Heat Transfer of MHD Flow of Casson Fluid due to Stretching Sheet with PST and PHF Heating Conditions

Mahantesh M. Nandeppanavar[1]* Veena P.H[2] Vinuta H.Deshapande[3]
1.Department of Studies and Research in Mathematics, Government College, Kalaburagi-585105, Karnataka, India
2.Department of Mathematics, Smt.V.G.Women's College, Kalaburagi-585102, Karnataka, India

Abstract
Here we have considered the two-dimensional flow of non-Newtonian MHD flow of Casson fluid, Using Nevier Stoke's Equations of Motion we have derived the momentum and energy equstions of Casson fluid, these governing equations of motion and temperature are non-linear partial differential equations which are tedious to solve as they are, hence these partial differential equations are converted into Ordinary differential equations using suitable similarity transformations. These ODE's are solved numerically and we have analyzed various effects of various governing parameters on flow and heat transfer profiles. The numerical Values of Wall temperature and Wall temperature Profiles are tabulated and discussed in detail.
Keywords: Convective Heat transfer, Non-Newtonian fluid, BVP, IVP, Numerical Solution

Nomenclature

b	stretching rate
x	horizontal coordinate
y	vertical coordinate
u	horizontal velocity component
v	vertical velocity component
T	temperature
c_p	specific heat
f	dimensionless stream function
Pr	Prandtl number
l	Characteristic length
Mn	Magnetic parameter
'	differentiation with respect to η

Greek symbols

η	similarity variable
θ	dimensionless temperature
k	thermal conductivity
μ	viscosity
υ	kinematic viscosity
ρ	density
α	thermal difusivity
β	Casson parameter

Subscripts

w	properties at the plate
∞	free stream condition

1. Introduction

On analyzing the various studies on boundary layer flow and Heat transfer of continuous moving surface , we came to know that, boundary layer flow is an important type of flow occurring in a many of engineering applications. Some of them are, In an Aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation process and a polymer sheet of filament extruded continuously from a die are examples of practical applications of continuous moving surfaces, cooling of an infinite metallic plate in a cooling path, Gas blowing, continuous casting and spinning of fibers also involve the flow due to a stretching sheet. we have looked towards some important studies as mentioned below:

Crane [1] investigated the flow due to stretching sheet, first time stretching sheet concept is used, which is very helpful in various engineering industries, this work is considered for a viscous fluid. But as considered the applications of Newtonian fluids the applications of non-Newtonian fluids are more hence many researchers have worked on flow, heat and mass transfer analysis of non-Newtonian fluids. Hayat et.al [2] studied soret and dufour effects on the magnetohydrodynamics of Casson fluids. Nadeem et. al[3] studied the MHD flow of non-Newtonian Casson fluid due to an exponential shrinking sheet. Pramanik [4] studied the flow analysis and heat transport analysis of Casson fluid using porous stretching sheet, author investigated the effects of suction and blowing effect on flow and heat transfer also. Bhattacharya et.al [5] investigated flow analysis of Casson fluid over non-porous stretching sheet with the slip effect. Further Bhattacharyya [6] studied the stagnation point flow of Casson fluid and heat transfer analysis due to stretching sheet and investigated the effect of thermal radiation on flow and temperature. Swati et .al [7] investigated the effect of unsteadiness on flow and heat transfer analysis. Qasim and Nooreer [8] Flow analysis of Casson fluid due to permeable shrinking sheet with viscous dissipation. Rizwan et.al [9] Studied the Flow of MHD Casson nano fluid due to a shrinking sheet. Hussain et.al [10] studied the flow analysis of Casson nanofluid with viscous dissipation and convective boundary conditions. Kamehswaran et.al [11] obtained dual solutions for Flow analysis of Casson fluid due to stretching or shrinking sheet. Nandeppanavar [12] studied the flow and heat transfer analysis with two heating conditions considering the non-Newtonian Casson fluid due to linear stretching sheet. The solution they obtained is by a power series method analystically, further Nandeppanavar [13-14] investigated the heat transfer analysis of Casson fluid due to stretching sheet with convective heating condition both Numerical and analytical results in terms of Kummer's function and Runge-Kutta flurth order method with shooting technique. Attia and Ahmed[15] studied the transient Coutte flow analysis of Casson fluid between parallel plates with heat transfer analysis. Bhattacharyya et.al[16] have given an analytical solution for magnetohydrodynamic boundary layer flow of Casson fluid, they also studied the effect of wall mass transfer analysis too. Swati[17] studied the effect of thermal radiation on the flow and heat transfer analysis of Casson fluid over an unsteady stretching sheet with effect of suction and blowing. Shehzad et.al[18] investigated the mass transfer of magnetohydrodyanic flow of Casson fluid with an chemical reaction.

