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Flow and Heat Transfer due to an a Shrinking Sheet with Second Order Slip

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Abstract
This paper considers the study of viscous flow and heat transfer over a shrinking sheet considering the effect of second order slip. The governing partial differential equations of the flow and heat transfer are transferred into nonlinear ordinary differential equations by using suitable similarity transformation. The exponential form of solution for momentum is assumed and governing heat transfer equation is solved analytically by power series method in terms of Kummer's Hypergeometric function. The effects of various physical parameters on flow and heat transfer are investigated with graphical illustrations.
Keywords: Shrinking sheet, Second Order Slip, Kummer's Function, Mass suction

1. Introduction

The flow induced by a moving boundary is important in the study of extrusion processes (Sakiadis B.C. 1961). (Sakiadis B.C.1961). (Crane L .J, 1970) and is a subject of considerable interest in the contemporary literature (Miklavcic M, Wang CY, 2006), for both permeable and impermeable stretching sheets. Miklavcic and Wang (Miklavcic M, Wang C.Y.2006).have reported an exact solution of the NS equations for flow over a shrinking sheet. The shrinking sheet problem was also extended to power-law shrinking velocity and other fluids.

In the past decade, fluid flow in micro-electro-mechanical systems (MEMS) has become a hot research topic. Because of the micro-scale dimensions of these devices, the flow behavior deviates significantly from the traditional no-slip flow (Gal-el-Hak M, 1999). Rarefied gas flows with slip boundary conditions are often encountered in micro-scale devices and low-pressure situation (Gal-el-Hak M, 1999). For the flow in the slip regime (Shidlovskiy VP.1967 and Pande GC, Goudas CL 1996)., the fluid motion still obeys the Navier-Stokes (NS) equations with slip velocity boundary conditions. In addition, partial slips over moving surface also occurs for fluids with particulate such as emulsions, suspensions, foams, and polymer solutions (Yoshimura A, proteome RK,1988),the slip flows under different flow configurations have been studied in recent years. However, in these papers, only the first order Maxwell slip condition was used. Recently, (Wu L.A, 2008) proposed a new second order slip velocity model, which matches with the Fukui-Kaneko results based on the direct numerical simulation of the linerized Boltzmann equation (Fukui S, Kaneko R. A, 2009.). (Tiegang Fang, Shanshan Yao, ji Zzhang, Abdul Aziz, 2009). Studied the slip flow over a permeable shrinking surface with the newly proposed Wu's slip velocity model with exact solutions of the governing NS equations. In the present study we have extended the work of (Tiegang Fang, Shanshan Yao, ji Zzhang, Abdul Aziz ,2009).considering heat transfer and also with boundary layer approximation.

2. Mathematical formulation and discussion

Consider a steady, two-dimensional laminar flow over a continuously shrinking sheet in a quiescent fluid. The sheet shrinking velocity is $U_w = - U_0 x$, with U_0 being a constant and the wall mass transfer velocity is $V_w = V_w (x)$, which will be determined later.

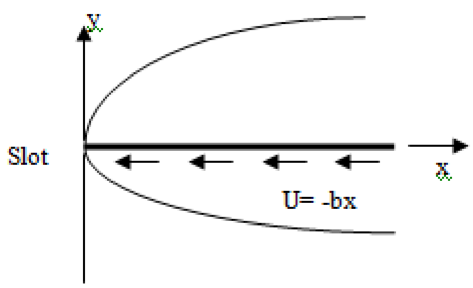


Fig (1): Schematic diagram of boundary layer slip flow past a shrinking sheet

The x-axis runs along the shrinking surface in the direction opposite to the sheet motion and the y-axis is perpendicular to it. The governing boundary layer equation for the proposed problem can be expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

