

Convective Heat & Mass Transfer through a Porous Medium in Vertical Wavy Channel with Chemical Reaction & Heat Sources

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Abstract

In this paper we deal with the two-dimensional laminar simultaneous heat and mass transfer flow of a viscous, incompressible, electrically conducting and chemically reacting fluid through a porous medium confined in a vertical wavy channel. The equations of continuity, linear momentum, energy and diffusion which govern the flow fields are solved by employing a regular perturbation technique with slope δ as the perturbation parameter. The behavior of the velocity, temperature and concentration, skin friction, Nusselt number and Sherwood Number has been discussed for variations in the governing parameters.

Keywords: Wavy channel, Porous medium, chemically reacting fluid.

1.INTRODUCTION

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair[2],Lai and Kulacki[6]. The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al[1]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably Nelson and Wood[15]. In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. Vajravelu and Nayfeh [11] have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. This problem has been extended to the case of wavy walls by Rao et al [9]. Hyan Goo Kwon et al[5] have analyzed that the Flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Comini et al [3] have analyzed the Convective heat and mass transfer in wavy finned-tube exchangers.

Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. A common area of interest in the field of aerodynamics is the analysis of thermal boundary layer problems for two dimensional steady and incompressible laminar flow passing a wedge. Simultaneous heat and mass transfer from different geometrics embedded in a porous media has many engineering and geophysical application such as geothermal reservoirs, drying of porous solids thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and under ground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection/conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in no conventional energy sources, such as the use of salt-gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial application such as geophysics, oceanography, drying process, solidification of binary alloy and chemical engineering. Kandaswamy et al[7] have discussed the Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection.

In this paper we deal with the two-dimensional laminar simultaneous heat and mass transfer flow of a viscous, incompressible, electrically conducting and chemically reacting fluid through a porous medium confined in a vertical wavy channel. The equations of continuity, linear momentum, energy and diffusion which govern the flow fields are solved by employing a regular perturbation technique with slope δ as the perturbation parameter. The behavior of the velocity, temperature and concentration, skin friction, Nusselt number and Sherwood Number has been discussed for variations in the governing parameters.

2. FORMULATION OF THE PROBLEM

We consider the coupled heat and mass transfer flow of a viscous electrically conducting fluid through a porous medium confined in a vertical channel bounded by wavy walls in the presence of constant heat sources, transverse magnetic field effects and a first order chemical reaction. The flow is assumed to be

steady, laminar and two-dimensional and the surface is maintained at constant temperature and concentration. It is also assumed that the applied magnetic field is uniform and that magnetic Reynolds number is small so that the induced magnetic field is neglected. In addition, there is no applied electric field and all of the Hall effect, viscous dissipation and Joule heating are neglected. All thermophysical properties are constant except the density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq approximation. We consider rectangular Cartesian coordinate system $O(x, y, z)$ with walls at

$y = \pm Lf \left(\frac{\delta x}{L} \right)$ with slope δ . Under these assumptions, the equations describing the physical situation are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \beta g (T - T_e) + \beta^* g (C - C_e) - \frac{\sigma B_0^2 u}{\rho} - \left(\frac{v}{k} \right) u \quad (2.2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \left(\frac{v}{k} \right) v \quad (2.3)$$

$$\rho_0 C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \lambda \left(\frac{\partial T}{\partial x^2} + \frac{\partial T}{\partial y^2} \right) + Q \quad (2.4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial C}{\partial y^2} - \gamma C + k_{11} \frac{\partial^2 T}{\partial y^2} \quad (2.4a)$$

where y is the horizontal or transverse co-ordinate, u is the axial velocity, v is the transverse velocity, T is the fluid temperature, C is the concentration, T_e is the ambient temperature, C_e is the ambient concentration and $\rho, g, \beta, \beta^*, \mu, \sigma, B_0, Q, D$ are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, dynamic viscosity, fluid electrical conductivity, magnetic induction, heat generation/absorption coefficient, mass diffusion coefficient and chemical reaction parameter respectively. The physical boundary conditions for the problem are

