Unsteady MHD Convective Heat and Mass Transfer of a Casson Fluid Past a Semi-infinite Vertical Permeable Moving Plate with Heat Source/Sink

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Abstract
In this paper, the effects of heat and mass transfer on an unsteady MHD flow of a Casson fluid past a semi-infinite moving vertical plate with heat source/sink are investigated. The governing equations are transformed into a system of linear partial differential equations using appropriate non-dimensional variables. The resulting equations are solved analytically using perturbation technique. Further the expressions for velocity, temperature and concentration are obtained with the help of boundary conditions. Finally the effects of various parameters on velocity, temperature and concentration are shown in graphs. It is found that velocity increases as Casson parameter increases and temperature increases as heat absorption coefficient decreases.
Keywords: Casson parameter, MHD, Heat source/sink, Heat and mass transfer.

1. INTRODUCTION

The analysis of boundary layer flow of a unsteady heat and mass transfer fluids has been the focus of extensive research by various scientists due to its importance in continuous casting, glass blowing, paper production, polymer extrusion and several others. One of my refer to recent investigations by Hayat and Qasim [1], Fang et al. [2], Khan and Pop [3], and Kandaswamy et al. [4]. On the other hand, mass transfer is important due to its appearance in many scientific disciplines that involve convective transfer of this phenomenon are evaporation of water, separation of chemicals in distillation processes, natural or artificial sources etc;

However, there is another model known as Casson model which is recently the most popular one. Casson [5] was the first who introduce this model for the prediction of the flow behavior of pigment oil suspensions of the printing ink type. Later on several researchers studied Casson fluid different flow situations and configurations. Amongst them, Mustafa et al. [6] studied the unsteady flow and heat transfer of a Casson fluid past a moving flat plate. Rao et al. [7] considered the thermal hydrodynamic slip conditions on heat transfer flow of a Casson fluid past a semi-infinite vertical plate. Heat transfer flow of a Casson fluid past a permeable shrinking sheet with viscous dissipation was considered by Qasim and Noreen [8].

The objective of this paper is consider unsteady MHD convective heat and mass transfer of a Casson fluid past a semi-infinite vertical permeable moving plate with heat source/sink in the presence of Casson parameter, heat source parameter effects. Most of previous works assumed that the semi-infinite plate is rest. In the present work, it is assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. Chamkha investigated [9] unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Recently Kim [10] discussed unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction.

2. PROBLEM FORMULATION

We consider unsteady two-dimensional flow of an incompressible, viscous, electrically conducting and heat-absorbing fluid past a semi-infinite vertical permeable plate embedded in a uniform porous medium which is subject to slip boundary condition at the interface of porous medium which is subject to slip boundary at the interface of porous and fluid layers. A uniform transverse magnetic field of strength B_0 is applied in the presence of radiation and concentration buoyancy effects in the direction of y^*-axis. The transversely applied magnetic field and magnetic Reynolds number are assumed to be very small so that induced magnetic field and Hall Effect are negligible. It is assumed that there is no applied voltage which implies the absence of electric field. Since the motion is two dimensional and the length of the plate is large enough so all the physical variables are independent of x^*. The wall is maintained at constant temperature T_w and concentration C_w, higher than the ambient temperature T_∞ and concentration C_∞, respectively. Also, it is assumed that there exists a homogeneous first-order Casson fluid, heat source and the fluid. It is assumed that the porous medium is homogeneous and present everywhere in local thermodynamic equilibrium. Rest of properties of the fluid and the porous medium are assumed to be constant. In the above assumptions the governing equations as follows:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \left(1+\frac{1}{\beta}\right)\nu\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T(T-T_\infty) + g\beta_c(C-C_\infty) - \nu\frac{u^*}{K^*} - \frac{\sigma}{\rho}B_0^2 u^* \tag{2}$$

$$\frac{\partial T}{\partial t^*}+v^*\frac{\partial T}{\partial y^*}=\alpha\frac{\partial^2 T}{\partial y^{*2}}-\frac{Q_0}{\rho C_p}(T-T_\infty)+\frac{Q_0}{\rho C_p}(C-C_\infty) \quad (3)$$

$$\frac{\partial C}{\partial t^*}+v^*\frac{\partial C}{\partial y^*}=D\frac{\partial^2 C}{\partial y^{*2}} \quad (4)$$