With the boundary conditions

$U(x, 0) = U_0\, x+U_{slip,}$ $V(x, 0) = U_w\,(x)$, and $u(x,\infty) = 0$, (3)

where u and v are the velocity components in the x and y directions. ν is the kinematic coefficient of viscosity, ρ is the fluid density, and U_{slip} is the velocity slip at the wall. The Wu's slip velocity model used in this paper is valid for arbitrary Knudsen numbers, $K_{n,}$ and is given as follows (Wu ,2008):

$$U_{slip}=\frac{2}{3}\left(\frac{3-\alpha l^3}{\alpha}-\frac{3}{2}\frac{1-l^2}{K_n}\right)\lambda\frac{\partial u}{\partial y}-\frac{1}{4}\left[l^4+\frac{2}{K_n^2}\left(1-l^2\right)\right]\lambda^2\frac{\partial^2 u}{\partial y^2}=A\frac{\partial u}{\partial y}+B\frac{\partial^2 u}{\partial y^2}, \quad (4)$$

where l = min [$\frac{1}{K_n}$, 1], α, is the momentum accommodation coefficient with $0\le \alpha \le 1$, and λ is the molecular mean free path. Based on the definition of l, it is noticed that any given value of $K_{n,}$ we have $0\le l\le 1$. The molecular mean free path is always positive. Thus we know that $B \prec 0$ and positive. The stream function and similarity variable can be assumed in the following form,

$$\psi(x,y)=f(\eta)x\sqrt{\nu U_{0,}}\quad \eta=y\sqrt{\frac{U_0}{\nu}} \quad (5)$$

With these transformations, the velocity components are expressed as

$u=U_0 x f'(\eta)$ and $v=-\sqrt{U_0\nu}\,f(\eta)$. (6)

The wall mass transfer velocity becomes $v_w(x)=-\sqrt{U_0\nu}\,f(0)$. (7)

Using equations (5) and (6) in equations (1) and (2) we obtain the transformed form of boundary layer equations of motion,

$$f'''+ff''-f'^2=0 \quad (8)$$

Similarly, the boundary conditions equation (3) takes the form

$f(0)=s,\quad f'(0)=-1+\gamma f''(0)+\delta f'''(0)=0$, and $f'(\infty)=0$, (9)

where s is the wall mass transfer parameter showing the strength of the mass transfer at the surface, γ is the first order velocity slip parameter with $0\prec\gamma=A\sqrt{\frac{U_0}{\nu}}$, and δ is the second order velocity slip parameter with $0\succ\delta=\frac{BU_0}{\nu}$. we derive a closed form exact solution of Eq.(8) subject to the BCs of Eq. (9). We assume a solution of the form $f(\eta)=a+be^{-\beta\eta}$. The application of boundary condition (9) gives the values for a and b as mentioned below.

$$b=\frac{1}{\beta+\gamma\beta^2-\delta\beta^3}, \quad (10)$$

$$a=S-\frac{1}{\beta+\gamma\beta^2-\delta\beta^3}. \quad (11)$$

Substituting the assumed solution into Eq.(9) yields a = β. The use of this relationship in Eq. (11) leads to the following fourth order algebraic equation for β,

$$\delta\beta^4-(\gamma+\delta s)\beta^3+(\gamma s-1)\beta^2+s\beta-1=0. \quad (12)$$

β should be least positive value.

Then the solution reads as

$$f(\eta) = s - \frac{1}{\beta + \gamma\beta^2 - \delta\beta^3} + \frac{1}{\beta + \gamma\beta^2 - \delta\beta^3} e^{-\beta\eta} \;, \tag{13}$$

and

$$f(\eta) = -\frac{1}{1 + \gamma\beta - \delta\beta^2} e^{-\beta\eta} \,, \tag{14}$$

Based on the results in Eq. (14), it is easy to show that

$$f^{//}(0) = \frac{\beta}{1 + \gamma\beta - \delta\beta^2} = \beta^2 (s - \beta) \tag{15}$$

3. Heat transfer analysis:

The thermal boundary layer equation, with work done by deformation, and internal heat generation or absorption is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{16}$$

where c_p is the specific heat, ρ is density, k is thermal conductivity

3.1 Constant surface temperature (CST)

The boundary conditions in case of CST is given by

$$\begin{aligned} &T = T_w \ \text{ at } y = 0; \\ &T \to T_\infty \text{ as } y \to \infty \end{aligned} \tag{17}$$

where T_W is the temperature of the sheet and T_∞ is the temperature of the fluid far away from the sheet.

