

# Effect of Thermal Radiation, Dissipation and Chemical Reaction on Mixed Convective Heat and Mass Transfer Flow Past a Stretching Sheet with Hall Effects in Slip Flow Regime

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## Abstract

The aim of this analysis is to discuss the combined influence of chemical reaction, dissipation on hydromagnetic mixed convective heat and mass transfer flow of a viscous electrically conducting fluid past a stretching sheet in the presence of non-uniform heat source and Hall effects. The equations governing the flow, heat and mass transfer have been solved numerically by using Runge-Kutta fourth order technique together with the shooting technique. The velocity, temperature and concentration have been analyzed. The rate of heat and mass transfer on the plate has been evaluated numerically for different parametric variations.

**Keywords:** Hall Currents; Heat & Mass Transfer; Thermal Radiation; Dissipation, Non-Uniform Heat Source; Soret and Dufour effects; Stretching sheet; Chemical reaction.

## Nomenclature

$A_1, B_1$	Non- uniform heat source
$A$	Slip flow velocity
$C$	Concentration of the fluid
$C_o$	Concentration of the fluid at outer cylinder
$C_i$	Concentration of the fluid at inner cylinder
$C_p$	Specific heat coefficient
$D_1$	Molecular diffusivity
$D^{-1}$	Inverse Darcy parameter
$Du$	Dufour parameter
$F$	Function depends on Reynolds number
$g$	Acceleration due to gravity
$G$	Grashof number
$H_0$	Constant applied magnetic field
$k$	Permeability of porous medium
$M$	Hartmann number
$Pr$	Prandtl number
$Rd$	Thermal Radiation
$Sc$	Schmidt number
$So$	Soret parameter

## Greek Symbols

$\beta$	Thermal expansion coefficient
$\beta c$	Coefficient of volume expansion of Concentration
$\rho$	density of the fluid friction
$\theta$	non-dimensional temperature
$\sigma$	electrical conductivity of the fluid
$\sigma^*$	Stefan-Boltzmann constant parameter
$\gamma$	Chemical reaction parameter
$(x, y, z)$	Cartesian coordinates
$(u, v)$	velocity components along $x$ and $y$ axes

## 1. Introduction

The effect of chemical reaction on free convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify [2004] in the presence of a transverse magnetic field. In all these investigations the electrical conductivity of the fluid was assumed to be uniform. However, in an ionized fluid where the density is low and/or magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the spiraling of electrons and ions about the magnetic lines of force before collisions take place and a current induced in a direction normal to both the electric and magnetic fields. This phenomenon available in the literature is known as Hall Effect. Thus the study of MHD viscous flows, heat and mass transfer with Hall currents has important bearing in the engineering applications. Hall effect on MHD boundary layer flow over a continuous semi-infinite flat plate moving with a uniform velocity in its own plane in an incompressible viscous and electrically conducting fluid in the presence of a uniform transverse magnetic field were investigated by Watanabe and Pop [1995]. Abo-Eldahab and Elbarbary [2001] have investigated free convective flows past a semi-infinite vertical plate with mass transfer. The effect of Hall current on the study MHD flow of an electrically conducting, incompressible Burger's fluid between two parallel electrically insulating infinite plane was studied by Rana et. al. [2008]. Samad and Mohebujjaman [2009] have studied MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation. Seddeek [2007] have studied the heat and mass transfer on a stretching sheet with a magnetic field in a visco-elastic fluid flow through a porous medium with heat source or sink. Veena et.al. [2007] have discussed the non-similar solutions for heat and mass transfer flow in an electrically conducting

visco-elastic fluid over a stretching sheet embedded in a porous medium. Hsiao [2008] has analysed the heat and mass transfer for electrical conducting mixed convection with radiation effect for visco-elastic fluid past a stretching sheet. Shit [2009] has studied Hall effects on MHD free convective flow on mass transfer over a stretching sheet. Raghavendra Rao [2012] has discussed the effect of chemical reaction, Hall effects on the convective heat and mass transfer flow past a stretching sheet.

The behavior of laminar boundary layer past a moving continuous and linearly stretching surface is a significant type of flow has considerable practical applications in engineering, electrochemistry (Chin [1975], Gorla [1978]) and polymer processing, (Griffith [1964], Erickson et. al. [1966]). For example, materials manufactured by extrusion process and heat treated materials traveling between a feed roll and a windup roll or on a conveyor belt possesses the characteristics of a moving continuous surface. The hydromagnetic flow and heat transfer problems have become important industrially. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. Mention may be made of drawing, annealing and tinning of copper wires. In all the cases the properties of the final product depend to a great extent on the rate of cooling. By drawing such strips in an electrically conducting fluid subjected to magnetic fluid, the rate of cooling can be controlled and a final product of desired characteristics can be achieved. Another interesting application of hydro magnetics to metallurgy lies in the purification of molten metals from nonmetallic inclusions by the application of a magnetic field. The study of heat and mass transfer is necessary for the determining the quantity of the final product. However, there are fluids, which react chemically with some other ingredients present in them. The effect of a chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux was studied by Das et.al. [1994], Anderson et. al. [1994], have studied the diffusion of a chemical reactive species from a linearly stretching sheet. Anjalidevi and Kandaswamy [1999] have investigated the effect of a chemical reaction on the flow along a semi infinite horizontal plate in the presence of heat transfer. Anjalidevi and Kandaswamy [2000] have studied the effect of a chemical reaction on the flow in the presence of heat transfer and magnetic field. Muthukumaraswamy and Ganesan [2000] have analyzed the effect of a chemical reaction on the unsteady flow past on impulsively started semi-infinite vertical plate, which is subject to uniform heat flux. McLeod and Rajagopal [1987] have investigated the uniqueness of the flow of a Navier Stoke's fluid due to a linear stretching boundary. Raptis et. al. [2006], have studied the viscous flow over a non-linearly stretched sheet in the presence of a chemical reaction and magnetic field. Several authors (Sakiadis [1961], Erickson et.al. [1966], Fox et.al. [1968], Chen and Char [1988], Ali [1995], Magyari et.al. [2001], Crane[1970]) have discussed fluid flow past a stretching sheet under different conditions.