Considering all above works, The flow and Heat transfer analysis with two heating conditions (Prescribed surface temperature and Prescribed Wall heat flux) with Numerical Solution. Hence this work is undertaken and the effects of all governing parameter are analysed on the flow and heat transfer analysis.

2. Mathematical Formulation:

Assuming rheological equation of non-newtonian Casson fluid

$$\tau_{ij} = \left[\mu_B + \left(\frac{P_y}{\sqrt{2\pi}} \right)^{\frac{1}{n}} \right]^n 2e_{ij} \tag{1}$$

Here μ is the dynamic viscosity, μ_B is the plastic dynamic viscosity of Casson fluid, P_y is the stress of Casson fluid. $\pi = e_{ij}e_{ji,}$, e_{ij} is the (i,j) th component of the deformation rate Casson fluid (π is the product of the component of deformation rate with itself). Considering $n \geq 1$ we have many applications.

Considering above rheology of Casson fluid. The governing equations of the fluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} \tag{3}$$

u and v are the components of velocity in x and y axis respectively. υ is the kinematic viscosity of Casson fluid $\beta = \mu_B \sqrt{2\pi_c / P_y}$ is Casson parameter.

Where *u* and *v* are the velocity components of the fluid in *x* and *y* directions respectively and is kinematic viscosity and β is the Casson parameter (non-Newtonian parameter).

The boundary conditions for the problem are

$$\left.\begin{aligned} u_w(x) = bx,\ v = 0, \quad & y = 0 \\ u \to 0, \quad as \quad & y \to \infty \end{aligned}\right\} \tag{4}$$

with *b* > 0, the stretching rate. The Eqns. (2) and (3), subjected to the boundary condition (3), admit a selfsimilar solution in terms of the similarity function f and the similarity variable η defined by

$$u = b\,x\,f'(\eta), \quad v = -\sqrt{b\upsilon}, \quad \eta = \sqrt{\frac{b}{\upsilon}}\,y. \tag{5}$$

It can be easily verified that Eq. (2) is identically satisfied and substituting the above transformations in Eq. (3) we obtain

$$\left. f'^2 - ff'' = \left(1+\frac{1}{\beta}\right) f''' - M_n f' \right\}. \tag{6}$$

Similarly the boundary conditions (4) can be written as:

$$\left.\begin{array}{llll} f'(\eta) = 1, & f(\eta) = 0 & at & \eta = 0 \\ f'(\eta) \to 0, & & as & \eta \to \infty \end{array}\right\}. \tag{7}$$

where

$M_n = \sqrt{\frac{\sigma\beta_0}{b}}$ - Magnetic field

3. Heat transfer analysis:

The Energy equations with boundary layer approximations can be written as:

$$\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

where k is the thermal conductivity, ρ is the density of the fluid, C_p is the specific heat at constant pressure . the heat transfer analysis is carried out due to following two heating conditions:

3.1 Prescribed surface temperature (PST Case):

The PST boundary conditions are:

$$\left.\begin{array}{lll} T = A\left(\frac{x}{l}\right)^2, & at & y = 0 \\ T \to T_\infty & as & y \to \infty \end{array}\right\}, \tag{9}$$

where T_∞ is the temperature of the fluid far away from the sheet. Defining the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{10}$$

Where T_w the temperature of the sheet.

Using Eqn. (10), Eqs. (8) and (9) can be written as

$$\theta'' + \Pr f\,\theta' - 2f\,\Pr\theta = 0, \tag{11}$$

$$\left.\begin{array}{lll} \theta(\eta) = 1 & \text{at} & \eta = 0, \\ \theta(\eta) \to 0 & \text{as} & \eta \to \infty. \end{array}\right\} \tag{12}$$

Where

$\Pr = \frac{\mu C_p}{k}$ is the Prandtl number.

3.2: Prescribed Surface Wall Heat Flux (PHF Case):

Consider the power law heat flux on the wall surface is considered as:

$$\left.\begin{array}{ll} -k\dfrac{\partial T}{\partial y} = B\left(\dfrac{x}{l}\right)^2 & at \quad y=0 \\ T \to T_\infty & at\ y \to \infty \end{array}\right\} \tag{13}$$

where B is a constant. The scaled temperature is defined as:

$$g(\eta) = \frac{T-T_\infty}{T_w - T_\infty} \tag{14}$$

Making use of the transformation (24) into Eqn(11) and (23) we get

$$g'' + \Pr f\, g' - 2f \Pr g = 0 \tag{15}$$

$$\left.\begin{array}{lll} g'(\eta) = -1 & \text{at} & \eta = 0, \\ g(\eta) \to 0 & \text{as} & \eta \to \infty. \end{array}\right\} \tag{16}$$

