Chemical and Process Engineering Research www.iiste.org
ISSN 2224-7467 (Paper) ISSN 2225-0913 (Online)
Vol.41, 2016

Effect of Heat Transfer on Free Surface Flow of a Jeffrey Fluid over a Deformable Permeable Bed

S. Sreenadh[1*] M. Krishna Murthy[1] M. Eswar Rao[1] E. Sudhakara[2]
1.Department of Mathematics, Sri Venkateswara University, Tirupati-517 502, A.P., INDIA
2.Department of Mathematics, Siddartha Institute of Engineering & Technology,Puttur, A.P., INDIA

Abstract
Free surface flow of a conducting Jeffrey fluid in a channel is investigated. The channel is bounded below by a finite deformable porous layer. The governing equations are solved in the free flow region and deformable porous layer. The expressions for the velocity field and solid displacement are obtained. The effects of the Jeffrey parameter, viscosity parameter, the volume fraction component of the fluid on the velocity displacement and mass flux are discussed. It is found that the velocity increases with the increase in the non-Newtonian Jeffrey parameter.
Keywords: Free surface flow; Jeffrey fluid; Porous layer; Permeable bed.

1. INTRODUCTION

The study of flow through and past porous media has attracted the attention of many research workers because of its potential applications in industrial, physical biological and hydrological problems. The pumping of petroleum from oil wells can be improved with the knowledge of physics of flow through porous media. Also when we deal with blood flow in an artery, it will be appropriate to model the tissue region as finite porous layer. Hence mathematical modelling of porous flows plays vital role in understanding practical situations.

Rajesekhara [1] has investigated plane Couettee flow in the presence of a pressure gradient and found slight deviation between his theoretical and experimental results. Channabasappa et al. [2] investigated analytically the effect of the thickness of the porous material on the parallel plate channel flow when the walls are provided with non-erodible porous lining. Chikh et al. [3] performed an analytical study on fully developed forced convection in an annular duct partially filled with a porous medium. Some important studies on the forced convection heat transfer in a parallel plate channel partially filled with porous media are made by Kuznetsov [4,5,6,7]. Morosuk [8] investigated entropy generation due to flow in a pipe and parallel plate channel partially filled with porous medium. All these works are done assuming the porous media to be non-deformable.

Nomenclature

μ_a Apparent viscosity of the fluid in the porous material.	M_d Mass flow rate in the deformable porous layer.
K Drag coefficient.	M_r Mass flow rate in the non deformable porous layer.
μ Lame constant.	M Mass flow rate in the channel
μ_f Coefficient of viscosity.	v Velocity of the fluid in the deformable porous layer.
q Fluid velocity in the free flow region	δ Viscous drag.
u Displacement in x -direction.	η Viscosity parameter in porous layer.
G_0 Typical pressure gradient.	λ_1 Jeffrey parameter.
ϕ^β Volume fraction of component β and $\beta = s, f$ for the binary mixture of solid and fluid phases with $\varphi^s + \varphi^f = 1$.	ε Porous layer thickness.
T_0 Temperature of the ambient fluid	K_T Thermal conductivity
m Temperature parameter $\left(T_2 - T_1\right)/\left(T_1 - T_0\right)$	T Temperature

The study of flow through deformable porous materials was initiated by Terzaghi [9] and later continued by Biot [9,10] into a successful theory of soil consolidation and acoustic propagation. Atkin and Craine [11], Bowen [12] and Bedford and Drumheller [13] made important contributions to the theory of mixtures. Similar theory was proposed by Mow et al. [14] for the study of biological tissue mechanics. Water transport in the artery wall is studied by Jayaraman[15] using theory of deformable porous media. Sreenadh et al. [16] analyzed the Couette flow of a viscous fluid in a parallel plate channel in which a finite deformable porous layer is attached to the lower plate. Sreenadh et al. [17] discussed the free convection flow of a Jeffrey fluid through a vertical deformable porous stratum.

Jeffrey model is one of the best non-Newtonian fluid models used by researchers to explain the biological fluid flow in living organisms. Nadeem et al. [18] examined the effects of thermal radiation on the boundary layer flow of a Jeffrey fluid over an exponentially stretching surface. Vajravelu et al. [19] studied the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Hayat et al.[20] investigated the boundary layer flow of a Jeffrey fluid with convective boundary conditions. Bhaskara Reddy et al. [21] studied the flow of a Jeffrey fluid between torsionally oscillating disks.

Motivated by these studies, the effect of heat transfer on free surface flow of a Jeffrey fluid over a deformable permeable bed is investigated. The fluid velocities, the temperature, the displacement of the solid matrix and the mass flux are obtained. The effects of various physical parameters on the flow quantities are discussed through graphs.

2. MATHEMATICAL FORMULATION

Consider a steady, fully developed free surface flow of a Jeffrey fluid in a channel bounded below by a deformable porous layer of finite thickness L (Fig.1.) the nominal surface of the porous layer is taken as $y = 0$ and the free surface is represented by $y = h$ .The temperature at $y = 0$ and $y = h$ are T_2 and $T_1 (< T_2)$ respectively. The fluid velocity in the free surface flow region and the porous flow region are assumed to be $(q, 0, 0)$ and $(v, 0, 0)$ respectively. The displacement due to the deformation of the solid matrix is taken as $(u, 0, 0)$. A pressure gradient $\frac{\partial p}{\partial x} = G_0$ is applied, producing an axially directed flow in the channel.