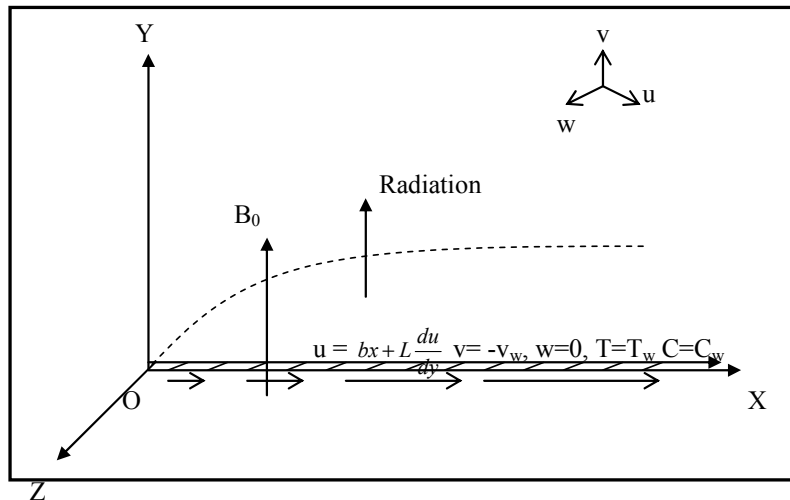
In flows through porous medium with combined buoyancy effects owing to heat and mass transfer, the coupling of heat and mass transfer takes place owing to the density variations with temperature and concentration. The direct coupling between temperature and concentration is possible when the cross diffusion (Soret and Dufour) effect is small. When mass flux is produced due to temperature gradients it is known as Soret effect and Dufour effect refers to the heat flux by concentration gradients. Knobloch [1980] and Taslim and Narusawa [1986] explained the relation between Dufour and Soret numbers. Dulal Pal and Mondal [2012] studied the combined effect of Soret and Dufour on unsteady MHD non-Darcy mixed convection over a stretching sheet embedded in a saturated porous medium in the presence of thermal radiation, viscous dissipation and chemical reaction. Anwar [2009] made a numerical study of MHD heat and mass transfer from a stretching surface in a porous medium with Soret and Dufour effects. Sallam [2010] investigated the thermo-diffusion and diffusion-thermo effects on mixed convection heat and mass transfer in a porous medium. Kuznetsov and Neild [2011] analysed the double diffusion natural convective boundary layer flow of a nanofluid past a vertical plate. Stanford Sateyi et.,al [2010] investigated the effect of Hall currents with Soret and Dufour effects on the flow past a vertical surface. Sarojamma et al [2015] have studied the influence of Hall currents on cross diffusive convection in a MHD boundary layer flow on stretching sheet in porous medium with heat generation.

In this paper, we study the combined influence of chemical reaction, Hall currents on convective heat and mass transfer flow of a viscous electrically conducting fluid past a stretching sheet in the presence of non-uniform heat source. The equations governing the flow of heat and mass transfer have been solved numerically employing Runge-Kutta fourth order technique together with the shooting technique. The velocity, temperature and concentration have been analyzed for different values of  $m$ ,  $N$ ,  $R_d$ ,  $A_1$ ,  $B_1$   $So$ ,  $Du$ ,  $Ec$  and  $\gamma$ . The rate of heat and mass transfer on the plate has been evaluated numerically for different variations.

## 2. Problem Formulation

We considered a steady convective flow of an incompressible, viscous and electrically conducting fluid past a plate through a porous medium in the vertical direction stretching with a velocity proportional to the distance from the origin 'O', of a stationary frame of reference  $O(x,y,z)$ . The positive x-coordinate is measured along the stretching sheet in the direction of motion and the positive y coordinate is measured normal to the sheet in the

outward direction towards the fluid, the x-axis coincides with the leading edge of the stretching sheet.



**Figure 1. Schematic Diagram of the Problem**

The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The flow is subjected to a strong transverse magnetic field with a constant intensity  $B_0$  along the positive y-direction. The magnetic Reynolds number is assumed to be small enough ( $Re \ll 1$ ), so that the induced magnetic field can be neglected. The governing equations for the magnetic field  $\nabla \cdot \vec{H} = 0$ , where  $\vec{H} = (H_x, H_y, H_z)$  gives  $H_y = H_0$  (constant) everywhere in the flow field, which gives  $\vec{H} = (0, 0, H_0)$ . If  $(j_x, j_y, j_z)$  are the components of current density  $\vec{j}$ , then the equation of conservation of electric charge  $\nabla \cdot \vec{j} = 0$  gives  $j_y = 0$  everywhere in the flow since the plate is electrically non-conducting. The generalized Ohm's law, in the absence of electric field is of the form

$$\vec{j} + \frac{\omega_e \tau_e}{H_0} (\vec{j} + \vec{H}) = \sigma (\mu_e \vec{v} \times \vec{H}) + \frac{1}{en_e} \nabla p_e \quad (1)$$

Where  $\vec{v}$ ,  $\sigma$ ,  $\mu_e$ ,  $\tau_e$ ,  $e$ ,  $n_e$  and  $p_e$  are the velocity, the electrical conductivity, the magnetic permeability, the cyclotron frequency, the electron collision time, the electric charge, the number density of the electron and the electron pressure, respectively. Under the usual assumption, the electron pressure (for a weakly ionized gas), the thermoelectric pressure, and ion slip are negligible, so we have from the Ohm's law

$$j_x + \omega_e \tau_e j_y = (\sigma \mu_e H_0) v$$

$$j_y - \omega_e \tau_e j_x = -(\sigma \mu_e H_0) w$$

From these equations, we obtain that

$$j_x = (\sigma \mu_e H_0) (uw + v)$$

$$j_y = (\sigma \mu_e H_0) (mw + u) \quad (2)$$

The flow has significant thermal radiation, chemical reaction, heat source and Hall, Soret and Dufour effects. By using Boussinesq approximation the basic boundary layer equations of flow, heat and mass transfer are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \beta_T g (T - T_\infty) + \beta_c g (C - C_\infty) - \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)} (u + mw) - \left(\frac{\mu}{\rho k}\right) u \quad (4)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)} (mu - w) - \left(\frac{\mu}{\rho k}\right) w \quad (5)$$

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_f \frac{\partial^2 T}{\partial y^2} + \frac{Dk_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} + \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)} (u^2 + w^2) + q''' \quad (6)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{Dk_T}{T_m} \frac{\partial^2 T}{\partial y^2} - k_r (C - C_\infty) \quad (7)$$

Where  $u$  and  $v$  are velocity components in the  $x$ -direction and  $y$ -direction respectively,  $w$  is the velocity component in the  $z$ -direction,  $\nu$  is the kinematic coefficient of viscosity,  $g$  is the Acceleration due to gravity,  $\beta_T$  is the coefficient of thermal expansion,  $T$  is the temperature of the fluid inside the thermal boundary layer,  $T_w$  is the plate temperature,  $T_\infty$  is the temperature of the fluid in the free stream with  $T_w > T_\infty$ ,  $\beta_c$  is the volume coefficient of expansion with concentration,  $C$  is the concentration,  $\sigma$  is the electrical conductivity of the fluid,  $\mu_e$  is the magnetic permeability of the fluid,  $e$  is the electron charge,  $n_e$  is the electron number density,  $\tau_e$  is the electron collision time,  $m_e$  is the mass of electron,  $B_0$  is the Magnetic field of constant strength,  $\rho$  is the density of the fluid,  $m(\omega_e \tau_e)$  is the Hall parameter,  $\mu$  is the dynamic viscosity,  $k$  is permeability of the porous medium,  $C_p$  is the specific heat constant bpressure,  $k_f$  is the thermal conductivity,  $C_s$  is the concentration susceptibility,  $D$  is the mass diffusivity,  $K_T$  is the thermal-diffusivity rate,  $q_R$  is the radiative heat flux in the  $y$ -direction.  $T_m$  is the mean fluid temperature,  $C_w$  is the uniform concentration,  $C_\infty$  is the free stream concentration with  $C_w > C_\infty$ , and  $k_r$  is the chemical reaction parameter.

