Entropy Generation Analysis for Porous Channel Flow with Asymmetric Slip and Thermal Boundary Conditions

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Abstract
This study investigates the inherent irreversibility in the steady flow of viscous incompressible fluid through a channel with infinite parallel porous plates. The system is assumed to exchange heat with the ambient surrounding following the Newton's law of cooling. Exact solution of the dimensionless governing equations are obtained and used to compute the entropy generation rate and the irreversibility ratio. The effects of slip and convective cooling parameters on the velocity, temperature and entropy generation profiles are presented and discussed extensively based on the physics of the fluid.
Keywords: asymmetric slip flow, asymmetric convective heating, entropy generation, irreversibility ratio, suction/injection, Poiseuille flow

1. Introduction
In a recent times, there has been a renewed interest in the thermodynamics analysis of a moving non-Newtonian fluid through a channel, due to its numerous applications in everyday lives. Interested readers can see the work [1]-[5]. The advent of nanotechnology has brought about reduction in sizes of many electronics devices thus giving room for possibility for flows through a micro-channel. It is well known that the no-slip condition is not valid in micro-channels and also when surfaces are electroplated with hydrophobic octadecyltrichlorosilane to avoid rusting and corrosion.

In actual fact literature is rich on studies related to slip flows ([6]-[21]) through a channel with the entropy analysis, hence, the objective of this paper is to investigate entropy generation in the porous micro-channel which exchange heat with the ambient following Newtonian cooling law. Basically, heat transfer to the thin fluid layer is irreversible. Therefore, by using the second law of thermodynamics, the entropy generation within the fluid layer can be minimized so as to enhance the exergy of the thermal system.

In the following section, the equations governing the fluid flow are formulated, non-dimensionalized and solved. The solution for the velocity and temperature fields are used to compute expressions for the entropy generation rate and the irreversibility ratio. In section three, the method of solution is described while section four deals with the discussion of results based on the physics of the problem while section five concludes the paper.

2. Mathematical Analysis
Consider the steady flow of viscous incompressible fluid through porous channels of distance $2L$ apart. The x - axis is taken horizontally in the direction of the flow while the y-axis is chosen normal to it. The flow is induced by change in the fluid pressure. And heat is generated by viscous dissipation and heating of the fluid. It is further assumed that the flow is thermodynamically and hydrodynamically developed, therefore, the temperature and velocity fields are functions of y alone. Based on these assumptions, the basic equations governing the slip flow fluid can be written as

$$\left.\begin{aligned} &-\frac{dP}{dx}+\rho v_0\frac{du'}{dy'}+\mu\frac{d^2u'}{dy'^2}=0\\ &k\frac{d^2T}{dy'^2}+v_0\rho C_P\frac{dT}{dy'}+\mu\left(\frac{du'}{dy'}\right)^2=0\end{aligned}\right\} \tag{1}$$

subject to the asymmetrical slip and convective boundary conditions

$$\begin{aligned} &u'=\beta_0\frac{du'}{dy'},\ k\frac{dT'}{dy'}=\gamma_0\left(T-T_0\right)\quad on\quad y'=-L\\ &u'=-\beta_1\frac{du'}{dy'},\ -k\frac{dT'}{dy'}=\gamma_1\left(T-T_f\right)\quad on\quad y'=L\end{aligned} \tag{2}$$

where $\beta_{0,1}$ are the Navier slip coefficients at the walls, μ is the fluid viscosity, u' is the axial velocity, P is the fluid pressure, ρ - is the fluid density, v_0 measures the channel porosity due to suction and injection. C_P

measures the specific heat capacity of the fluid, (T, k) are the fluid dimensional fluid temperature and thermal conductivity of the material respectively, (T_0, T_1) are referenced fluid temperatures and $\gamma_{0,1}$ measures the Newtonian cooling rate at the walls. Introducing the following dimensionless variables

$$y=\frac{y'}{a},\ u=\frac{u'}{U},\ G=\frac{-L^2}{\mu U},\quad S=\frac{v_0 L}{\upsilon},\ \theta=\frac{T-T_0}{T_f-T_0},\ Pe=\frac{v_0\rho C_P L}{k},\ Br=\frac{\mu U^2}{k(T_1-T_0)} \tag{3}$$

we obtain the following nonlinear ordinary differential equations with appropriate boundary conditions

$$\begin{gathered}1+S\frac{du}{dy}+\frac{d^2u}{dy^2}=0\\ \frac{d^2\theta}{dy^2}+Pe\frac{d\theta}{dy}+Br\left(\frac{du}{dy}\right)^2=0\end{gathered} \tag{4}$$