Where x^* and y^* are the dimensional distances along to the plate. u^* and v^* are the components of dimensional velocities along x^* and y^* directions. g is the gravitational acceleration, T^* is the dimensional temperature of the fluid near the plate,T_∞ is the stream dimensional temperature, C^* is the dimensional concentration, C_∞ is is the stream dimensional concentration. β_T and β_C are the thermal and concentration expansion coefficients, respectively. p^* is the pressure ,C_p is the specific heat of constant pressure, B_0 is the magnetic field coefficient, μ is viscosity of the fluid, ρ is the density, K is the thermal conductivity, σ is the density magnetic permeability of the fluid, $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity, D is the molecular diffusivity,Q_0 is the dimensional heat absorption coefficient and β is the Casson parameter. The fourth and fifth terms of RHS of the momentum Eq. (2) denote the thermal and concentration buoyancy effects, respectively. The second and third term on the RHS of Eq. (3) denote the inclusion of the effect of thermal radiation and heat absorption effects, respectively.

$$u^*=u_p^*,\ T=T_w+\varepsilon(T_w-T_\infty)e^{n^*t^*}, C=C_w+\varepsilon(C_w-C_\infty)e^{n^*t^*}\ at\ y^*=0 \quad (5)$$

$$u^*=U_\infty^*=U_0\left(1+\varepsilon e^{n^*t^*}\right), T\to T_\infty,\ C\to C_\infty\ as\ y\to\infty \quad (6)$$

Where $U_p, C_w\ and\ T_w$ are the wall dimensional velocity, concentration and temperature, respectively. $U_\infty^*, C_\infty, and\ T_\infty$ are the free stream dimensional velocity, concentration and temperature, respectively $U_0\ and\ n^*$are constants.

It is clear from Eq.(1) that the suction velocity at the plate surface is a function of time only. Assuming that it takes the following exponential form:

$$v^*=-V_0(1+\varepsilon A e^{n^*t^*}) \quad (7)$$

Where A is a real positive constant, ε and εA are small less than unity, and V_0 is a scale of suction velocity which has non-zero positive constant. Outside the boundary layer, Eq. (2) gives

$$-\frac{1}{\rho}\frac{dp^*}{dx^*}=\frac{dU_\infty^*}{dt^*}+\frac{\sigma}{\rho}B_0^2U_\infty^*+\frac{\nu}{K^*}U_\infty^* \quad (8)$$

Introducing the non-dimensional quantities

$$u=\frac{u^*}{U_0},\ v=\frac{v^*}{V_0},\ \eta=\frac{v_0 y^*}{\nu},\ U_\infty=\frac{U_\infty^*}{U_0},\ t=\frac{V_0^2t^*}{\nu},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ Q_1=\frac{Q_1^*\nu^2(C_w-C_\infty)}{V_0^2(T_w-T_\infty)},$$
$$C=\frac{C-C_\infty}{C_w-C_\infty}, U_p=\frac{u_p^*}{U_0},\ Sc=\frac{\nu}{D},\ K=\frac{V_0^2K^*}{\nu^2},\ \emptyset=\frac{Q_0\nu}{\rho C_p V_0^2}, \quad (9)$$
$$G_T=\frac{\rho g\nu(T_w-T_\infty)\beta_T}{U_0V_0^2}, G_c=\frac{\rho g\nu(C_w-C_\infty)\beta_C}{U_0V_0^2},\ M=\frac{\sigma\nu B_0^2}{\rho V_0^2},\ Pr=\frac{\nu C_p}{K}=\frac{\nu}{\alpha}$$

