

Effect of Non-Linear Density Variation on Non-Darcy Convective Heat and Mass Transfer with Newtonian Cooling

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Abstract
We investigate the effect of Non-linear density temperature relation on convective heat and mass transfer flow past stretching sheet with Soret and Dufour effects. The Non-linear coupled governing equations have been solved by fourth –order Runge-Kutta method. The velocity, temperature and concentration, skin friction and rate of heat and mass transfer have been discussed for different parametric variations. We observed that an increase in the density ration γ reduces the velocity, temperature and concentration.
Keywords: Heat and Mass transfer, Non-linear temperature relation, Chemical Reaction, Soret and Dufour Effects, Heat sources.

1. INTRODUCTION

In recent years, the problems of free convective heat and mass transfer flows through a porous medium under the influence of magnetic field have been attracted the attention of a number of researchers because of their possible applications in many branches of science and technology, such as its applications in transportation cooling of re-entry vehicles and rocker boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. Magneto hydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. In light of these applications, steady MHD free convective flow past a heated vertical flat plate has been studied by many researchers as Lykoudis [21] and Nanda and Mohanty [27].

Chaudhary and Sharma [7] considered combined heat and mass transfer by laminar mixed convection flow from a vertical surface with induced magnetic field. El-Amin [8] studied the MHD free convection and mass transfer flow in a micro polar fluid over a stationary vertical plate with constant suction. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. Many researchers have studied MHD free convective heat and mass transfer flow in a porous medium. Some of them are Raptis and Kafoussias [33] investigated heat and mass transfer effects on steady MHD over a porous medium bounded by an infinite vertical porous plate with constant heat flux. Kim [16] found that the effects of heat and mass transfer on MHD micro polar flow over a vertical moving porous plate in a porous medium.

Coupled heat and mass transfer by free convective in porous media has been widely studied in the recent years due to its wide applications in engineering as post accidental heat removal in nuclear reactors, solar collectors, drying processes, heat exchangers, geothermal and oil recovery, building construction, etc. A comprehensive review of the studies of convective heat transfer mechanism through porous medium has been made by Nield and Bejan [28]. Hiremath and Patil [12] studied the effect on free convection currents on the oscillatory flow through a porous medium which is bounded by vertical plane surface of constant temperature. Fluctuating heat and mass transfer on three-dimensional flow through porous medium with variable permeability has been discussed by Sharma et al. [34]. A comprehensive account of the available information in this field is provided in books by Pop and Ingham [29] , Vafai [37], Vadasz [36] etc.

Combined heat and mass transfer problems in presence of chemical reaction are of importance in many processes and thus have received considerable amount of attention in recent times. In processes such as drying, distribution of temperature and moisture over agricultural field and groves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body, energy transfer in a wet cooling tower and flow in a desert cooler, heat and mass transfer occur simultaneously. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. Therefore, the study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists. Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single-phase volume reaction. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself. In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. In view of heat and mass transfer and chemical reactions numerous and wide-ranging applications in various fields like polymer processing industry in particular in manufacturing process of artificial film and artificial fibers and in some applications of dilute polymer solution. Many researchers have studied chemical reaction effects on steady MHD flow with combined heat and mass transfer; (Gangadhar [9], Mohammed Ibrahim [25], Rajashekar et al. [31], Kishan and Srinivas [17], Anjalidevi and David [4]).

In all the studies cited above, the flow is driven either by prescribed surface temperature or by a prescribed surface heat flux. Here, a relatively different driving mechanism for unsteady free convection along a vertical surface is considered where it is assumed that the flow is also start up by Newtonian heating from the surface. Heat transfer characteristics are dependent on the thermal boundary conditions, In general, there are four common heating processes specifying the wall-to-ambient temperature distributions, namely, prescribed wall temperature distributions (power-law wall temperature distribution along the surface has been usually used); prescribed surface heat flux distributions; conjugate conditions, where heat is supplied through a bounding surface of finite thickness and finite heat capacity. The interface is not known a priori but depends on the intrinsic properties of the system, namely the thermal conductivities of the fluid and solid, respectively. Finally there is Newtonian heating, where the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature and which is usually termed conjugate convective flow. This last configuration occurs in many important engineering devices, for example,

(i) In heat exchangers where the conduction in solid tube wall is greatly influenced by the convection in the fluid flowing over it;
(ii) For conjugate heat transfer around fins where the conduction within the fin and the convection in the fluid surrounding it must be simultaneously analyzed in order to obtain vital design information;
(iii) In convective flows set up when the bounding surfaces absorbs heat by solar radiation.

Therefore, we conclude that the conventional assumption of no interaction of conduction-convection coupled effects is not always realistic and it must be considered when evaluating the conjugate heat transfer processes in many practical engineering applications.

Merkin [23] was the first to consider a somewhat different but practically relevant driving mechanism for the natural convection boundary layer flow near a vertical surface in which it was assumed that the flow was setup by the Newtonian heating from the bounding surface, i.e. the heat transfer from the surface was taken to be proportional to the local surface temperature. The situation with Newtonian heating arises in what are usually termed conjugate convective flows, where the heat is supplied to the convicting fluid through a bounding surface with a finite heat capacity. Lesnic et al. [19] considered free convection boundary layer flow along vertical and horizontal surfaces in a porous medium generated by Newtonian heating. The steady free convection boundary layer along a semi-infinite plate, slightly inclined to the horizontal and embedded in a porous medium with the flow generated by Newtonian heating was investigated by Lesnic et al. [20].Madhusudhan Rao et al. [22] has studied Soret and Dufour effects on Hydro-Magnetic heat and mass transfer over a vertical plate in a porous medium with a convective surface boundary condition and chemical reaction. Lavanya et al. [18] has studied Dufour and Soret effects on steady MHD free convective flow past a vertical porous plate embedded in a porous medium with chemical reaction heat generation and viscous dissipation. Chaudhary and Jain [6] presented an exact solution for the unsteady free convection boundary layer flow of an incompressible fluid past an infinite vertical plate with the flow generated by Newtonian heating and impulsive motion of the plate. Rajesh et al. [32] studied unsteady convective flow past an exponentially accelerated infinite vertical porous plate with Newtonian heating and viscous dissipation.