Defining the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty} \tag{18}$$

Using (18), Eq. (16) can be written in the form

$$\theta''(\eta) + \Pr f(\eta)\theta'(\eta) = 0 \tag{19}$$

where $\Pr = \dfrac{\mu Cp}{k}$ is the Prandtl number.

Consequently the boundary conditions (17) take the form

$$\left.\begin{aligned} &\theta(\eta) = 1 \ at \ \eta = 0 \\ &\theta(\eta) \to 0 \ as \ \eta \to \infty \end{aligned}\right\} \tag{20}$$

Introducing the new independent variable

$\xi = -\Pr e^{-\beta\eta}$

and substituting in Eq. (19) we obtain

$$\xi\theta''(\xi) + \theta'(\xi) - P_{11}\theta'(\xi) + P_{12}\xi\theta'(\xi) = 0 \tag{21}$$

where $P_{11} = \dfrac{a\Pr}{\beta}$, $P_{12} = \dfrac{b}{\beta}$ $and \ \beta = \Pr e^{-\beta\eta}$

The corresponding boundary conditions are

$$\left.\begin{aligned} &\theta(\xi) = 1 \ at \ \xi = -\Pr e^{-\beta\eta} \\ &\theta(\xi) \to 0 \ as \qquad \xi \to 0 \end{aligned}\right\} \tag{22}$$

The solution of Eq. (21) subject to the boundary conditions (22) is given by

$$\theta(\eta)=C_1\left[-\Pr e^{-\beta\eta}\right]^{\Pr} M\left[\left(\frac{-a\Pr}{\beta}-k+1\right)\left(\frac{b}{\beta}\right)^k,\left(\frac{a\Pr}{\beta}+k\right);-\Pr e^{-\beta\eta}\right] \quad (23)$$

where

$$C_1=\frac{1}{\left[-\Pr e^{-\beta\eta}\right]^{\Pr} M\left[\left(\frac{-a\Pr}{\beta}-k+1\right)\left(\frac{b}{\beta}\right)^k,\left(\frac{a\Pr}{\beta}+k\right)\right]}$$

3.2 The Prescribed surface temperature (PST case)

The boundary conditions in case of PST are given by

$$\left.\begin{aligned} &T=T_W=T_\infty+A\left(\frac{x}{l}\right)^2 && at\ \ y=0 \\ &T=T_\infty && at\ y\to\infty \end{aligned}\right\} \quad (24)$$

where A is constant. T_W is temperature at the wall. T_∞ is temperature away from the sheet.
We define non-dimensional temperature as

$$\phi(\eta)=\frac{T-T_\infty}{T_W-T_\infty} \quad (25)$$

So that the equation (16) reduces to the form

$$\phi''(\eta)=\Pr 2f'(\eta)\phi(\eta)-\Pr f(\eta)\phi'(\eta) \quad (26)$$

the corresponding boundary conditions (24) reduces to

$$\left.\begin{aligned} &\eta=0 \quad \phi=1 \\ &\eta=\infty \quad \phi=0 \end{aligned}\right\} \quad (27)$$