In the view of the above non-dimensional variables, the basic field of Eqs. (2)-(4) can be expressed in non-dimensional form as

$$\frac{\partial u}{\partial t}-(1+\varepsilon A e^{nt})\frac{\partial u}{\partial\eta}=\frac{dU_\infty}{dt}+\frac{\partial^2 u}{\partial\eta^2}(1+\frac{1}{\beta})+G_T\theta+G_cC+N(U_\infty-u) \quad (10)$$

$$\frac{\partial\theta}{\partial t}-(1+\varepsilon A e^{nt})\frac{\partial\theta}{\partial\eta}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2}-\emptyset\theta+\emptyset C \quad (11)$$

$$\frac{\partial C}{\partial t}-(1+\varepsilon A e^{nt})\frac{\partial C}{\partial\eta}=\frac{1}{Sc}\frac{\partial^2 C}{\partial c^2} \quad (12)$$

Where, $N=\left(M+\frac{1}{K}\right)$

The corresponding boundary conditions (5) and (6) in dimensionless form are

$$u=U_p, \theta=1+\varepsilon e^{nt}, C=1+\varepsilon e^{nt}\ \text{at}\ \eta=0 \quad (13)$$

$$u\to U_\infty=(1+\varepsilon e^{nt}), \theta\to 0, C\to 0\ as\ \eta\to\infty \quad (14)$$

3. PROBLEM SOLUTION

Eqs. (10)-(12) represent a set of partial differential equations that cannot be solved in closed-form. However, it can be reduced to a set of ordinary differential equations in dimensionless form that can be solved analytically. This can be done by representing the velocity, temperature and concentration as

$$u=f_0(\eta)+\varepsilon e^{nt}f_1(\eta)+O(\varepsilon^2) \quad (15)$$

$$\theta=g_0(\eta)+\varepsilon e^{nt}g_1(\eta)+O(\varepsilon^2) \quad (16)$$

$$C=h_0(\eta)+\varepsilon e^{nt}h_1(\eta)+O(\varepsilon^2) \quad (17)$$

Substituting (15)-(17) into Eqs.(10)-(12) and equating the harmonic and non-harmonic terms, and neglecting the higher order $O(\varepsilon^2), and$ simplifying to get the following pairs of equations for f_0, g_0, h_0 and f_1, g_1, h_1.

$$(1+\frac{1}{\beta})f_0''+f_0'-Nf_0=-N-Gr_Tg_0-Gr_ch_0 \quad (18)$$

$$(1+\frac{1}{\beta})f_1''+f_1'-(N+n)f_1=-(N+n)-Af_0'-Gr_Tg_1-Gr_ch_1 \quad (19)$$