The study of heat generation in moving fluids is important as it changes the temperature distribution and the particle deposition rate particularly in nuclear reactor cores, fire and combustion modeling, electronic chips and semi conductor wafers. Heat generation is also important in the context of exothermic or endothermic chemical reaction. Vajravelu and Hadjinicolaou [38] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Hossain et al [13] studied problem of the natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption. Kesavaiah et.al [15] reported that the effects of the chemical reaction and radiation absorption on unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction. Mohammed Ibrahim and Bhaskar Reddy [26] investigated heat and mass transfer effects on steady MHD free convective flow along a stretching surface with dissipation, heat generation and radiation.

But in the above mentioned studies, Dufour and Soret terms have been neglected from the energy and concentration equations respectively. It has been found that energy flux can be generated not only by temperature gradient but also by concentration gradient as well. The energy flux caused by concentration gradient is called Dufour effect and the same by temperature gradient is called the Soret effect. These effects are very significant when the temperature and concentration gradient are very high. Anghel et al. [3] studied the Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. Postelnicu [30] analyzed the influence of magnetic field on heat and mass transfer from vertical surfaces in porous media considering Soret and Dufour effects. Alam et al. [1] investigated the Dufour and Soret effects on steady MHD mixed convective and mass transfer flow past a semi-infinite vertical plate. Chamkha and Ben-Nakhi [5] analyzed MHD mixed convection-radiation interaction along a permeable surface immersed in a porous medium in the presence of Soret and Dufour effects. M.S. Alam and M.M Rahman [2] investigated Chemical reaction and radiation effects on MHD free convection flow along a stretching surface with viscous dissipation and heat generation.

In all the above a linear density variations is considered in the equation of state. This is valid for temperature variation at 20^0C. But this analysis is not applicable to the study of the flow of water at 4^0C. The density of water is Maximum at atmospheric pressure and the modified form of the equation to water at 4^0c is given by

$$\Delta\rho = - \rho\gamma (\Delta T)^2$$

Where $\gamma = 8 \times 10^{-6} (0c)^{-2}$. Taking this fact into account, Goren [10] showed in this case similarity solution for the free convection flow of water at 4^0c past a semi-finite vertical plate exists. Govindarajulu [1] showed that a similarity solution exists for the free convection flow of water at 4^0C from vertical and horizontal plates in the presence of suction and injection. Several authors [14, 24, and 35] have investigated the effect of non-uniform density-temperature relation on convective heat /mass transfer problems.

The aim of this paper is to discuss the Non-linear density temperature effect on MHD free convection flow past a vertical porous plate placed in porous medium in the presence of chemical reaction, viscous dissipation and heat source. The equations governing the flow heat and mass transfer have been solved by fourth order Runge-Kutta-Fehlberg integration scheme. The effects of various parameters on the flow characteristics have been analyzed.

2. MATHEMATICAL ANALYSIS

A steady two-dimensional flow of an incompressible and electrical conducting viscous fluid, along an infinite vertical porous plate embedded in a porous medium is considered. The *x*- axis is taken on the infinite plate, and parallel to the free-stream velocity which is vertical and the *y*- axis is taken normal to the plate. A magnetic field B_0 of uniform strength is applied transversely to the direction of the flow. Initially the plate and the fluid are at same at temperature T_∞ in a stationary condition with concentration level C_∞ at all points. The plate starts moving impulsively in its own plane with velocity U_0, its temperature is raised to T_w and the concentration level at the plate is raised to C_w. A homogeneous first order chemical reaction between fluid and the species concentration is considered, in which the rate of chemical reaction is directly proportional to the species concentration. The flow configuration and coordinate system are shown in the Figure 1**.** The fluid is assumed to be slightly conducting, and hence the magnetic field is negligible in comparison with the applied magnetic field. It is further assumed that there is no applied voltage, so that electric field is absent. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussinesq approximation). Then, under the above assumptions, the governing boundary layer equations are

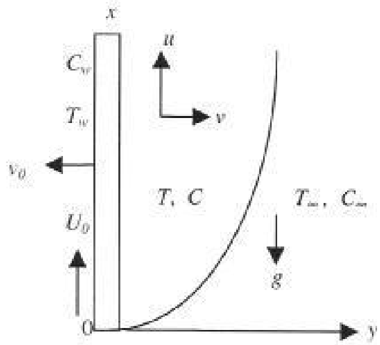

Fig.1. Flow configuration and coordinate system

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta_0(T - T_\infty) + g\beta_1(T - T_\infty)^2 + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho}u - \frac{\upsilon}{K}u - \frac{b}{K}u^2 \quad (2)$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{D_m}{c_s}\frac{k_T}{c_p} + \frac{Q_0}{\rho c_p}(T - T_\infty) + \frac{\mu}{\rho c_p}(\frac{\partial u}{\partial y})^2 \quad (3)$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}-kr'(C-C_\infty) \quad (4)$$