$$u(-f) = 0, v(-f) = 0, T(-f) = T_1, C(-f) = C_1$$

$$u(+f) = 0, v(+f) = 0, T(+f) = T_2, C(+f) = C_2 \quad (2.5)$$

where T_1, T_2 and C_1, C_2 are the surface temperature and concentrations on $y = \pm L$ respectively. The flow is maintained by a constant volume flux for which a characteristic velocity is defined by

$$q = \frac{1}{L} \int_{-Lf}^{Lf} u \, dy \quad (2.6)$$

In view of the equation of continuity we define the stream function ψ as

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x} \quad (2.7)$$

the equation governing the flow in terms of stream function ψ are

$$\frac{\partial \psi}{\partial x} \frac{\partial (\nabla^2 \psi)}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial (\nabla^2 \psi)}{\partial x} = \mu \nabla^4 \psi + \left(\frac{\nu}{k} \right) \nabla^2 \psi + \left(\frac{\sigma \mu_e^2 H_e^2}{\rho_o} \right) \frac{\partial^2 \psi}{\partial y^2} + \beta g \frac{\partial}{\partial y} (T - T_e) + \beta^* g \frac{\partial}{\partial y} (C - C_e) \quad (2.8)$$

$$\rho_0 C_p \left(-\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q \quad (2.9)$$

$$\left(-\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y}\right) = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - \gamma C + k_{11} \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right) \quad (2.10)$$

and

$$\psi(+f) - \psi(-f) = -1 \quad \frac{\partial \psi}{\partial y} = 0 \quad \frac{\partial \psi}{\partial x} = 0$$

$$T(+f) = T_1 \quad T(-f) = T_2$$

$$C(+f) = C_1 \quad C(-f) = C_2$$

In order to write the governing equations and boundary conditions in the dimensionless form the following non-dimensional quantities are introduced

$$x' = \frac{x}{L}, \quad y' = \frac{y}{L}, \quad \psi' = \frac{\psi}{(v)}, \quad \theta = \frac{T - T_1}{T_2 - T_1}, \quad C' = \frac{C - C_1}{C_2 - C_1}$$

(2.11)

the equations after dropping the dashes are

$$\left(\frac{\partial \psi}{\partial x} \frac{\partial (\nabla^2 \psi)}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial (\nabla^2 \psi)}{\partial x}\right) = \nabla^4 \psi + D^{-1} \frac{v}{k} \nabla^2 \psi + M^2 \frac{\partial^2 \psi}{\partial y^2} - G \left(\frac{\partial \theta}{\partial y} + N \frac{\partial C}{\partial y}\right) \quad (2.12)$$

$$P \left(-\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}\right) = \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + \alpha \quad (2.13)$$

$$Sc \left(-\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y}\right) = \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - K C + \frac{ScS}{N} \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \quad (2.14)$$

and

$$\psi(+f) - \psi(-f) = -1 \quad \frac{\partial \psi}{\partial y} = 0 \quad \frac{\partial \psi}{\partial x} = 0$$

$$\theta(+f) = 1_1 \quad \theta(-f) = 0$$

$$C(+f) = 1 \quad C(-f) = 0 \quad (2.15)$$

where

$$G = \frac{\beta g (T_2 - T_1) L^4}{v^2} \quad (\text{Grashof Number}), \quad M^2 = \frac{\sigma \mu_e^2 H_o^2 L^2}{v^2} \quad (\text{Hartman Number})$$

$$D^{-1} = \frac{L^2}{k} \quad (\text{Darcy parameter}), \quad P = \frac{\mu C_p}{\lambda} \quad (\text{Prandtl Number}),$$

$$Sc = \frac{v}{D} \quad (\text{Schmidt Number}), \quad K = \frac{\mathcal{N}^2}{v} \quad (\text{Chemical reaction parameter})$$

$$N = \frac{\beta^* \Delta C}{\beta \Delta T} \quad (\text{Buoyancy ratio}), \quad \alpha = \frac{QL^2}{\lambda} \quad (\text{Heat source parameter})$$