The boundary conditions are

$$u = U_s + L \frac{du}{dy} = bx + L \frac{du}{dy}, v = -v_w w = 0, T = T_w, C = C_w \quad \text{at} \quad y = 0 \quad (7)$$

$$u \rightarrow 0, w \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as} \quad y \rightarrow \infty$$

Where  $U_s$  is the velocity of the surface,  $b$  is a constant with dimension  $(\text{time})^{-1}$  and  $v_w$  is the suction ( $>0$ ) or injection ( $<0$ ) of the velocity at the plate.  $L$  is the velocity slip factor.

Employing Rosseland diffusion approximation (Raptis[23]), the radiative heat flux is given by

$$q_R = -\frac{4\sigma^* \partial T'^4}{3\beta_R \partial y} \quad (8)$$

Where  $\sigma^*$  is the Stefan-Boltzmann constant and  $\beta_R$  is the mean absorption coefficient, temperature difference within the flow are assumed to be sufficiently small so that  $T'^4$  may be expressed as a linear function of temperature  $T$  using a truncated Taylor series about the free stream temperature  $T_\infty$ , i.e

$$T'^4 = 4T_\infty^3 T - 3T_\infty^4 \quad (9)$$

From equations (8)&(9), we obtain

$$T'^4 \frac{\partial q_R}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3\beta_R} \frac{\partial^2 T}{\partial y^2} \quad (10)$$

The coefficient  $q'''$  is the rate of internal heat generation ( $>0$ ) or absorption ( $<0$ ).

The internal heat generation/absorption  $q'''$  is modeled (Dulal pal[2012]) as

$$q''' = \left(\frac{ku_s}{x\nu}\right) [A^* (T_w - T_\infty) f''(\eta) + B^* (T - T_\infty)] \quad (11)$$

Where  $A^*$  and  $B^*$  are coefficients of space-dependent and temperature-dependent heat generation or absorption respectively. It is noted that the case  $A^* > 0$ , and  $B^* > 0$ , corresponds to internal heat generation and that  $A^* < 0$ , and  $B^* < 0$ , the case corresponds to the internal heat absorption.

Introducing the similarity variables as

$$\eta = \sqrt{(b/\nu)}y, \quad u = bx f'(\eta), \quad v = -\sqrt{b\nu} f(\eta) \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (12)$$

Substituting (12) in equations(4)-(7) results in to the non-linear ordinary differential equations of the form

$$f'''' + f f'' - (f')^2 + G(\theta + N\phi) - \frac{M^2}{1+m^2} (f' + mg) - D^{-1} f' = 0, \quad (13)$$

$$g'' + f g' - f' g + \frac{M^2}{1+m^2} (m f' - g) + D^{-1} g = 0 \quad (14)$$

$$\frac{1}{Pr} \left(1 + \frac{4}{3} Rd\right) \theta'' + f \theta' + Du \phi'' + \frac{M^2 Ec}{1+m^2} (f'^2 + g^2) + \frac{1}{Pr} (A^* f' + B^* \theta) = 0 \quad (15)$$

$$\phi'' + Sc(f\phi' + Sr\theta'' - \gamma\phi) = 0 \quad (16)$$

The transformed boundary conditions are

$$\begin{aligned} f'(\eta) = 1 + Af'(0), f(\eta) = f_w, g(\eta) = 0, \theta(\eta) = 1, \phi(\eta) = 1 & \quad \text{at } \eta = 0 \\ f'(\eta) = 0, g(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0 & \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad (17)$$

Where  $f_w = \frac{v_w}{\sqrt{b\nu}}$  is the mass transfer coefficient such that  $f_w > 0$  represents suction and  $f_w < 0$  represents injection at the surface.

The local Skin friction coefficient  $C_{fx}$  along the z-direction is defined by

$$C_{fx} = \frac{2\tau_{wx}}{\rho(ax)^2} \quad (18)$$

Where  $\tau_{wx}$  is the wall shearing stress given by  $\tau_{wx} = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}$  (19)

By using the relation (12), equations (17)-(19) take the form

$$C_{fx} \text{Re}_x^{-1/2} = 2f''(0) \quad \text{and} \quad (20)$$

$$C_{fz} \text{Re}_x^{-1/2} = 2g'(0)$$

The heat flux is given by

$$q_w = -k_f \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad (21)$$

The local Nusselt number is given by

$$Nu_x = \frac{xq_w}{k_f} \quad (22)$$

By using the non-dimensional variables equations (12) in equations (21)&(22), we get

$$Nu_x \text{Re}_x^{-1/2} = -\theta'(0) \quad (23)$$

The mass transfer coefficient may be written as follows:

$$q_m = -D \left( \frac{\partial C}{\partial y} \right)_{y=0} \quad (24)$$

Where D is the molecular diffusivity.

The local Sherwood number is given by

$$Sh_x = \frac{xq_m}{D} \quad (25)$$

By using the equations (12) in equations (24)&(25), we get

$$Sh_x \text{Re}_x^{-1/2} = -\phi'(0) \quad (26)$$

### 3. Numerical procedure

An exact solution for f from equations(13)-(16) together with the boundary conditions(17) in the absence of Hall currents, thermal buoyancy and solutal buoyancy can be obtained as

$$\begin{aligned} f(\eta) &= f_w + \frac{1}{a}(1 - \exp(-a\eta)) \\ a &= \frac{f_w + \sqrt{f_w^2 + 4(M^2 + D^{-1} + 1)}}{2} \end{aligned}$$

The set of non-linear differential equations (13)-(16) with the boundary conditions (17) are solved numerically employing Runge\_Kutta fourth order technique together with the shooting technique. The boundary conditions as  $\eta \rightarrow \infty$  enable us to use value of  $f', g, \theta, \phi$ . So that the velocity, temperature and concentration fields can be obtained for different variations in the physical parameters such as Grashof number (G), Buoyancy ratio (N), Radiation parameter (Rd), chemical reaction parameter( $\gamma$ ), Hall parameter(m), Magnetic parameter (M), Eckert Number(Ec), Heat source parameters( $A^*$ ,  $B^*$ ).