Using the new independent variable defined as

$$t=\frac{-\Pr e^{-\beta\eta}}{\beta^2}$$

and substituting in equation (26) we obtain

$$t\phi''(t)+\left[1-P_{11}+P_{12}t\right]\phi'(t)-2\beta=0 \quad (28)$$

Where $P_{11}=\dfrac{a\Pr}{\beta}$, $P_{12}=b\beta$

The corresponding boundary conditions will be

$$\left.\begin{aligned} &\phi(t)=1\ at\, t=-\Pr e^{-\beta\eta} \\ &\phi(t)\to 0\ as \qquad t\to 0 \end{aligned}\right\} \quad (29)$$

We obtain the solution of above equation (26) by using power series method and in terms of Kummer's function is as mentioned below,

$$\phi(\eta)=\frac{e^{-\Pr a\eta} M\left[2\beta-b\beta\left(2+\frac{\Pr a}{\beta}\right),\left(1+\frac{\Pr a}{\beta}\right),\frac{-\Pr}{\beta}e^{-\Pr}\right]}{M\left[2\beta-b\beta\left(2+\frac{\Pr a}{\beta}\right),\left(1+\frac{\Pr a}{\beta}\right),\frac{-\Pr}{\beta^2}\right]} \quad (30)$$

4. Results and Discussion

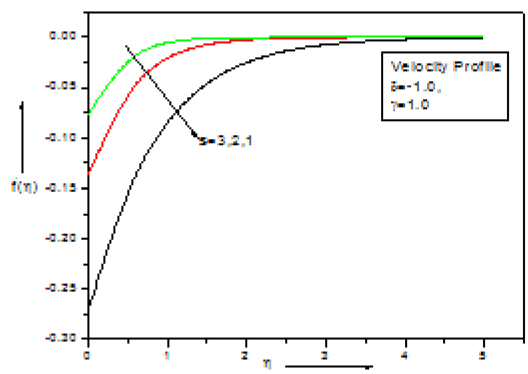
In this problem, we proposed to investigate the flow and heat transfer characteristics of a viscous fluid with second order slip. The governing equations for momentum and heat transfer are partial differential equations which are converted into ordinary differential equations by using suitable similarity transformations.

An analytical solution [exponential solution] for flow has been assumed, and this assumed solution is used to solve the heat transfer equations by power series method and expressed in terms of Kummer's hyper geometric functions. The results are depicted graphically from graph Fig 2 to 12.

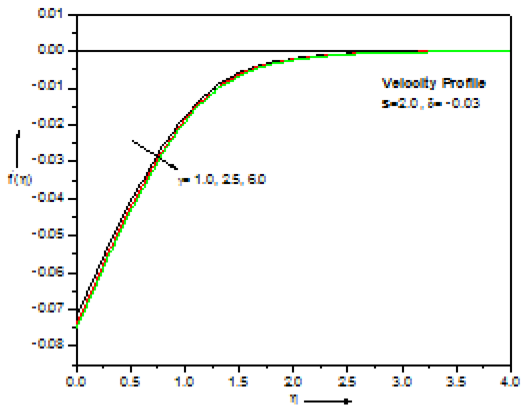
Fig 2. Shows the effect of mass suction parameter s on considered flow. It shows that as there is increase in the parameter value of's' velocity f' is decreases.

Fig 3. Shows the effect of first order slip parameter γ on velocity profile f'. It is noticed that as first order slip parameter γ increases velocity profile f' decreases.

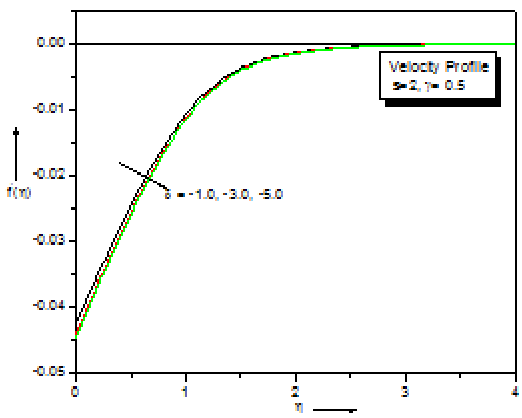
Similarly in Fig 4, we notice that the effect of second order slip parameter δ is to sustain velocity profile f' in the boundary layer.