$g_0'' + Prg_0' - \text{Pr}\emptyset g_0 = -\emptyset h_0$ (20)
$g_1'' + Prg_1' - \text{Pr}(\emptyset + n)g_1 = -Pr\emptyset h_1 - PrAg_0'$ (21)
$h_0'' + Sch_0' = 0$ (22)
$h_1'' + Sch_1' - nSch_1 = -ASch_0'$ (23)
Where the prime denotes ordinary differentiation with respect to y. The corresponding boundary conditionsare
$f_0 = U_p,\ f_1\ g_0 = 1, g_1 = 1, h_0 = 1, h_1 = 1 \quad at\ \eta = 0$ (24)
$f_0 = 1,\ \ f_1 = 1, g_0 \to 0,\ g_1 \to 0, h_0 \to 0, h_1 \to 0$ as $\eta \to \infty$. (25)
Without going into the details, the solutions of Eqs. (18)- (23) With the help of boundary conditions (24) and (25), we get
$f_0 = 1 + K_{13}e^{-m_9\eta} - K_{11}e^{-m_5\eta} - K_{12}e^{-Sc\eta}$ (26)
$f_1 = 1 + K_{22}e^{-m_{11}\eta} + K_{18}e^{-m_9\eta} + K_{19}e^{-m_5\eta} + K_{20}e^{-Sc\eta} + K_{21}e^{-m_3\eta}$ (27)
$g_0 = K_2e^{-m_5\eta} + K_1e^{-Sc\eta}$ (28)
$g_1 = K_9e^{-m_7\eta} + K_6e^{-m_5\eta} + K_7e^{-Sc\eta} + K_8e^{-m_3\eta}$ (29)
$h_{0= e^{-Sc\eta}}$ (30)
$h_1 = e^{-m_3\eta} + \frac{ASc}{n}(e^{-m_3\eta} - e^{-Sc\eta})$ (31)
In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become
$u(y,t) = (1 + K_{13}e^{-m_9\eta} - K_{11}e^{-m_5\eta} - K_{12}e^{-Sc\eta}) + \varepsilon e^{nt}(1 + K_{22}e^{-m_{11}\eta} + K_{18}e^{-m_9\eta} + K_{19}e^{-m_5\eta} + K_{20}e^{-Sc\eta} + K_{21}e^{-m_3\eta})$ (32)
$\theta(y,t) = (K_2e^{-m_5\eta} + K_1e^{-Sc\eta}) + \varepsilon e^{nt}(K_9e^{-m_7\eta} + K_6e^{-m_5\eta} + K_7e^{-Sc\eta} + K_8e^{-m_3\eta})$ (33)
$C(y,t) = e^{-Sc\eta} + \ \varepsilon e^{nt}(e^{-m_3\eta} + \frac{ASc}{n}(e^{-m_3\eta} - e^{-Sc\eta}))$ (34)
The Skin-friction coefficient, the Nusselt number and the Sherwood number are important physical parameters for this type of boundary-layer flow. These parameters can be defined and determined as follows:
$C_{fx} = \frac{\tau_w{}^*}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial \eta}\right)_{at\ \eta=0}$
$=(-m_9K_{13} + K_{11}m_5 + K_{12}Sc) + \varepsilon e^{nt}\left(-m_{11K_{22}} - m_9K_{18} - m_5K_{19} - ScK_{20} - m_3K_{21}\right)$(35)
$Nu_x = x\frac{\left(\frac{\partial T}{\partial y^*}\right)_{at\ \eta=0}}{(T_w - T_\infty)} \Rightarrow {Nu_x}/{Re_x} = \left(\frac{\partial \theta}{\partial \eta}\right)_{at\ \eta=0} = (-m_5K_2 - K_1Sc) + \varepsilon e^{nt}(-m_7K_9 - m_5K_6 - ScK_7 - m_3K_8)$ (36)
$Sh_x = \ x\frac{\left(\frac{\partial C}{\partial \eta^*}\right)_{at\ \eta=0}}{(C_w - C_\infty)} \Rightarrow {Sh_x}/{Re_x} = \left(\frac{\partial C}{\partial \eta}\right)_{at\ \eta=0} = -Sc + \varepsilon e^{nt}(-m_3 + \frac{ASc}{n}(-m_3 + Sc))$ (37)

4. RESULTS AND DISCUSSION

Numerical evaluation of the analytical results reported in the previous section was performed and a representative set of results is reported graphically in Figs. 1-7. These results are obtained to illustrate the influence of the solutal Grashof numberG_c, the heat absorption coefficient $\emptyset$, Schmidt number Sc, thermal Grashof number G_T and Casson parameter β on the velocity, temperature and concentration profiles.

Fig.1 shows that species concentration profiles for different values of Schmidt number Sc. It is clear that the concentration boundary layer thickness decreases with Sc, concentration decreases exponentially and attains free stream condition for large values of Sc. The temperature profiles for different values of heat absorption parameter are depicted in fig. 2. It is noticed that the temperature decreases significantly with the increasing values of $\emptyset$, because when heat is absorbed, the buoyancy force decreases the temperature profiles.