### 4. Results and Discussion

To get a physical insight of the problem, the profiles of velocity, temperature and concentration are graphically presented. The values of the parameters are fixed throughout the computations as  $M=m=1, G=1, N=1, D^{-1}=1,$

$Pr=0.71, Rd=0.5, Ec=0.01, Sr=2.0, Du=0.03, sc=1.3, \gamma=0.5, fw=0.1, A1=0.01=B1, A=0.1.$

Figs. 2-5 show the effect of Hall current on the velocity, temperature and concentration. It can be seen from the velocity profiles that an increase in the Hall parameter ( $m$ ) enhances both the primary velocity and cross flow velocity. The temperature and concentration distributions depreciate in the boundary layer with increase in the Hall parameter ( $m$ ) (Figs. 4&5). Figs. 6-9 represent the effect of buoyancy ratio ( $N$ ) on the velocity, temperature and concentration. It is observed from the profiles that when the molecular buoyancy force dominates over the thermal buoyancy force the primary velocity and Cross flow velocity components enhances when the buoyancy forces are in the same direction and for the forces acting in opposite directions the components reduce in the boundary layer. The temperature and concentration reduces with increase in  $N>0$  and enhances with  $N<0$  in the boundary layer. (Figs. 8&9).

Figs. 10-13 show the variation of chemical reaction effect on the velocity, temperature and concentration. From Figs. (10&11) we observe that the primary velocity ( $f'$ ) and cross flow velocity ( $g$ ) reduces with increase in the chemical reaction parameter ( $\gamma$ ) in both degenerating and generating chemical reaction cases. The variation of temperature with  $\gamma$  shows that the temperature reduces in the degenerating chemical reaction case and enhances in the generating chemical reaction case (Fig.12) while the concentration decreases with increasing values of the chemical reaction parameter in both degenerating and generating chemical reaction cases. In fact the concentration of the species steadily falls from its higher value on the surface to lower value eventually attaining the free stream concentration as  $\eta \rightarrow \infty$  (Fig. 13).

Figs. 14-17 show the variation of velocity, temperature and concentration with thermal radiation parameter ( $Rd$ ). It is found that there is a significant rise in the primary velocity in the presence of thermal radiation throughout the boundary layer. A similar behavior for the cross flow velocity is also noticed with  $Rd$ . It can be seen from the profiles of the cross flow velocity ( $g$ ) that the rise in the cross flow velocity  $g$  is relatively lesser than that of the primary velocity ( $f'$ ) in the boundary layer. The radiation parameter is found to enhance the hydrodynamic boundary layer along the  $x$  and  $z$ -directions. The presence of the thermal radiation is very significant on the variation of temperature. It is seen from the profiles the temperature increases rapidly in the presence of the thermal radiation parameter in the entire thermal boundary layer. This may be attributed to the fact that as the Rosseland radiative absorption parameter  $Rd$  diminishes the corresponding heat flux divergence and thus raising the rate of radiative heat transfer to the fluid causing a rise in the temperature of the fluid. The thickness of the boundary layer increases in the presence of  $Rd$  (Fig.16). The effect of  $Rd$  on mass concentration is not significant as compared to the velocity and temperature. However, the concentration reduces in the solutal boundary layer with increase in  $Rd$  (Fig.17). Figs. 18-21 depict the influence of the Eckert number ( $Ec$ ) on velocities, temperature and concentration. It is pointed out that the presence of Eckert number increases the temperature. This is due to the fact the thermal energy is reserved in the fluid on account of friction heating. Hence, the temperature distribution rises in the entire thermal boundary layer (Fig. 20). The primary and the cross flow velocity components are also found to enhance for increasing the values of  $Ec$  due to the energy release which increases the momentum. However, the mass concentration reduces with increase in  $Ec$  (Fig. 21). Figs. 22-25 exhibits the influence of Soret and Dufour effects on the velocity, temperature and concentration. It is found that increasing the Soret parameter  $So$  (or decreasing Dufour parameter  $Du$ ) results in an increase in the primary and cross flow velocity components (Figs. 22&23). Increasing  $So$  (or decreasing  $du$ ) leads to a depreciation in the temperature and enhancement in the mass concentration in the boundary layer (Fig. 24&25).

The effect of non-uniform heat source on the velocity, temperature and concentration is exhibited in Figs. 26-29. From Figs. 26&27 we find that the presence of the heat source generates energy in the thermal boundary layer and as a consequence the temperature rises. Near the plate an overshoot in the temperature is also noticed. Increasing the value of  $A1>0, B1>0$  (heat generating source) produce further rise in the temperature. In the case of  $A1<0, B1<0$ , the temperature falls with decreasing values of  $A1<0, B1<0$  due to the absorption of energy in the boundary layer. The primary and cross flow velocity enhances marginally with increase in heat generation ( $A1>0, B1>0$ ) while we notice a depreciation in them in the case of heat absorption ( $A1<0, B1<0$ ). The temperature enhances with heat generation and reduces with heat absorption (Fig. 28). The effect of heat generation/absorption parameter is to enhance the mass concentration marginally (Fig. 29).

Figs. 30-33 represent the influence of slip velocity parameter  $A$  on the velocity components, temperature and concentration. It can be seen from the profiles the primary and cross flow velocities decreases in the boundary layer with increase in slip velocity parameter  $A$ . This is due to the fact that in increase in  $A$  leads to a decrease in the thickness of the momentum boundary layer. From Figs. 32&33, we find that the temperature and mass concentration enhances with increase in  $A$ . This can be attributed to the fact that thermal boundary layer and the solutal boundary layer increases with increase in the velocity slip parameter  $A$ .



