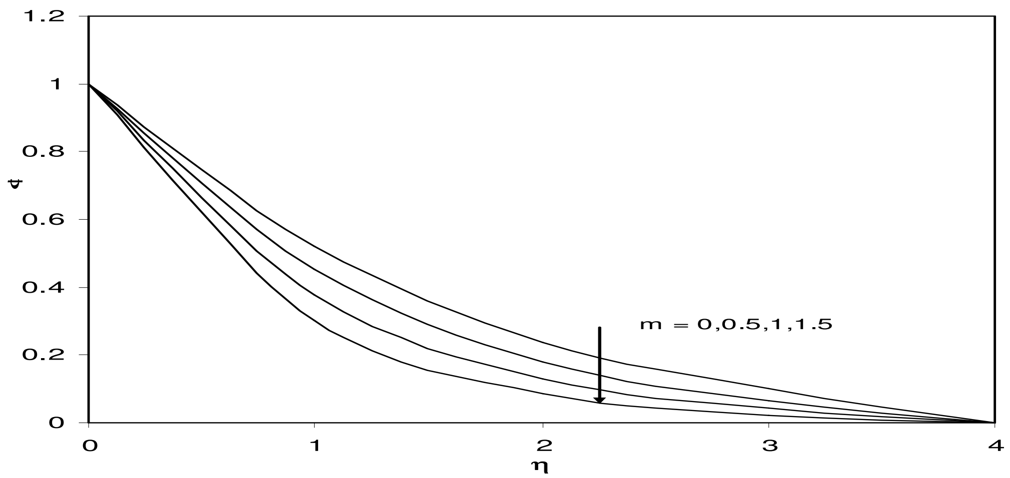

Fig. 9 Variation of ϕ with m

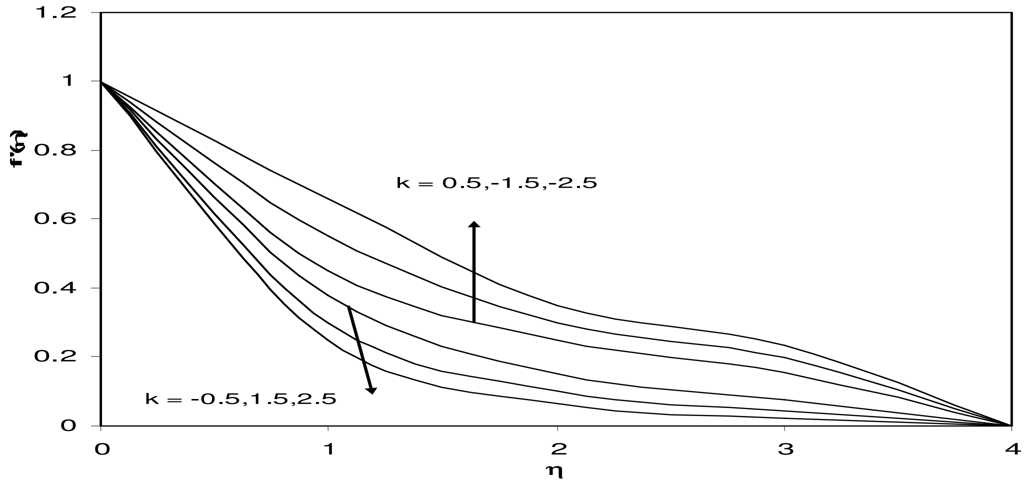

Fig. 10 Variation of f ′(η) with k

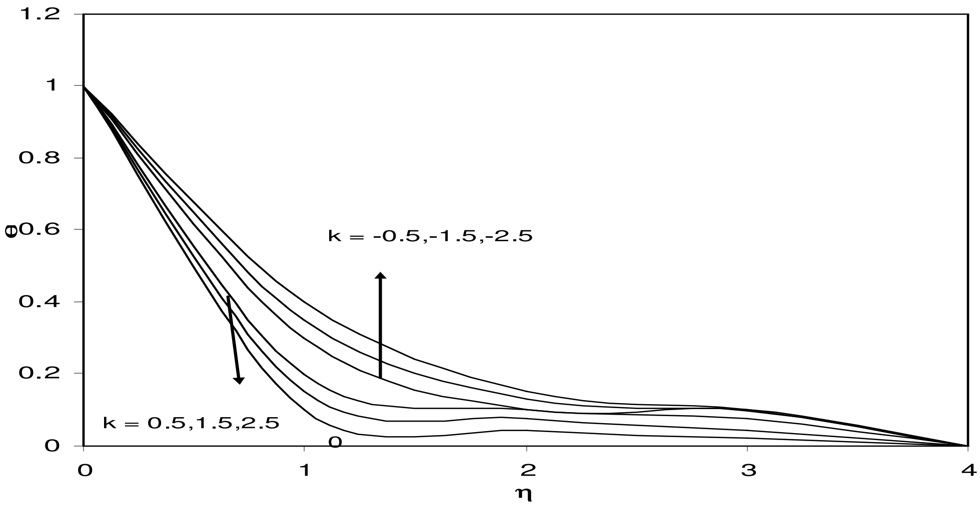

Fig. 11 Variation of θ with k

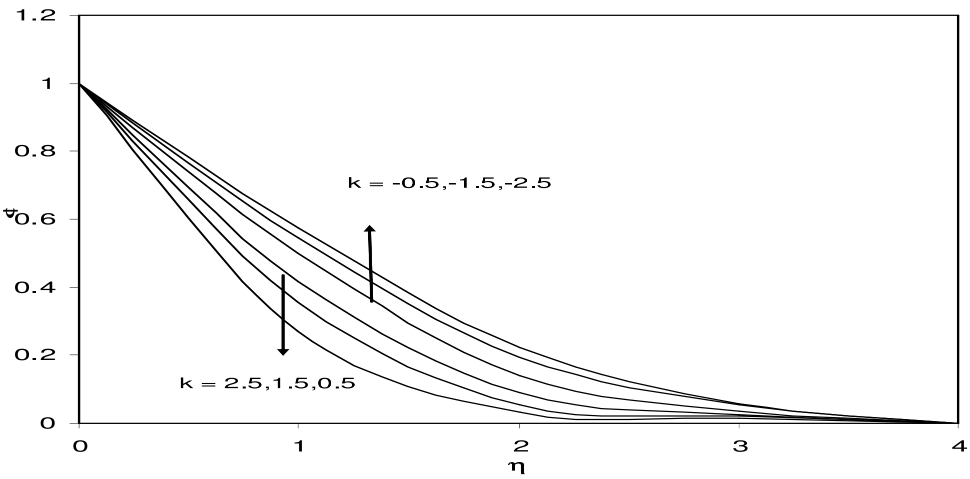

Fig. 12 Variation of ϕ with k

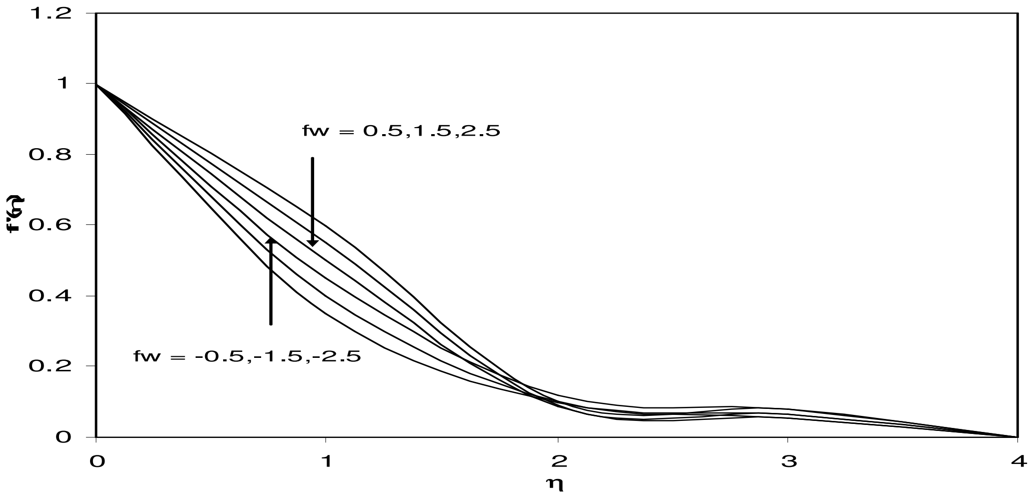

Fig. 13 Variation of f ′(η) with fw

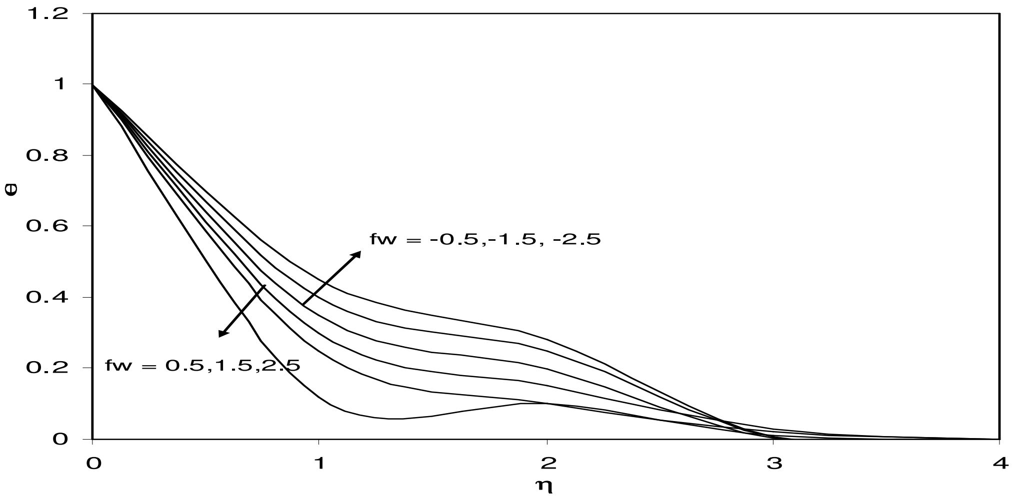

Fig. 14 Variation of θ with fw

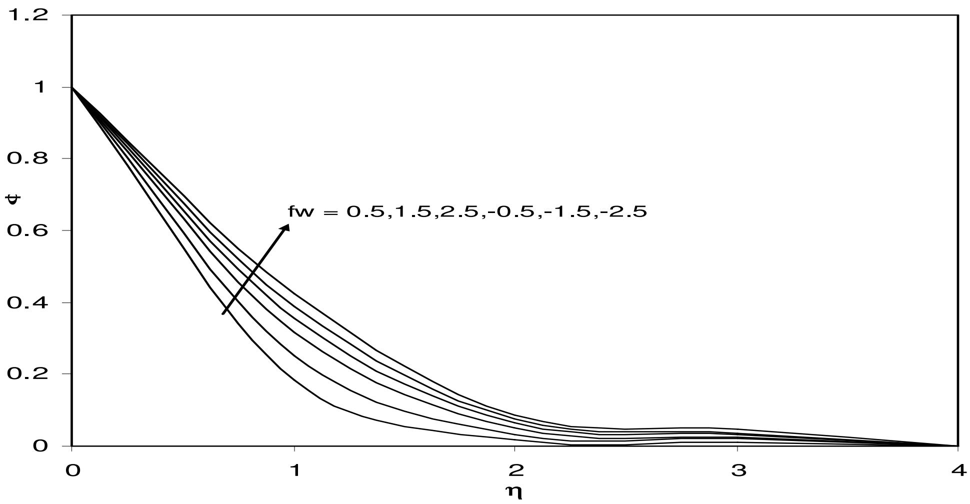

Fig. 15 Variation of ϕ with fw

The skin- friction (τ) at η=0 is shown is tables 1-3 for different values of M, Sc, D*u*, Sr, α, k, m and Pr. It is found that the skin-friction at the plate enhances with increase in the Hartman number M< 1.5 and depreciates with M≥ 2.5. The variation of τ with Schmidt number Sc shows that lesser the