Fig.3 represents the decreases in temperature profiles when the Schmidt number Sc is increases. Also we observe that for low values of Sc (0.5) the temperature is very high comparing with higher values Sc (3.0). Fig.4 represents the decrease in fluid velocity when the heat absorption parameter $\emptyset$ is increased, it is clear that the hydro magnetic boundary layer decreases as the heat absorption effect increase also observed that in the absence of heat absorption the velocity attains maximum peak value.

Velocity distribution for various values G_T and solutal buoyancy force parameter G_C are plotted in fig.5 and fig.6. As seen from this figures that the maximum peak value is observed in the absence of buoyancy force, this is due to fact that buoyancy force enhances fluid velocity and increase the buoyancy layer thickness with increase in the values of G_T and G_C.

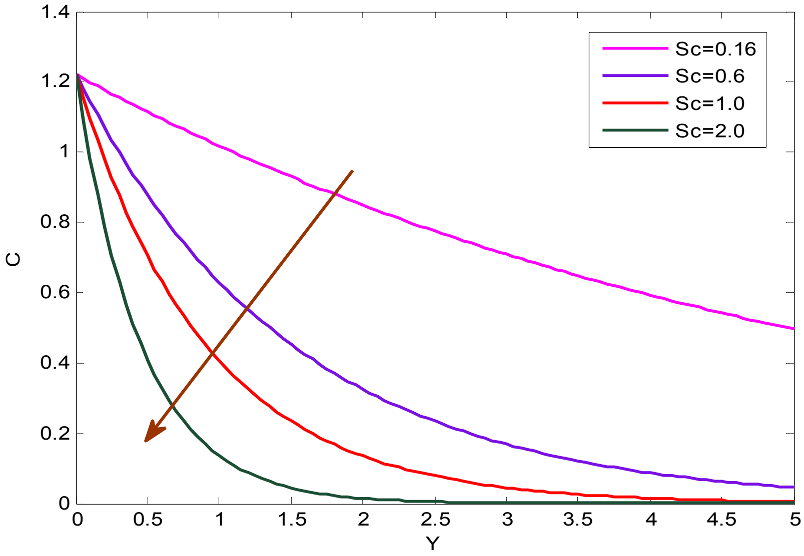

Fig 1 Effects of Sc on concentration profiles
A=0.50, E=0.2, n=0.1, t=1.0.

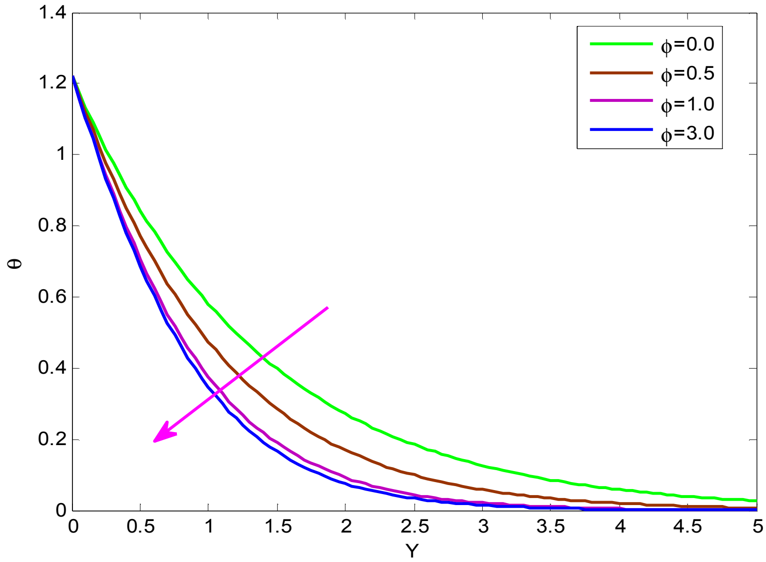

Fig 2 Effects of Ø on temperature profiles.
Sc=2, A=0.50, ε=0.2, n=0.1, t=1.0, Pr=0.71.