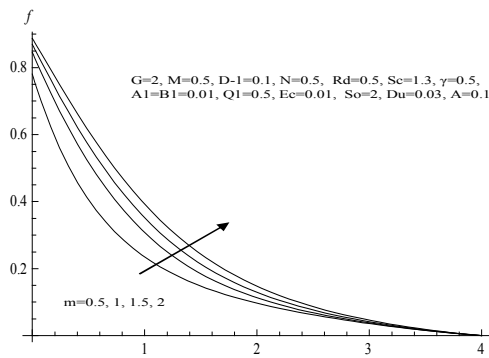


Figure 2. Effect of Hall current  $m$  on axial velocity  $f'$

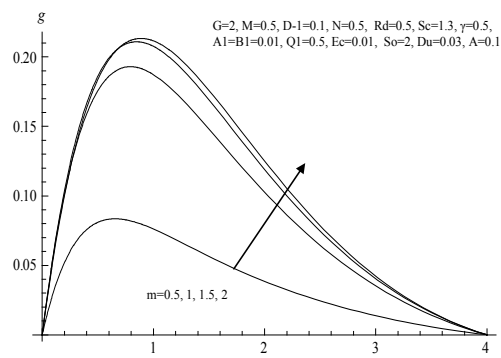


Figure 3. Effect of Hall current  $m$  on cross flow velocity  $g$

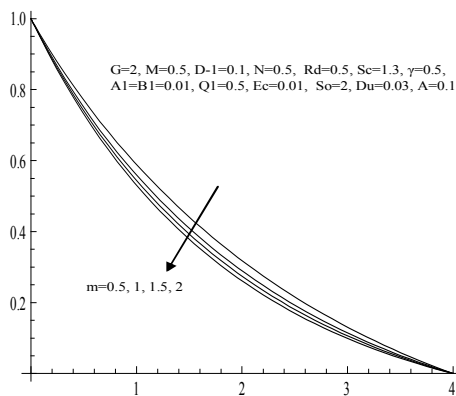


Figure 4. Effect of Hall current  $m$  on Temperature  $\theta$

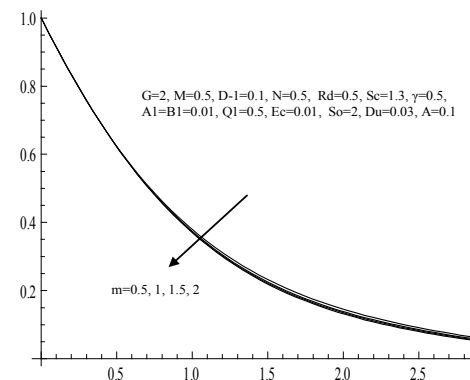


Figure 5. Effect of Hall current  $m$  on Concentration  $\phi$

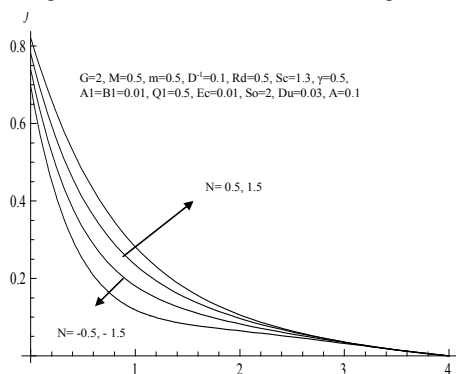


Figure 6. Effect of Buoyancy Ratio  $N$  on axial velocity  $f'$

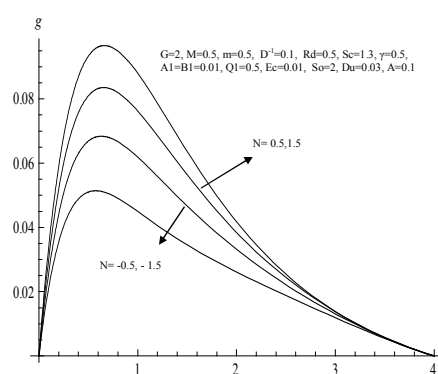


Figure 7. Effect of Buoyancy Ratio  $N$  on cross flow velocity  $g$

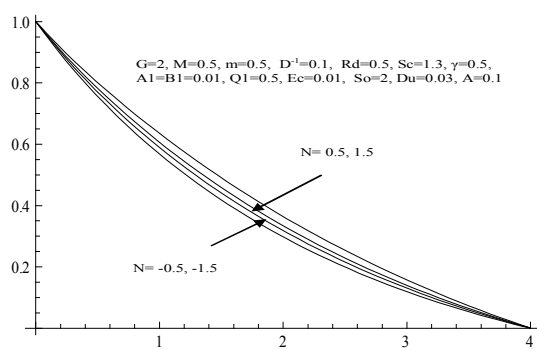


Figure 8. Effect of Buoyancy Ratio  $N$  on Temperature  $\theta$

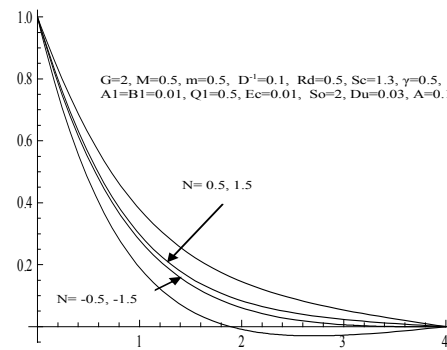


Figure 9. Effect of Buoyancy Ratio  $N$  on Concentration  $\phi$

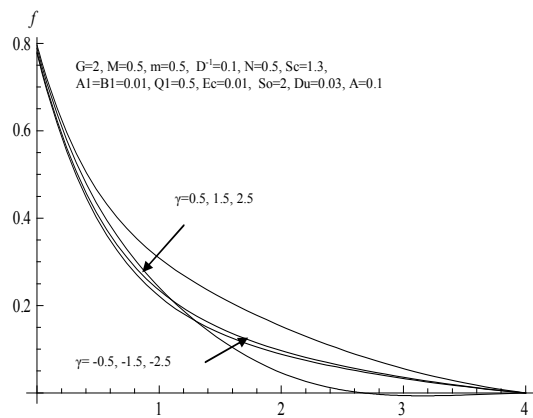


Figure 10 Effect of chemical reaction  $\gamma$  on axial velocity  $f'$

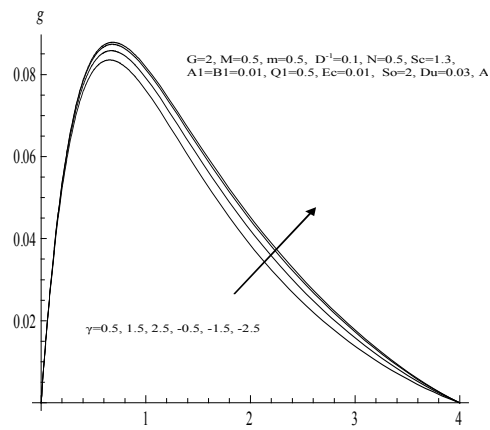


Figure 11. Effect of chemical reaction  $\gamma$  on cross flow velocity  $g$

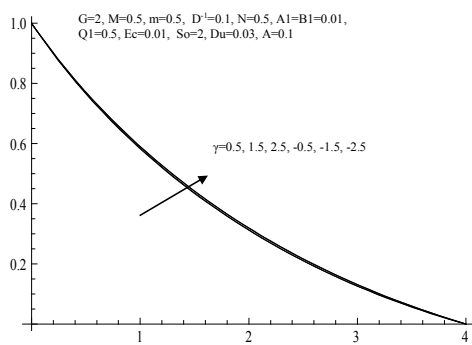


Figure 12. Effect of chemical reaction  $\gamma$  on Temperature  $\theta$

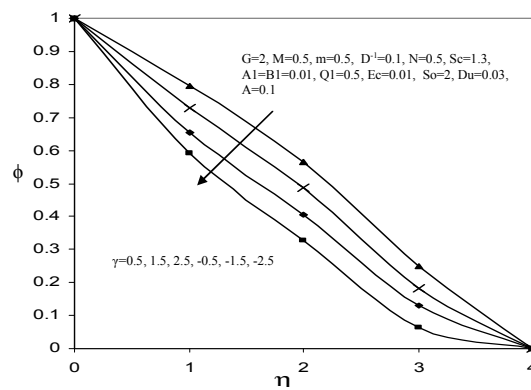