where u, v are the Darcian velocities components in the x and y directions respectively, υ is the kinematic viscosity, g is the acceleration due to gravity, ρ is the density, μ is the dynamic viscosity, β_0^2, β_1 are the coefficients of volume

expansion with temperature, β^* is the volumetric coefficient of expansion with concentration, b is the empirical constant, T, T_w and T_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, C, C_w and C_∞ are the corresponding concentrations. K is the Darcy permeability. σ is the electric conductivity. $\alpha=\frac{k_f}{\rho_0 c_p}$ is the thermal diffusivity, c_p is the specific heat at constant pressure . D_m is the coefficient of mass diffusivity, k_T is thermal diffusion ratio, c_s is the concentration susceptibility, the $Q_0(T-T_\infty)$ is assumed to be amount of heat generated or absorbed per unit volume and Q_0 is a constant, which may take on either positive or negative values, kr' is chemical reaction parameter.

The boundary conditions for velocity, temperature and concentration fields are given by

$$u=U_0,\quad v=v_0(x),\quad -k_f\frac{\partial T}{\partial y}=h[T-T_w(x,0)],\quad C=C_W \quad \text{at } y=0$$

$$u=0,\quad v=0,\quad T=T_\infty,\quad C=C_\infty \quad \text{at } y\to\infty \quad (5)$$

Where U_0 is the uniform velocity and $v_0(x)$ is the velocity of suction at the plate...

The equation (2) and (4) are coupled, parabolic and nonlinear partial differential equations and hence analytical solution is not possible. Therefore numerical technique is employed to obtain the required solution. Numerical computations are greatly facilitated by non-dimensionalization of the equations. Proceeding with the analysis, we introduce the following similarity transformations and dimensionless variables which will convert the partial differential equations from two independent variables (x, y) to a system of coupled, non-linear ordinary differential equations in a single variable (η) i.e., coordinate normal to the plate.
In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\eta=y\sqrt{\frac{U_0}{2\upsilon x}},\qquad \psi=\sqrt{2\upsilon x U_0}\ f(\eta)\qquad \theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty}\qquad \phi(\eta)=\frac{C-C_\infty}{C_W-C_\infty} \quad (6)$$

Where $f(\eta)$ is the dimensionless steam function and ψ is the dimensional stream function defined in the usual way

$$u=\frac{\partial\psi}{\partial y}\quad \text{and}\quad v=-\frac{\partial\psi}{\partial x}$$

Cleary the continuity equation (1) is identically satisfied.
Then introducing the relation (6) into equation (1) we obtain

$$u=U_0 f'(\eta)\quad \text{and}\quad v=\sqrt{\frac{\upsilon U_0}{2x}}(\eta f'-f) \quad (7)$$

Further introducing equation (6) and (7) in to momentum equation (2), Energy equation (3) and Concentration equation (4) we obtain the following local similarity equations.

$$f'''+ff''-Mf'-\frac{1}{Da\,\mathrm{Re}}f'-\frac{Fs}{Da}f'^2+Gr(\theta+\gamma\theta^2+N\phi)=0 \quad (8)$$

$$\theta'' + \Pr f\theta' + \Pr Du\phi'' + \Pr Q \operatorname{Re}\theta + \Pr Ec(f'')^2 = 0 \quad (9)$$

$$\phi'' + Scf\phi' + ScSr\theta'' - Kr\phi = 0 \quad (10)$$

Where

$Gr = \dfrac{g\beta(T_w - T_\infty)2x}{U_0^2}$ is the Grashof number $N = \dfrac{\beta^*(C_W - C_\infty)}{\beta(T_W - T_\infty)}$ is the Buoyancy ratio

$M = \dfrac{2\sigma B_0^2 x}{\rho U_0}$ is the Magnetic field parameter $Da = \dfrac{K}{2x^2}$ is the Darcy number,

$\operatorname{Re} = \dfrac{U_0 x}{\upsilon}$ is the Reynolds number, $Fs = \dfrac{b}{x}$ is the Forchheimer number,

$\Pr = \dfrac{\upsilon}{\alpha}$ is the Prandtl number, $Du = \dfrac{D_m K_T (C_w - C_\infty)}{c_s c_p \upsilon (T_w - T_\infty)}$ is the Dufour number,

$Sr = \dfrac{D_m K_T (T_w - T_\infty)}{\upsilon T_m (C_w - C_\infty)}$ is the Soret number $Ec = \dfrac{U_0^2}{c_p (T_w - T_\infty)}$ is the Eckert number,

$Q = \dfrac{Q_0 \upsilon}{\rho c_P U_0^2}$ is the heat generation parameter, $Sc = \dfrac{\upsilon}{D_m}$ is the Schmidt number,

$\gamma = \dfrac{\beta_1 \Delta T}{\beta_0}$ is the density ratio, $Kr = \dfrac{2K_r' \upsilon x}{D_m U_0}$ is the Chemical reaction parameter,

$Bi = (\dfrac{h}{k})\sqrt{\dfrac{2\upsilon x}{U_0}}$ is the Convective heat transfer parameter

The corresponding boundary conditions are

$$f = f_w,\ f' = 1,\ ,\ \phi = 1 \text{ at } \eta = 0 \qquad \theta'(0) = Bi[\theta(0) - 1]$$
$$f' = 0,\ \theta = 0,\ \phi = 0 \text{ at } \eta \to \infty \quad (11)$$

Where $f_w = -\upsilon_0 \sqrt{\dfrac{2x}{\upsilon U_0}}$ the dimensionless suction velocity and primes is is denote partial differentiation with respect to the variable η.