4. SOLUTION OF THE PROBLEM

Solving the equations subject to the boundary conditions we obtain,

$$\psi_0(\eta) = a_9 \operatorname{Ch}(\beta_2 \eta) + a_{10} \operatorname{Sh}(\beta_2 \eta) + a_{11} \eta + a_{12} + \phi_1(\eta)$$

$$\phi_1(\eta) = a_7 \operatorname{Ch}(\beta_1 \eta) + a_8 \operatorname{Sh}(\beta_1 \eta) + -a_5 \eta^2 - a_6 \eta^3$$

$$\theta_0(\eta) = 0.5 \alpha f^2(\eta^2 - 1) + 0.5(\eta + 1)$$

$$C_0(\eta) = \frac{a_1}{\beta_1^2} \left(\frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} - 1 \right) + 0.5 \left(\frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} + 1 \right)$$

$$\begin{aligned} \theta_1(\eta) = & a_{65}(\eta^2 - 1) + a_{66}(\eta^3 - \eta) + a_{67}(\eta^4 - 1) + a_{68}(\eta^5 - \eta) + \\ & + (a_{70} + \eta a_{74})(\operatorname{Ch}(\beta_3 \eta) - \operatorname{Ch}(\beta_2)) + a_{71}(\operatorname{Sh}(\beta_2 \eta) - \eta \operatorname{Sh}(\beta_2)) + \\ & + (a_{72} + \eta a_{76})(\operatorname{Ch}(\beta_1 \eta) - \operatorname{Ch}(\beta_1)) + a_{73}(\operatorname{Sh}(\beta_1 \eta) - \eta \operatorname{Sh}(\beta_1)) + \\ & + (a_{75} + \eta a_{79})(\eta \operatorname{Sh}(\beta_2 \eta) - \operatorname{Sh}(\beta_2)) + (a_{77} + \eta a_{81})(\eta \operatorname{Sh}(\beta_1 \eta) - \operatorname{Sh}(\beta_1)) + \\ & + a_{78}(\eta^2 \operatorname{Ch}(\beta_2 \eta) - \operatorname{Ch}(\beta_2)) + a_{80}(\eta^2 \operatorname{Ch}(\beta_{21} \eta) - \operatorname{Ch}(\beta_1)) \end{aligned}$$

$$\begin{aligned} C_1(\eta) = & b_{62} \left(1 - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \right) + b_{64} \left(\eta^2 - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \right) + b_{66} \left(\eta^4 - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \right) + \\ & + b_{68} \left(\operatorname{Ch}(\beta_2 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Ch}(\beta_2) \right) + b_{70} \left(\eta^2 \operatorname{Ch}(\beta_2 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Ch}(\beta_2) \right) + \\ & + b_{72} \left(\eta \operatorname{Sh}(\beta_2 \eta) - \operatorname{Ch}(\beta_2 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Sh}(\beta_2) \right) + b_{74} \left(\eta^3 \operatorname{Sh}(\beta_2 \eta) - \operatorname{Ch}(\beta_2 \eta) - \right. \\ & \left. - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Sh}(\beta_2) \right) + b_{77} \left(\eta^2 - 1 \right) \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} + b_{80} \left(\eta \operatorname{Sh}(\beta_1 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Sh}(\beta_1) \right) + \\ & + b_{82} \left(\eta^3 \operatorname{Sh}(\beta_1 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Sh}(\beta_1) \right) + b_{84} \left(\operatorname{Ch}(\beta_3 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Ch}(\beta_3) \right) + \\ & + b_{86} \left(\operatorname{Ch}(\beta_4 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Ch}(\beta_4) \right) + b_{89} \left(\eta \operatorname{Sh}(\beta_3 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Sh}(\beta_3) \right) + \\ & + b_{90} \left(\eta \operatorname{Sh}(\beta_4 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Sh}(\beta_4) \right) + b_{92} \left(\operatorname{Ch}(2\beta_1 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Ch}(2\beta_1) \right) + \\ & + b_{93} \left(\eta \operatorname{Sh}(2\beta_1 \eta) - \frac{\operatorname{Ch}(\beta_1 \eta)}{\operatorname{Ch}(\beta_1)} \operatorname{Sh}(2\beta_1) \right) + b_{63} \left(\eta - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \right) + b_{65} \left(\eta^3 - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \right) + \\ & + b_{65} \left(\eta^5 - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \right) + b_{67} \left(\eta \operatorname{Ch}(\beta_2 \eta) - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Ch}(\beta_2) \right) + b_{71} \left(\eta^3 \operatorname{Ch}(\beta_2 \eta) - \right. \\ & \left. - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Ch}(\beta_2) \right) + b_{73} \left(\eta^2 \operatorname{Sh}(\beta_2 \eta) - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Sh}(\beta_2) \right) + b_{76} \left(\eta \operatorname{Ch}(\beta_1 \eta) - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Ch}(\beta_1) \right) + \\ & + b_{78} \left(\eta^3 \operatorname{Ch}(\beta_1 \eta) - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Ch}(\beta_1) \right) + b_{81} \left(\eta^2 - 1 \right) \operatorname{Sh}(\beta_1 \eta) + b_{83} \left(\operatorname{Sh}(\beta_3 \eta) - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Sh}(\beta_3) \right) + \\ & + b_{84} \left(\operatorname{Sh}(\beta_4 \eta) - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Sh}(\beta_4) \right) + b_{87} \left(\eta \operatorname{Ch}(\beta_3 \eta) - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Ch}(\beta_3) \right) + b_{88} \left(\eta \operatorname{Ch}(\beta_4 \eta) - \right. \\ & \left. - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Ch}(\beta_4) \right) + b_{91} \left(\operatorname{Sh}(2\beta_1 \eta) - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Sh}(2\beta_1) \right) + b_{94} \left(\eta \operatorname{Ch}(2\beta_1 \eta) - \frac{\operatorname{Sh}(\beta_1 \eta)}{\operatorname{Sh}(\beta_1)} \operatorname{Ch}(2\beta_1) \right) \end{aligned}$$