Subject to the boundary conditions

$$\left.\begin{aligned}u=\alpha_0\frac{du}{dy},\qquad \frac{d\theta}{dy}=Bi_0\theta\quad on\quad y=-1\\ u=-\alpha_1\frac{du}{dy},\qquad \frac{d\theta}{dy}=-Bi_1\theta\quad on\quad y=1\end{aligned}\right\} \tag{5}$$

where u is the dimensionless fluid velocity, θ is the dimensionless fluid temperature, $\alpha_{0,1}$ are the dimensionless slip parameters at the walls, s is the fluid suction/injection parameter due to channel porosity, *Pe* is the Peclet number, *Br* is the Brinkman number while $Bi_{0,1}$ are the Biot's numbers.

In view of the foregoing, the exact solution for the velocity profile can be written as

$$\begin{aligned}u(y)=\frac{e^{sy}}{s\left(-1+e^{2s}(1+s\alpha_0+s\alpha_1)\right)}\Big\{&-2e^{s}-(y-1)e^{s(2+y)}+e^{2y}(1+y)-\alpha_1\left(e^{s}-e^{s(2+y)}\right)\\ &-\alpha_1 se^{sy}(1+y)+\alpha_0\left(-e^{s}+e^{sy}-s(y-1)e^{s(2+y)}\right)+s\alpha_1e^{sy}\left(-1+e^{2s}\right)\Big\}\end{aligned} \tag{14}$$

Observe that the limiting case as $\alpha_{0,1}\to\infty$ corresponds to the perfect lubricated plate surface

Substituting (14) in the dimensionless energy equation (7) and implemented on a computer algebra package-MATHEMATICA, a huge size of symbolic solution is obtained. As a result, only the graphical solution will be presented in section 4 of the paper.

3. Entropy generation analysis

The general equation for the entropy generation per unit volume is given by

$$S^m=\frac{k}{T_0^2}(\nabla T)^2+\frac{\mu}{T_0}\left\{2\left[\left(\frac{\partial u'}{\partial x'}\right)^2+\left(\frac{\partial v'}{\partial y'}\right)^2\right]+\left[\frac{\partial u'}{\partial y'}+\frac{\partial v'}{\partial x'}\right]^2\right\} \tag{15}$$

Where the first term is due to irreversibility due to heat transfer while the second term represents the irreversibility due to fluid friction. However, for a thermally and hydrodynamically developed flow is given by

$$Q^m=\frac{k}{T_0^2}\left(\frac{dT}{dy'}\right)^2+\frac{\mu}{T_0}\left(\frac{du'}{dy'}\right)^2 \tag{16}$$

In dimensionless from, we have

$$N_S=\frac{Q^m T_0^2L^2}{k(T_1-T_0)^2}=\left(\frac{d\theta}{dy}\right)^2+\frac{Br}{\Omega}\left(\frac{du}{dy}\right)^2 \tag{17}$$

Where N_S, $\Omega=\dfrac{T_1-T_0}{T_0}$ represents the dimensionless entropy generation and temperature difference parameters respectively.

The total entropy generated within the flow channel can then be written as

$$N_T = \int_{-1}^{1}\left(\left(\frac{d\theta}{dY}\right)^2 + \frac{Br}{\Omega}\left(\frac{du}{dY}\right)^2\right)dY \tag{18}$$

As shown by Bejan, the irreversibility distribution ratio (Φ) is the ratio of entropy generation due to fluid friction and entropy generation due to heat transfer. Hence, we denote

$$N_1 = \left(\frac{d\theta}{dy}\right)^2, \quad N_2 = \frac{Br}{\Omega}\left(\frac{du}{dy}\right) \tag{19}$$

Such that $N_S = N_1 + N_2$, where N_1, N_2 represents the irreversibility due to heat transfer and fluid friction respectively.