3. SKIN FRICTION, NUSSELT NUMBER AND SHERWOOD NUMBER

The parameters of engineering interest for the present problem are the skin-friction coefficient, the Nusselt number and the Sherwood number, which are given respectively by the following expressions. Knowing the velocity field the skin-friction at the plate can be obtained, which in non-dimensional form is given by

$$\frac{1}{2}\operatorname{Re}^{\frac{1}{2}} C_f = f''(0) \quad (13)$$

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form in terms of Nusselt number, is given by

$$Nu \ \operatorname{Re}^{\frac{1}{2}} = -\theta'(0) \quad (14)$$

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in non-dimensional form, in terms of Sherwood number, is given by

$$Sh \operatorname{Re}^{\frac{1}{2}} = \phi'(0) \quad (15)$$

Where $\mathrm{Re} = \dfrac{U_0 x}{\upsilon}$ is the Reynolds Number.

4. DISCUSSION OF THE NUMERCIAL RESULTS:
In order to analyze the effects of various physical parameters on the flow, heat and mass transfer of a viscous fluid over a vertical plate, the equations 8-11 have been solved numerically by Runge-Kutta-Fehlberg integration scheme. The effects of various physical parameters on the velocity, temperature and concentration profiles have been discussed and are shown graphically in figures 2-28.

Figures 2, 3, 4 represents f^1, θ , and ϕ with buoyancy ratio N. It is found that when the molecular buoyancy force dominates over the thermal buoyancy force the velocity and concentration reduces and temperature enhances in the flow region concentration reduces in the flow field.

Figures 5, 6, 7 represents f^1, θ , and ϕ with chemical reaction parameter Kr. It can be seen from the profiles that the velocity, temperature and concentration reduces in the case of Kr>0 and for Kr<0 we notice a depreciation in θ and ϕ and enhancement in f^1 in entire fluid region.

Figures 8, 9, 10 depict f^1, θ , and ϕ with heat source parameter Q. It is found that an increase in the strength of heat generating /absorption reduces the velocity, temperature and concentration in the boundary layer.