$$\begin{aligned} \psi_1(\eta) &= e_1 Ch(M_1\eta) + e_2 Sh(M_1\eta) + e_3\eta + e_4 + \phi_2(\eta) \\ \phi_2(\eta) &= -b_{166}\eta^2 - b_{167}\eta^3 - b_{168}\eta^4 - b_{169}\eta^5 - b_{170}\eta^6 - b_{171}\eta^7 + (b_{172} + \eta^2 b_{189} + \\ &\quad + \eta^4 b_{191})Ch(\beta_1\eta) + (b_{173} + \eta^2 b_{191} + \eta^3 b_{197} + \eta^4 b_{198})Ch(\beta_2\eta) + (b_{174} + \eta b_{202} \\ &\quad + \eta^2 b_{203})Ch(\beta_3\eta) + (b_{175} + \eta b_{207} + \eta^2 b_{209})Ch(\beta_4\eta) + (b_{176} + \eta b_{210})Ch(2\beta_1\eta) + \\ &\quad + (b_{177} + b_{212} + \eta^2 b_{214})Ch(2\beta_2\eta) + (b_{178} + \eta^2 b_{188} + \eta^3 b_{186} + \eta^4 b_{187})Sh(\beta_1\eta) + \\ &\quad + (b_{180} + \eta^2 b_{193} + \eta^3 b_{194})Sh(\beta_2\eta) + (b_{183} + \eta b_{200} + \eta^2 b_{201})Sh(\beta_3\eta) + (b_{183} + \eta b_{205} \\ &\quad + \eta^2 b_{206})Sh(\beta_4\eta) + (b_{176} + \eta b_{210})Ch(2\beta_1\eta) + (b_{179} + b_{212} + \eta^2 b_{211})Sh(2\beta_2\eta) \end{aligned}$$

In the case of $\beta = 0$ and $S_0=0$ the results are in good agreement with that of Sudha[10]

5. STRESS ,NUSELT NUMBER AND SHERWOOD NUMBER

$$\tau^* = \left(\mu \frac{du}{dy}\right)_{y=\pm L}$$

The shear stress on the boundaries $y = \pm 1$ are given by

$$\begin{aligned} \tau &= \frac{\tau^*}{(v^2 / L^2)} = \left(\frac{du}{dy}\right)_{y=\pm 1} \\ &= \left(\frac{du_0}{dy} + \delta \frac{du_1}{dy} + \delta^2 \frac{du}{dy}\right)_{y=\pm 1} \end{aligned}$$

which in the non-dimensional form reduces to

$$\tau_{y=+1} = b_3 + \delta b_5 + \delta^2 b_7 + \dots$$

and the corresponding expressions are

$$\tau_{y=-1} = b_4 + \delta b_6 + \delta^2 b_8 + \dots$$

The rate of heat transfer(Nusselt Number) on the boundaries $y = \pm 1$ are given by

