

Numerical Investigation of Buoyancy Effect Associated with a Continuously Moving Horizontal plate

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Abstract

This work presents numerical studies of the effects of buoyancy force parameter, $B_{f,}$ coupled with viscous dissipation on the convective heat transfer in a fluid-filled rectangular cavity. The cavity is bounded by a hot horizontal plate maintained at temperature T_w and moving continuously at velocity $U_{w.}$ It is also bounded on the left vertical and lower horizontal sides by cold isothermal walls and an adiabatic vertical wall on the right. The cavity was filled with quenching medium. The study was carried out for different quenching media such as oil with Prandtl number, $Pr = 10$, air with Prandtl number, $Pr = 0.7$ and liquid metal with Prandtl number, $Pr = 0.01$, for various bouyancy parameters in the range $5 \times 10^{-3} \leq B_f \leq 10^{-2}$, fixed Eckert numbers, $Ec = 1.0$ and mixed convection parameter, $Gr/Re^2 = 1.0$, in order to characterize the nature of the flow patterns and energy distribution. The flow governing equations including the momentum and energy equations were thereby solved using the finite difference method. The results are presented in the form of profiles for temperature, velocity and local Nusselt numbers. The results show that the buoyancy force parameter has significant influence on the velocity and temperature profile for a Prandtl number higher than unity at fixed viscous dissipation. Further results show that an increase in the buoyancy force parameter for a Prandtl number greater than unity leads to a significant increase in the maximum velocity attainable in the cavity. The results would be useful as baseline data for manufacturing and material processing industries involved with wire drawing, continuous rolling and glass fiber productions.

Keywords: Buoyancy effect, Finite difference scheme, Heat transfer, Isotherms, Mixed Convection

1. Introduction

Heat transfer in the boundary layer adjacent to continuous moving surfaces has many significant applications in many manufacturing processes including the cooling and drying of paper and textiles, wire drawing, continuous casting, metal extrusion, glassfiber production and hot rolling (Sami et al., 2003; Chen, 2000; Ali and Al-Yousef, 1998). The flow over a material moving continuously through a fluid is induced by the movement of the solid materials and by thermal buoyancy. Hence surface motion and buoyancy effect will determine the momentum and thermal transport processes. Thermal buoyancy effect due to the heating or cooling of a continuously moving surface, under some circumstances may alter significantly the flow pattern, thermal field and heat transfer behaviour in the manufacturing process (Chen, 2000).

Many researchers have investigated the effects of buoyancy force caused by continuously moving surfaces on quiescent fluid for different orientations. The numerical simulation of thermal transport associated with a continuously moving flat sheet in materials processing was carried out by Karwe and Jaluria (1991, 1998). Ali and Al-Yousef (1998) examined this effect on vertical surfaces and Chen (2000) examined its effect on vertical and inclined surfaces. They concluded that the buoyancy force has significance on the velocity and temperature distribution and hence on the heat transfer rate from the surface. Wong (2007) investigated the effect of the combined buoyancy- and lid-driven convection in a square cavity in which the influence of pressure on the flow was studied. Al-Sanea and Ali (2000) investigated the effect of buoyancy parameter on moving plate in rolling and extrusion processes. The laminar mixed convection adjacent to a vertical, continuously stretching sheet was studied numerically by Chen (1998). Fan et al. (1997) carried out the numerical investigation of the mixed convective heat and mass transfer over a horizontal plate. A numerical study of the flow and heat transfer characteristics associated with a heated continuously stretching surface being cooled by a mixed convection flow was carried out by Chen (2000). They concluded that the buoyancy force has pronounced effects on the flow field, the local Nusselt number and friction coefficient.

In all the above cited works, the effect of viscous dissipation on heat transfer at high temperature was not considered. However, high speed and temperature have appreciable effects on the energy distributions and flow field (Oosthuizen and Naylor, 1999). From the search of literature, it is clear that the effect of buoyancy force coupled with viscous dissipation on temperature profile, heat transfer rate and flow field in a rectangular enclosure bounded by a continuously moving heated horizontal plate have not been considered. This fact motivates the present study.

The aim of this work is thus to present the effect of varying buoyancy force coupled with viscous dissipation on the flow patterns, energy distribution and heat transfer rate within rectangular cavity.