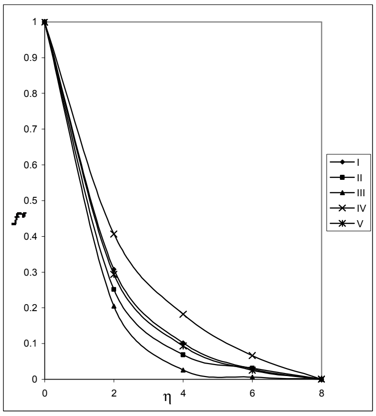

Fig.2 Variation of f' with N
I II III IV V
N 0.5 1.5 2 -0.5 -0.8

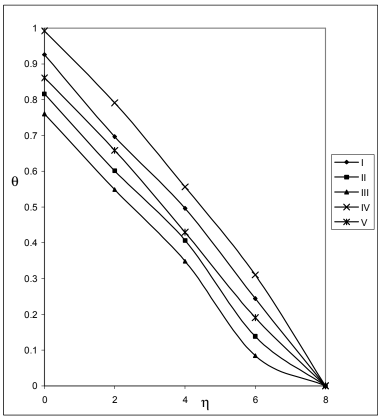

Fig.3 Variation of θ with N
I II III IV V
N 0.5 1.5 2 -0.5 -0.8

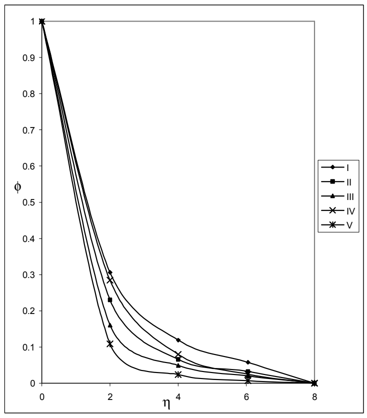

Fig.4 Variation of ϕ with N
I II III IV V
N 0.5 1.5 2 -0.5 -0.8

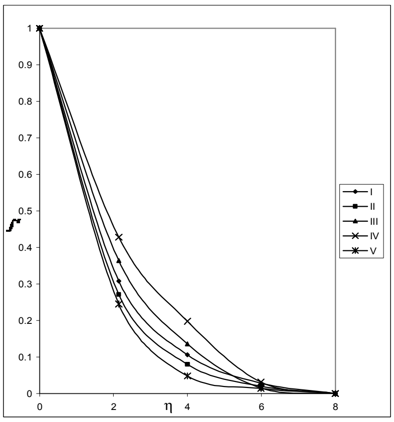

Fig.5 Variation of f' with Kr
I II III IV V
Kr 0.5 1.5 2.5 -0.5 -1.5

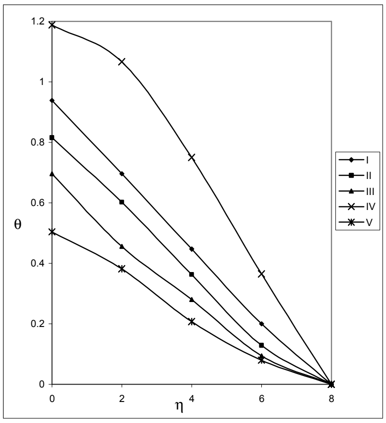

Fig.6 Variation of θ with Kr
I II III IV V
Kr 0.5 1.5 2.5 -0.5 -1.5

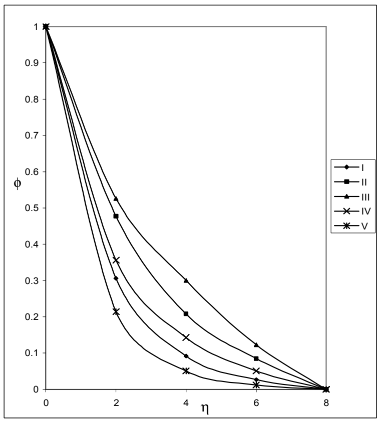

Fig.7 Variation of ϕ with Kr
I II III IV V
Kr 0.5 1.5 2.5 -0.5 -1.5

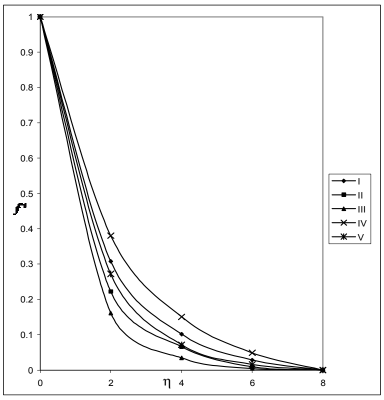

Fig. 8 Variation of f' with Q
I II III IV V
Q 0.5 1.5 2.5 -0.5 -1.5

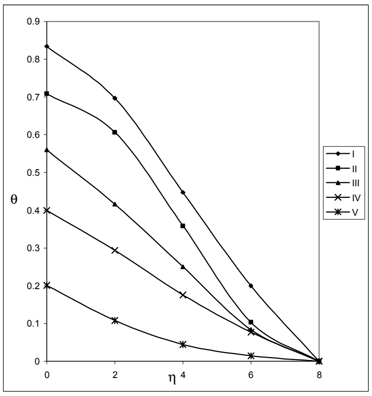

Fig.9 Variation of θ with Q
I II III IV V
Q 0.5 1.5 2.5 -05 -1.5

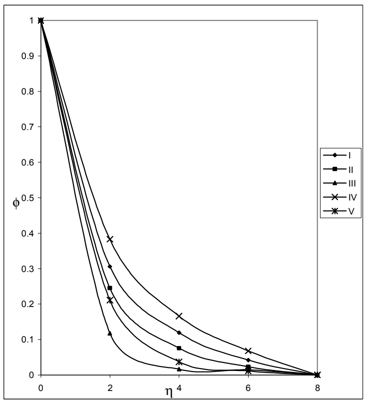

Fig.10 Variation of ϕ with Q
I II III IV V
Q 0.5 1.5 2.5 -0.5 -1.5

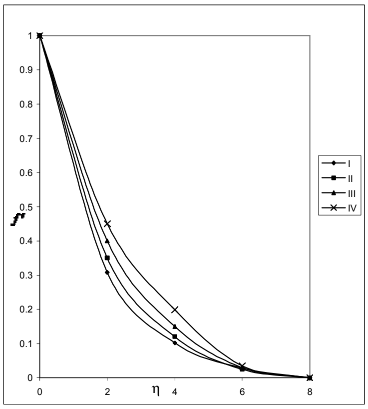

Fig. 11 Variation of f' with E_c
I II III IV
E_c 0.01 0.03 0.05 0.07

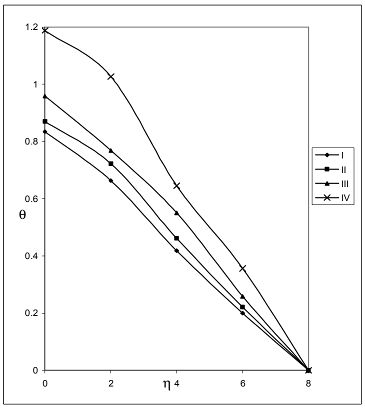

Fig.12 Variation of θ with Ec
I II III IV
Ec 0.01 0.03 0.05 0.07

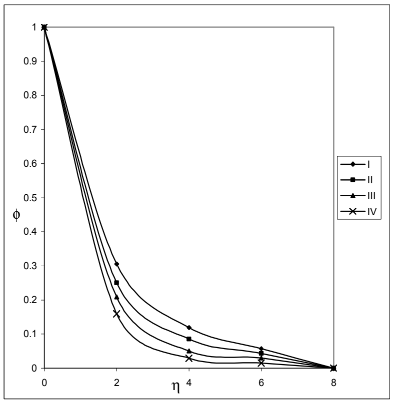

Fig.13 Variation of ϕ with Ec
I II III IV
Ec 0.01 0.03 0.05 0.07

Figs 11, 12, 13 represents f^1, θ , and ϕ with Eckert number Ec. Higher the dissipative heat larger the temperature and smaller the velocity and concentration in the boundary layer.