$$Nu_{y=\pm 1} = \left(\frac{d\theta_0}{dy} + \delta \frac{d\theta_1}{dy} + \delta^2 \frac{d\theta_2}{dy}\right)_{y=\pm 1}$$

and the corresponding expressions are

$$Nu_{y=+1} = b_7 \text{ \& } Nu_{y=-1} = b_8$$

The rate of mass transfer(Sherwood Number) on the boundaries $y = \pm 1$ are given by

$$Sh_{y=\pm 1} = \left(\frac{dC_0}{dy} + \delta \frac{dC_1}{dy} + \delta^2 \frac{dC_2}{dy}\right)_{y=\pm 1}$$

and the corresponding expressions are

$$Sh_{y=+1} = b_9 \quad Sh_{y=-1} = b_{10}$$

Where b_1, b_2, \dots, b_{10} are constants

6. RESULTS & DISCUSSIONS

In this analysis we investigate the convective heat and mass transfer of a viscous, electrically conducting, chemically reacting fluid in a vertical wavy channel. The walls are taken in the form $\eta=1+\beta\exp(-x^2)$. The channel is converging or diverging according as $\beta>0$ or $\beta<0$. We confine our attention for $\beta>0$. The velocity, temperature and concentration are shown in figs 1-15 for different values of $G, M, D^{-1}, Sc, So,$ and x . It is found that from fig.1 that the actual axial flow is negative and hence $u>0$ represents the reversal flow. The velocity exhibits reversal flow for $G<0$ and no such flow exists for $G>0$. The region of reversal flow enlarges with increase in $|G|(<0)$. Also $|u|$ enhances with increase in $|G|(<0)$. The variation of u with D^{-1} and M shows that lesser the permeability of the porous medium higher the Lorentz force smaller $|u|$ in the entire flow region (fig-2). From fig 3 the variation of u with S_c & S_o shows that lesser the molecular diffusivity larger $|u|$ in the region. An increase in $|S_o|(\geq 0)$ results in an enhancement in $|u|$. $|u|$ experiences a depreciation with increase in $k\leq 1$ and enhances with $k\geq 2$. Also we notice reversal flow in the entire flow region for $k=2$. Moving along the axial direction the axial velocity depreciates in the flow region (fig-4).

The secondary velocity (v) is towards the mid region for all $|G|$, except for $|G|\geq 2 \times 10^2$, at which it is towards the boundary. $|v|$ enhances with increase in $G>0$ while it depreciates with $|G|\leq 3 \times 10^2$ and enhances with higher $|G|\geq 5 \times 10^3$ (fig5). Lesser the permeability of the porous medium / higher the Lorentz force smaller $|v|$ in

the flow region (fig-6). Lesser the molecular diffusivity larger $|v|$ and also $|v|$ enhances marginally with $|S_o|(\leq 0)$ (fig-7). An increase in the chemical parameter k leads to a depreciation in $|v|$. Moving along the axial direction the secondary velocity reduces marginally in the flow region (fig-8).

It is found from fig.9 that for $G>0$ θ is positive except in the mid- region and this region enlarges with increase in $G>0$ while for $G<0$ θ is positive in the entire flow region. The actual temperature depreciates with increase in $G>0$ and enhances with $|G|$ in the entire flow region. Fig.10 represents the variation of θ with different M and D^{-1} . The region of transition from positive to negative spreads towards the boundary with increase in M and D^{-1} . Also lesser the permeability of the porous medium larger the actual temperature and higher the Lorentz force smaller the actual temperature in the flow region. From fig-11, we find that lesser the molecular diffusivity, smaller the actual temperature in the flow region. An increase in the solet parameter $S_o>0$ enlarges the transition region while no such phenomenon is observed with $S_o<0$. An Increase in $S_o>0$ results in a depreciation of actual temperature while it enhances with $|S_o|(\leq 0)$. The variation of θ with chemical reaction parameter k and axial distance 'x' is shown in fig-12. The actual temperature experiences an enhancement with increase in chemical reaction parameter k . Moving along the axial direction, the actual temperature enhances except near the boundaries $\eta = \pm 1$ with $x \leq \pi/2$ while it depreciates with higher $x \geq \pi$.