molecular diffusivity larger is $|\tau|$ =0 for all M and for further lowering of the molecular diffusivity smaller is $|\tau|$ at the plate. With respect to Soret parameter Sr, it can be seen that $|\tau|$ depreciates for M ≤ 0.5 and enhances with higher M≤1.5 with increase in Sr≤1 and for higher Sr≥1, we notice a depreciation in $|\tau|$ for M≤1.5 and enhancement for M≥2.5. An increase in the Dufour parameter *Du* leads to an enhancement in $|Du|$ for M≤1.5 and depreciates with *Du* for higher M≥2.5. With respect to heat source parameter α we find that $|\tau|$ depreciates with α≤2 and for higher α≥4, $|\tau|$ enhances for M≤1.5 and depreciates for M≥2.5 (Table-1).Table-2 represents the variation of τ with chemical reaction parameter k. It is found that $|\tau|$ enhances for M≤1.5 and depreciates for M=2.5 with increase in k≤1.5, while for higher k≥2.5, $|\tau|$ depreciates for M≤1.5 with increase in k≥0 and for higher M≥2.5 it enhances on the plate. An increase in the index parameter m≤1 depreciates $|\tau|$ at η=0 and for higher m≥2, $|\tau|$ enhances for M≤0.5 and depreciates with M≥2.5. With respect to Prandtl number Pr, we observed that the skin-friction enhances for M≤0.5 and depreciates for M≥1.5. For Pr≤1 and for higher Pr≥2 we notice an enhancement in $|\tau|$ for all M(Table 3).

Table-1: Skin-friction (τ) at η=0

M	I	II	III	IV	V	VI	VII	VIII	IX
0.5	-299314	-3.26014	-3.41614	-2.93116	-2.99061	-2.93975	-2.84909	-1.37434	-3.16283
1.5	-3.0674	-3.38072	-3.78255	-3.60194	-3.67278	-3.04739	-3.08259	-1.85158	-2.69304
5.0	-1.99197	-2.45788	-3.12666	-0.90173	-2.02315	-2.09111	-2.16198	-2.73668	-2.29999