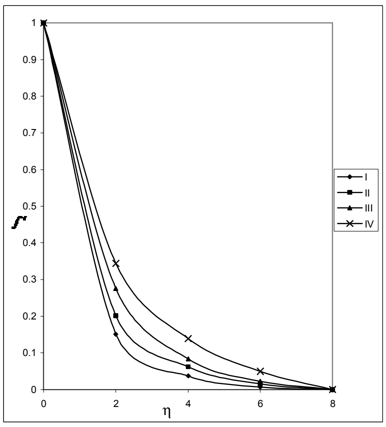

Fig.14 Variation of f' with Sr&Du
I II III IV
Sr 1.5 0.8 0.4 0.2
Du 0.4 0.075 0.15 0.3

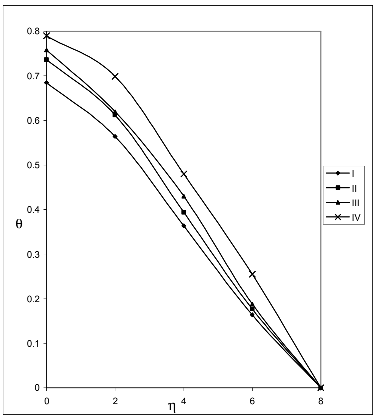

Fig.15 Variation of θ with Sr & Du
I II III IV
Sr 1.5 0.8 0.4 0.2
Du 0.4 0.075 0.15 0.3

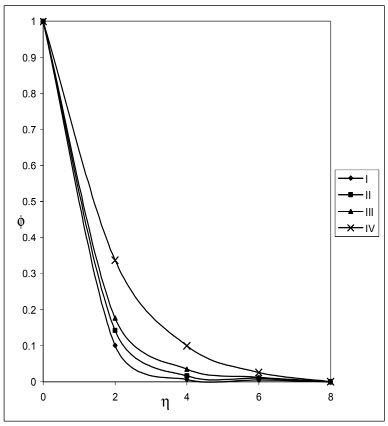

Fig.16 Variation of ϕ with Sr & Du
I II III IV
Sr 1.5 0.8 0.4 0.2
Du 0.4 0.075 0.15 0.3

Figures 14, 15, 16 illustrate the effect of Soret and Dufour parameter on the variations of the fluid velocity, temperature and concentration respectively. It is found that increase in the Soret parameter Sr (or decrease in the Dufour parameter Du) reduces the velocity, the temperature and concentration in the boundary layer.

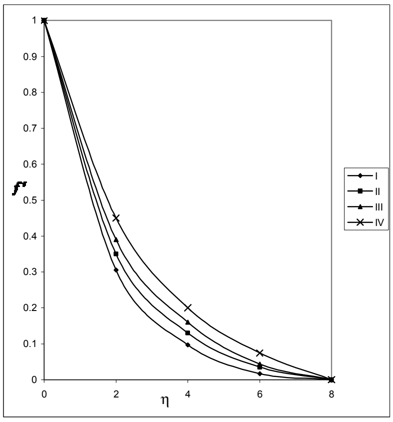

Fig.17 Variation of f' with B_i
I II III IV
B_i 0.1 0.5 1 2

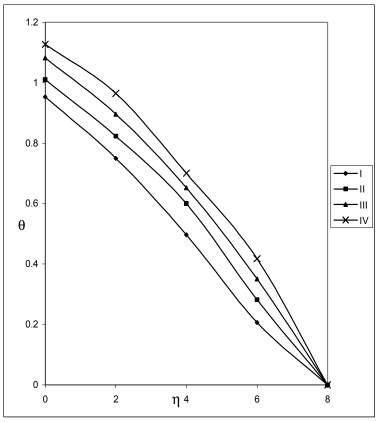

Fig.18 Variation of θ with Bi
I II III IV
Bi 0.1 0.5 1 2

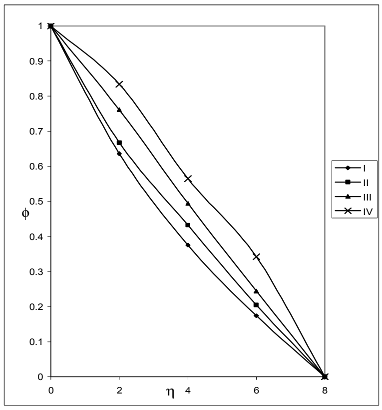

Fig.19 Variation of ϕ with Bi
I II III IV
Bi 0.1 0.5 1 2

Figures17, 18, 19 represents f^1, θ , and ϕ with convective heat transfer coefficient Bi. It is found that an increase in Bi leads to an enhancement in f^1, θ , and ϕ.

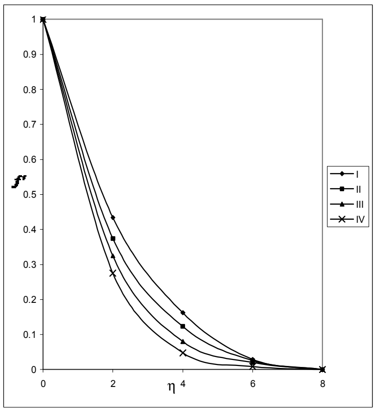

Fig.20 Variation of f' with f_s
I II III IV
f_s 0.1 0.3 0.5 1

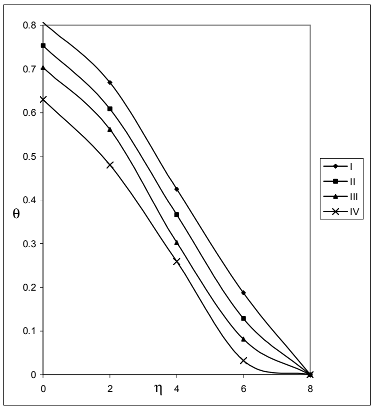

Fig.21 Variation of θ with fs
I II III IV
fs 0.1 0.3 0.5 1

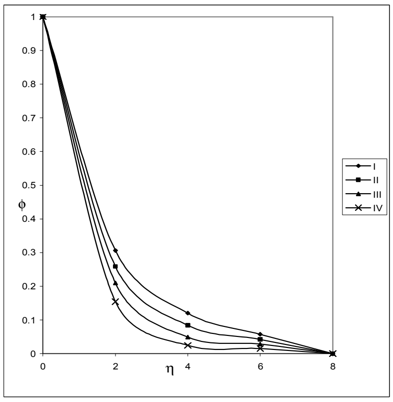

Fig.22 Variation of ϕ with fs
I II III IV
fs 0.1 0.3 0.5 1

Figures 20, 21, 22 depict the effect of Forchheimer number f_s on velocity, temperature and concentration. It is observed from fig.11that the velocity of the fluid decreases with increase in f_s. Since Forchheimer number f_s represent the inertial drag, thus an increase in the Forchheimer number f_s, increases the resistance to the flow and so a decrease in the fluid velocity ensues. It is noticed from figs 23 & 35 that the temperature and concentration reduces with increase in f_s .