2 The Physical and the Mathematical Models

Figure 1 shows a continuously moving horizontal plate emerging from a slot at a velocity U_w and temperature T_w into an otherwise quiescent fluid. The plate forms the upper wall of the rectangular enclosure under consideration. The enclosure is also bounded by a fixed horizontal isothermal wall on the lower part, a fixed isothermal vertical wall bordering the extrusion die surface on the left and an adiabatic vertical wall on the right. The temperature T_w of the upper horizontal wall is higher than that of the lower horizontal wall (i.e. $T_w > T_\infty$) as a result of which free convective motion ensued in the enclosure.

The flow is assumed steady, incompressible, laminar, and two-dimensional and the fluid Newtonian. The heat transfer by radiation and the internal heat generation are assumed negligible, while viscous-dissipation effect is considered. The fluid properties are assumed independent of temperature except for the buoyancy term in the momentum equation for which the Boussinesq approximation is used. The extrusion die wall is stationary and impermeable for which the non-slip boundary conditions applied.

The flow governing equations at every point of the continuum comprise the expressions for the conservation of mass, momentum and energy with viscous dissipation term included. These equations for a two-dimensional rectangular domain are (Ozisik, 1985):

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The Navier-Stokes equations in the *x*- and *y*-directions:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

where $\beta g(T - T_\infty)$ is the body force per unit volume in *y* direction.

The thermal energy transport equation

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\phi \tag{4}$$

where ϕ is the viscous-energy-dissipation function, defined as:

$$\phi = 2\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right) + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{5}$$

The consideration of this function becomes important if either the fluid viscosity or the flow velocities are high (Oosthuizen and Naylor, 1999).

The prescribed boundary conditions for the velocities and the temperature are:

$$u = U_w, \quad v = 0, \quad T = T_w \;\; at\; y = H, \;\; 0 \le x \ge L;$$
$$u = 0, \quad v = 0, \quad T = T_\infty \;\; at\; y = 0, \;\; 0 \le x \ge L;$$
$$u = 0, \quad v = 0, \quad T = 0 \;\; at\; x = 0, \;\; 0 \le y \ge H;$$
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial T}{\partial x} = 0 \quad at\;\; x = L, \;\; 0 \le y \ge H \tag{6}$$

3 Method of Analysis and the Solution Techniques

The Navier-Stokes equations are class of partial differential equations that could be classified as elliptic, parabolic or hyperbolic depending on the problem under consideration. These equations in their incompressible form can be solved by using either the vorticity-stream function approach or in their primitive-variable form. In this work, the former approach is adopted and so equations (2) and (3) are reduced to vorticity transport equation by eliminating the pressure gradient terms between them using the continuity equation (1), and the expression for scalar value of the vorticity, ω, in the two-dimensional Cartesian coordinate system defined as

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{7}$$

The resulting expression is the dimensional vorticity transport equation:

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = -\beta g\frac{\partial T}{\partial x} + v\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) \tag{8}$$

The velocity components in a two-dimensional Cartesian coordinates are defined as the derivatives of the stream-function, ψ, as follows:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{9}$$

which on substitution in equation (7) gives the Poisson equation for the stream function

$$\omega = -\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) \tag{10}$$

The derived and energy equations and the prescribed boundary conditions were cast in non-dimensional form so that the results obtained could be generalized for a wide range of physical situations using $L, (T_w - T_\infty), U_w$ $U_w L,$ and U_w/L respectively for length, temperature, velocity, stream function and vorticity following Sami et al. (2003):

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ U = \frac{u}{U_w},\quad V = \frac{v}{U_w},$$

$$\theta = \frac{(T - T_\infty)}{(T_w - T_\infty)},\ \Psi = \frac{\psi}{U_w L},\quad \Omega = \frac{\omega}{U_w/L} \tag{11}$$

The normalized form of the X- and Y-velocity components, stream function, vorticity and energy transport equations are:

$$U = \frac{\partial\psi}{\partial Y},\ V = -\frac{\partial\psi}{\partial X} \tag{12}$$

$$-\Omega = \frac{\partial^2\Psi}{\partial X^2} + \frac{\partial^2\Psi}{\partial Y^2} \tag{13}$$

$$U\frac{\partial\Omega}{\partial X} + V\frac{\partial\Omega}{\partial Y} = \frac{1}{Re}\left(\frac{\partial^2\Omega}{\partial X^2} + \frac{\partial^2\Omega}{\partial Y^2}\right) - \frac{Gr}{Re^2}\frac{\partial\theta}{\partial X} \tag{14}$$

$$U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y} = \frac{1}{PrRe}\left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right) + \left(\frac{Ec}{Re}\right)\left[2\left(\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right) + \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right)^2\right] \tag{15}$$