Then the irreversibility ratio can be written as

$$Be = \frac{N_1}{N_1 + N_2} = \frac{1}{1+\Phi} \tag{20}$$

From (20), it is evident that Φ is bounded i.e. $0 \le \Phi \le 1$. i.e. when Be = 0, the fluid friction irreversibility dominates over irreversibility due to heat transfer and when Be = 1 corresponds to the case when irreversibility due to heat transfer dominates over irreversibility due to fluid friction. The special case when Be = 0.5 implies equal contributions to the entropy generation rate.

4. Discussion of result

In this section, the numerical result of the solution of (14), (17) and (20) are presented graphically and discussed. Fig. 1a-d represents the velocity profile of the fluid with variations in some of the flow parameters. In Fig 1a the result shows that an increase in the lower Navier slip parameter enhances the flow at the lower plate. Similar behaviour is seen as the upper slip parameter is increased while in Fig 1c the result of increase in the suction parameter is seen on the flow symmetry. Fig 2a-g shows the response of the fluid temperature distributions to variation of parameters. In all these variations, the graphical solutions are well behaved. The entropy generation rate is shown in Fig. 3 with variation in different fluid parameters. From the result is observed that both slip and convective parameters have significant influence on the flow and cannot be neglected while the irreversibility in the heat flow is show in Fig. 4 a-e.

