Convective Heat Transfer Analysis of Non-Newtonian Fluid due to a Linear Stretching Sheet

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Abstract
In this Present paper a two dimensional boundary layer flow and heat transfer of a non-Newtonian fluid due to stretching sheet with convective boundary condition is considered. The flow of non-Newtonian Casson fluid and the heat transfer equations are nonlinear partial differential equations with variable coefficients, these PDE's are transformed into non-linear ordinary differential equations by means of similarity transformations. These BVP's are converted into IVP's and are solved numerically using Runge-Kutta Fehlberg method with shooting technique. The effects of various governing parameters on flow, heat transfer are plotted and discussed the obtained results.
Keywords: Convective Heat transfer, Non-Newtonian fluid, BVP, IVP, Numerical Solution

Nomenclature

c	stretching rate
Bi	Biot number
x	horizontal coordinate
y	vertical coordinate
u	horizontal velocity component
v	vertical velocity component
T	temperature
c_p	specific heat
f	dimensionless stream function
Pr	Prandtl number
l	Characteristic length
'	differentiation with respect to η

Greek symbols

η	similarity variable
θ	dimensionless temperature
k	thermal conductivity
μ	viscosity
υ	kinematic viscosity
ρ	density
α	thermal difusivity
β	Casson parameter

Subscripts

w	properties at the plate
∞	free stream condition

1. Introduction

As we Know the Boundary layer flow and Heat transfer due to continuous moving surface is an important type of flow occurring in a many of engineering processes. In an Aerodynamic extrusion of plastic sheets, cooling of an infinite metallic plate in a cooling path, the boundary layer along a liquid film in condensation process and a polymer sheet of filament extruded continuously from a die are examples of practical applications of continuous moving surfaces. Gas blowing, continuous casting and spinning of fibers also involve the flow due to a stretching sheet.

On observing the literature Crane [1] has given the closed form of solution for steady two-dimensional flow incompressible viscous boundary layer flow generated by a stretching surface. Further Crane's works has been extended under various diverse physical aspects. Here we refer only some recent studies on flow heat and mass transfer of non-Newtonian Casson fluid over stretching surfaces. There are so many works available on studies of various fluid flows due to stretching surfaces. But there are few works on casson fluid and heat transfer on stretched surfaces which are solved analytically. Hayat et.al [2] Studied Soret and Dufour effects on MHD flow of casson fluid solution of governing equations are found by homotopy analysis method, here the only PST heating condition is used to analyze heat transfer characteristics. Nadeem et.al [3] studied the MHD flow of casson fluid due to an exponentially shrinking sheet where they used adomain decomposition method (with Pade'sapproximation)for obtaining the solution and they studied only flow analysis. Pramanik [4] studied flow of Casson fluid and Heat transfer past an exponentially porous stretching sheet with thermal radiation, to analyze Heat transfer characteristic author used constant surface temperature condition (CST) and used numerical method for obtaining solution. Bhattacharyya et.al [5] studied exact solution of casson fluid over a permeable stretching/shrinking sheet. But these authors ignored the heat transfer analysis which was very important.

Bhattacharyya [6] studied the MHD stagnation point flow of casson fluid and heat transfer over a stretching sheet with thermal radiation. The author used the constant surface temperature (CST) heating condition to analyze heat transfer and numerical method is used to solve the connected BVP's. Swati et.al [7] studied Casson fluid flow over an unsteady stretching surface. The authors used numerical Method used to obtain the solution. Quasim and Noreem [8] studied flow and heat transfer of casson fluid due to permeable shrinking sheet with viscous dissipation but the CST heating condition is used to analyse heat transfer analysis and Runge-Kutta numerical method is used to obtain solution of governing equations.

Where as Haq et.al [9]studied the convective heat transfer of Casson fluid for nanofluid model and studied the heat transfer analysis for shrinking sheet problem. Hussain et.al [10] studied the flow of Casson nanofluid with viscous dissipation and convective heating boundary condition. Ramesh et.al [11] studied the heat transfer of dusty fluid with convective heating boundary condition. Rahaman et.al[12]studied the mixed convection boundary layer flow past vertically stretching sheet with convective heating condition. Ishak et.al [13] investigated the radiation effects of thermal boundary layer flow with convective heating condition. Rahaman [14] and Rahaman et.al[15] studied the radiative heat transfer in nanofluid with convective heating boundary conditions with variable fluid properties. Pantokratoras [16] investigated the effect of Grashof number on thermal boundary layer past vertical plate with convective heating boundary condition. Merkin et.al [17] investigated the mixed convection effects on the boundary layer flow over vertical plate in a porous medium in a constant convective boundary condition. Kameshwaran et.al [18] obtained the dual solutions of flow and heat transfer of casson fluid due to stretching or shrinking sheet. Makinde [19] studied the effects of variable viscosity on the thermal boundary layer over a permeable plate with radiation and convective surface boundary condition. Makinde and Aziz[21] studied boundary layer flow of nanofluid past a stretching sheet with convective boundary condition. Alsaedi et.al [22] studied the effects of heat generation/absorption on a stagnation point flow of a nanofluid over surface with convective boundary condition. Nandeppanavar [21] studied the flow and heat transfer analysis for two different heating conditions (PST and PHF) analatically.