In view of the assumptions mentioned above, the equations of motion in the deformable porous region and energy equation in the free flow region are [22,23]

$$\mu \frac{\partial^2 u}{\partial y^2} - \phi^s G_0 + K v = 0, \tag{1}$$

$$\frac{2\mu_a}{1+\lambda_1} \frac{\partial^2 v}{\partial y^2} - \phi^f G_0 - K v = 0 \tag{2}$$

$$\frac{\mu_f}{1+\lambda_1} \frac{\partial^2 q}{\partial y^2} = G_0 \tag{3}$$

$$K_T \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{1+\lambda_1} \left(\frac{\partial q}{\partial y} \right)^2 = 0 \tag{4}$$

The boundary conditions are

at $y = -L : v = 0,\ u = 0$ (5a)

at $y = 0 :\ q = \phi^f v$

$$\phi^f \mu_f \frac{dq}{dy} = 2\mu_a \frac{dv}{dy}$$

$$\mu_f \frac{dq}{dy} = \frac{\mu}{\phi^s} \frac{du}{dy}$$

$$T = T_2 \tag{5b}$$

at $y = h : \frac{dq}{dy} = 0$

$$T = T_1 \tag{5c}$$

3. NON-DIMENSIONALIZATION OF THE FLOW QUANTITIES

It is convenient to introduce the following non-dimensional quantities.

$$y = hy,\ u = -\frac{h^2 G_0}{\mu} u,\ v = -\frac{h^2 G_0}{\mu_f} v,\ q = -\frac{h^2 G_0}{\mu_f} q,\ \varepsilon = \frac{L}{h}, \theta = \frac{\hat{T} - T_1}{T_2 - T_1}$$

In view of the above dimensionless quantities, the equations (1) – (5) takes the following form after neglecting the hats $(\wedge)$ are neglected.

$$\frac{d^2u}{dy^2} = -\phi^s - \delta v \tag{6}$$

$$\frac{d^2v}{dy^2} - \delta\eta(1+\lambda_1)v = -\phi^f(1+\lambda_1)\eta \tag{7}$$

$$\frac{d^2q}{dy^2} = -(1+\lambda_1) \tag{8}$$

$$\frac{d^2\theta}{dy^2} + \frac{Ec\Pr}{1+\lambda_1}\left(\frac{dq}{dy}\right)^2 = 0 \tag{9}$$

where $\delta = \frac{Kh^2}{\mu_f}$, $\hat{G} = \frac{G}{G_0}$, $\eta = \frac{\mu_f}{2\mu_a}$, $G_0 = \frac{dp}{dx}, Ec = \frac{\left(\frac{h^2G_0}{\mu_f}\right)^2}{c_p\left(T_1 - T_0\right)}, \Pr = \frac{\mu c_p}{K_T}$

we note that δ is a measure of the viscous drag of the outside fluid relative to drag in the porous medium and η is the ratio of the bulk fluid viscosity to the apparent fluid viscosity in the porous layer.

The boundary conditions are

at $y = -\varepsilon$: $v = 0, u = 0$ (10a)

at $y = 0$: $q = \phi^f v$

$$\frac{dq}{dy} = \frac{1}{\eta\phi^f}\frac{dv}{dy}$$

$$\frac{dq}{dy} = \frac{1}{\phi^s}\frac{du}{dy}.$$

$$\theta = 1 + m \tag{10b}$$

at $y = 1$: $\frac{dq}{dy} = 0$

$$\theta = 1 \tag{10c}$$

4. SOLUTION OF THE PROBLEM

Equations (6) - (9) are coupled differential equations that can be solved by using the boundary conditions (10). The solid displacement in the deformable porous region and fluid velocities, temperature in the free surface flow region are obtained as:

$$u(y) = -\frac{\delta c_1 e^{a y}}{a^2} - \frac{\delta c_2 e^{-a y}}{a^2} - \frac{y^2}{2} + c_5 y + c_6 \quad \left(-\epsilon \le y \le 0\right) \tag{11}$$

$$v(y) = c_1 e^{a y} + c_2 e^{-a y} + \frac{\varphi^f}{\delta} \quad \left(-\epsilon \le y \le 0\right) \tag{12}$$

$$q(y) = c_3 + c_4 y - \frac{y^2\left(1+\lambda_1\right)}{2} \quad \left(0 \le y \le 1\right) \tag{13}$$

$$\theta(y) = c_7 + c_8 y - Ec\Pr\left(1+\lambda_1\right)\left(\frac{y^2}{2} + \frac{y^4}{12} - \frac{y^4}{2}\right) \quad \left(0 \le y \le 1\right) \tag{14}$$

where $a = \sqrt{\delta\eta(1+\lambda_1)}$. The constants c_1, c_2, c_3, c_4, c_5 , c_6 , c_7 and c_8 are

$$c_1 = \frac{1}{e^{a\varepsilon}+e^{-a\varepsilon}}\left[\frac{e^{a\varepsilon}\eta\phi^f(1+\lambda_1)}{a}-\frac{\phi^f}{\delta}\right], c_2 = \frac{-\left[c_1e^{-a\varepsilon}+\frac{\phi^f}{\delta}\right]}{e^{a\varepsilon}}, c_3 = (c_1+c_2)\phi^f + \frac{(\phi^f)^2}{\delta},$$

$$c_4 = 1+\lambda_1 \quad , \quad c_5 = (1-\phi^f)c_4 - \frac{\delta(c_2-c_2)}{a} \quad , \quad c_6 = \frac{\delta\left[c_1e^{-a\varepsilon}+c_2e^{a\varepsilon}\right]}{a^2}+\frac{\varepsilon^2}{2}+c_5\varepsilon,\; c_7 = 1+m \quad \text{and}$$

$$c_8 = 1-c_7+\frac{Ec\Pr(1+\lambda_1)^2}{12}.$$

The dimensionless total mass flow rate M per unit width of the channel in the deformable porous flow region and free flow region is given by

$$M = \int_{-\varepsilon}^{0} v\,dy + \int_{0}^{1} q\,dy = \frac{c_1(1-e^{-a\varepsilon})-c_2(1-e^{a\varepsilon})}{a}+\frac{\phi^f\varepsilon}{\delta}+\frac{c_4}{2}-\frac{(1+\lambda_1)}{6}+c_3 \tag{15}$$

5. RESULTS AND DISCUSSIONS

In this paper free surface flow of a Jeffrey fluid past a deformable finite porous permeable bed is investigated. The solutions for the fluid velocities q, v and temperature θ in the free flow region and deformable porous region and solid displacement of solid matrix u are evaluated numerically for different values of physical parameters such as the volume fraction of component ϕ^f, the viscous drag parameter δ, the viscosity parameter η and the thickness of lower wall ε, Jeffrey parameter λ_1, Eckert number Ec and Prandtl number $\Pr$.