Figure 13. Effect of chemical reaction  $\gamma$  on Concentration  $\phi$

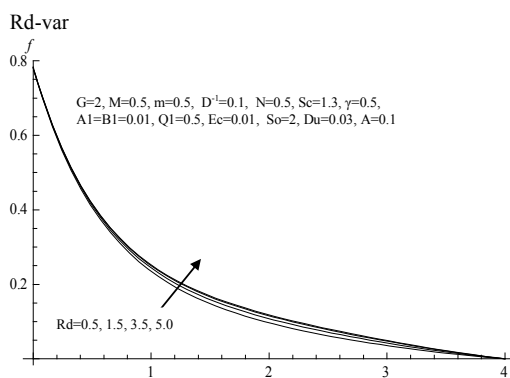


Figure 18. Effect of Radiation  $R_d$  on axial velocity  $f'$

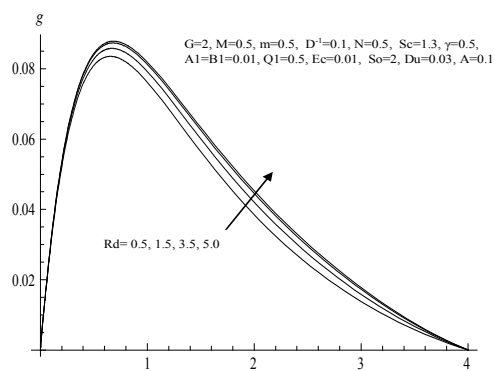


Figure 19. Effect of Radiation  $R_d$  on cross flow velocity  $g$



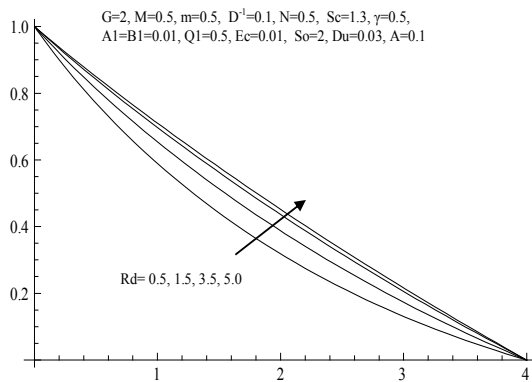


Figure 20. Effect of Radiation  $Rd$  on Temperature  $\theta$

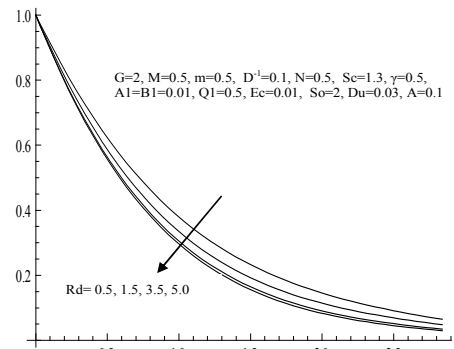


Figure 21. Effect of Radiation  $Rd$  on Concentration  $\phi$

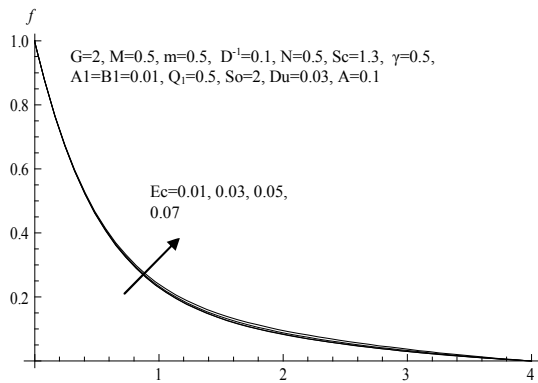


Figure 18. Effect of on Dissipation parameter  $Ec$  on axial velocity  $f'$

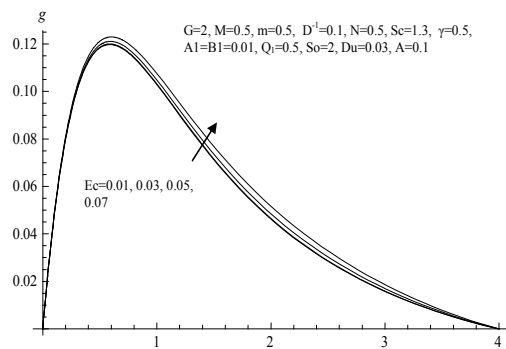


Figure 19. Effect of on Dissipation parameter  $Ec$  on cross flow velocity  $g$

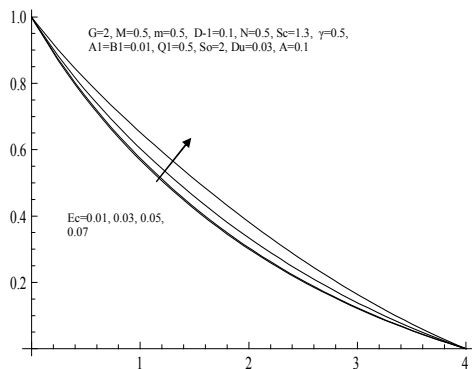


Figure 20. Effect of on Dissipation parameter  $Ec$  on Temperature  $\theta$

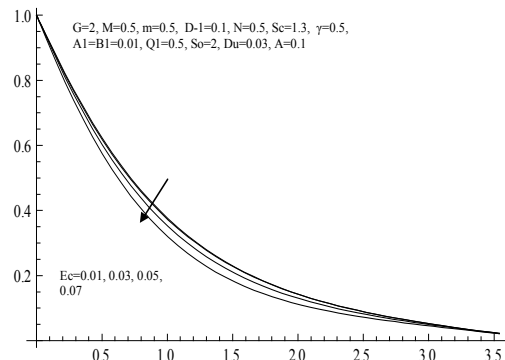


Figure 21. Effect of on Dissipation parameter  $Ec$  on Concentration  $\phi$

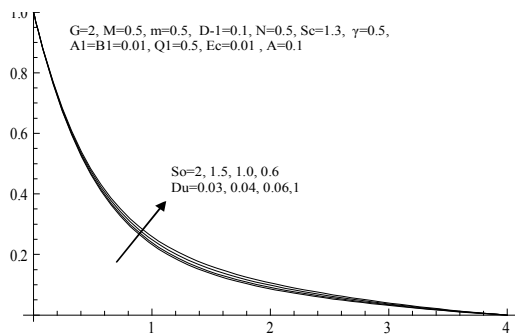


Figure 22. Effect of on Soret  $So$  and Dufour  $Du$  effects on axial velocity  $f'$

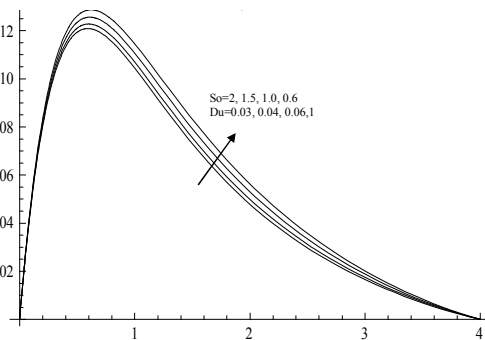


Figure 23. Effect of on Soret  $So$  and Dufour  $Du$  effects on cross flow velocity  $f'$