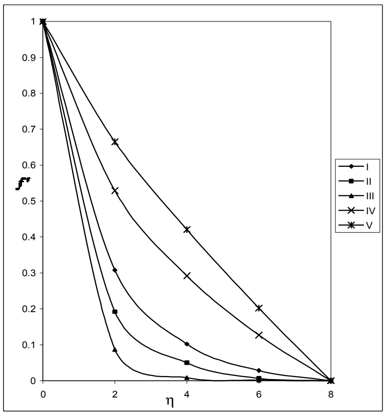

Fig.23 Variation of f' with f_w
I II III IV V
f_w 0.5 1.5 2.5 -0.5 -1.5

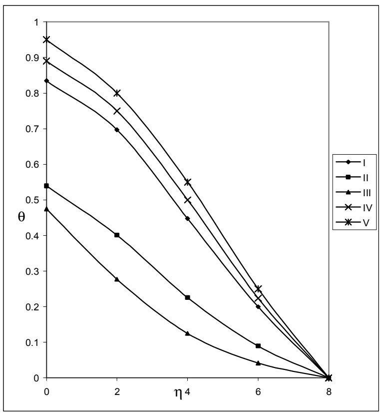

Fig.24 Variation of θ with fw
I II III IV V
fw 0.5 1.5 2.5 -0.5 -1.5

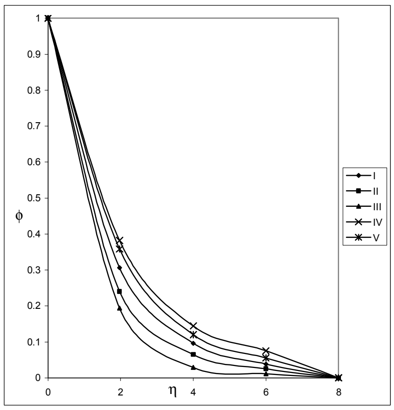

Fig.25 Variation of ϕ with fw
I II III IV V
fw 0.5 1.5 2.5 -0.5 -1.5

The effect of suction parameter f_w on the velocity profiles is shown in fig23. It is found from the figure that the velocity profiles decrease monotonically with the increase in the suction parameter indicating the fact that suction parameter stabilizes the boundary layer growth. The effect of suction parameter on the temperature and concentration is shown in figures 24 & 25. An increase in $|f_w|$ (<0) leads to an enhancement in f^1 and depreciation in θ , andϕ.

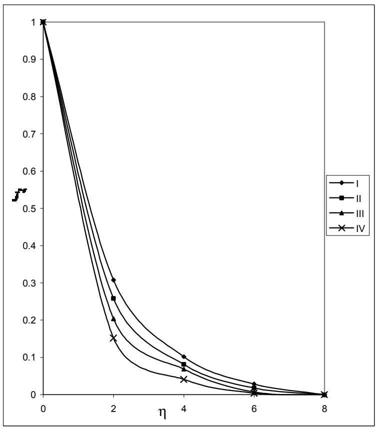

Fig.26 Variation of f' with γ
I II III IV
γ 0.01 0.03 0.05 0.07

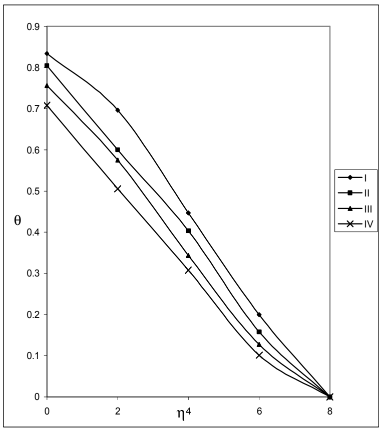

Fig.27 Variation of θ with γ
I II III IV
γ 0.01 0.03 0.05 0.07

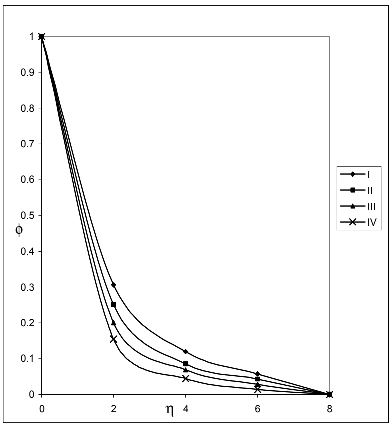

Fig.28 Variation of ϕ with γ
I II III IV
γ 0.01 0.03 0.05 0.07

The effect of density ratio γ on f^1, θ , and ϕ is shown in figs 26, 27, 28. From figures we find that the velocity f^1 , θ , and ϕ reduces with increase in the density ratio. Thus the non-linearity in the density-temperature relation results in depreciation in f^1, θ , and ϕ.