From fig-13, we notice that the concentration is negative for $G>0$ and positive for $G<0$. The actual concentration experiences an enhancement with increase in $|G|(\leq 0)$. Lesser the permeability of the porous medium / higher the Lorentz force smaller the actual concentration in the entire flow region. The depreciation in the actual concentration with D^{-1} is more predominant than that with M (fig-14).

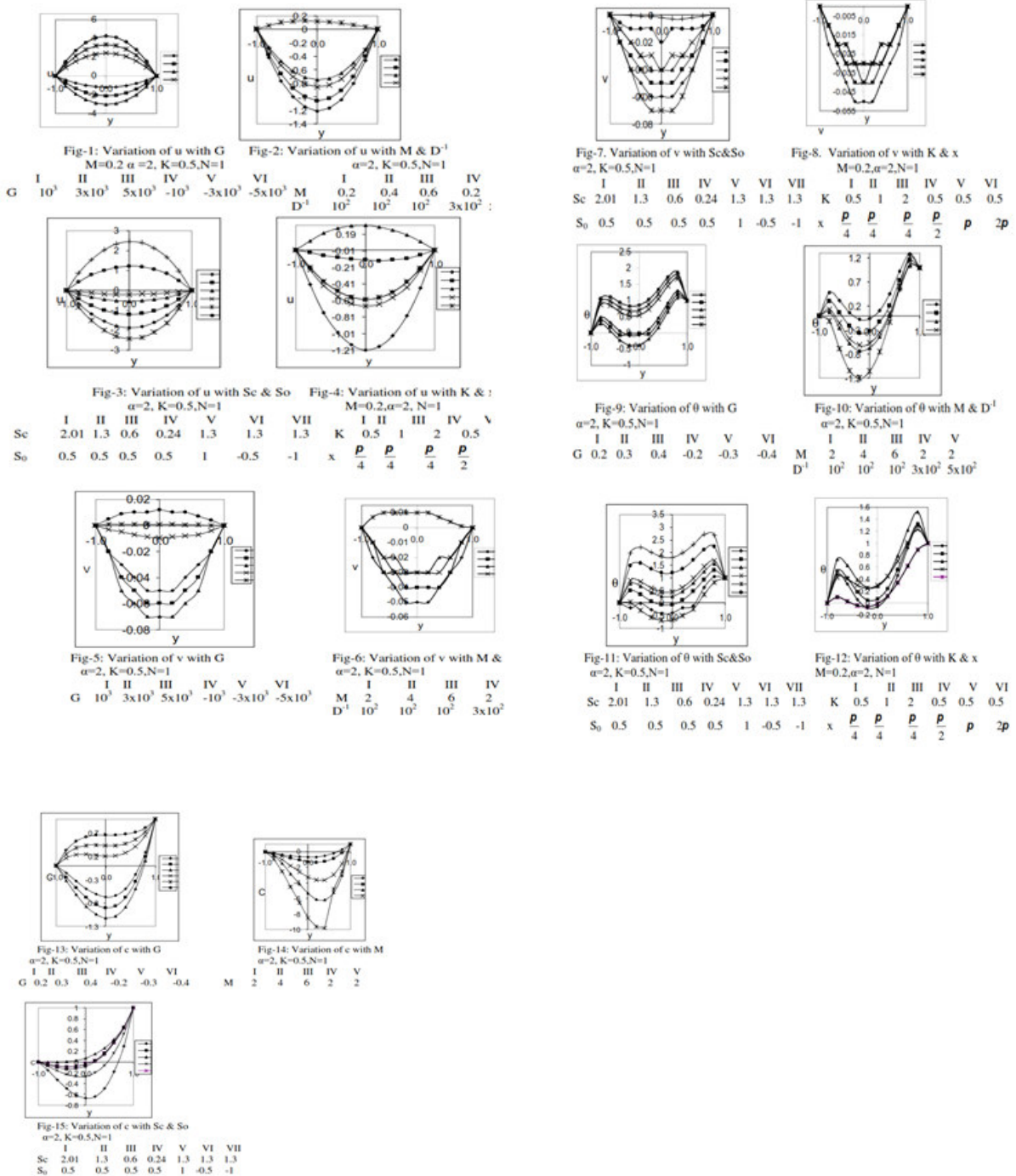
Fig-15 represents the variation of concentration with Schmidt number Sc and solet parameter S_o . For smaller values of $Sc \leq 0.6$ the concentration is positive in the flow region and $Sc > 1.3$ the concentration is negative in entire flow region. Also for $S_o > 0$ the concentration is negative in entire flow region and for $S_o < 0$ the region of transition which appears in vicinity of $\eta = -1$ extends towards mid-region with increase in $|S_o|$. Lesser the molecular diffusivity smaller the actual concentration in the entire flow field. An increase in the solet parameter $|S_o|$ leads to a depreciation in the actual concentration everywhere in the flow region.