The variation of fluid velocities q, v solid displacement u and temperature θ in the channel is calculated from equation (11) – (14) for different values of Jeffrey parameter λ_1 and is shown in figures 2, 3, 4 and 5for fixed δ =2.0, η=0.5, ϕ^f =0.5, ε=0.2, $Ec\Pr = 2$ and $m = 2$ .We observe that the velocities q, v and solid displacement increases whereas the temperature θ decreases with the increase Jeffrey parameter λ_1.

The variation of temperature θ in the channel is calculated from equation (14) for different values of m and is shown in figure 6 for fixed λ_1=1.0 and $Ec\Pr$ =2. We observe that the temperature θ increases with the increase m.

The variation of temperature θ in the channel is calculated from equation (14) for different values of $Ec\Pr$ and is shown in figure 7 for fixed λ_1=1.0 and m =2. We observe that the temperature θ decreases with the increase $Ec\Pr$.

The variation of fluid velocities in the channel q , v is calculated from equation (12) – (13) for different values of viscosity parameter η and is shown in figures 8 and 9 for fixed δ =2.0, ϕ^f =0.5, $\lambda_1 = 0.5$ and ε=0.2. We observe that the velocities q, v increases with the increase in viscosity parameter η.

The variation of fluid velocities in the channel q , v and solid displacement u in the channel is calculated from equation (11) – (14) for different values of volume fraction of component ϕ^f and is shown in figures 10, 11 and 12 for fixed δ =2.0, η=0.5, $\lambda_1 = 0.5$ and ε=0.2. We observe that the velocities q , v increase with the increasing ϕ^f whereas the solid displacement u decreases with the increase in ϕ^f.

The variation of total mass flow rate for M in the free flow and deformable region is calculated from equation (15) for different values of Jeffrey Parameter λ_1 are shown in figure13 for fixed δ =2.0, η=0.5 and ε =0.2. We observe that the mass flow rate increases with increase in the Jeffrey parameter.

6. CONCLUSIONS

The present study deals with free surface flow of a Jeffrey fluid over a deformable porous layer. The results are analyzed for different values of the pertinent parameters, namely, Jeffrey parameter, viscosity parameter, volume fraction component, Prandtl number, Eckert number, temperature parameter and the mass flux. The findings of the problem find applications in understanding the blood (modelled as Jeffrey fluid) flow behavior near the tissue layer (modelled as a deformable porous layer). Some of the interesting findings are as follows:

1. The velocity of the fluid in the free flow region and the deformable porous layer and solid displacement increases with increase Jeffrey parameter whereas the temperature decreases with increasing Jeffrey parameter.
2. The temperature in the free flow region increases with the increase in m whereas it will be decreases with increasing $Ec\,\Pr$.
3. An increase in the volume fraction component ϕ^f is to enhance in free flow fluid velocity between the parallel plates. But opposite behavior is observed in the case of deformable fluid flow velocity and solid displacement.
4. The total mass flux in the free flow and porous flow region increases with an increase in the Jeffrey parameter.

References

[1] Rajasekhara B.M., Rudraiah N. and Ramaiah B.K. Couettee flow over a naturally permeable bed. Journal of Mathematical and Physical Sciences, 1975, 9, 49-56.
[2] Channabasappa M.N., Umapathy K.G and Nayak I.V. The effect of the thickness of the porous material on the parallel plate channel flow when the walls are provided with non erodible porous lining. Appl.Sci.Res, 1976, 32,606-617.
[3] Chikh S., Boumedien A., Bouhadef K. and Lauriat G. Analytical solution of non darcian forced convection in an annular duct partially filled with a porous medium. Int.J.Heat Mass Transf. 1995, 38, 1543-1551.
[4] Kuznetsov A.V., Analytical investigation of the fluid flow in the interface region between a porous medium and a clear fluid in channels partially filled with a porous medium. Appl. Sci. Res. 1996, 56, 53–67.
[5] Kuznetsov A.V., Influence of the stress jump boundary condition at the porous medium clear-fluid interface on a flow at a porous wall. Int. Commun. Heat Mass Transf. 1997, 24, 401– 410.
[6] Kuznetsov, A.V. Forced convective heat transfer in a parallel-plate channel with a porous core. Int. J. Appl. Mech. Eng. 1999, 4, 271– 290.
[7] Kuznetsov, A.V. Analytical studies of forced convection in partly porous configurations. In Vafai, K. (ed.) Handbook of porous media, Marcel Dekker, New York. 2000, 269–312.
[8] Morosuk T.V. Entropy generation in conduits filled with porous medium totally and partially. Int. J. Heat Mass Transf., 2005, 48, 2548–2560.
[9] Biot MA. General theory of three-dimensional consolidation. J. Appl. Phys. 1941, 12, 155-164.
[10] Biot MA. Theory of elasticity and consolidation for porous anisotropic solid. J. Appl. Phys.1955, 26, 182-185.
[11] Biot MA. Mechanics of deformation and acoustic propagation in porous media. J. Appl. Phys.1956, 27, 240-253.
[12] Bowen RM. Incompressible porous media models by the theory of mixtures. Int. J. Engng. Sci. 1980, 18, 1129-1148.
[13] Drumheller D S, Bedford A, Recent advances, theory of immiscible and structured mixtures. Int. J. Engng. Sci. 1983, 21, 863-960.
[14] Mow VC, Holmes MH, Lai M. fluid transport and mechanical properties of articular cartilage: a review. J. Biomechanics. 1984, 17, 377-394.
[15] Jayaraman G. Water transports in the arterial wall- A theoretical study. J. Biomechanics 1983, 16, 833- 840.
[16] Sreenadh S, Krishnamurthy M, Sudhakara E, Gopi Krishna.G. Couete flow over a deformable permeable bed, (IJIRSE) International Journal of Innovative Research in Science & Enginering 2014; ISSN (Online) 2347-3207.
[17] Sreenadh S, Mohammad Mehdi Rashidi, Kumara swamy naidu k, Parandhama A. free convection flow of a Jeffrey fluid through a vertical deformable porous stratum, (JAFM) Journal of Applied Fluid Mechanics. 2015
[18] Nadeem S, Zaheer S, Fang T. Effects of thermal radiation on the boundary layer flow of a Jeffrey fluid over an exponentially stretching surface, Numerical Algorithms 2011;57: 187–205.
[19] Vajravelu K, Sreenadh S, Lakshminarayana P. The influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Commun Nonlinear Sci Numer Simulat 2011; 16: 3107–3125.
[20] Hayat T, Asad S, Qasim M, Hendi AA. Boundary layer flow of a Jeffrey fluid with convective boundary conditions. Int J Numer Methods Fluids 2012; 69:1350–62.
[21] Bhaskara Reddy G, Sreenadh S, Hemadri Reddy R, Kavitha A. Flow of a Jeffrey fluid between torsionally oscillating disks, Ain Shams Engineering Journal 2014.(article in press).