Hence in this paper we obtained the numerical Solution for casson fluid flow and heat transfer governing equations with convective boundary condition.

2. Mathematical Formulation:

Let us Consider the flow of an incompressible casson fluid past a stretching sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. Assuming the rheological equation of Casson fluid. Considering the rheological equation of stress transfer (τ) for an incompressible and isotropic flow of non-Newtonian Casson fluid can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}\,, \tag{2}$$

Where u and v are the velocity components of the fluid in x and y directions respectively and υ is kinematic viscosity and β is the Casson parameter (non-Newtonian parameter).

The boundary conditions for the problem are

$$\left.\begin{array}{ll} u_w(x)=cx,\ v=0, & y=0 \\ u\to 0, \quad as & y\to\infty \end{array}\right\} \tag{3}$$

with $c > 0$, the stretching rate. The Eqns. (1) and (2), subjected to the boundary condition (3), admit a self-similar solution in terms of the similarity function f and the similarity variable η defined by

$$u = c\,x\,f'(\eta), \quad v = -\sqrt{c\upsilon}, \quad \eta = \sqrt{\frac{c}{\upsilon}}\,y. \tag{4}$$

It can be easily verified that Eq. (1) is identically satisfied and substituting the above transformations in Eq. (2) we obtain

$$f'^2 - f''f = \left(1+\frac{1}{\beta}\right)f'''. \tag{5}$$

Similarly the boundary conditions (3) can be written as:

$$\left.\begin{array}{llll} f'(\eta)=1, & f(\eta)=0 & at & \eta=0 \\ f'(\eta)\to 0, & & as & \eta\to\infty \end{array}\right\}. \tag{6}$$

The exact solution of (5), satisfying the boundary conditions (6) is given by:

$$f = \sqrt{1+\frac{1}{\beta}}\left(1-e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right) \quad (\beta \text{ is positive}) \tag{7}$$

3. Heat transfer analysis

The Energy equations with boundary layer approximations can be written as:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

where k is the thermal conductivity, ρ is the density of the fluid, C_p is the specific heat at constant pressure .

3.1 Convective temperature boundary condition:

It is assumed that the bottom surface of the plate is heated by convection from a hot fluid of uniform temperature T_f which provides a heat transfer coefficient h_f. Under this assumption the thermal boundary conditions may be written as, i.e

The CTBC (Convective temperature boundary condition) is:

$$\left.\begin{array}{lll} -k\dfrac{\partial T}{\partial y} = h_f\left(T_f - T\right) & at & y=0 \\ T\to T_\infty & as & y\to\infty \end{array}\right\}, \tag{9}$$

where T_∞ is the temperature of the fluid far away from the sheet (temperature of ambient cold fluid).

T is the uniform temperature on the top surface of the plate. Hence we have $T_f > T > T_\infty$.

Defining the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta) = \frac{T-T_\infty}{T_f - T_\infty}. \tag{10}$$

Where T_f the temperature of the sheet.

Using Eqn. (10), Eqs. (8) and (9) can be written as

$$\theta'' + \Pr f\,\theta' = 0, \tag{11}$$

$$\left.\begin{array}{lll} \theta'(\eta) = B_i\left(1-\theta(\eta)\right) & \text{at} & \eta=0, \\ \theta(\eta)\to 0 & \text{as} & \eta\to\infty. \end{array}\right\} \tag{12}$$

Where

$\Pr = \dfrac{\mu C_p}{k}$ is the Prandtl number.