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3	1.3	1.3
Du	0.03	0.03	0.03	0.03	0.15	0.3	0.03	0.03	0.03
Sr	0.4	0.4	0.4	0.4	0.6	1	2	0.4	0.4
α	2	2	2	2	2	2	2	4	6

Table-2: Skin-friction (τ) at η=0

M	I	II	III	IV	V	VI
0.5	-3.41614	-3.15434	-3.04229	-2.92208	-2.88136	-2.85371
1.5	-3.78255	-3.36668	-3.20068	-3.04071	-3.02531	-3.01401
5.0	-3.12666	-2.22051	-3.53872	-2.00507	-2.02858	-2.95976
k	0.5	1.5	2.5	-0.5	-1.5	-2.5

Table-3: Skin-friction (τ) at η=0

M	I	II	III	IV	V	VI	VII	VIII	IX	X
0.5	-3.36167	-2.61614	-2.87864	-3.24253	-299314	-4.08074	-4.08174	-1.71008	-1.32862	-1.20782
1.5	-3.52823	-2.78255	-1.75811	-1.42858	-3.0674	-2.51877	-2.68879	-2.57788	-2.12712	-1.65988
5.0	-3.97729	-3.12666	-1.39356	-1.49981	-1.99197	-1.59096	-1.81088	-2.14275	-1.37544	-0.719319
m	0	1	2	1	1	1	1	1	1	1
Pr	0.71	0.71	0.71	1	2	0.71	0.71	0.71	0.71	0.71