4. Numerical Solution

The set of non-linear differential equations (5 and 11) subject to the boundary conditions (5and 12) are integrated numerically using a very efficient method known as Runge-Kutta Fehlberg method with shooting technique. The most important factor of this method is to choose the appropriate finite values of $\eta \to \infty$ in order to determine η_∞ for the boundary value problem stated by Eq.(5 &11), we start with some initial guess value for some particular set of physical parameters to obtain $f''(0)\ \&\ \theta'(0)$. The solution procedure is repeated with another large value of η_∞ until two successive values of $f''(0)\ \&\ \theta'(0)$ differ only by the specified significant digit. The last value of η_∞ is finally chosen to be the most appropriate value of the limit $\eta \to \infty$ for that particular set of parameters. The value of η may change for another set of physical parameters. Once the finite value of η is determined then the coupled boundary value problem given by Eq. (5) - (11) are solved numerically using the method of superposition. In this method the third order Non-linear Eqs. (5) and second order Eqs.(11) have been reduced to five simultaneously ordinary differential equations as follows:

Let us call

$$\left.\begin{array}{l} y_1 = f \\ y_2 = f' \\ y_3 = f'' \\ y_4 = \theta \\ y_5 = \theta' \end{array}\right\} \tag{13}$$

The Boundary value problem is given by

$$\left.\begin{array}{l} \dfrac{dy_1}{d\eta} = y_2 \\ \dfrac{dy_2}{d\eta} = y_3 \\ \dfrac{dy_3}{d\eta} = \dfrac{(y_2{}^2 - y_1 y_3) + M\, y_2}{(1+\dfrac{1}{\beta})} \\ \dfrac{dy_4}{d\eta} = y_5 \\ \dfrac{dy_5}{d\eta} = -\Pr y_1 y_5 \end{array}\right\} \tag{14}$$

The boundary conditions now becomes for PST Case as:

$$y_1(0)=0, y_2(0)=1, y_3(0)=s_1, y_4(0)=1? \; y_5(0)=s_2, y_2(\infty)=0, y_4(\infty)=0 \;\}$$ (15)

Where s_1 & s_2 determined such that it satisfied $y_2(\infty)=0$ & $y_4(\infty)=0$. Thus, to solve this resultant system, we need five initial conditions, but we have only two initial conditions on f and one initial condition on θ.The third condition on f (i. e. f''(0))and second condition on θ (i,e $\theta'(0)$) are not prescribed which are to be determined by shooting method by using the initial guess values s_1 & s_2 until the boundary conditions $f_2(\infty)=0, f_4(\infty)=0$ (or $y_2(\infty)=0, y_4(\infty)=0$) are satisfied. In this way, we employ shooting technique with Runge-Kutta Fehlberg scheme to determine two more unknowns in order to convert the boundary value problem to initial value problem. Once all the five initial conditions are determined the resulting differential equations can then be easily integrated, without any iteration by initial value solver. For this purpose, Runge kutta scheme has been used In this manner any non lin solution of eqear equation involved in boundary value problem can easily be solved by this technique.

The Same procedure is followed to find the solution of Eq.(15) with PHF boundary conditions.

To study the behavior of the velocity and temperature profiles, curves are drawn for various values of the parameters that describe the flow.

5. Results and Discussion:

We have considered the flow and heat transfer of MHD Casson fluid due to linear stretching sheet. Here we have considered two types of heating conditions they are namely, prescribed surface temperature and prescribed wall heat flux. The Governing equation which are partial differential equations are solved numerically using Runge-Kutta method with efficient shooting techniques by converting them into ordinary differential equations by means of suitable similarity transformations.

Fig1: shows the geometry of the considered problem, which shows the heated plate, flow direction etc.

Fig.2: shows the influences of Casson parameter β on flow profile. We observe that the magnitude of flow in the boundary layer decreases with an increase in the Casson fluid parameter β. Also the effect of Casson parameter on velocity is also seen in Fig 3. Which depicts that, when Casson parameter β approaches infinity, the problem will reduce to a Newtonian case. Hence increasing value of Casson parameter β , decreases the velocity and boundary layer thickness. Hence velocity is decreases with increase in parametric value of Casson parameter β because of resistance created by Casson parameter in the fluid flow.

Fig (4) and (5) shows the effect of magnetic parameter Mn on the flow and velocity profiles respectively, on observing them we can notice that the effect of the magnetic parameter Mn on flow and the velocity profiles, As the magnetic parameter Mn increases, the flow and the velocity deceases with η. Thus the presence of the magnetic field reduces the momentum boundary layer thickness. Physically, the presence of a transverse magnetic (applied normally) field gives rise to a drag force which results in the flow and velocity retardation

Fig. (6): shows the effect of the Casson parameter β on the temperature distribution for PST case. The effect of increasing Casson parameter leads to an enhancement of the temperature due to increase in the velocity stress parameter, the same effect is observed in fig (9) for PHF case too.