In the above equations, *Ec* stands for Eckert number, *Pr* Prandtl number, *Re* Reynolds number and *Gr* Grashof number. The Eckert number relates the flow viscous-dissipation term to the energy distributions. The number is a criterion for deciding whether the viscous-energy dissipation effect should be considered in the heat transfer analysis or not. The Prandtl number relates the rates of diffusion of heat and momentum. The Grashof number is a dimensionless parameter representing the ratio of the buoyancy force to the viscous force in the free-convection flow problem. It indicates whether the flow is laminar or turbulent and the dynamic process that is dominant.

The boundary conditions in non-dimensional form are:

$$\Omega \neq 0;\ \Psi \neq 0;\ V = 0;\ U = \theta = 1 \text{ at } Y = 1;\ 0 \leq X \leq 1;$$

$$\Omega \neq 0;\ \Psi = U = V = \theta = 0 \text{ at } Y = 0;\ 0 \leq X \leq 1;$$

$$\Omega \neq 0;\ \Psi = U = V = \theta = 0 \text{ at } X = 0;\ 0 \leq Y \leq 1;$$

$$\Omega \neq 0;\ \Psi = \frac{\partial U}{\partial X} = \frac{\partial V}{\partial X} = \frac{\partial\theta}{\partial X} = 0 \text{ at } X = 1;\ 0 \leq Y \leq 1; \tag{16}$$

The vorticity and energy transport equations (14) and (15) are non-linear in character. There are no known general analytical solutions for these coupled equations. One of the most fruitful approaches to the solution of equations (12) – (15) is the finite difference method, which involves the approximation of each term of the differential equations by their corresponding differential quotient. The resulting linear equations were solved simultaneously by adopting the relaxation technique.

The convective heat transfer into the enclosure is computed in terms of the Nusselt number, which is a dimensionless number that describes the ratio of the heat transfer by convection and conduction across the fluid layer. The temperature gradient that would result from the heat exchange process between the fluid and the wall can be related to the local Nusselt number, Nu_x, through the following expression:

$$Nu_x = \frac{h_X X}{k} = -\left(\frac{\partial\theta}{\partial Y}\right)_{Y=1} \tag{17}$$

The average Nusselt number is obtained by the integration of the local Nusselt number over the entire length of the heated wall:

$$\overline{Nu} = \frac{\dot{Q}_{conv}}{\dot{Q}_{cond}} = -\int_0^1 \frac{\partial\theta}{\partial Y}\bigg|_{Y=0\ or\ 1} dX \tag{18}$$

The flow steady state was determined by monitoring the convergence of the temperature and vortex field, using the following criterion:

$$\frac{\sum_{i=2}^{N}\sum_{j=2}^{M}\left|\phi_{ij}^{n+1} - \phi_{ij}^{n}\right|}{\sum_{i=2}^{N}\sum_{j=2}^{M}\left|\phi_{ij}^{n+1}\right|} < \delta \tag{19}$$

The parameter ϕ stands for Ψ, θ or Ω and n denotes the number of iterations before convergence of the results. The value of δ used in different literatures varies between 10^{-3} and 10^{-8} (Chung, 2002).

4 Discussion of Numerically Generated Results

The effect of the convergence criterion on the numerical results was studied by computing the average Nusselt number at different value of convergence parameter, δ, between 10^{-1} and 10^{-8}. The results which are presented in Figure 2 show that a value of $\delta = 10^{-4}$ was adequate for convergence. The results of the grid independence tests show that a 41 by 41 grid system is sufficient for good numerical stability, field resolution and accurate results as reported in a similar work carried out by Waheed (2009). In order to ascertain the validity of the code used in this work and the accuracy of the present simulation, the Nusselt number for a convective flow of a classical problem was computed for a Prandtl number, Pr = 0.7 and Rayleigh number, Ra = 1000. The Nusselt number computed with the help of the program used in this work is Nu = 1.1210 which compared very well with Nu = 1.132 computed by Waheed (2006) for the same Rayleigh and Prandtl numbers with about 2% discrepancy. Further validation of the results was done as follow: for $Ra = 10^5$ and Pr = 0.7, the computed Nusselt number by Hong (1992) is Nu = 4.5885 and the one computed by Waheed (2006) is 4.6201. The computed Nusselt number from the present simulation is 4.7438 which is in good agreement with the above two results.