The rate of heat transfer (Nusselt number) at $\eta = 0$ is depicted in the tables 4-6 for different parametric values. It is found that the rate of heat transfer enhances with M≤1.5. With respect to Sc we observe that lesser the molecular diffusivity larger is the rate of mass transfer. An increase in the Soret parameter Sr≤2 leads to an enhancement in |Nu| and for further higher Sr>2 leads to a depreciation in |Nu|. An increase in the temperature heat source parameter α≤2, we notice a depreciation and enhances with higher α≥4 (Table 4).The rate of variation of the Nu with chemical reaction parameter k shows that |Nu| depreciates with k≤1.5 and enhances with higher k≥2.5,while it depreciates in the generating chemical reaction case (Table-5). The rate of heat transfer enhances with increase in index parameter m≤1 and depreciates with higher m≥2. With respect to Prandtl number Pr it can be seen that the rate of heat transfer depreciates with increase in Pr≤1 and enhances with Pr≥2 (Table-6).

Table-4: Nusselt Number (Nu) at η=0

M	I	II	III	IV	V	VI	VII	VIII	IX
0.5	0.805123	1.03796	2.78221	0.62783	0.802003	0.755662	0.678484	0.385703	1.11399
1.5	0.673882	1.14509	2.78522	0.71746	0.646231	0.349213	0.145019	0.532384	1.57865
5.0	0.080914	2.19554	2.85417	3.07346	0.08272	0.059175	0.032566	0.704249	2.93566
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3	1.3	1.3
Du	0.03	0.03	0.03	0.03	0.15	0.3	0.03	0.03	0.03
Sr	0.4	0.4	0.4	0.4	0.6	1	2	0.4	0.4
α	2	2	2	2	2	2	2	4	6

Table-5: Nu at η=0

M	I	II	III	IV	V	VI
0.5	2.78221	0.943906	1.94579	0.76044	0.739743	0.728126
1.5	2.78522	1.00291	1.88167	0.89236	0.0793347	0.071661
5.0	2.85417	0.12299	1.80142	0.05333	0.0526645	0.027529
k	0.5	1.5	2.5	-0.5	-1.5	-2.5

Table-6: Nu at η=0

M	I	II	III	IV	V	VI	VII	VIII	IX	X
0.5	-0.119399	2.782216	0.450028	0.383053	-1.84375	0.653155	0.654155	1.30717	1.69187	-0.502899
1.5	-0.040446	2.768522	0.053628	0.286182	-2.65711	-0.039721	-0.04364	-1.39674	-1.70002	-0.885369
5.0	0.156568	2.854173	0.102313	0.745518	-0.84698	-0.007373	-0.080049	-1.49727	1.826805	1.64757
m	0	1	2	1	1	1	1	1	1	1
Pr	0.71	0.71	0.71	1	2	0.71	0.71	0.71	0.71	0.71

The rate of mass transfer (Sherwood Number) is shown in the tables 7-9 for different parametric values. It is found that the rate of mass transfer enhances with increase in M≤1.5 and depreciates with higher M≥5. With respect to Sc, we find that lesser the molecular diffusivity larger is the rate of mass transfer and for further lowering of molecular diffusivity (Sc≥2.01) smaller is |Sh| at η=0.An increase in Soret parameter Sr≤ 1 leads to a depreciation in |Sh| for all M,while for higher Sr≥2 we notice an enhancement in |Sh|. The variation of Sh with Dufour parameter Du shows that the rate of mass transfer depreciates with increase in Du. An increase in the strengths of heat generating sources results in a depreciation in |Sh| at η=0 (Table-7).Table-8 represents variation of rate of mass transfer with chemical reaction parameter k. The variation of Sh in the degenerating chemical reaction case (k>0) shows that |Sh| depreciates with increase in k≤1.5 and enhances with higher k≥2.5,while in the generating chemical reaction case |Sh| enhances for all M.An increase in the index parameter m≤1 enhances |Sh| and depreciates with higher m≥2. $|Sh|$ depreciates with increase in Pr≤1 and enhances with higher Pr≥2 (Table-9).