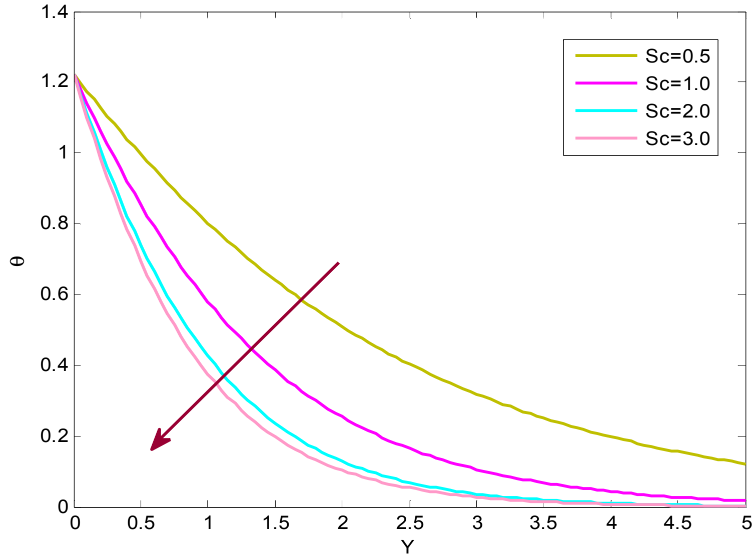

Fig 3 Effects of Sc on temperature profiles.
A=0.50, ε=0.2, n=0.1, t=1.0, Pr=0.71, Ø=1.

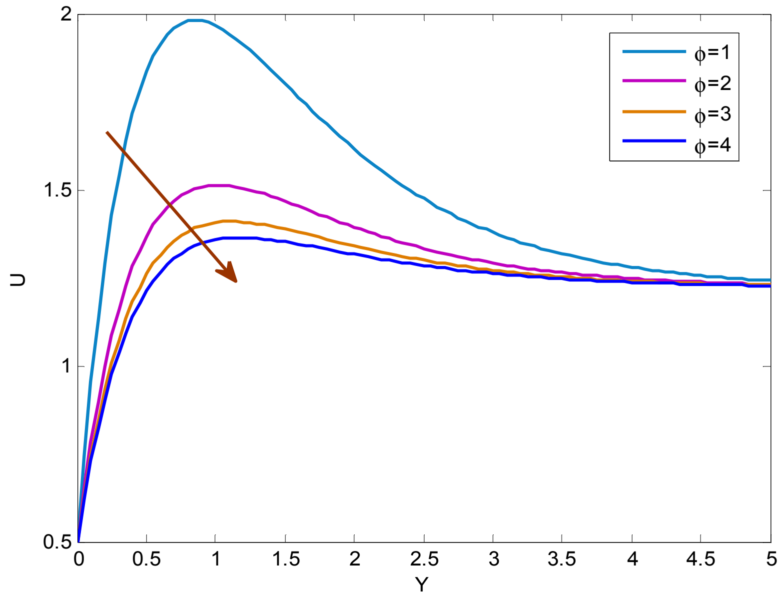

Fig 4 Effects of Ø on velocity profiles.
A=0.5, Pr=0.71,M=2, K=0.5, Gt=2, Gc=1, Up=0.5, β=2.