$B_i = \frac{h}{k}\sqrt{\frac{\upsilon}{a}}$ *is the thermal Biot number*

4. Numerical Solution

The set of non-linear differential equations (5 and 11) subject to the boundary conditions (5and 12) are integrated numerically using a very efficient method known as Runge-Kutta Fehlberg method with shooting technique. The most important factor of this method is to choose the appropriate finite values of $\eta \to \infty$ in order to determine η_∞ for the boundary value problem stated by Eq.(5 &12), we start with some initial guess value for some particular set of physical parameters to obtain $f''(0)$ & $\theta'(0)$. The solution procedure is repeated with another large value of η_∞ until two successive values of $f''(0)$ & $\theta'(0)$ differ only by the specified significant digit. The last value of η_∞ is finally chosen to be the most appropriate value of the limit $\eta \to \infty$ for that particular set of parameters. The value of η may change for another set of physical parameters. Once the finite value of η is determined then the coupled boundary value problem given by Eq. (5) - (11) are solved numerically using the method of superposition. In this method the third order Non-linear Eqs. (5) and second order Eqs.(11) have been reduced to five simultaneously ordinary differential equations as follows:

Let us call

$$\left.\begin{aligned} y_1 &= f \\ y_2 &= f' \\ y_3 &= f'' \\ y_4 &= \theta \\ y_5 &= \theta' \end{aligned}\right\} \quad (13)$$

The Boundary value problem is given by

$$\left.\begin{aligned} \frac{dy_1}{d\eta} &= y_2 \\ \frac{dy_2}{d\eta} &= y_3 \\ \frac{dy_3}{d\eta} &= \frac{(y_2{}^2 - y_1 y_3)}{(1+\frac{1}{\beta})} \\ \frac{dy_4}{d\eta} &= y_5 \\ \frac{dy_5}{d\eta} &= -\Pr y_1 y_5 \end{aligned}\right\} \quad (14)$$

The boundary conditions now becomes

$$y_1(0)=0, y_2(0)=1, y_3(0)=s_1, y_4(0)=1, y_5(0)=s_2, y_2(\infty)=0, y_4(\infty)=0 \ \} \quad (15)$$

Where s_1 & s_2 determined such that it satisfied $y_2(\infty)=0$ & $y_4(\infty)=0$. Thus, to solve this resultant system, we need five initial conditions, but we have only two initial conditions on f and one initial condition on θ.The third condition on f (i. e. f''(0))and second condition on θ(i,e $\theta'(0)$) are not prescribed which are to be determined by shooting method by using the initial guess values s_1 & s_2 until the boundary conditions $f_2(\infty)=0, f_4(\infty)=0$ (or $y_2(\infty)=0, y_4(\infty)=0$) are satisfied. In this way, we employ shooting technique with Runge-Kutta Fehlberg scheme to determine two more unknowns in order to convert the boundary value problem to initial value problem. Once all the five initial conditions are determined the resulting differential equations can then be easily integrated, without any iteration by initial value solver. For this purpose, Runge kutta scheme has been used In this manner any non linear equation involved in boundary value problem can easily be solved by this technique. To study the behavior of the velocity and temperature profiles, curves are drawn for various values of the parameters that describe the flow.

5. Results and Discussion:

The flow and the heat transfer differential equation of a non-Newtonian fluid are non-linear differential equations and are solved Numerially using Runge-Kutta Fehlberg method with shooting technique. Geometry of considered problem is given by the Figure (1). The velocity distribution is presented in Figs. 2. The temperature distribution is presented through the plots Fig. (2) to Fig.(7).

Fig1: shows the geometry of the considered problem, which shows the heated plate, flow direction etc.

Fig.2: shows the influence of Casson parameter β on velocity profile. We observe that the magnitude of velocity in the boundary layer decreases with an increase in the Casson fluid parameter β. It is noticed that when Casson parameter approaches infinity, the problem will reduce to a Newtonian case. Hence increasing value of Casson parameter β , decreases the velocity and boundary layer thickness.

Fig. 3: shows the effect of the Casson parameter β on the temperature profiles. The temperature and the thermal boundary layer thickness are increasing as decreasing function of β. Effect of casson parameter it leads to increase the temperature field, it also cause the thickening of the thermal boundary layer due to increase in the elastic stress parameter.

Fig.4 shows the effect of the Prandtl number $\Pr$ on temperature profile. On observing this plot we can conclude that the temperature and the thermal boundary layer thickness decrease as the Prandtl number increase.