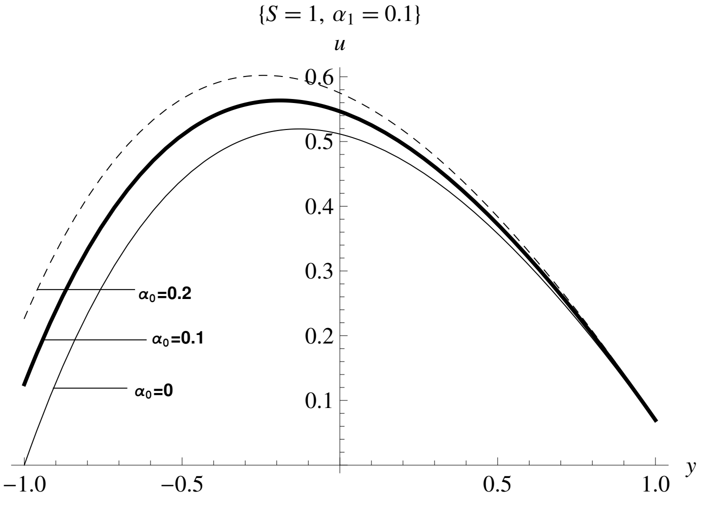

Fig. 1a: Effect of lower slip parameter on the fluid flow

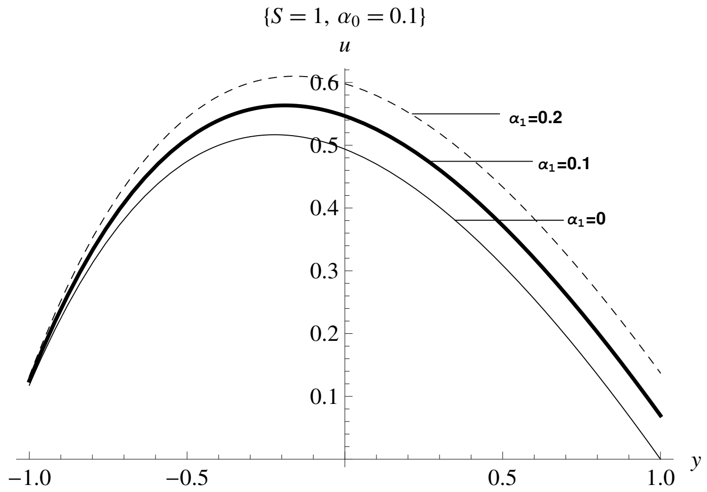

Fig. 1b: Effect of upper slip parameter on the fluid flow

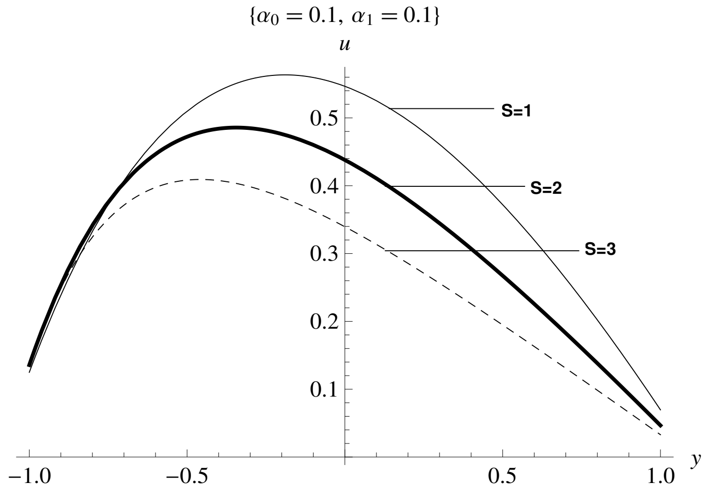

Fig. 1c: Effect of suction parameter on the fluid flow

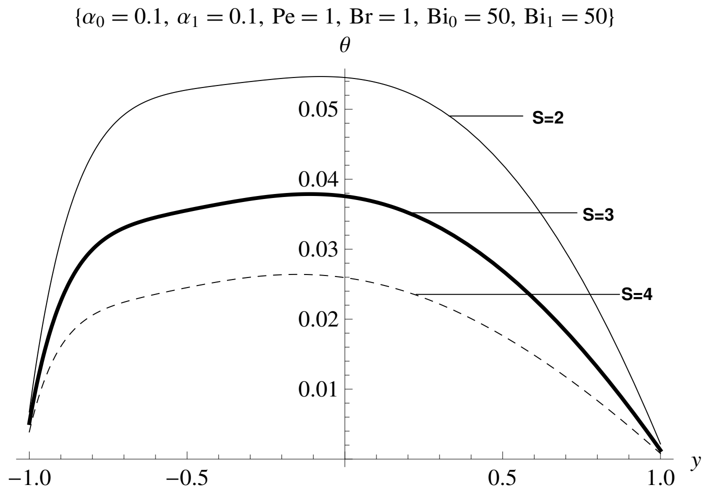

Fig. 2a: Effect of suction parameter on the temperature distribution

	