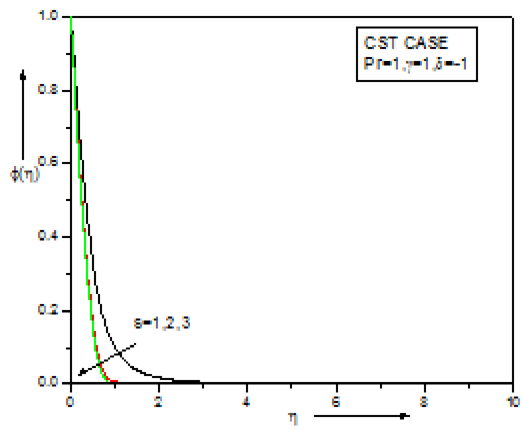
Fig(2) Variation of f'(η) versus η for different values of s



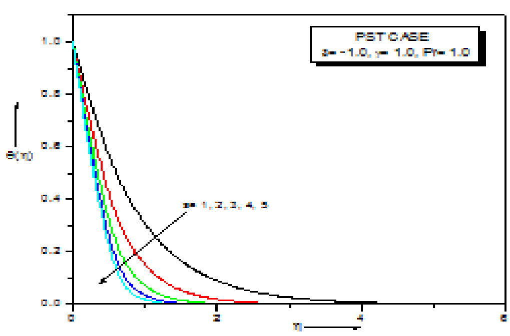
Fig(3) Variation of f'(η) versus η for different values of γ



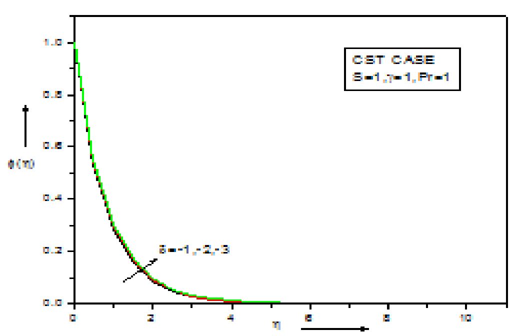
Fig(4) Variation of f'(η) versus η for different values of δ



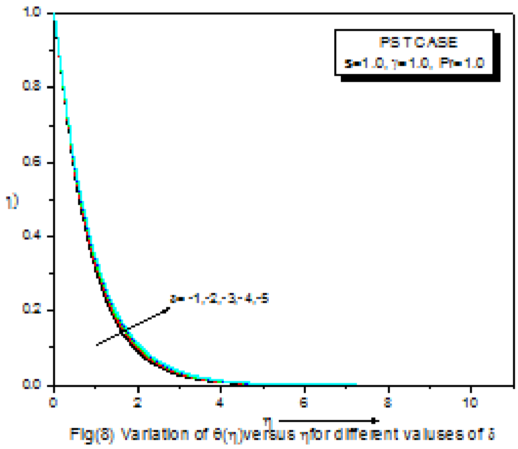
Fig(5) Variation of ϕ(η) versus η for different values of s



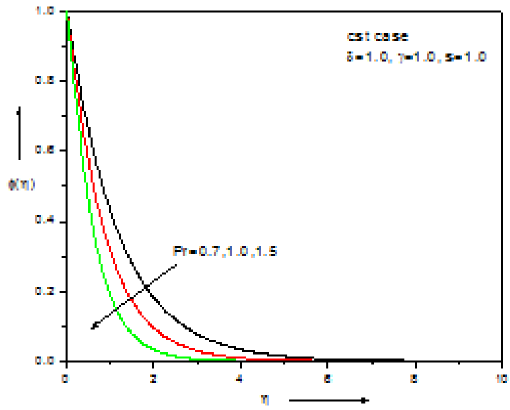
Fig(6) Variation of ϕ(η)versus η for different values of s