Fig. (5), shows the effect of the thermal Biot number Bi on the temperature profile, Fig.(5) is plotted for the different Biot number parameter on observing the temperature profile increases with increasing values of thermal Biot number

6. Conclusions

- Here numerical Solutions for flow and convective heat transfer problems are obtained.
- The effects of the Casson fluid parameter β on flow and temperature are quite opposite.
- The thermal boundary layer thickness decreases with increasing Prandtl number on convective heat transfer phenomenon
- When β tends to infinity, as reduce to results the Newtonian case
- Skin friction decreases with an increase in the Casson fluid parameter

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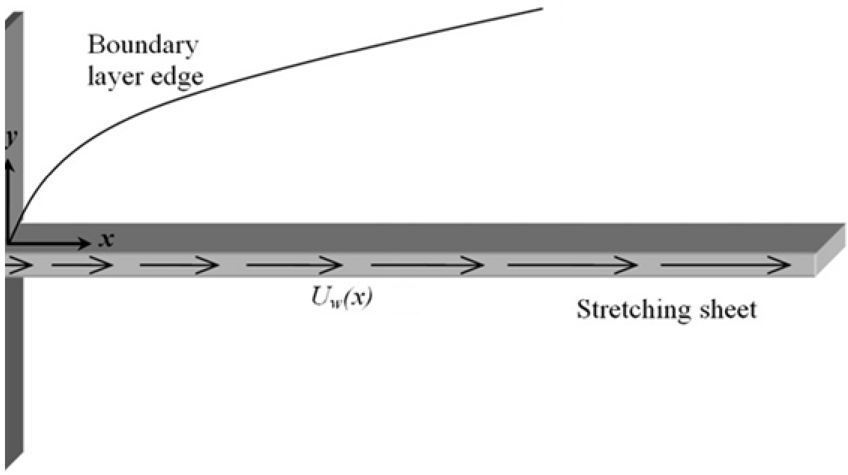