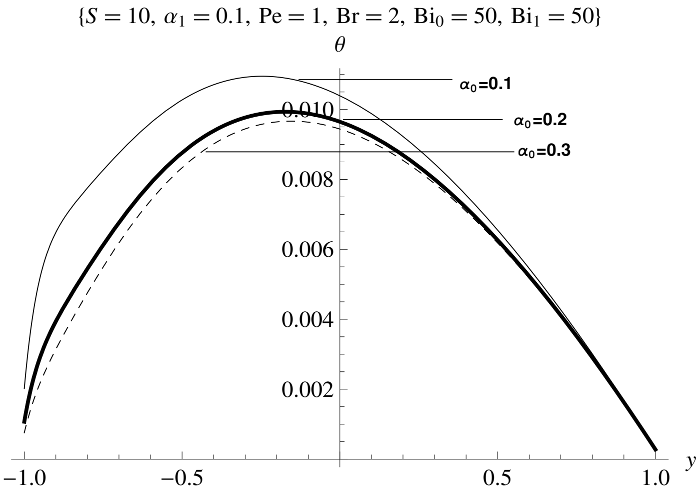

Fig. 2b: Effect of lower slip parameter on the temperature distribution

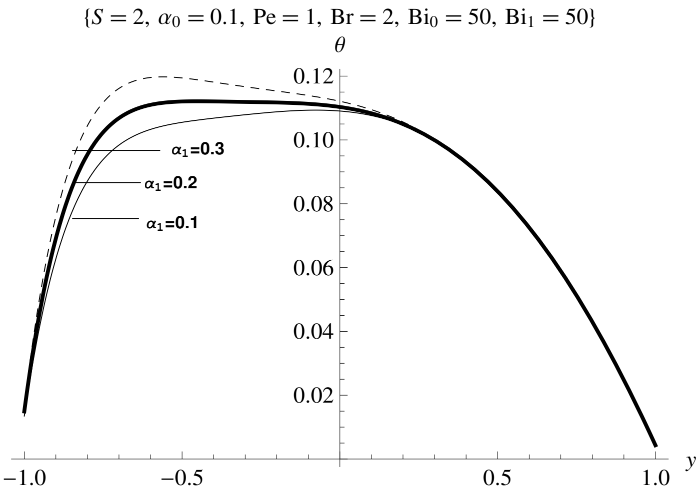

Fig. 2c: Effect of upper slip parameter on the temperature distribution

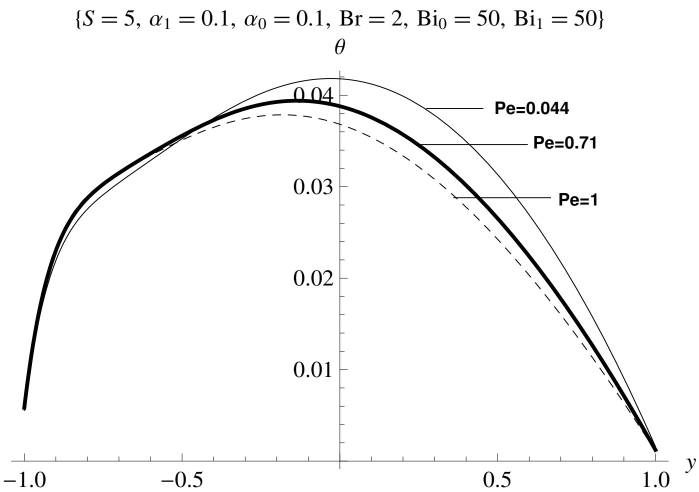

Fig. 2d: Effect of Peclet number on the temperature distribution

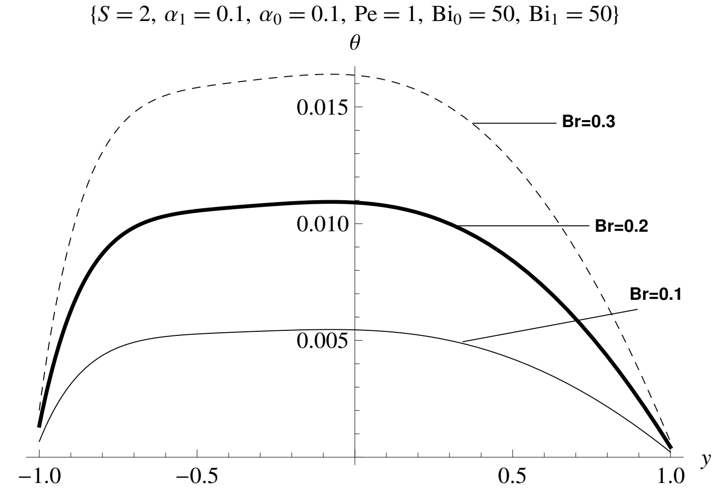

Fig. 2e: Effect of Brinkman number on the temperature distribution

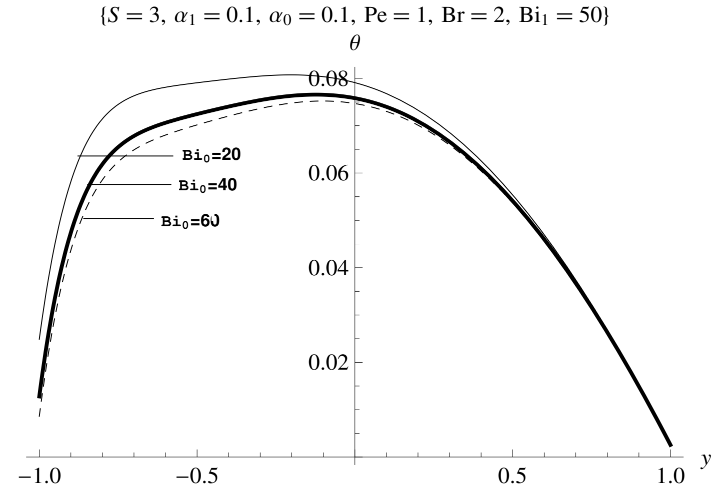

Fig. 2f: Effect of lower Biot number on the temperature distribution