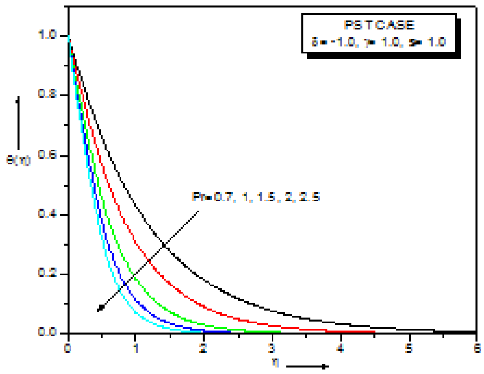
Fig(7) Variation of ϕ(η)versus η for different values of δ



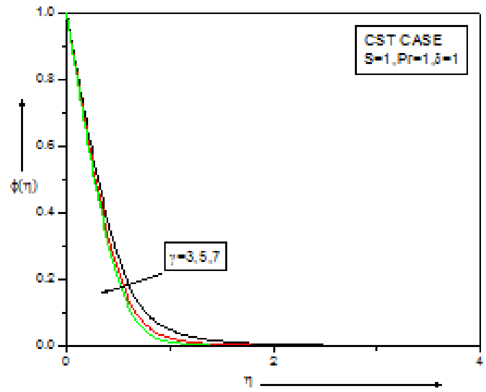
Fig(8) Variation of $\theta(\eta)$ versus η for different valuses of δ



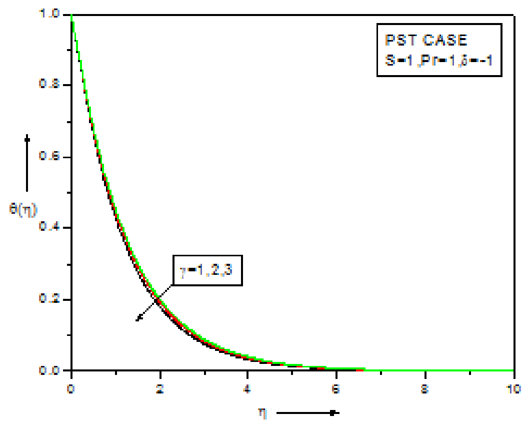
Fig(9) Variation of $\phi(\eta)$ versus η for different valuses of Pr



Fig(10) Variation of $\theta(\eta)$ versus η for different valuses of Pr



Fig(11) Variation of $\phi(\eta)$ versus η for different valuses of γ



Fig(12) Variation of $\theta(\eta)$ versus η for different valuses of γ

Fig (5) and (6), Shows the effect of mass suction parameter on temperature profile in CST and PST cases. It is observed from these two graphs that as mass suction parameter s increases temperature decreases.

Fig (7) and (8), Shows the effect of second order slip parameter δ in CST and PST cases. It is observed from these two graphs that as δ decreases temperature increases.

Fig. (9) and (10), Shows the effect Pr in CST and PST cases. It is observed from these two graphs that as Pr increases temperature decreases.

Fig (11) and (12), Shows the effect of first order slip parameter γ .in CST and PST cases. It is observed from these two graphs that as γ increases temperature decreases.

Nomenclature:

x	flow directional coordinate along the stretching sheet
y	distance normal to the stretching sheet
u, v	velocity components along x and y direction

a, b	constants
β	root value
γ	first order velocity slip parameter
δ	second order velocity slip parameter
A	prescribed constants
c_p	specific heat at constant pressure
k	thermal Conductivity
T	fluid temperature of the moving sheet
T_w	wall temperature
Pr	Prandtl number
T_∞	temperature far away from the plate
τ_w	wall shearing stress
s	mass suction
M	Kummer's Function
Greek symbols	
θ	dimensionless temperature
η	dimensionless space variable
υ	Kinematic viscosity
ρ	density
μ	coefficient of viscosity
Subscripts	
w	properties at the plate
∞	free stream condition
η	differentiation with respect to η

References

Sakiadis B.C., 1961. Boundary-layer behavior on continuous solid surface: I. Boundary-layer equations for two-dimensional and axisymmetric flow Aiche :26-8.