In Fig (7), The increase in magnetic field parameter Mn then it reduces the boundary layer thickness and hence it enhances the temperature profile. same effect is observed in fig (10) for PHF case too.

Fig.(8) shows the effect of the Prandtl number Pr on temperature profile. On observing this plot we can conclude that the temperature and the thermal boundary layer thickness decrease as the Prandtl number increase, same effect is observed in fig (11) for PHF case too.

6. Conclusions

- Here numerical Solutions for MHD flow Casson fluid and heat transfer problems (for PST and PHF cases) are obtained.
- The effects of the Casson fluid parameter β on velocity and temperature are quite opposite.
- The thermal boundary layer thickness decreases with increasing Prandtl number on heat transfer phenomenon for both PST and PHF cases.
- When β tends to infinity, as reduce to results the Newtonian case

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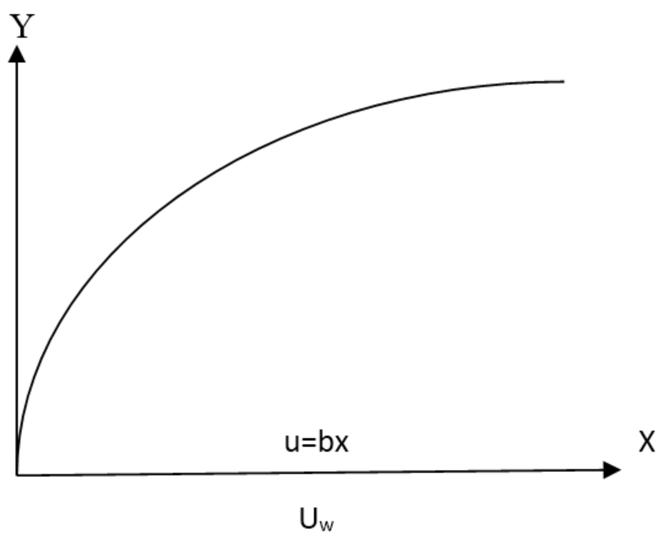