[22] Barry SI, Parker KH, Aldis GK. Fluid flow over a thin deformable porous layer. Journal of Applied Mathematics and Physics (ZAMP) 1991; 42: 633-648.
[23] Ranganatha TR, Siddagamma NG. Flow of Newtonian fluid a channel with deformable porous walls. Proc. Of National Conference on Advances in fluid mechanics 2004; 49-57.

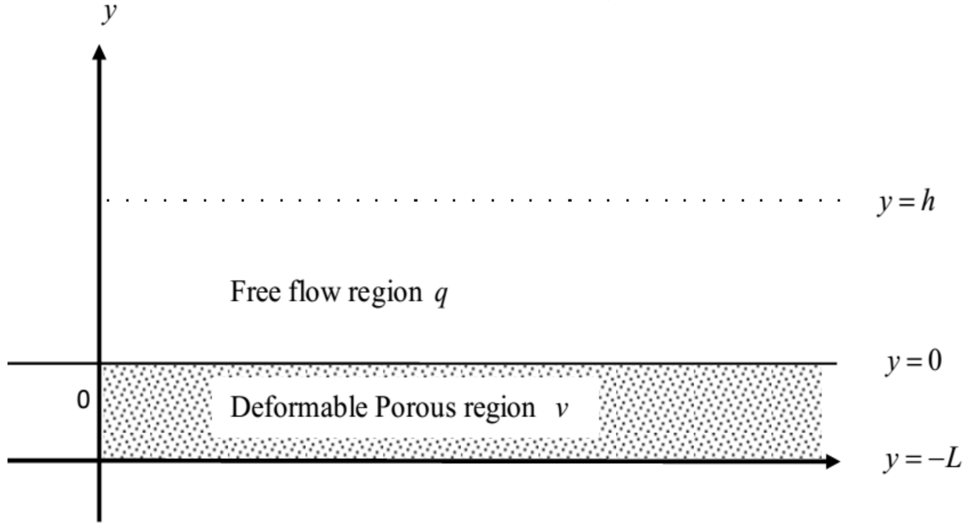

Fig.1. Physical Model

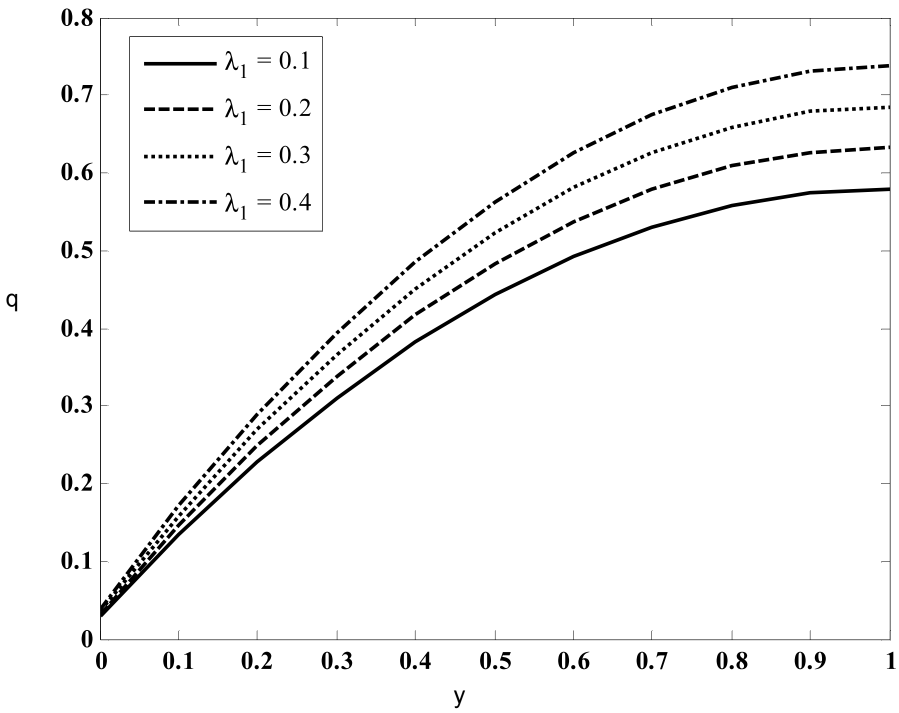

Fig 2. Velocity profiles in free flow surface for different values of λ_1 for fixed values of ϕ^f =0.5, η =0.5, δ =2 and ε =0.2.