TABLE -1

		τ(0)	Nu(0)	Sh(0)
	1	-1.5255	0.0210	0.4834
	2	-1.3971	0.0232	0.4822
N	-0.5	-1.6145	0.0194	0.4449
	-0.8	-1.6372	0.0220	0.6344
	0.5	-1.5255	0.0210	0.4834
	1.5	-1.5321	0.0227	0.5939
Kr	-0.5	-1.5157	0.0184	0.3081
	-1.5	-1.4895	0.0104	0.0560
	0.24	-1.5255	0.0210	0.4834
Sc	0.66	-1.5251	0.0801	0.8127
	1.3	-1.4838	0.0980	1.4758
	0.01	-1.5255	0.0210	0.4834
Ec	0.03	-1.5106	0.0679	0.4430
	0.05	-1.4854	0.0736	0.3652
	2	-1.5255	0.0210	0.4834
Q	4	-1.6390	0.0932	0.9423
	-2	-1.5342	0.4250	0.8062
	-4	-1.6657	0.8074	1.8864
	2/0.03	-1.5318	0.0261	0.4756
Sr/Du	1.5/0.4	-1.5303	0.0253	0.4332
	1/0.6	-1.5241	0.0151	0.4066
	0.1	-1.5255	0.0210	0.4834
Bi	0.3	-1.5183	0.0432	0.4441
	0.5	-1.4938	0.0613	0.3914
	0.01	-1.5277	0.0209	0.4835
γ	0.03	-1.5262	0.0211	0.4452
	0.05	-1.5257	0.0238	0.4049

The skin friction (τ) on the surface η =0 is shown in tables 1 for different values of N, Sc, Kr, Q, Ec, Sr &Du Bi, & γ. With reference to the buoyancy ratio N we find that when the molecular buoyancy force dominates over the thermal buoyancy force, | τ | reduces on the wall when the buoyancy forces are in the same direction and for the forces acting in the opposite directions it enhances on the wall. The variation of τ with the chemical reaction parameter Kr shows that | τ | enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case. An increase in the Schmidt number Sc, convective heat transfer coefficient Bi (or) Eckert number Ec reduces | τ | on the wall. With reference to the heat source parameter Q we find that | τ | | enhances with increase in the strength of heat generating / absorbing source. Increase in the soret parameter S_0 (or decrease in the Dufour parameter Du) leads to an enhancement in | τ |. An increase in the density ratio γ reduces | τ |. Thus the non-linearity in the density-temperature relation leads to a depreciation in | τ | on the wall.

The rate of heat transfer (Nusselt number) is exhibited in the table 1 for different variation of parameters. With reference to buoyancy ratio N it is observed that |Nu| enhances with |N| irrespective of the directions of the buoyancy forces. |Nu| enhances on the wall in the degenerating chemical reaction case and reduces in the generating chemical reaction case. An increase in Sc or Ec or Bi leads to an enhancement in the rate of heat transfer. The variation of Nu with chemical reaction parameter Kr shows that |Nu| enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case. |Nu| enhances on the wall $\eta=0$ with increase in the strength of the heat generating / absorption source. Increase in the soret parameter Sr (or decrease in Du) results an enhancement in |Nu|. An increase in the density ratio γ enhances |Nu|. Thus the non-linearity in the density temperature relation leads to an enhancement in the rate of heat transfer on the wall.

The rate of mass transfer (Sherwood Number) on the wall $\eta=0$ is exhibited in the table 1 for different variations. It is found that the rate of mass transfer reduces with increase in convection heat transfer coefficient Bi. With reference to buoyancy ratio N it is found that the rate of mass transfer reduces when the buoyancy forces are in the same direction and for the forces acting in opposite directions it enhances on the wall. Also |Sh| enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case. Lesser the molecular diffusivity / higher the strength of the heat source, larger |Sh| on $\eta=0$. Higher the dissipative heat smaller |Sh| on the wall. Increasing Sr (or decreasing Du) results in an enhancement in |Sh| on the wall. An increase in the density ratio γ leads to a depreciation in |Sh|. Then the non linearity in the density-temperature relation leads to a reduction in |Sh| on the wall.

COMPARISION

Comparison values of the Skin friction, the rate of heat and mass transfer at the plate $\eta=0$ is shown in the table 2. In the case of Linear density temperature relation ($\gamma=0$), Cf, Nu and Sh are in good agreement with Alam et al. with G=12, M=1, f_w-0.5, N=1, $D^{-1}=1$ Pr=0.71, $f_s=1$ Re=100 and Sc=0.22.

TABLE-2

		Cf_x		Nu		Sh	
Sr	**Du**	**Alam & Rahman**	**Present**	**Alam & Rahman**	**Present**	**Alam & Rahman**	**Present**
2.0	**0.03**	**3.4231141**	**3.42305**	**1.0283189**	**1.02829**	**0.1296854**	**0.12967**
1.0	**0.06**	**3.3457474**	**3.34584**	**1.0155338**	**1.0155319**	**0.2992750**	**0.29926**
0.5	**0.12**	**3.3162482**	**3.316246**	**1.0019868**	**1.0019795**	**0.3844602**	**0.384458**
0.4	**0.15**	**3.3141130**	**3.314098**	**0.9965224**	**1.996498**	**0.4017999**	**0.401798**
0.2	**0.30**	**3.3287043**	**3.328709**	**0.9718535**	**0.9718485**	**0.4381199**	**0.438108**
0.1	**0.60**	**3.3828661**	**3.382794**	**0.9248360**	**0.9247316**	**0.4602605**	**0.460259**

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