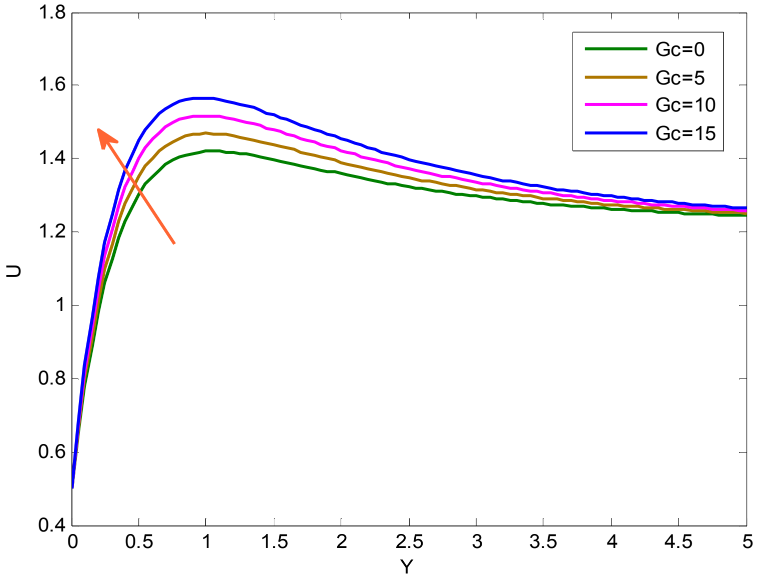

Fig 5 Effects of Gc on velocity profiles.
Sc=0.6, ε=0.2, Pr=0.71, M=5, K=0.5, Gt=2, Up=0.5, β=5.

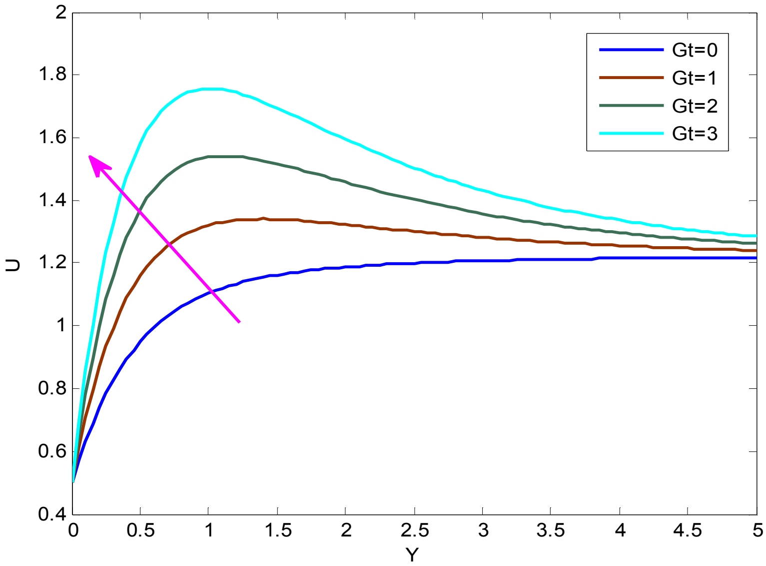

Fig 6 Effects of Gt on velocity profiles.
Sc=0.6, ε=0.2, Ø=2, M=2, K=0.5, Gc=1, Up=0.5, β=5.

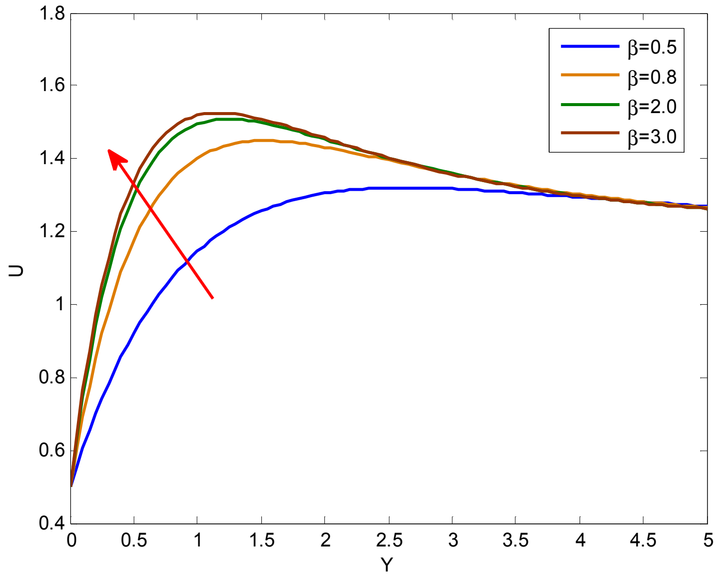

Fig 7 Effects of β on velocity profiles.
Sc=0.6, ε=0.2, Ø=2, M=2, K=0.5, Gt=2, Gc=1, Up=0.5.