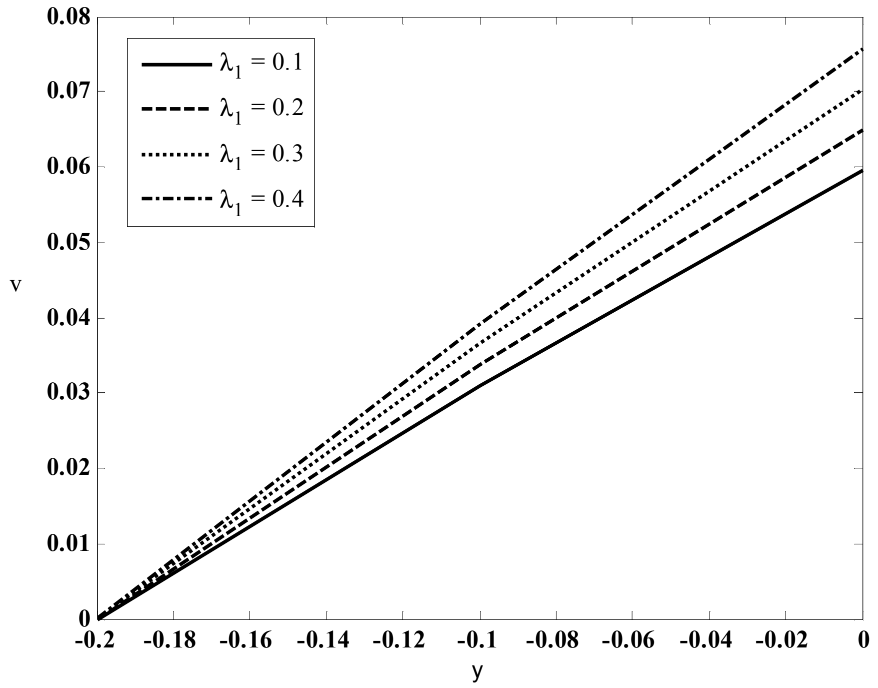

Fig 3.Velocity profiles in deformable porous region for different values of λ_1 for fixed values of ϕ^f =0.5, η =0.5, δ =2 and ε =0.2.

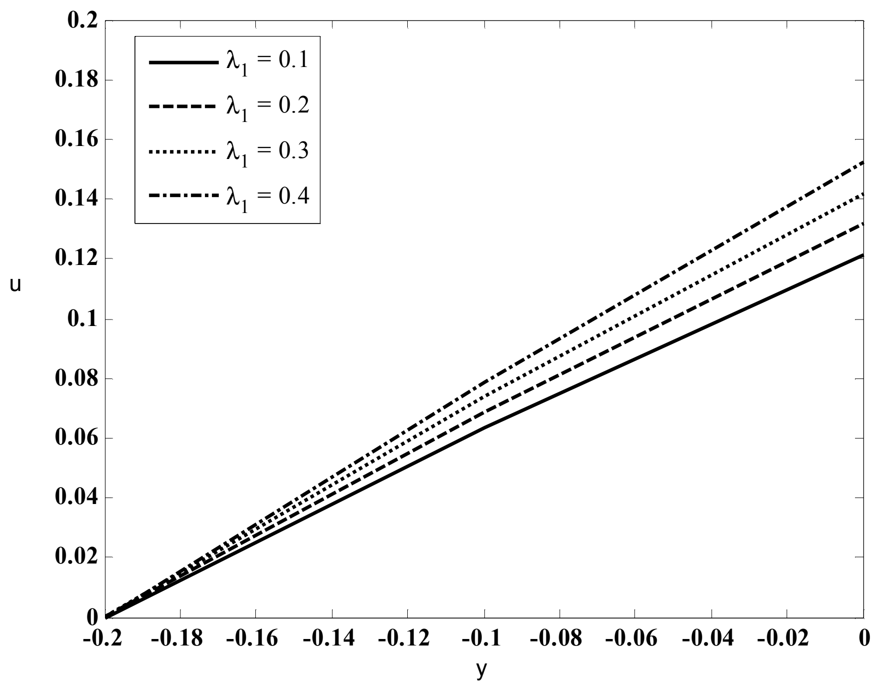

Fig 4. Displacement profiles in deformable porous region for different values of λ_1 for fixed values of ϕ^f =0.5, η =0.5, δ =2 and ε =0.2.