The Nusselt number (Nu) at $\eta = \pm 1$ are evaluated for different values of G , D^{-1} , M , α , S_c , S_o , β , k & x . The variation of Nu with Grashoff number G shows that $|Nu|$ experiences a depreciation with increase in $|G|(\geq 0)$. Lesser the permeability of the porous medium smaller the magnitude of the rate of heat transfer at both the walls in the heating case while in the cooling case smaller $|Nu|$ and for further lowering of the permeability larger $|Nu|$ at $\eta = \pm 1$. (tables 1&6). Tables 2 & 7 represent the variation of Nu with heat source parameter α . It is found that $|Nu|$ experience a depreciation with increase in the strength of the heat source parameter in $\alpha \leq 4$ and enhancement with higher $\alpha \geq 6$ at both the walls in the heating case while it enhances with α in the cooling case. An increase in the solet parameter $S_o(0)$ enhances $|Nu|$ at $\eta = \pm 1$ and reduces it at $\eta = -1$ for $G > 0$ while it reduces at both the walls for $G < 0$. An increase in $|S_o|(\leq 0)$ reduces $|Nu|$ at $\eta = \pm 1$ in the heating case while it reduces in the cooling case (tables 3&8). The influence of the surface geometry on Nu is exhibited in tables 4& 9. It is found that higher dilation smaller the rate of heat transfer at $\eta = \pm 1$ in the heating case while in the cooling case larger $|Nu|$ and for higher dilation smaller $|Nu|$ at $\eta = \pm 1$. At $\eta = -1$, $|Nu|$ depreciates with k at $G = 10^3$ and at higher $G \geq 3 \times 10^3$ it enhances with $k \leq 1.0$ and depreciates with higher $k \geq 2.0$ and $G > 0$ and it experiences an enhancement in the cooling case. Moving along the axial direction the rate of heat transfer depreciates with x at $G = 10^3$ and enhances at higher $G \geq 3 \times 10^3$ in the heating case and reduces with $x \leq \pi/2$ and enhances with $x \geq \pi$ in the cooling case. At $\eta = -1$, it experiences an enhancement with x in both heating and cooling cases (Table 5 & 10).