Fig. 1. Physical Configuration of Considered Problem

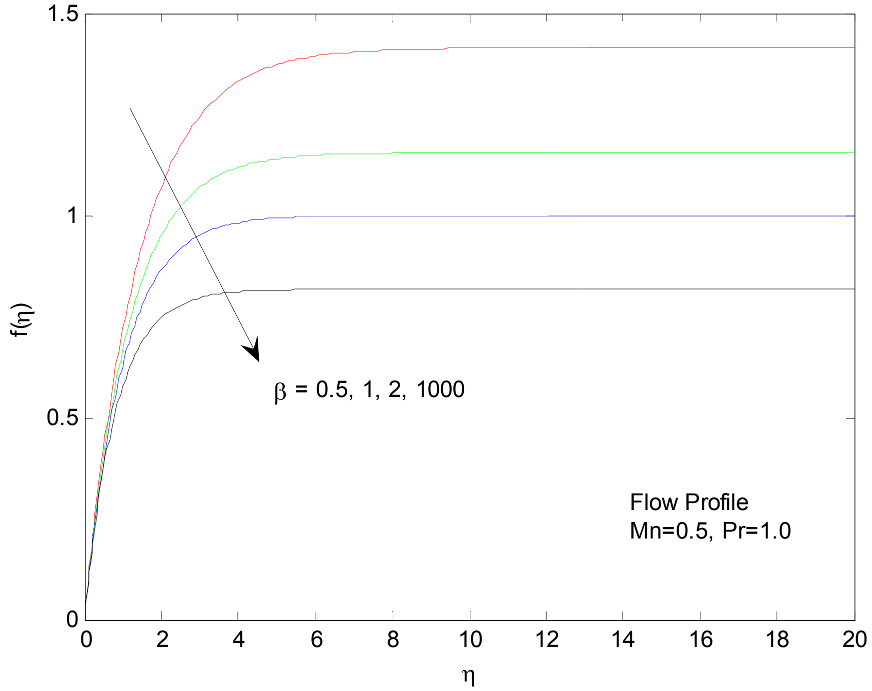

Fig 2. : Flow Profile for different values of Casson parameter

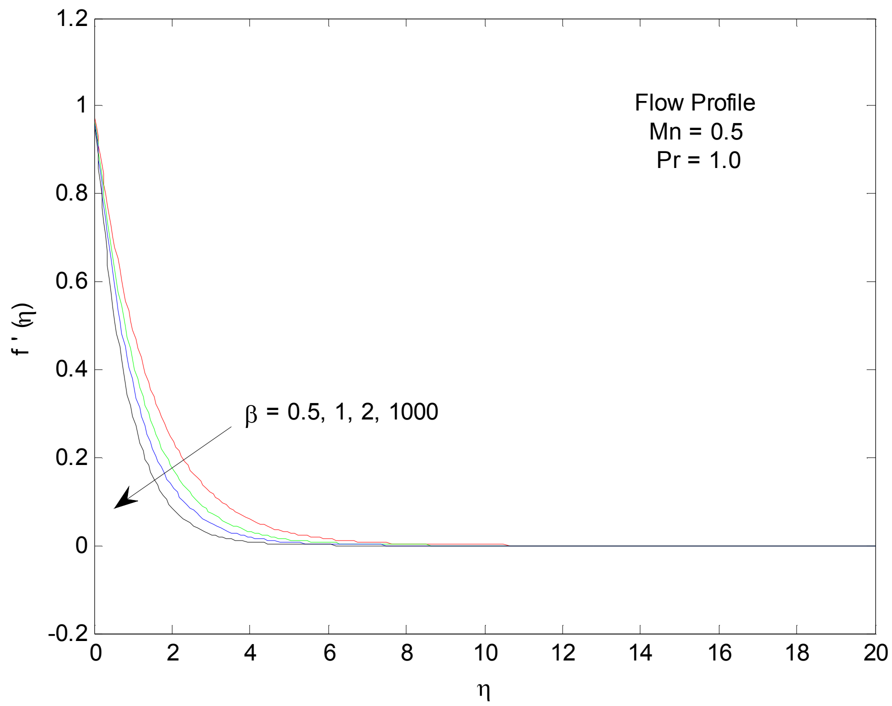

Fig 3. : Velocity Profile for different values of Casson parameter

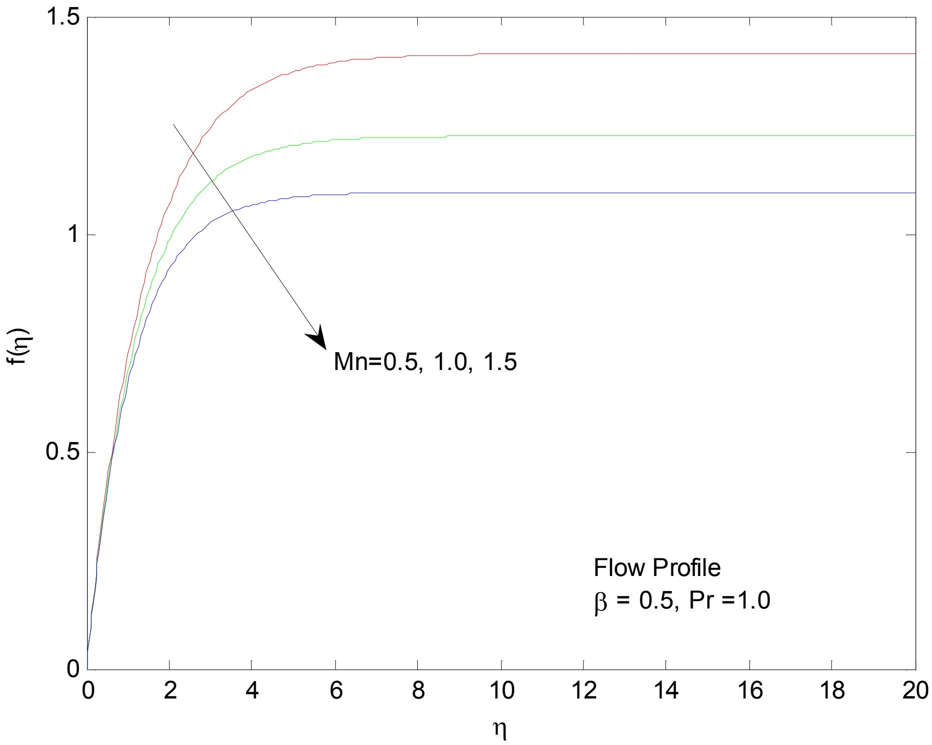

Fig 4: Flow Profile for Different values of Magnetic field parameter.