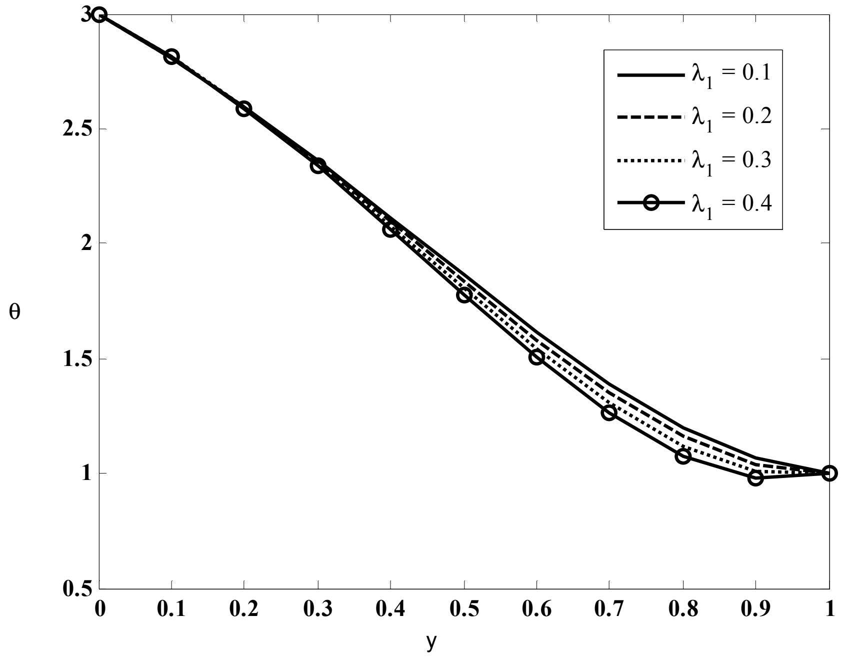

Fig 5. Temperature profiles for different values of λ_1
for fixed values of $Ec\,\mathrm{Pr}$ =2 and m =2.

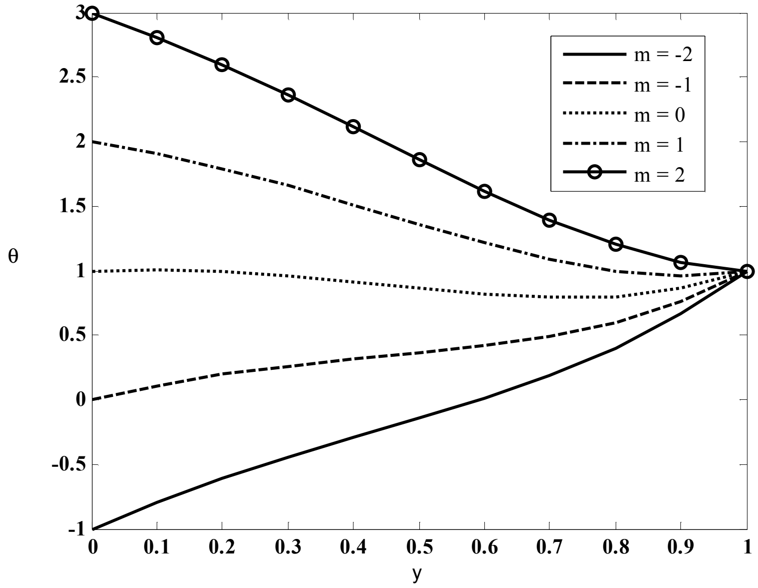

Fig 6. Temperature profiles for different values of m
for fixed values of λ_1=0.1 and $Ec\,\mathrm{Pr}$ =2.

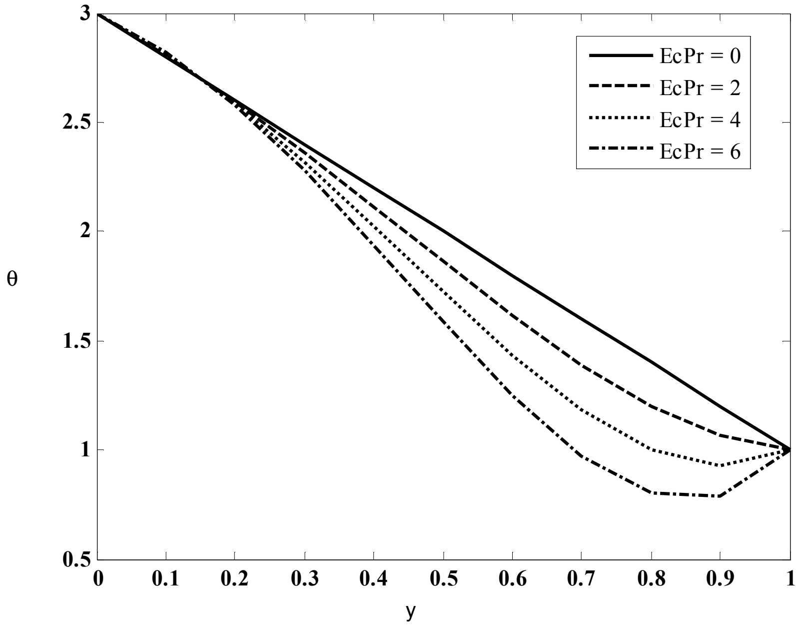

Fig 7. Temperature profiles for different values of $Ec\,\mathrm{Pr}$ for fixed values of λ_1=0.1and m=2.

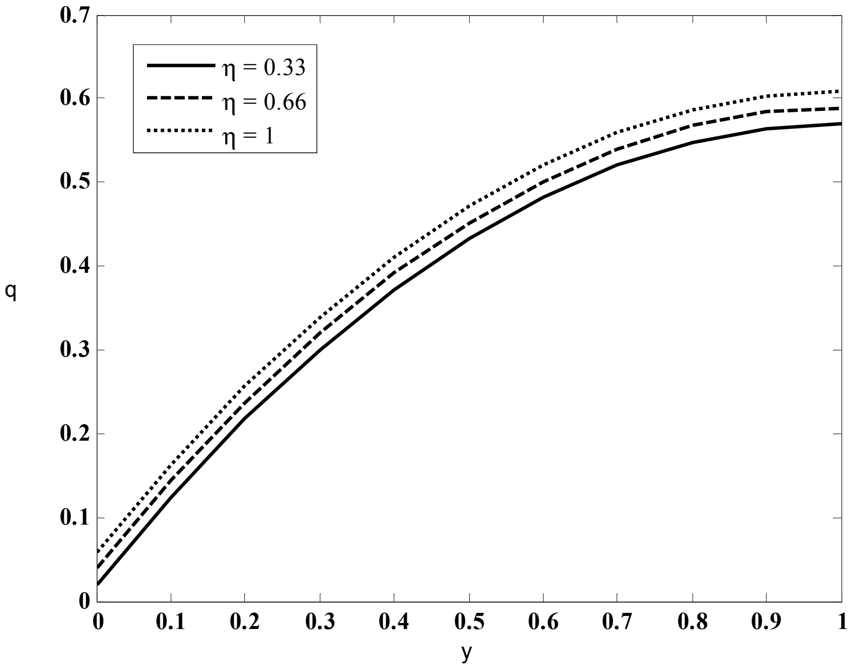

Fig 8.Velocity profiles in free flow surface for different values of η for fixed values of ϕ^f=0.5, λ_1=0.1, δ=2 and ε=0.2.