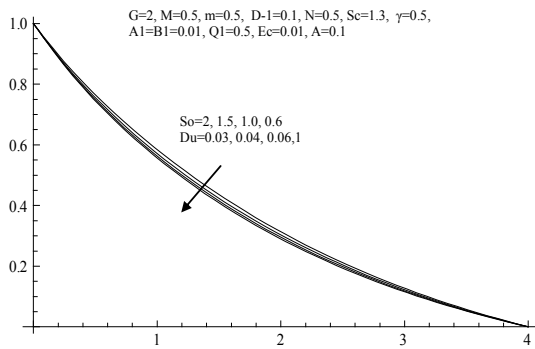


Figure 24. Effect of on Soret  $So$  and Dufour  $Du$  effects on Temperature  $\theta$

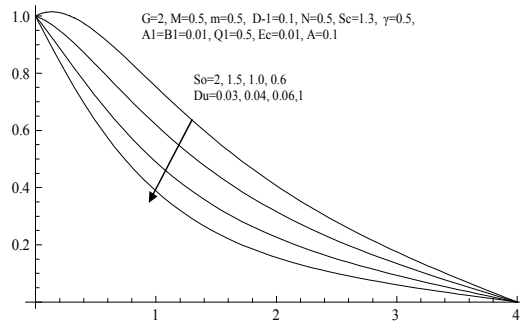


Figure 25. Effect of on Soret  $So$  and Dufour  $Du$  effects on Concentration  $\phi$

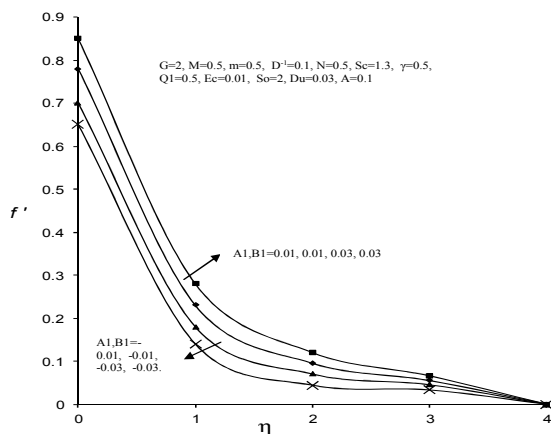


Figure 26. Effect of Non-uniform heat source  $A_1, B_1$  on axial velocity  $f'$

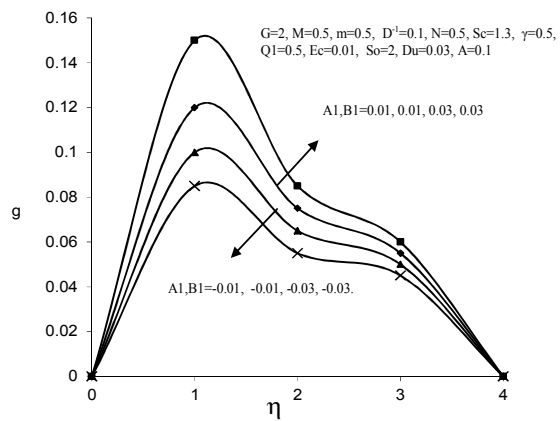


Figure 27. Effect of Non-uniform heat source  $A_1, B_1$  on Cross flow velocity  $g$

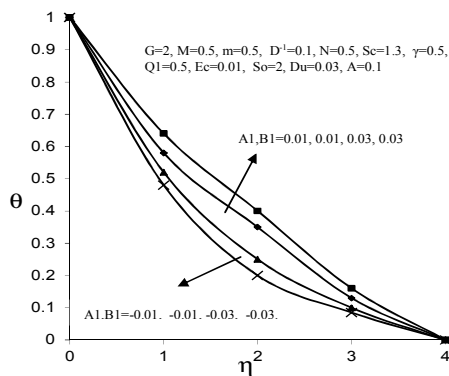


Figure 28. Effect of Non-uniform heat source  $A_1, B_1$  on Temperature  $\theta$

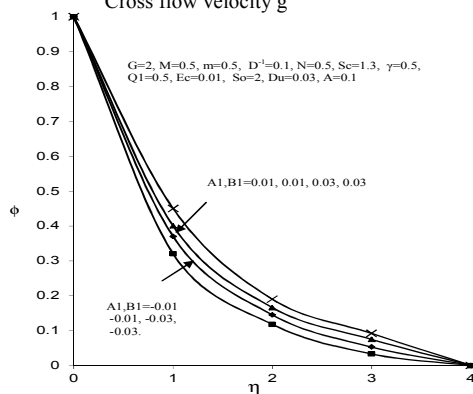


Figure 29. Effect of Non-uniform heat source  $A_1, B_1$  on Concentration  $\phi$