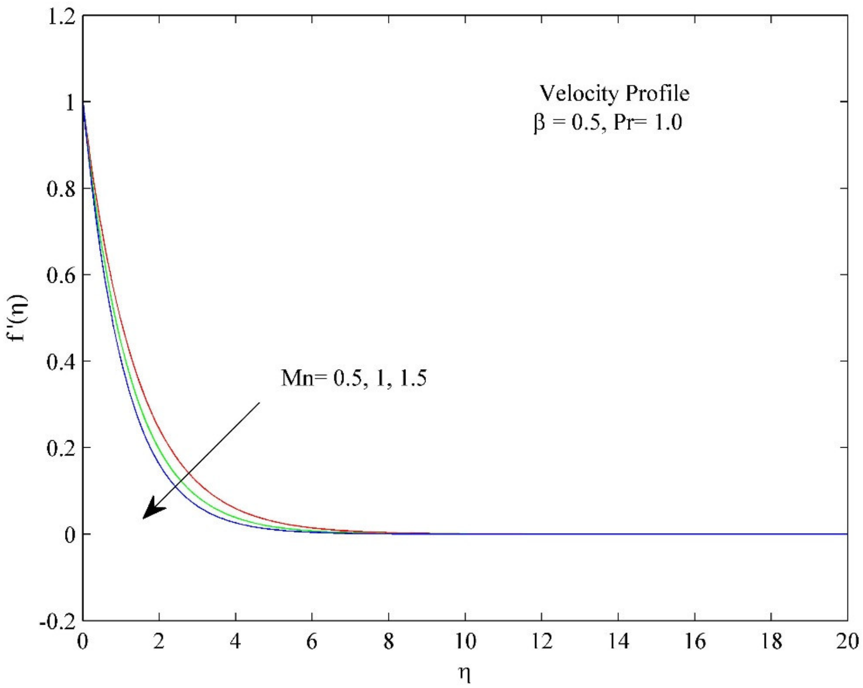

Fig 5: Velocity Profile for Different values of Magnetic field parameter

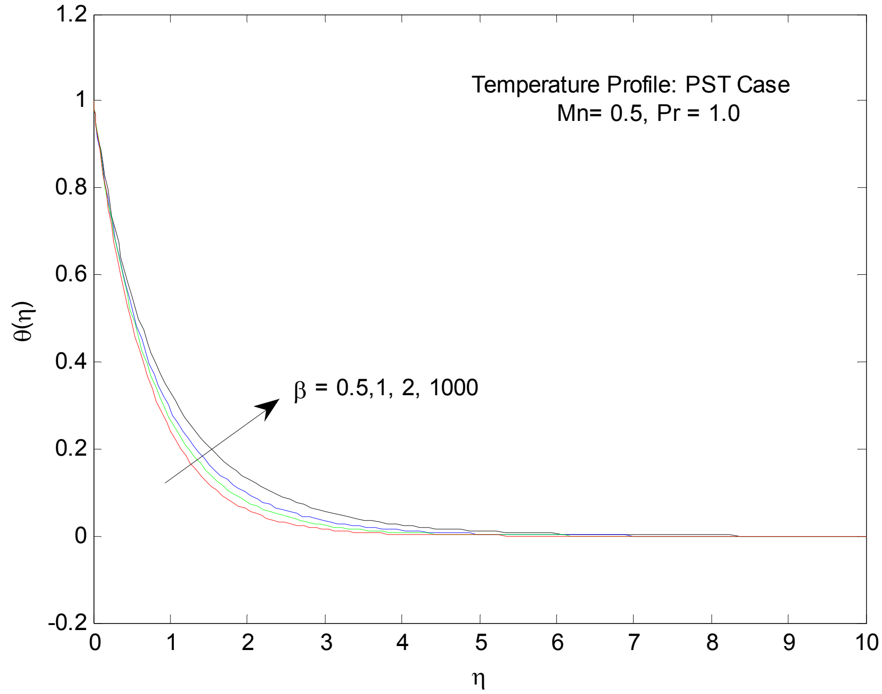

Fig 6.: Temperature Profile for different values of Casson parameter PST Case