Table-7: Sherwood Number (Sh) at η=0

M	I	II	III	IV	V	VI	VII	VIII	IX
0.5	-0.016719	-1.20205	-4.04615	-3.73408	0.13301	0.323973	0.167391	0.140782	0.123456
1.5	-0.070298	-1.74124	-4.14242	-4.11347	-0.05169	-0.087039	0.159823	0.145886	0.125678
5.0	0.06711	-0.72693	-4.54617	-2.29476	0.04997	0.081959	0.130372	0.122387	0.106584
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3	1.3	1.3
Du	0.03	0.03	0.03	0.03	0.15	0.3	0.03	0.03	0.03
Sr	0.4	0.4	0.4	0.4	0.6	1	2	0.4	0.4
α	2	2	2	2	2	2	2	4	6

Table-8: Sh at η=0

M	I	II	III	IV	V	VI
0.5	-4.04615	-0.841769	-4.90584	0.326946	0.534072	0.692002
1.5	-4.14242	-1.37388	-5.36777	0.245887	0.470688	0.644529
5.0	-4.54617	-0.64068	-6.53652	0.375843	0.582326	0.741053
k	0.5	1.5	2.5	-0.5	-1.5	-2.5

Table-9: Sh at η=0

M	I	II	III	IV	V	VI	VII	VIII	IX	X
0.5	-0.150945	-4.0465	-0.228715	-0.05374	-0.231356	-0.111398	-0.111398	0.22919	0.35909	0.171644
1.5	-0.156515	-4.1624	0.0907285	-0.10783	-0.280032	-0.003515	-0.034588	-0.14514	-0.24478	0.171289
5.0	-0.132862	-4.54617	0.101827	0.169854	0.2971196	0.001802	-0.045397	-0.04364	0.28116	0.256365
m	0	1	2	1	1	1	1	1	1	1
Pr	0.71	0.71	0.71	1	2	0.71	0.71	0.71	0.71	0.71

6. Conclusions

This paper presents the effects of Soret, Dufour and Chemical reaction on convective heat and mass transfer of an incompressible, electrically conducting fluid over a stretching sheet in the presence of heat generating sources. The similarity solutions are obtained by using scaling transformations. Furthermore, these similarity equations are solved numerically by using shooting technique with fourth-order Runge-Kutta integration scheme. A comparison with previously published work is performed and the results are found to be in good agreement. Numerical results of the local skin-friction coefficient, the local Nusselt number and the local Sherwood number as well as the velocity, the temperature and the concentration profiles are presented for different physical parameters. The numerical observations are as follows:

- An increase in the magneticfiled parameter depreciates the friction factor and heat transfer rate.
- A raise in the heat source parameter depreciates the velocity, temperature and concentration profiles of the flow.
- An increase in Dufour number enhances the velocity, temperature and concentration profiles of the flow.
- Soret number have tendency to decrease the velocity, temperature and concentration boundary layers.
- An increase in Prandtl number depreciates the mass transfer rate.

References

Afify, A.A. (2009). Similarity solution in MHD effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection, *Commun Nonlinear SciNumerSimulat* 14, 2202-2214.

Alam, M.S., Rahman, M.M., Abdul Maleque M. (2005). Local Similarity solutions for unsteady MHD free convention and mass transfer flow past an impulsively started vertical porous plate with Dufour and Soret effects. *ThammasatInt J Sci Tech* 10,1-18.

Alam, M.S., Rahman, M.M., Samad M A (2006). Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. *Non-linear Anal Modell Contr*, 11, 217-26.

Angirasa D, Peerson G P, Pop I (1997). Combined heat and mass transfer by natural convection with buoyancy effects in a fluid saturated porous medium. *Int. J. Heat and Mass Transfer* 40(12), 2755-2773.

Chakrabarti,A, Gupta A S (1979). Hydromagnetic flow and heat transfer over a stretching sheet. *Q Appl Math* 37, 73-8.