Sakiadis B.C.1961. Boundary-layer behavior on continuous solid surface: II. Boundary-layer equations for two-dimensional and axisymmetric flow A .I .C .H. E :7:221-5.

Crane L ,J. 1970. Flow past a stretching plate. Z Angew Maths phys .21 {4}; 645.

Miklavcic M, Wang CY.2006. Viscous flow due to a shrinking sheet. *Quart Appl Math 64(2); 283-90*

Fang T. 2008.Boundary layer flow over a shrinking sheet with power- law velocity.*Int. J Heat Mass Transfer 51(25/26); 5838-43.*

Fang T, Zhang J. 2009.Closed-form exact solutions of MHD Viscous flow over a shrinking sheet. *Comm.Nonlin .Sci.numer. simul 14(7); 2853-2857.*

Fang T, Zhang J, Yao S.2009. Viscous flow over an unsteady shrinking sheet with mass transfer. *Chin Phys Lett 26(1); 014703.*

Fang T, Liang W, Lee CF. 2008.A new solutions branch for the blasius equation – a shrinking sheet problem. *Comp Math Appl56 (12); 3088-95.*

Fang T, Zhang J, 2009.Heat transfer over a shrinking sheet – an analytical solution. *Acta Mech. Doi; 10.1007/s00707-009-0183-2;*

Hayat T, Abbas Z, Sajid M.2007. On the analytic solution of magneto hydrodynamic flow of a second grade fluid over a shrinking sheet *Appl Math Trans ASME 74(6); 1165-71.*

Sajid M, Hayat T, Jived T. 2008. MHD rotating flow of a viscous fluid over a shrinking surface. *Nonlinear Dyn 51(1-2); 259-65.*

Sajid M, Hayat T, 2007. The application of homotopy analysis method for MHD viscous flow due to a shrinking sheet. *Chaos, solutions & Fractals doi;10,1016/j.chaos.2007.06.019.*

Elbashbeshy EMA,1998. Heat transfer over a stretching with variable surface heat flux. *J. Phys D Appl. Phys.31; 1951-4.*

Magyari E, Keller B1999. Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. *J Phys D Appl Phys 32 ;(5); 577- 585.*

Magyari E, Keller B. 2000.Exact solutions for self-similar boundary-layer flows induced by permeable stretching walls. *Eur J mech B Fluids 19(1); 109-22.*