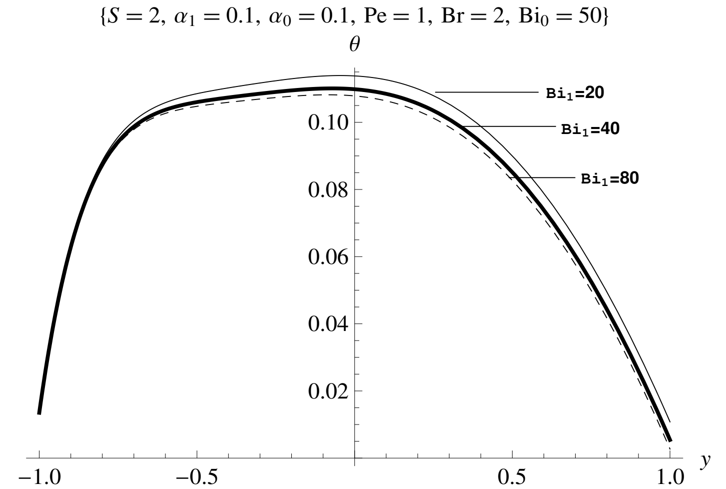

Fig. 2g: Effect of upper Biot number on the temperature distribution

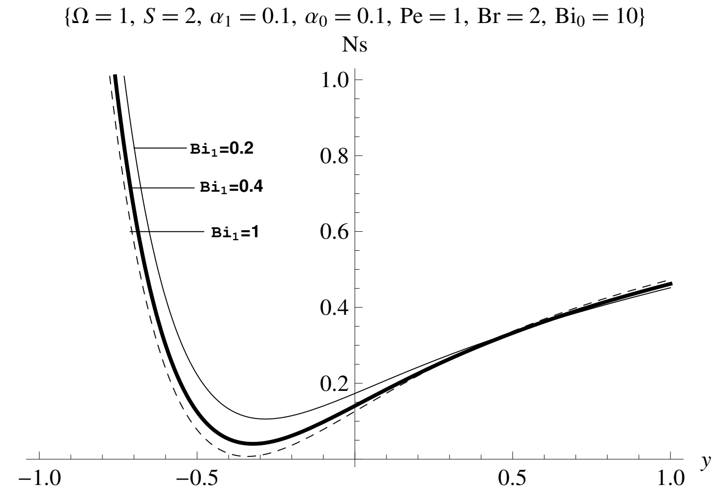

Fig. 3a: Effect of upper Biot number on the Entropy generation rate

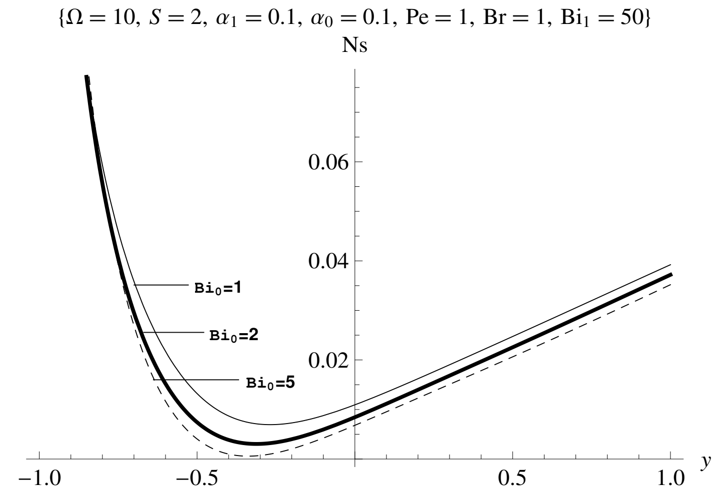

Fig. 3b: Effect of lower Biot number on the Entropy generation rate

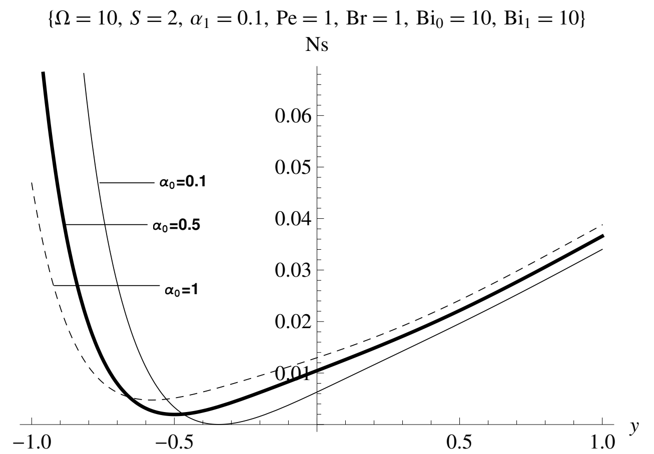

Fig. 3c: Effect of lower slip parameter on the Entropy generation rate

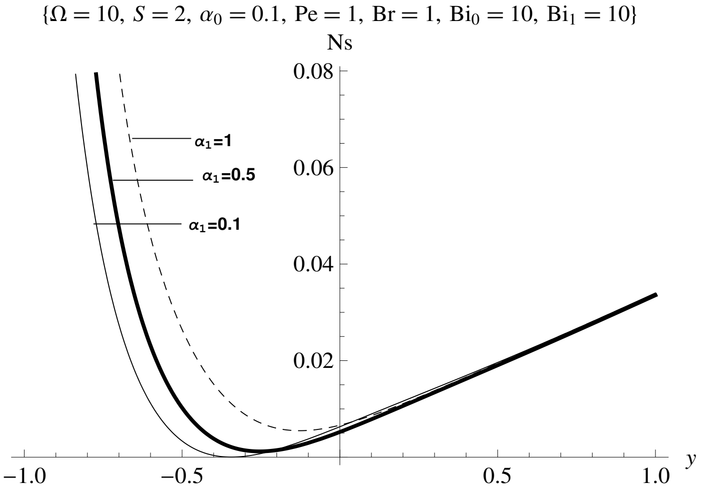

Fig. 3d: Effect of upper slip parameter on the Entropy generation rate

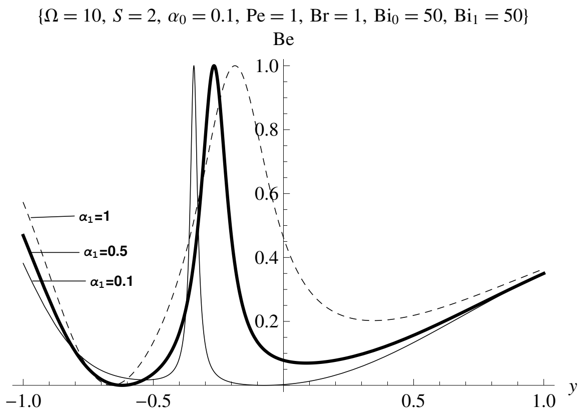