Chapman S, Cowling T G (1952). The mathematical theory of non-uniform gases. Cambridge. UK: *Cambridge University Press.*

Chen C K, Char M I (1998). Heat transfer of a continuous stretching surface with suction or blowing. *J Math Anal Appl* 135, 568-80.

Ching-Yang-Cheng (2011). Soret and Dufour effects on free convective boundary layer over inclined wavy surface in a porous medium. *Int. Communications in Heat and Mass Transfer* 38, 1050-1055.

Christofer Philips G (1990). Heat and Mass Transfer from a film into steady shear flow. *J. I. Mech. Application Mathematics* 431-41.

Crane L J (1970). Flow past a stretching sheet , *ZAMP* 21, 645-7.

Dulal pal, Hiranmoy Mondal (2011). Effect of Soret, Dufour, chemical reaction and thermal radiation on MHD, non-darcy, unsteady mixed convective heat and mass transfer over stretching sheet. *Common Non linearSciNumerSimulat* 16, 1942-1958.

Dulal pal, Hiranmoy Mondal (2012). Effects on MHD non-darcian mixed convective heat and mass transfer over a stretching sheet with non-uniform heat source/sink. *Physica B* 407 642-651.

Gebhart B, Pera D (1971). The natural convection flows resulting from the combined buoyancy effects of natural mass diffusion. *Int. J. Heat Mass Transfer* 15, 2025-2050.

Gorla RSR, Abboud D E, Sarmah A (1998). Magnetohydrodynamic flow over a vertical stretching surface with suction and blowing. *Heat Mass Transfer*, 34, 121-5.

Gupta P S, Gupta A S (1977). Heat and mass transfer on a stretching sheet with suction or blowing. Can J

ChemEng, 55, 744-6.

Ibrahim F.S., Mansoor, M.A., Hamad, M.A.A. (2005). Lie group analysis of radiative and magnetic field effects on free convection and mass transfer flow past a semi- infinite vertical flat plate. *Electronic J. Differential Equations*, 39, 1-17.

Kafoussias N G, Williams EW (1995). Thermal-diffusion and diffusion-thermo effects on mixed free-force convective and mass transfer boundary layer flow with temperature dependent viscosity. *Int J EngSci* 33, 1369-84.

Kalpakides V K, Balassas K G (2004). Symmetry groups and similarity solutions for a free convective boundary layer problem. *Int. J. Non-linear Mech*. 39, 1659-1670.

Kafousias N, Raptis A (1981). Letters in heat and mass transfer 8, 417.

Lakshminarayan P A, Sibanda, (2010). Soret and Dufour effects on free convection along vertical wavy surface in a fluid saturated darcy porous medium.*Int. J. of Heat and Mass Transfer* 53, 3030-3034.

Layek GC,MukhopadhyayS,Samad SA (2007). Heat and mass transfer analysis for boundary layer stagnation point flow towards a heated porous stretching sheet with heat absorption/generation and suction/blowing. *Int. J Commun Heat Mass Transfer* 34, 347-56.

Morgan AJA (1952). the reduction by one of the number of independent variable in some systems of partial differential equations. *Quart J Math* 3, 250-9.

Mukhopadhyay S, Md. S. Uddin, Layek G C (2012). Lie group analysis of MHD boundary layer slip flow past a heated stretching sheet in presence of heat source/sink . *Int. J Appl Math and Mech* 8(16), 51-66.

Mukhopadhyay S,Layek G C, Samad S A (2005). study of MHD boundary layer flow over a heated stretching sheet with variable viscosity.*Int. J. Heat Mass Transf* 48, 4460- 4466.

Na T Y Hasen (1979). Computational methods in engineering boundary value problems. *New York :* Academic Press.

Na TY,Hasen A G (1967). Similarity solutions of a class of laminar, three-dimensional, boundary layer equations of power law fluids. *Int J Non-Linear Mech* 2, 373-85.