Fig. 1: Geometry of the Problem

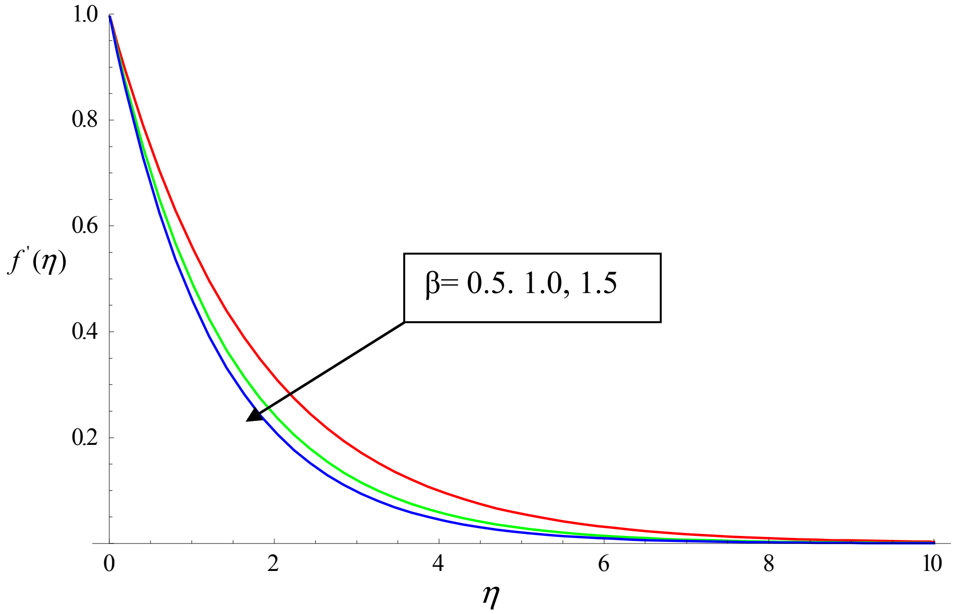

Fig 2: Velocity Profile for different values of Casson parmeter β

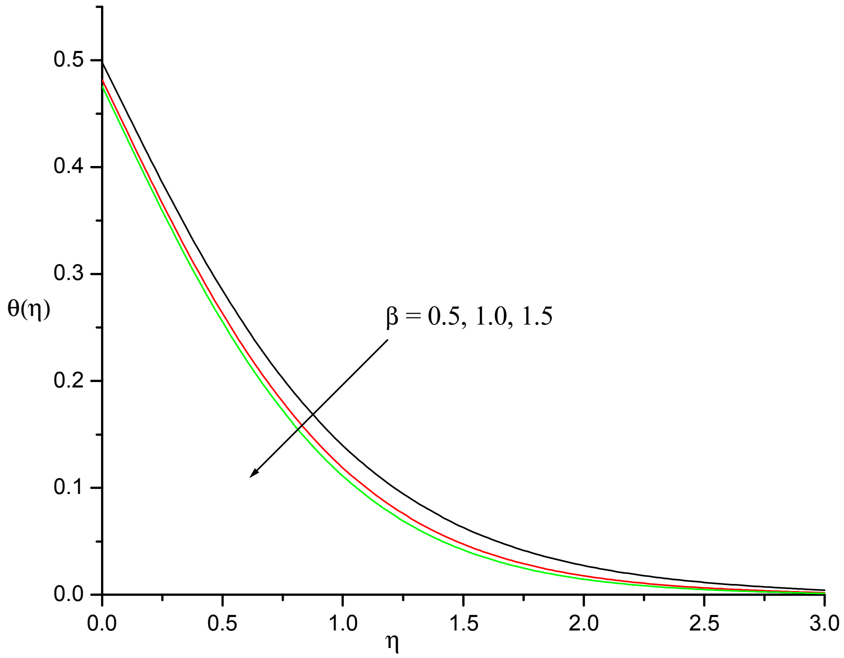

Fig. 3: Temperature Profile for different values of Casson parmeter β when Pr=3.0 and Bi=1.0

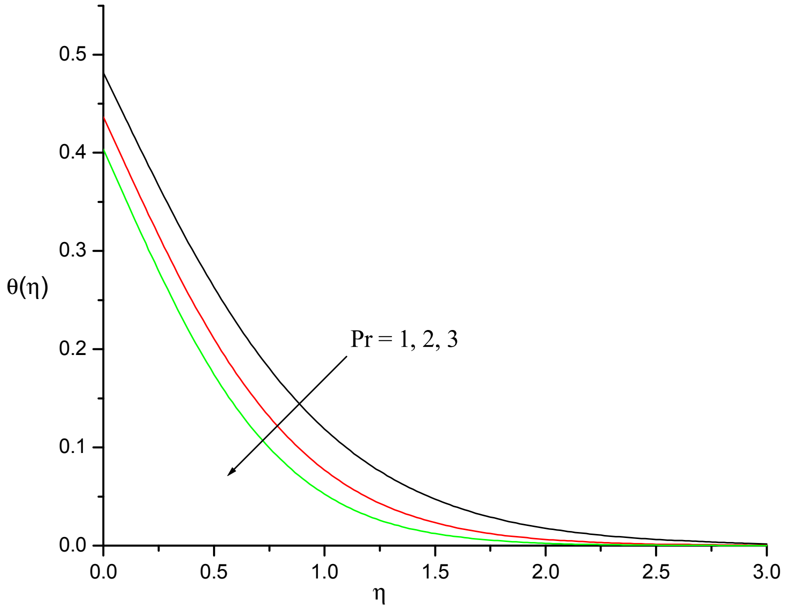

Fig. 4: Temperature Profile for different values of Prandtl number Pr when β =1.0 and Bi=1.0

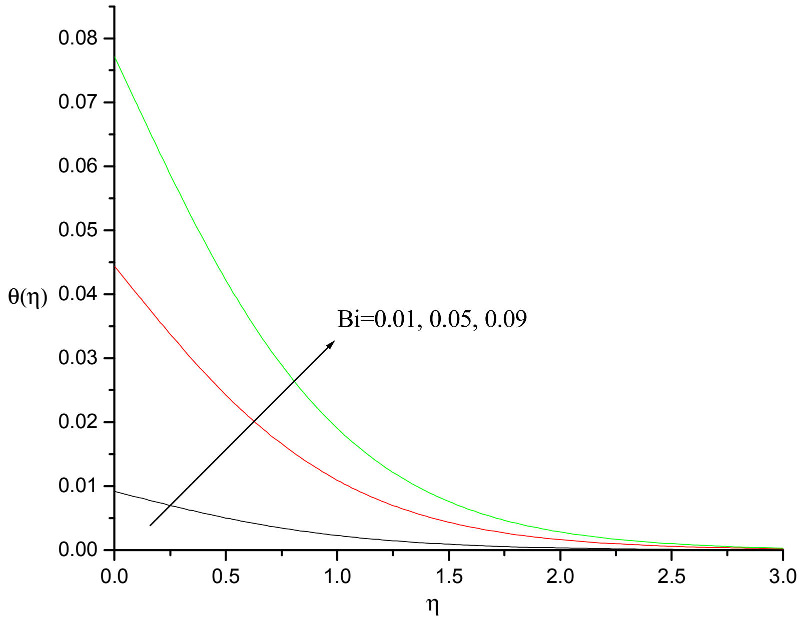

Fig. 5: Temperature Profile for different values of Biot number (Bi<1.0) when β =1.0 and Pr =3.0