Fig. 4a :Effect of lower slip parameter on the irreversibility ratio

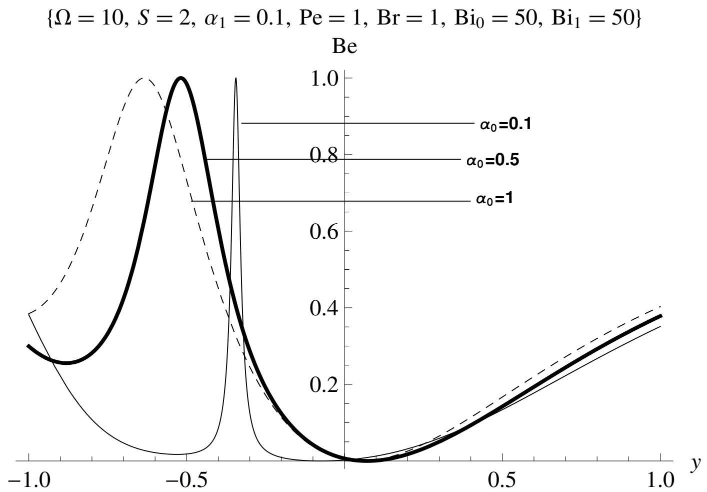

Fig. 4b: Effect of upper slip parameter on the irreversibility ratio

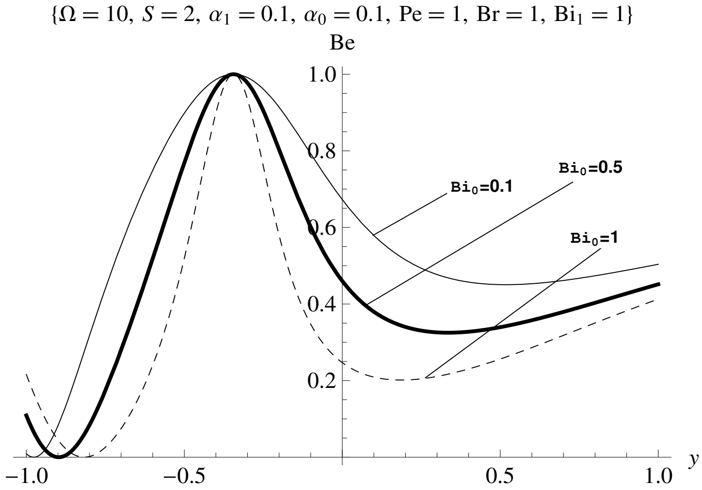

Fig. 4c: Effect of lower Biot number on the irreversibility ratio

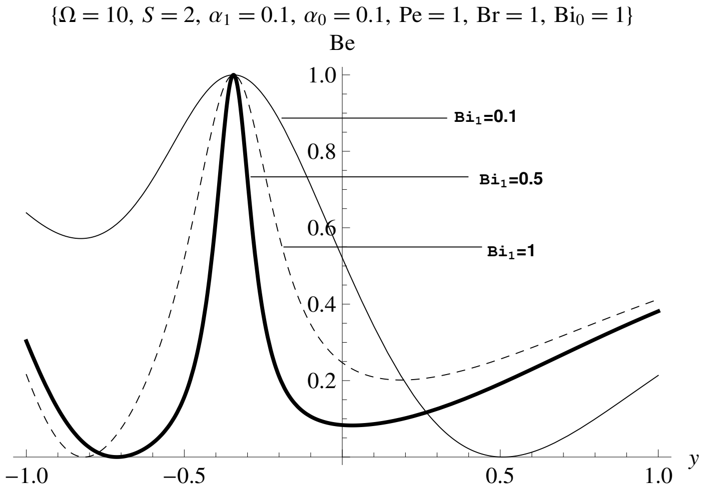

Fig. 4d: Effect of upper Biot number on the irreversibility ratio

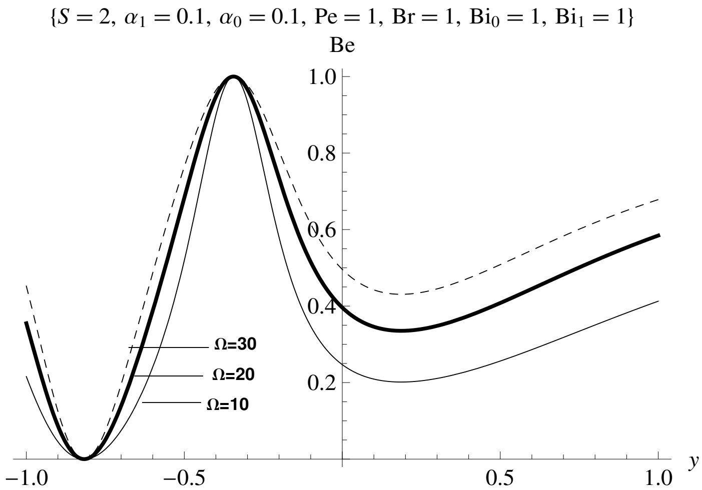

Fig. 4e: Effect of temperature difference parameter on the irreversibility ratio

5. Conclusion

The steady flow of Newtonian fluid flow through a porous channel with slip and thermal boundary condition is considered. The equations governing the fluid flow are formulated, non-dimensionalized and solved. Expression for the entropy generation rate due to fluid friction and heat transfer is also formulated. From the result it is found that slip and convective conditions do not only have significant effect on the flow and temperature fields, it also affects the entropy generation rate and the irreversibility of heat within the flow channel.

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