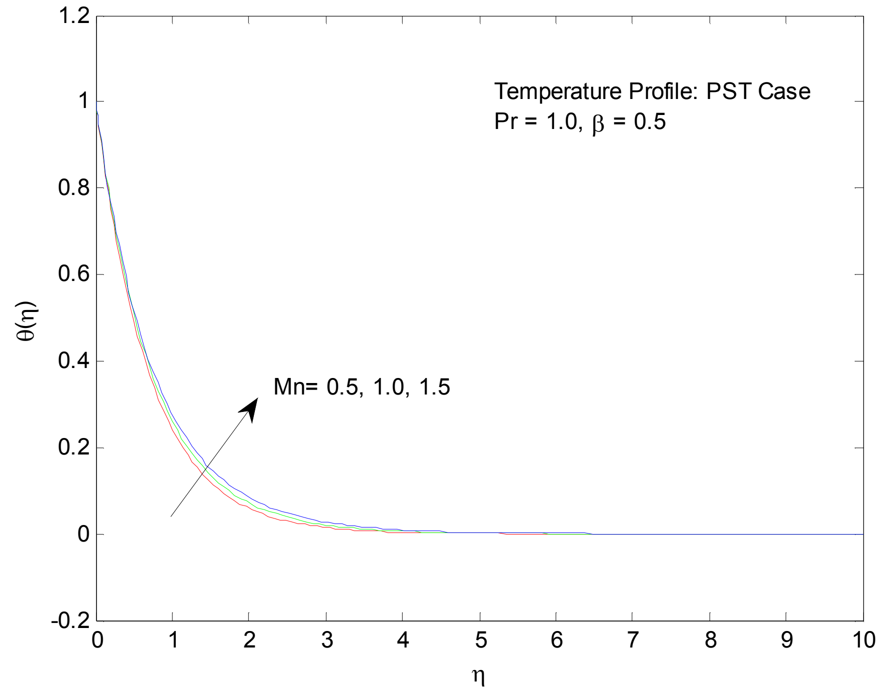

Fig 7.: Temperature Profile for different values of Magnetic field parameter in PST Case

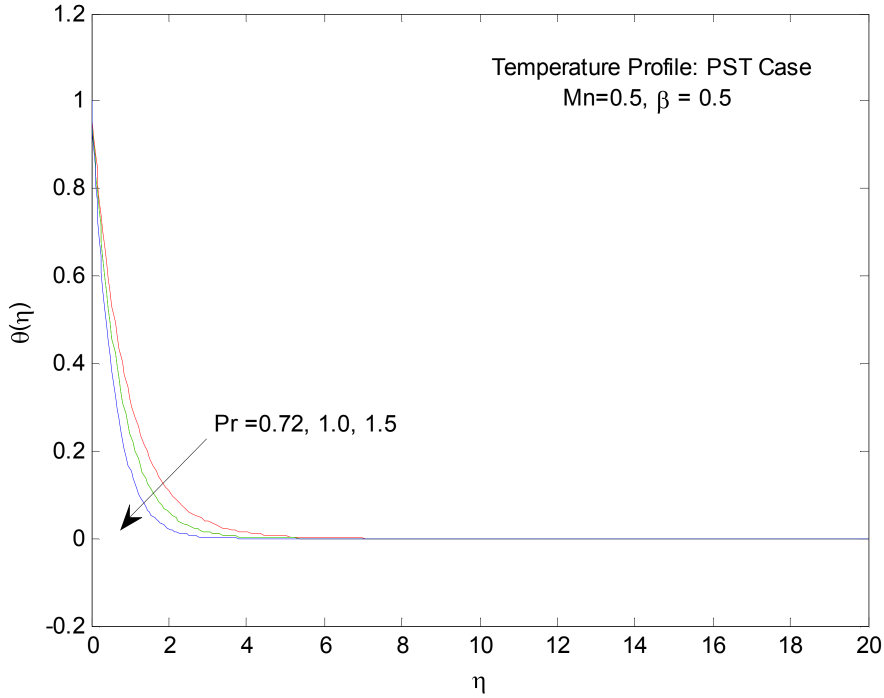

Fig 8.: Temperature Profile for different values of Prandtle number parameter in PST Case