Fig. 7. represents the velocity profiles for different values of Casson parameter β. From this figure we observe that the velocity profiles increases significantly with an increase in the Casson parameter β.

Table:1 :Numerical values of Solutal Grashof number G_C on C_f, ${Nu_x}/{Re_x}$, ${Sh_x}/{Re_x}$ for the reference values $G_t = 2, t = 1, Sc = 0.6, \emptyset = 2, \gamma = 0.5, \beta = 2$.

G_C	C_f	${Nu_x}/{Re_x}$	${Sh_x}/{Re_x}$
0	2.7200	-1.7167	-0.8098
1	3.2772	-1.7161	-0.8098
2	3.8343	-1.7161	-0.8098
3	4.3915	-1.7161	-0.8098
4	4.9487	-1.7161	-0.8098

The effects of Solutal Grashof number G_C on the skin-friction coefficient C_f, Nusselt number and Sherwood number respectively are presented in table 1. From this table it is seen that the effect of G_C is to increase the skin-friction coefficient C_f , where as no effect of G_C is observed on nusselt number and Sherwood number (see table-1).

5. APPENDIX

$$N = \left(M + \frac{1}{K}\right), m_3 = \frac{Sc + \sqrt{(Sc^2 + 4nSc)}}{2}, m_5 = \frac{Pr + \sqrt{Pr^2 + 4\emptyset Pr}}{2}, m_7 = \frac{Pr + \sqrt{Pr^2 + 4(\emptyset + n)Pr}}{2},$$

$$q = \left(1 + \frac{1}{\beta}\right),$$

$$m_9 = \frac{1+\sqrt{1+4Nq}}{2q}, m_{11} = \frac{1+\sqrt{1+4(N+n)q}}{2q}, K_1 = \frac{-\emptyset}{Sc^2 - PrSc - Pr\emptyset}, K_2 = (1 - K_1), K_3 = APrK_2m_5,$$

$$K_4 = \left(APrScK_1 - \frac{ASc}{n}\right), K_5 = \left(\frac{ASc}{n} - \emptyset Pr\right), K_6 = \frac{K_3}{m_5{}^2 - m_5Pr - Pr(n+\emptyset)}, K_7 = \frac{K_4}{Sc^2 - ScPr - Pr(n+\emptyset)},$$

$$K_8 = \frac{K_5}{m_3{}^2 - m_3Pr - Pr(n+\emptyset)}, K_9 = (1 - K_6 - K_7 - K_8), K_{10} = G_TK_1 + G_C, K_{11} = \frac{G_TK_2}{qm_5{}^2 - m_5 - N}$$

$$K_{12} = \frac{K_{10}}{qSc^2 - Sc - N}, K_{13} = (Up - 1 + K_{11} + K_{12}), K_{14} = Am_9K_{13}, K_{15} = -(Am_5K_{11} + G_TK_6),$$

$$K_{16} = -\left(AK_{12}Sc + G_TK_7 - \frac{AScG_C}{n}\right), K_{17} = -\left(G_TK_8 + G_C + \frac{ASc}{n}\right), K_{18} = \frac{K_{14}}{qm_9{}^2 - m_9 - (n+N)},$$

$$K_{19} = \frac{K_{15}}{qm_5{}^2 - m_5 - (n+N)}, K_{20} = \frac{K_{16}}{qSc^2 - Sc - (n+N)}, K_{21} = \frac{K_{17}}{qm_3{}^2 - m_3 - (n+N)}, K_{22} = (-1 - K_{18} - K_{19} - K_{20} - K_{21})$$

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