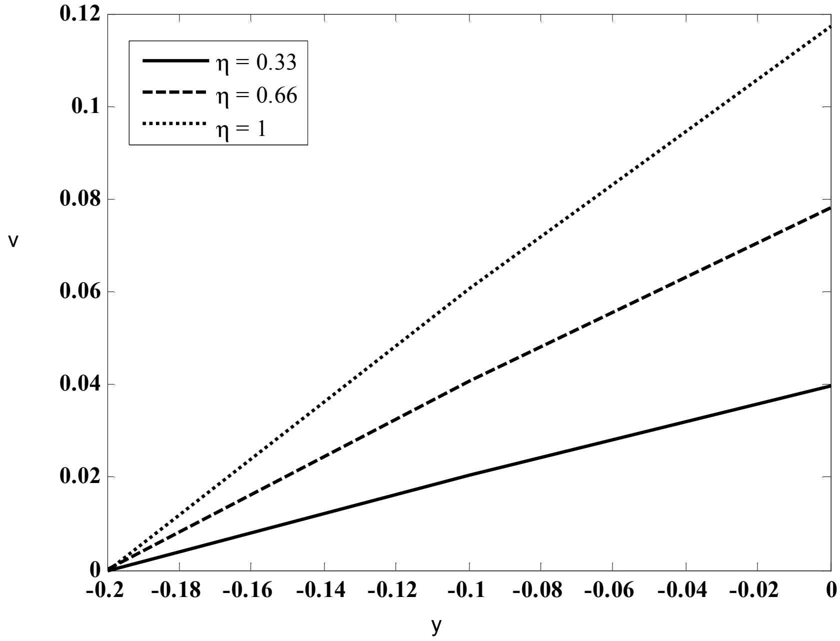

Fig 9.Velocity profiles in deformable porous region for different values of η for fixed values of ϕ^f =0.5, λ_1=0.1, δ =2 and ε =0.2.

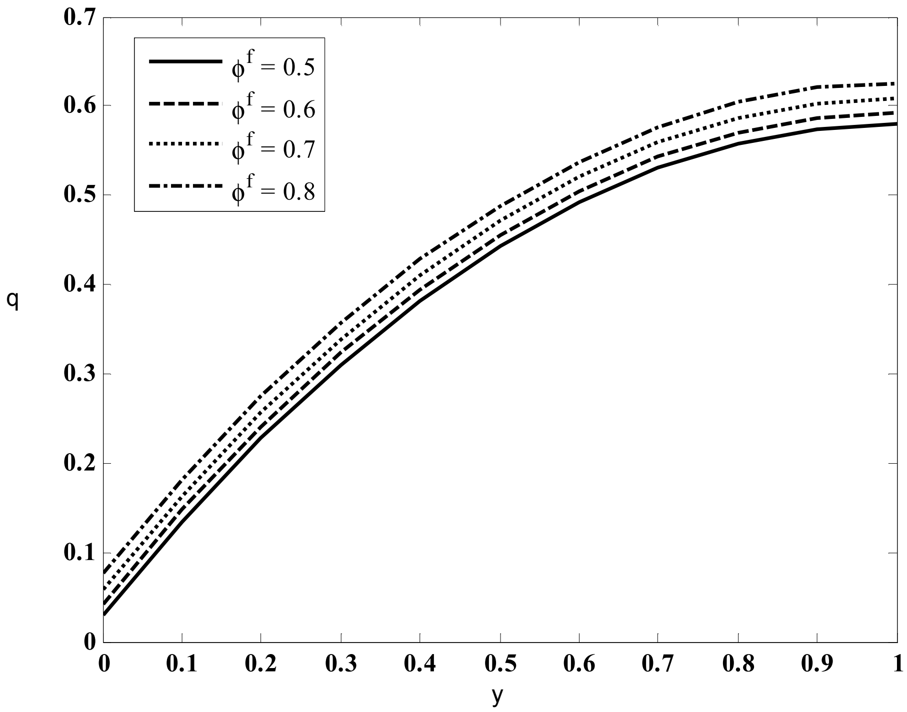

Fig 10.Velocity profiles in free flow region for different values of ϕ^f for fixed values of η =0.5, λ_1=0.1, δ =2 and ε =0.2.