The Sherwood number (Sh) which measures the rate of mass transfer at the boundaries is shown in tables 11-14 for different parameters G , D^{-1} , M , α , S_c , S_o , k and x . The variation of Sh with Grashoff number G shows that the rate of mass transfer depreciates with increase in $G > 0$ at $\eta = \pm 1$ while an increase in $G < 0$ enhances $|Sh|$ at $\eta = +1$ and reduces at $\eta = -1$. Lesser the permeability of the porous medium lesser $|Sh|$ at $\eta = +1$ for $G > 0$ and for $G < 0$, lesser the magnitude of Sh and for further lowering of the permeability smaller $|Sh|$. (table 11&17). The variation of Sh with α is exhibited in tables 12 & 18. An increase in the strength of the heat source enhances $|Sh|$ at $\eta = +1$ in both heating and cooling of the channel walls and at $\eta = -1$, it depreciates with $\alpha > 0$ for $G > 0$ and enhances while α for $\alpha < 0$. Also an increment in $\alpha < 0$ enhances $|Sh|$ for $G > 0$ and for $G < 0$, it depreciates $|Sh|$ at $\eta = 1$, while at $\eta = -1$, it depreciate with $|\alpha| \leq 4$ and enhances with $|\alpha| \geq 6$ for all $|G|(\leq 0)$. With reference to variation of Sh with Schmidt number S_c we find that the rate of heat transfer at $\eta = 1$, enhances with $S_c \leq 0.6$ and depreciates with higher $S_c \geq 2.01$. at $G = 10^3$ and higher $G \geq 3 \times 10^3$, $|Sh|$ reduces with $S_c \leq 0.6$ and enhances with higher $S_c \geq 2.01$ in the heating case while in the cooling case it reduces with $S_c \leq 1.3$ and enhances with $S_c \geq 2.01$. At $\eta = -1$, lesser the molecular diffusivity smaller $|Sh|$ and for further lowering of the diffusivity smaller $|Sh|$ for $G > 0$ and for $G < 0$, small $|Sh|$. An increase in $S_o > 0$ enhances $|Sh|$ at $\eta = 1$ and reduces at $\eta = -1$ for all $|G|(\geq 0)$ while an increase in $|S_o|(\leq 0)$ enhances $|Sh|$ at $\eta = \pm 1$ for $G > 0$ and depreciates it for $G < 0$. (Tables 13&19). When the molecular buoyancy force dominates over the thermal buoyancy force the rate of mass transfer reduces at $\eta = +1$ and

enhances at $\eta=-1$ for $G>0$ and for $G<0$ enhances at $\eta= \pm 1$ when the buoyancy forces act in the same direction and for the forces acting in opposite directions $|Sh|$ enhances for $G>0$ and depreciates for $G<0$ at $\eta=+1$ and at $\eta=-1$, it reduces for all G (table 14&20). The effect of waviness of the boundary on Sh is shown in tables 15&21. Higher the dilation larger $|Sh|$ at $\eta=1$ for all G while at $\eta=-1$ smaller $|Sh|$ at $G=10^3$ and at higher $G\geq 2\times 10^3$, smaller $|Sh|$ and for higher dilation larger $|Sh|$ and for $G<0$, smaller $|Sh|$. For an increase in the chemical reaction parameter k we find that $|Sh|$ at $\eta=+1$ enhances with $k\leq 1.0$ and depreciates with $k\geq 2.0$ for all G and for $G<0$, $|Sh|$ experiences an enhancement with k for all G . Moving along the axial direction the rate of mass transfer at $\eta=+1$ depreciates for $G>0$ and for $G<0$ it depreciates with $x\leq \pi/2$ and enhances with $x\geq \pi$ at $|G|=10^3$ and at higher $|G|\geq 2\times 10^3$, depreciates with x . At $\eta= -1$ it enhances with $x\leq \pi/2$ and reduces with higher $x\geq \pi$ in the heating case and enhances with x in the cooling case (Tables 16&20).

Figures&Tables:



6. Lai,F.C and Kulacki,F.A (1991): Coupled heat and mass transfer by natural convection from vertical surfaces in porous medium.,*Int.J.Heat Mass Transfer*,V.34,pp.1189- 1194(1991)
7. Kandaswamy .P,Abd.Wahid B.Md.Raj ,azme B.Khamis(2006): Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection, *Theoret.Appl.Mech.*,V.33.No.2,pp.123-148
8. Nelson,D.J and Wood,B.D(1989):Combined heat and mass transfer by natural convection between vertical plates ,*Int.J.,Heat Mass transfer*,V.82,pp.1789-1792.
9. Prasada Rao, D.R.V, Krishna, D.V and Debnath, L (1983). Free convection in Hydromagnetic flows in a vertical wavy channel, *Int. Engg. Sci*, V.21, No.9, pp1025-1039.
10. Sudha Mathew(2009): Hydro magnetic mixed convective heat and mass transfer through a porous medium in a vertical channel with thermo-diffusion effect. Ph.D thesis, S,K.University, Anantapur, India.
11. Vajravelu K and Ali Neyfeh (1981): Influence of wall waviness on friction and pressure drop in channels, *Int. J. Maths and Math. Sc.*, V.4, pp.805-818.
12. Wei-Mon Yan(1996):Combined buoyancy effects of thermal and mass diffusion on laminar forced convection in horizontal rectangular ducts,*Int,J,Heat Mass transfer*,V.39,pp.1479-1488.