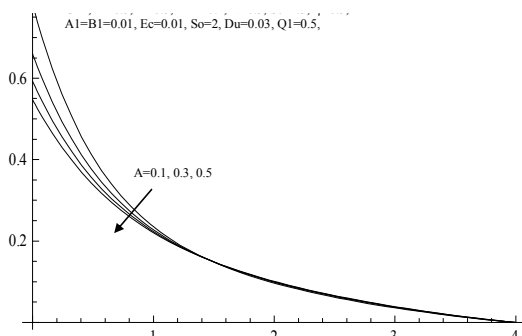


Figure 30. Effect of Slip velocity parameter  $A$  on axial velocity  $f'$

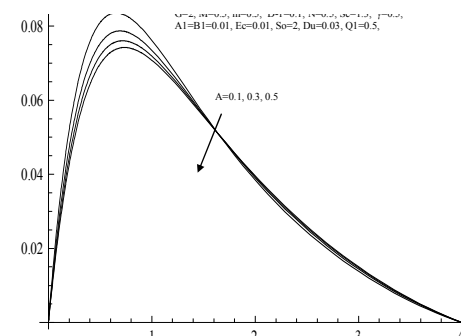


Figure 31. Effect of Slip velocity parameter  $A$  on cross flow velocity  $g$

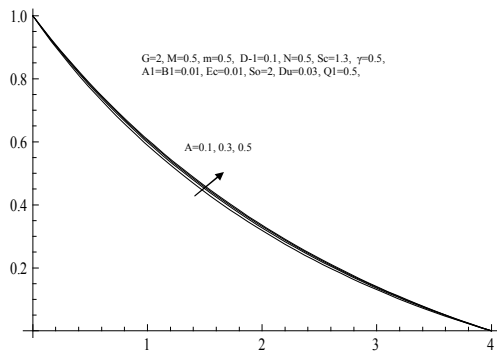


Figure 32. Effect of Slip velocity parameter A on Temperature  $\theta$

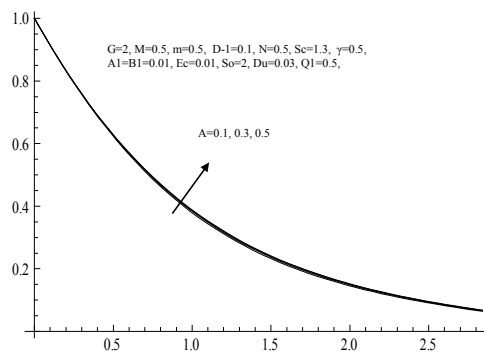


Figure 33. Effect of Slip velocity parameter A on Concentration  $\phi$

From Table.1 an increase in the Hall parameter (m) enhances the skin friction coefficient and enhances the rate of heat and mass transfer at the wall. The variation with respect to the buoyancy ratio (N) shows that when the molecular buoyancy force dominates over the thermal buoyancy force the skin friction coefficient reduces and the rate heat and mass transfer at the wall increases when the buoyancy forces are in the same direction and for the forces acting in opposite directions, an opposite effect is noticed in the behavior of  $C_{fx}$ , Nu and Sh. An increase in the strength of the heat generation ( $A1>0, B1>0$ ) reduces the skin friction coefficient, Nusselt number and enhances the Sherwood number at the wall while in case of heat absorption ( $A1<0, B1<0$ ) case, we find a reduction in  $C_{fx}$  and enhancement in Nu and Sh. Increasing the Soret parameter So (or decreasing Dufour parameter Du) leads to a reduction in  $C_{fx}$  and Sh and enhancement in Nu. Higher the dissipative heat larger  $C_{fx}$  and Sh and lesser Nu. The skin friction enhances and Nusselt number reduces in both the degenerating and generating chemical reaction cases. The Sherwood number enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case. With respect to radiation parameter Rd, we notice an increment in  $C_{fx}$  and Sh and reduction in Nu at the wall.

		$\tau_x$	$\tau_z$	Nu	Sh
m	0.5	-1.06575	0.373644	0.53571	0.1041667
	1.0	-0.723902	0.682746	0.593247	-0.130219
	1.5	-0.594201	0.685923	0.621219	-0.208006
	2.5	-0.494957	0.652992	0.644729	-0.270559
N	0.5	-1.06575	0.373644	0.53571	0.1041667
	1.5	-0.863159	0.416433	0.567315	0.779123
	-0.5	-1.27632	0.327071	0.498739	2.09272
	-1.5	-1.4982	0.274631	0.454261	1.37095
Rd	0.5	-1.06575	0.373644	0.53571	0.1041667
	1.5	-1.07161	0.372126	0.409532	0.544668
	3.5	-1.07527	0.371071	0.334705	0.836618
	5	-1.07637	0.370735	0.312657	0.921793
A1,B1	0.01,0.01	-1.06575	0.373644	0.53571	0.0416671
	0.03,0.03	-1.06637	0.373509	0.52197	0.0969671
	-0.01,-0.01	-1.06514	0.373776	0.549286	-0.013054
	-0.03,-0.03	-1.06455	0.373906	0.562702	-0.067213
So/Du	2.0/0.03	-1.06575	0.373644	0.53571	0.1041667
	1.5/0.04	-1.07658	0.368827	0.529849	0.336632
	1.0/0.06	-1.08698	0.364181	0.518605	0.631516
	0.6/1.0	-1.09421	0.360956	0.496734	0.867529
Ec	0.01	-1.06575	0.373644	0.53571	0.1041667
	0.03	-1.06619	0.373571	0.523712	0.0946876
	0.5	-1.06662	0.373498	0.511721	0.14767
	0.7	-1.06727	0.373388	0.493748	0.227074
A	0.1	-1.06575	0.373644	0.53571	0.1041667
	0.2	-0.789175	0.335588	0.511819	0.12434
	0.3	-0.639654	0.31433	0.498429	0.170779
	0.5	-0.538163	0.299609	0.489138	0.203044
$\gamma$	0.5	-1.06575	0.373644	0.53571	0.1041667
	1.5	-1.09668	0.360421	0.519457	0.918138
	-0.5	-0.931626	0.433374	0.587967	-2.89758
	-1.5	-1.09474	0.34473	0.54307	-0.54721

Table 2. The variations of Skin friction, Nusselt number, Sherwood number for different parametric values

## 5. Conclusions

The problem relating to the influence of Hall effect, Thermal radiation parameter  $R_d$ , Soret and Dufour effects on mixed convective flow and heat and mass transfer flow past on a stretching sheet in the presence of non-uniform heat source, Hall effects and chemical reaction have been analyzed in slip flow regime. The obtained numerical results were discussed graphically. Graphical results for various parametric conditions were presented and discussed for different values. The main findings are summarized below:

- An increase in the Hall parameter ( $m$ ) enhances both the primary velocity and cross flow velocities, temperature and concentration distributions depreciate in the boundary layer. Increase in the Hall parameter ( $m$ ) enhances the skin friction coefficient and enhances the rate of heat and mass transfer at the wall.
- An increase in Buoyancy ratio ( $N$ ) enhances both the velocities when the buoyancy forces are acting in the same direction and the opposite effect is observed when the buoyancy forces are acting in the opposite direction. Temperature and concentration reduces with increase in  $N > 0$  and enhances with  $N < 0$ .
- The primary velocity ( $f'$ ) and cross flow velocity ( $g$ ) reduces with increase in the chemical reaction parameter ( $\gamma$ ) in both degenerating and generating chemical reaction cases. The temperature reduces in the degenerating chemical reaction case and enhances in the generating chemical reaction case. Whereas the concentration decreases with increasing values of the chemical reaction parameter in both degenerating and generating chemical reaction cases.
- The skin friction enhances and Nusselt number reduces in both the degenerating and generating chemical reaction cases. The Sherwood number enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case.
- Higher the Thermal radiation parameter  $R_d$  larger the cross flow velocity and lesser the primary velocity. The presence of the thermal radiation, temperature increase rapidly in the boundary layer. The concentration reduces in the solutal boundary layer with increase in  $R_d$ .
- The presence of Eckert number  $Ec$  increases the temperature, both velocity components and reduces the concentration.
- The influence of increasing Soret parameter ( $So$ ) ( or decreasing Dufour parameter  $Du$ ) results in an increase in both velocity components, concentration and leads a depreciation in temperature in the boundary layer.
- The effect of heat generation/absorption parameter is to enhance the mass concentration marginally. The temperature enhances with heat generation and reduces with heat absorption. The primary and cross flow velocity enhances marginally with increase in heat generation ( $A_1 > 0, B_1 > 0$ ), while both the velocities depreciates in the case of heat absorption ( $A_1 < 0, B_1 < 0$ ).
- The primary and cross flow velocities decreases in the boundary layer with increase in slip velocity parameter  $A$ . The temperature and mass concentration enhances with increase in  $A$ . This can be attributed to the fact that thermal boundary layer and the solutal boundary layer increases with increase in the velocity slip parameter  $A$ .

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