Figure 3 shows the non-dimensional temperature profile for various Eckert numbers, Ec, at mid-plane, x = 0.5, for Ra = 1000, Pr = 0.7, and Re = 100. It can be seen from the figure that Eckert number has significant effect on the temperature profile such that an increase in number enhances temperature gradient. The results imply that viscous dissipation is more significant at high temperature and velocity.

Figures 4, 5 and 6 show the temperature profiles respectively for Pr = 0.01, Pr = 0.7 and Pr = 10.0 at Eckert number, Ec = 1.0 and Richardson number, Gr/Re^2 = 1.0 for various values of buoyancy force parameter, B_f. It is evident from these figures that an increase in the buoyancy force parameter results in an increase in the wall temperature gradient close to the moving plate, and hence produces an increase in the surface heat transfer rate. Also the effects of buoyancy force are found to be more pronounced for a fluid with a high Prandtl number.

Figure 7 shows the variations of Nusselt number with the buoyancy force parameter. It is observed that for a particular value of Pr, the Nusselt number increases with the buoyancy force parameter. Also an increase in the Prandtl number results in an increase in the Nusselt number. It is clear from Figure 8 that a positive buoyancy force parameter at high Prandtl number produces an increase in the velocity gradient that enhances the momentum transport, which in turn increases the flow rate. For liquid metal, i.e. fluid with $Pr \ll 1$, the effect of convection on the fields is very weak. Small Pr could result from very small momentum diffusivity (i.e. very weak convection) or very high thermal diffusivity. These diffusion rates are precisely the quantities that determine the thickness of the boundary layers for a given external flow field. Large momentum or thermal diffusivity means that the viscous or temperature influence is felt farther out in the flow field. It implies that positive increase in the buoyancy force parameter accelerates the flow at a particular Prandtl number.

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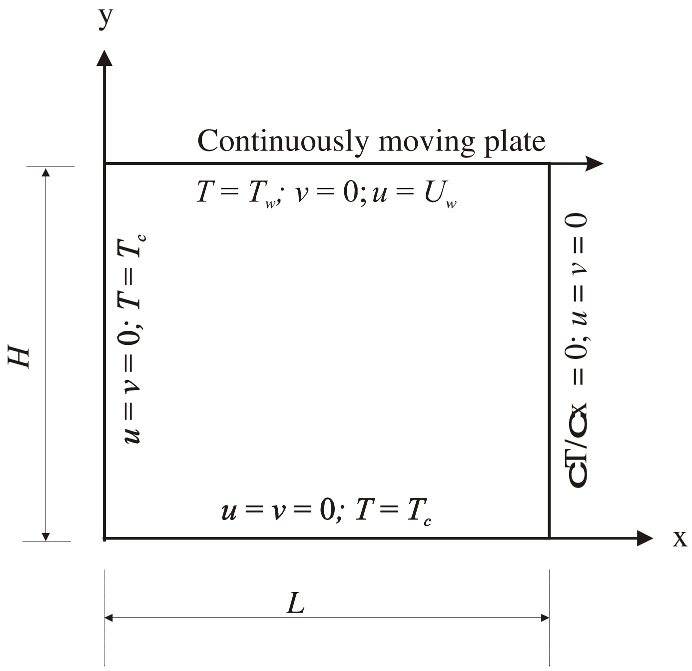

Figure 1: Schematic representation of the physical model with the boundary constraints and the coordinate axes.