Magyari E, Ali ME, Keller B. 2001.Heat and mass transfer characteristics of the self-similar boundary-layer flows induced by continuous surfaces stretched with rapidly decreasing velocities. *Heat Mass Transfer 38(1-2); 65-74.*
Liao S.J.2005. A new branch of solutions of boundary-layer flows over a stretching flat plate. *Int Heat Mass Transfer 49(12); 2529-39.*
Liao S.J, 2007. A new branch of solutions of boundary-layer flows over a permeable stretching plate. *Int J Nonlinear Mech 42; 819-30*
Fang T, Zhang J, 2008.Flow between two stretchable disks. *Int common Heat Mass transfer 35(8); 892-5.*
Fang T.2007. Flow over a stretchable disk. *Phys Fluids 19; 128105.*
Wang C.Y.1991. Exact solutions of the steady state navier-stokes equations. *Ann Rew Fluid Mech 23:159-77.*
Miklavcic M, Wang C.Y.2006. Viscous flow due to a shrinking sheet. *Quart Appl Math 64(2); 283-90*
Fang T.2008. Boundary layer flow over a shrinking sheet with power- law velocity. *Int J Heat Mass Transfer 51(25/26); 5838-43.*
Fang T, Zhang J. 2009. Closed-form exact solutions of MHD Viscous flow over a shrinking sheet. *Commun Nonlinear Sci numer simulate 14(7); 2853-7.*
Fang T, Zhang J, Yao S.2009. Viscous flow over an unsteady shrinking sheet with mass transfer. *Chin Phys Lett 26(1); 014703.*
Fang T, Liang W, Lee CF. 2008.A new solutions branch for the blasius equation – a shrinking sheet problem. *Comut Math Appl 56(12); 3088-95.*
Fang T, Zhang J, 2009.Heat transfer over a shrinking sheet – an analytical solution. *Acta Mech. Doi; 10.1007/s00707-009-0183-2.*
Hayat T, Abbas Z, Sajid M.2007.on the analytic solution of magneto hydrodynamic flow of a second grade fluid over a shrinking sheet. *Apple Meth Trans ASME 74(6); 1165-71.*
Sajid M, Hayat T, jived T. 2008.MHD rotating flow of a viscous fluid over a shrinking surface. *Nonlinear Dyn 51(1-2); 259-65.*
Sajid M, Hayat T, 2007.The application of homotopy analysis method for MHD viscous flow due to a shrinking sheet. *Chaos, solutions & Fractsls doi;10,1016/j.chaos.*
Gal-el-Hak M.1999. The fluid mechanics of micro-devices-the freeman scholar. *ASME Trans J fluids Eng 121; 5-33.*
Shidlovskiy V.P.1967. Introduction to the dynamics of rarefied gases. *New York; American Elsevier publishing company Inc*
Pande GC, Goudas CL.1996. Hydro magnetic Rayleigh problem for a porous wall in slip flow regime. *Astrophys Space Sci 243:285-9.*
Yoshimura A, proteome RK. 1998. Wall slips correction for coquette and parallel disc viscometers. *J Rheol 32; 53-67.*
Martin MJ, Boyd ID.2001. Blasius boundary layer solution with slip flow conditions. In: AIP conference proceedings, rarefied gas dynamics: *22nd international symposium, vol. 585, .p. 518-23.*
Anderson HI.2002. Slip flow past a stretching surface. *Acta Mech 158:121-5.*
Wang CY.2002. Flow due to a stretching boundary with partial slip-an exact solution of the Navier-stokes equations. *Chem Eng Sci 57(17); 3745-7.*
Fang T, Lee CF.2005. A moving-wall boundary layer flow of a slightly rarefied gas free stream over a moving flat plate. Apple Math Lett. 18 ;(5); 487-95.
Fang T, Lee CF.2006. exact solutions of incompressible coquette flow with porous walls for slightly rarefied gases. Heat mass transfer .42(3); 255-62.
Fang T, Zhang J, Yao S.2009. Slip MHD viscous flow over a stretching sheet – an exact solution. *Commum Nonlinear Sci numer simulat 14(11); 3731-7*
Wang CY., 2006**.** Stagnation slip flow and heat transfer on a moving plate**.** *Chem. eng sci. 61(23); 7668-72.*
Wang CY2007.stagnation flow on a cylinder with partial slip – an exact solution of the Navier-stokes equations. *IMA J Apple Math 72(3); 271-7.*
Wang CY.2009. Analysis of viscous flow due to a stretching sheet with surface slip and suction**.** *Nonlinear Anal Real world Apple, 10(1); 375-80.*
Aziz A. 2009.Hydrodynamic and thermal slip flow boundary layers over a flat plate with constant heat flux boundary condition. *Commun Nonlinear sci numer simulate. Doi; 10.1016/j.cnsns*
Wu L.A slip model for rarefied gas flows at arbitrary Knudsen number. Appl Phys lett 2008; 93:253103.
Tiegang Fang, Shanshan Yao, ji Zzhang, Abdul Aziz 2009, viscous flow over a shrinking sheet with a second order slip flow model in press.