Naga Radhika V, Prasad Rao D R V (2010). Dissipative and radiation effects on heat transfer flow of a viscous fluid in a vertical channel. *J. of Pure and Applied Phy.* 22(2), 255- 267.

Pakdemirli M (1994). Similarity analysis of boundary layer equations of a class of non- Newtonian fluids. *Int J Non-Linear Mech* 29, 187-96.

Raju, C.S.K, Sandeep, N., Sulochana, C., Sugunamma, V., Jayachandra Babu, M. (2015). Radiation, Inclined Magnetic field and Cross-Diffusion effects on flow over a stretching surface. *Journal of Nigerian Mathematical Society (In Press)*

Raju, C.S.K., Sandeep, N., Jayachandra Babu,M., Sugunamma, V., (2015). Radiation and chemical reaction effects on thermophoretic MHD flow over an aligned isothermal permeable surface with heat source. *Chemical and Process Engineering Research* 31, 27-42.

Raju, C. S. K., Jayachandra Babu, M., Sandeep, N., Sugunamma, V., Reddy, J.V.R (2015). Radiation and soret effects of MHD nanofluid flow over a moving vertical plate in porous medium, *Chemical and Process Engineering Research* 30, 9-23.

Ramana Reddy, J.V., Sugunamma,V., Sandeep, N., Mohan Krishna, P. (2014). Thermal diffusion and chemical reaction effects on unsteady MHD dusty viscous flow. *Advances in Physics Theories and Applications* 38, 7-21.

Sakiadis B C (1961). Boundary layer behavior on continuous solid surfaces I. Boundary layer equations for two-dimensional and axisymmetric flow. *AJCHE J* 7, 26-8.

SeddeekM A (2007) Heat and mass transfer on a stretching sheet with a magnetic field in a viscoelastic fluid flow through a porous medium with heat source or sink. *Comput Mater Sci* 38, 781-7.

Sandeep, N.,Vijaya Bhaskar Reddy, A., and Sugunamma, V., (2012). Effect of Radiation and Chemical Reaction on Transient MHD Free Convective Flow over a Vertical Plate through Porous Media, *Chemical and Process Engineering Research*, 2, 1-10.

Sandeep N., Sugunamma V., and Mohankrishna P.,(2014). Aligned Magnetic Field, Radiation, and Rotation Effects on Unsteady Hydromagnetic Free Convection Flow Past an Impulsively Moving Vertical Plate in a Porous Medium, *International Journal of Engineering Mathematics,*1-7.

Sandeep, N., and Sugunamma, V. (2013). Effect of Inclined Magnetic Field on Convective Flow of Dusty Viscous Fluid through Porous Medium with Heat Source. *International Journal of applied mathematics and modeling*, 1, 15-32.

Sivasankaran, S., Bhuvaneswari, M., Kandaswamy, P., Ramasami, E.K. (2006). Non-linear Analysis: Lie group analysis of natural convective heat and mass transfer in an inclined surface. *Modlling and Control* 11(1), 201-212.

Subhas Abel M, MonayyaMareppa (2013). MHD flow and transfer of mixed hydrodynamic/thermal slip over a linear vertically stretching sheet. International Journal of Mathematical Archive 4(5), 156-163.

Sulochana, C., Sandeep, N., (2015). Stagnation-point flow and heat transfer behavior of Cu- water nanofluid towards horizontal and exponentially stretching/shrinking cylinders, *Applied Nanoscience* (In Press)

Sugunamma, V., Ramana Reddy, J.V., Raju, C.S.K., Jayachandra Babu, M., Sandeep, N. (2014). Effects of Radiation and Magnetic field on the flow and heat transfer of a nanofluid in a rotating frame. *Industrial Engineering Letters* 4(11), 8-20.

Veera Suneela Rani, A., Dr. V. Sugunamma., and Sandeep, N., (2012). Radiation Effects on Convective Heat and Mass Transfer Flow in a Rectangular Cavity, *International Journal of Innovation and Applied Studies,* 1(1), 118-152.