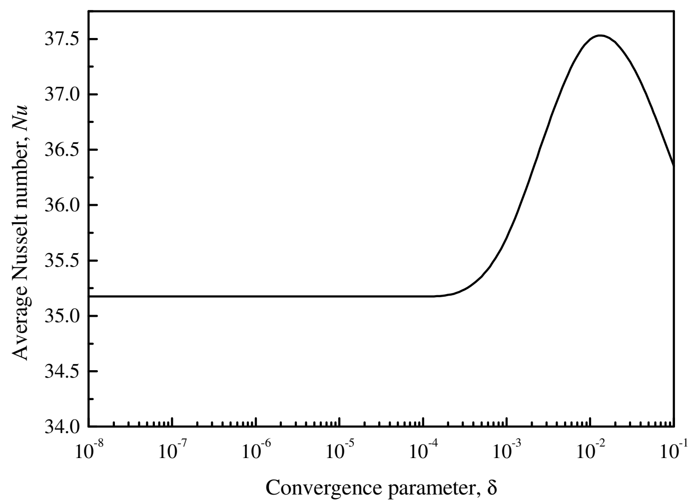

Figure 2: Plot of average Nusselt number, *Nu* versus the convergence parameter, δ

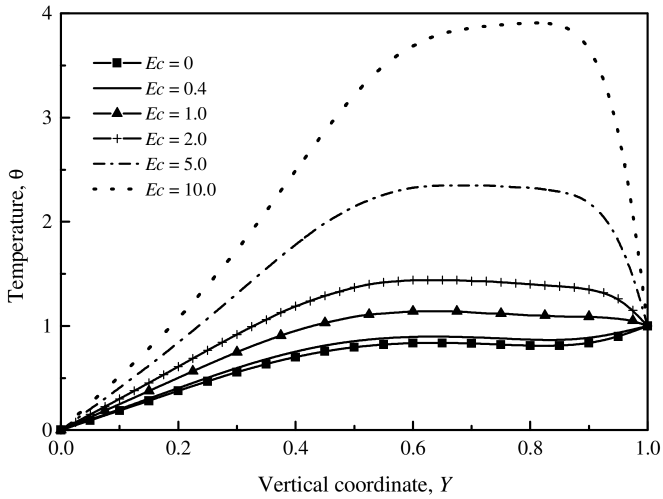

Figure 3: Plot of Temperature, θ versus vertical coordinate, *Y* for various Eckert numbers

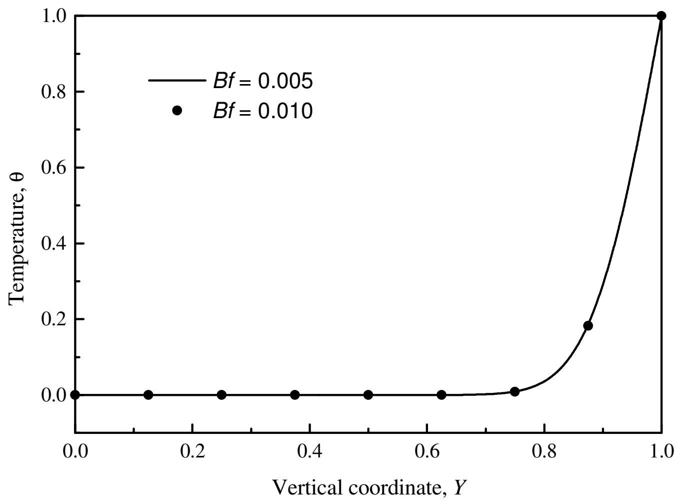

Figure 4: Plot of Temperature, θ versus vertical coordinate, *Y* for various Buoyancy factor, $Pr = 0.01$

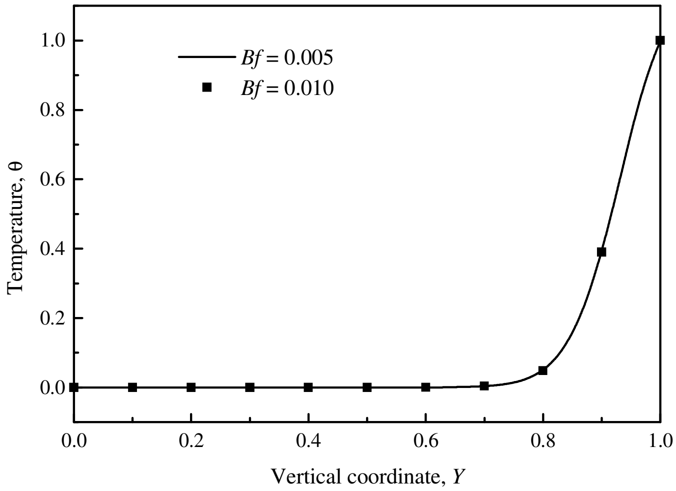

Figure 5: Plot of Temperature, θ versus vertical coordinate, *Y* for various Buoyancy factor, Pr = 0.7