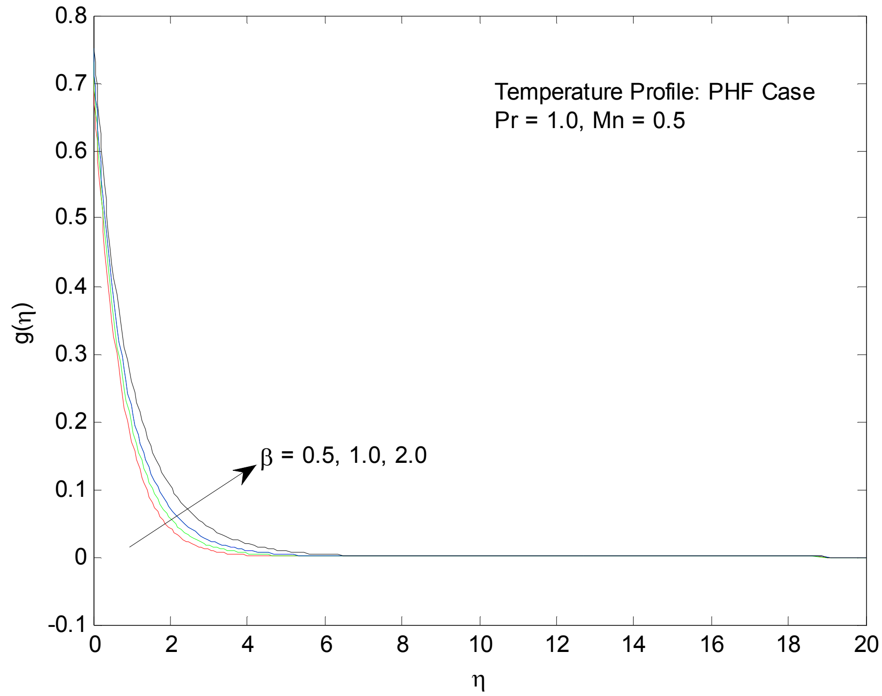

Fig 9.: Temperature Profile for different values of Casson parameter PHF Case

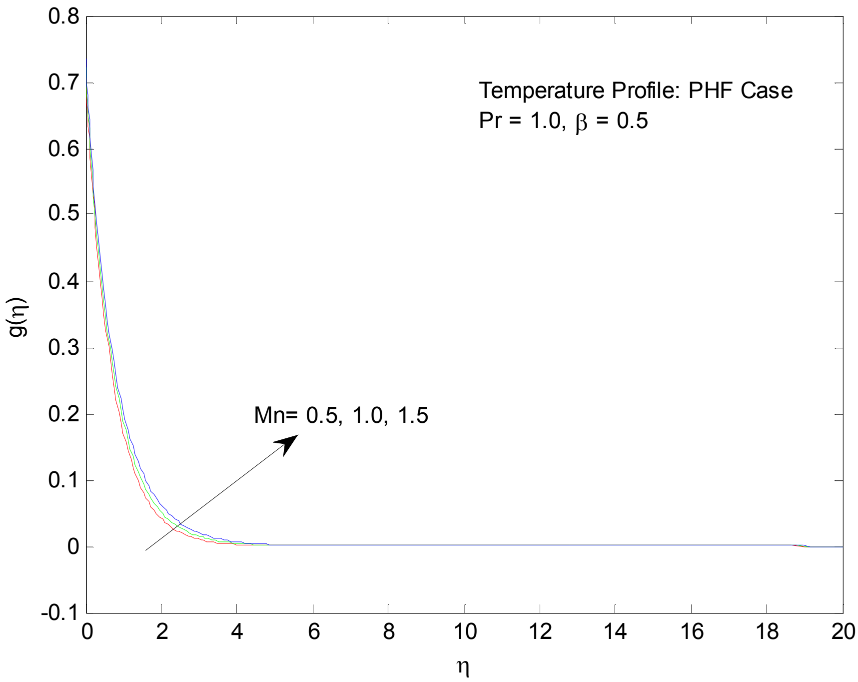

Fig 10.: Temperature Profile for different values of Magnetic field parameter in PHF case

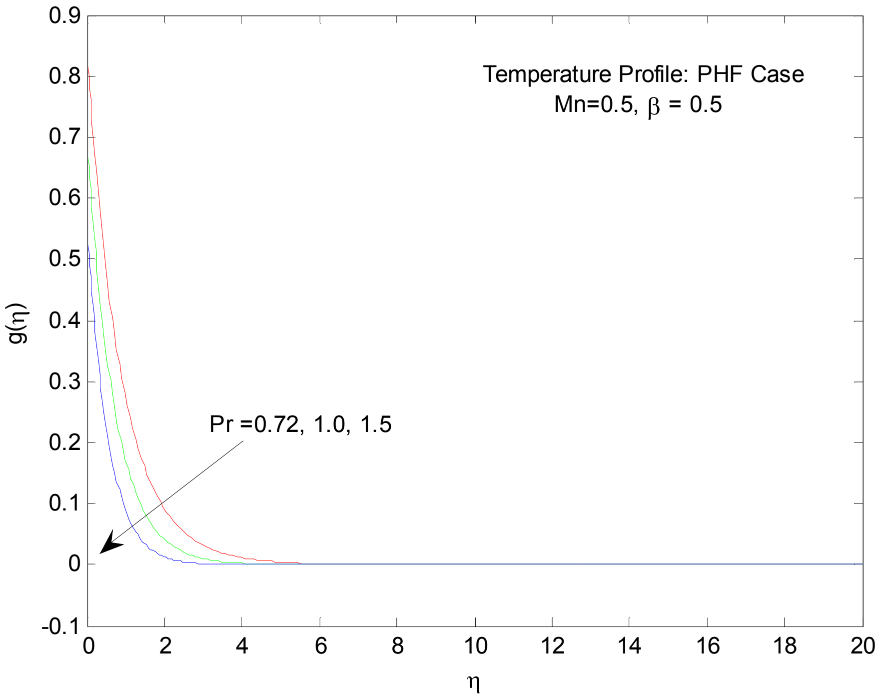

Fig 11: Temperature Profile for different values of Prandtle number parameter in PHF Case.