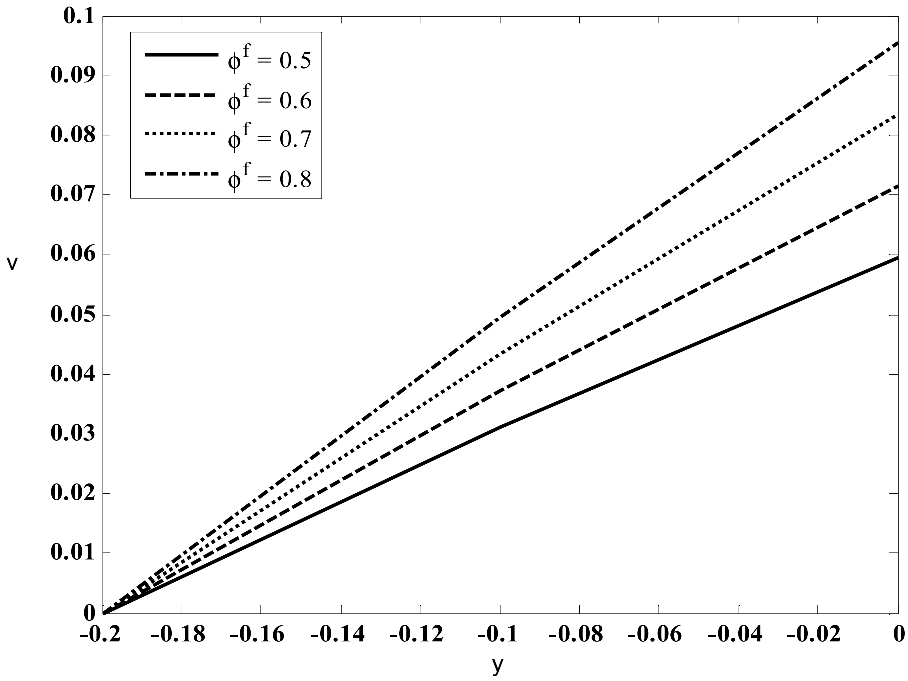

Fig 11. Velocity profiles in deformable porous region for different values of ϕ^f for fixed values of η=0.5, λ_1=0.1, δ=2 and ε=0.2.

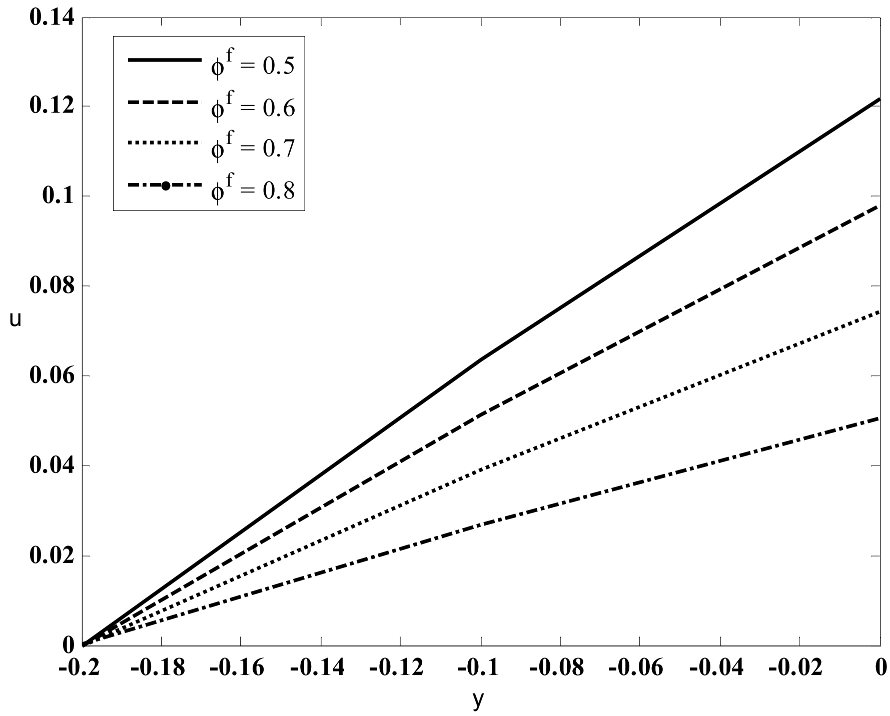

Fig 12. Displacement profiles in deformable porous region for different values of ϕ^f for fixed values of η=0.5, λ_1=0.1, δ=2 and ε=0.2.

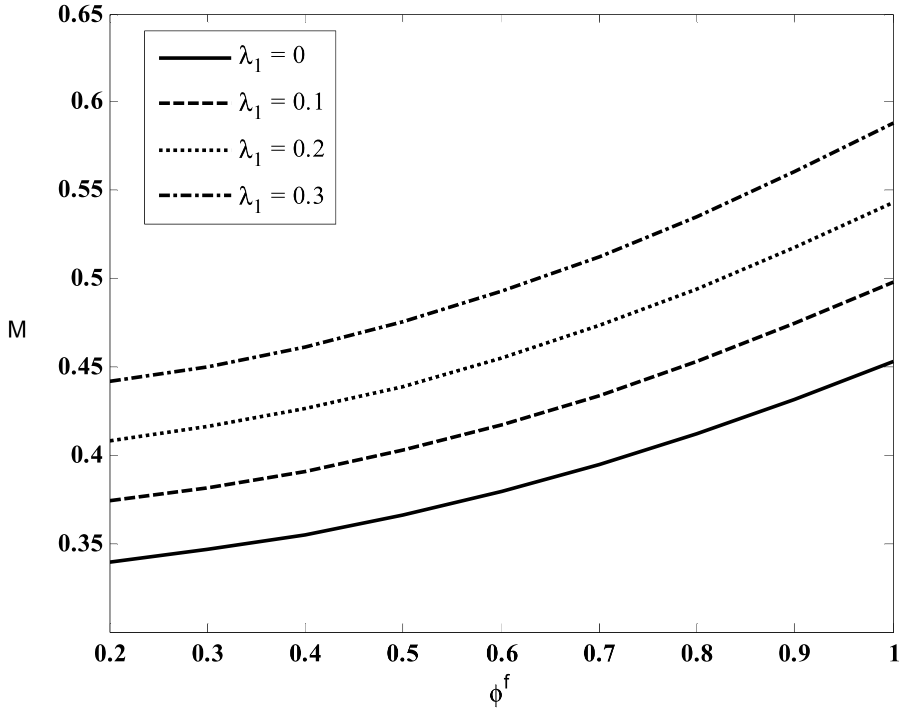

Fig 13.Total mass flux for different values of λ_1
for fixed values of η =0.5, δ =2 and ε =0.2.