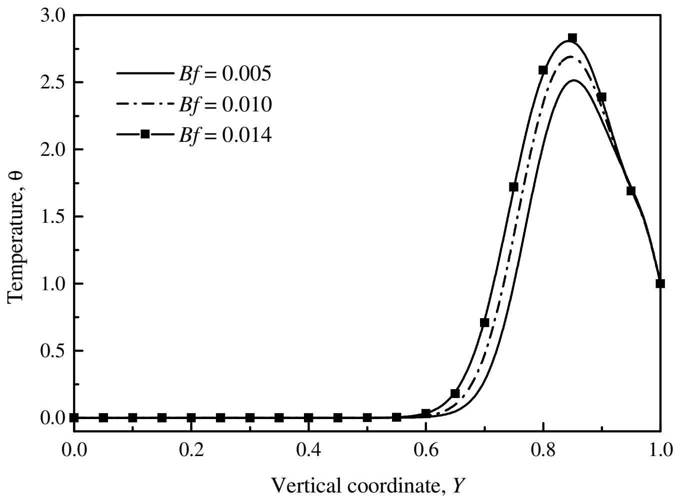

Figure 6: Plot of Temperature, θ versus vertical coordinate, *Y* for various Buoyancy factor, *Pr* = 10.0

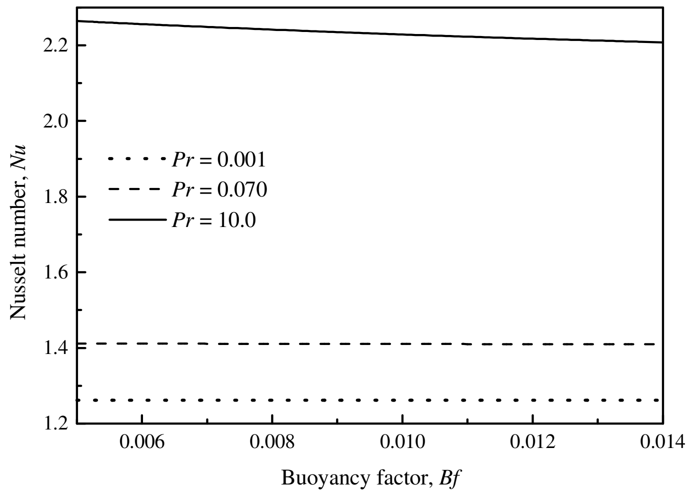

Figure 7: Plot of Nusselt number, *Nu* versus Buoyancy factor, *Bf*, for various Prandtl numbers, *Pr*

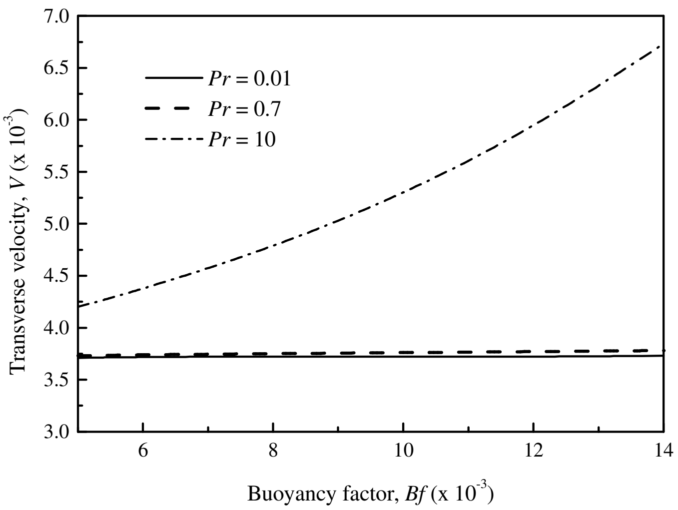

Figure 8: Plot of Transverse velocity, *V* versus Buoyancy factor, *Bf*, for various Prandtl numbers, *Pr*