MHD Free Convection Boundary Layer Flow of a Nanofluid over a Permeable Shrinking Sheet in the Presence of Thermal Radiation and Chemical Reaction

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Abstract
The objective of this work is to analyze the chemical reaction and thermal radiation effects on MHD free convection boundary layer low of a nanofluid over a permeable shrinking sheet. The model used for the nanofluid incorporates with Brownian and thermophoresis effects. A mathematical formulation has designed for momentum, temperature and nanofluid solid volume profiles. The highly nonlinear coupled partial differential equations are simplified with the help of suitable similarity transformations. The reduced partial differential equations are solved using the Method of line. The entire computation procedure is implemented using a program written and carried out using Mathematica computer language. The results for the dimensionless velocity, temperature, and nanofluid solid volume profiles are discussed with the help of graphs and tables. We have examined the effects of various controlling flow parameters namely unsteadiness parameter A, magnetic parameter M, thermal radiation parameter R, Prandtl number Pr, Brownian motion parameter Nb, thermophoresis parameter Nt, Radiation parameter Nr and Lewis number Le on the dimensionless velocity, temperature and nanoparticle volume fraction profiles..
Keywords: Nanofluid, Magentohydrodynamics (MHD), permeable shrinking sheet, chemical reaction, heat source, Thermal Radiation.

1. Introduction

Most interesting research in the field is that MHD free convection flow due to a shrinking sheet. The shrinking sheet situation occurs, for example, on a rising shrinking balloon. One of the common applications of shrinking sheet problems in engineering and industries is shrinking film. In packaging of bulk products, shrinking film is very useful as it can be unwrapped easily with adequate heat. Especially, the analysis of boundary layer flow of an unsteady nanofluid across a stretching sheet has gained attention of many researchers. Nowadays, a large amount of research work has been placed to focus on this topic in view of its several applications in engineering and industrial processes. The cooling of electronic devices by the fan and nuclear reactor, polymer extrusion, wire drawing, etc are examples of such flows in engineering and industrial processes. The list of importance of flows in fluid mechanics has motivated researchers to continue the study in different types of fluid as well as in different physical aspects. The boundary layer flow of an electrically conducting fluid in the presence of magnetic field has wide applications in many engineering problems such as MHD generator, plasma studies, nuclear reactors, geothermal energy extraction, and oil exploration. The term nanofluid has been introduced by Choi [1].This novel fluid have been used potentially in numerous application in heat and mass transfer as well as micro electronics, fuel cells, Pharmaceutical sections. Sakisdis [2] was the first another to analyze the boundary layer flow on a continuous moving surface. Buongiorno [3] was first who formulated the nanofluid model taking into account the effects of Brownian motion and thermophoresis.

The boundary layer flow over a shrinking surface is encountered in several technological processes. Such situations occur in polymer processing, manufacturing of glass sheets, paper production, in textile industries and many others. Crane [4] initiated a study on the boundary layer flow of a viscous fluid towards a linear stretching sheet. An exact similarity solution for the dimensionless differential system was obtained. Carragher and Carane [5] discussed heat transfer on a continuous stretching sheet. Afterwards, many investigations were made to examine flow over a stretching/shrinking sheet under different aspects of MHD, suction/injection, heat and mass transfer etc. [6–13]. In these attempts, the boundary layer flow, due to stretching/shrinking has been analyzed. Magyari and Keller [14] provided both analytical and numerical solutions for boundary layer flow over an exponentially stretching surface with an exponential temperature distribution. The combined effects of viscous dissipation and mixed convection on the flow of a viscous fluid over an exponentially stretching sheet were analyzed by Partha et al. [15], Elbashbeshy [16] numerically studied flow and heat transfer over an exponentially stretching surface with wall mass suction. Recently, the boundary layer flow near a shrinking sheet gets attention due to increasing engineering applications. The existence and uniqueness of steady viscous flow due to a shrinking sheet was established by Miklavcic and Wang [17] and they concluded that for some specific value of suction at the sheet, dual solutions exist and also in certain range of value of suction, no boundary layer solution exists.

Very recently, the unsteady flow and heat transfer over an unsteady shrinking sheet with suction in a nanofluid was analyzed by Rohni et al. [18] and it was found that dual solutions exist for a certain range of wall mass suction, unsteadiness and nanofluid parameters. Samir Kumar Nandy et al [19] discussed Unsteady MHD boundary-layer flow and heat transfer of nanofluid over a permeable shrinking sheet in the presence of thermal radiation.Srinivas et al [20] examined an unsteady MHD flow and heat transfer of nanofluid over a permeable shrinking sheet with thermal radiation and chemical reaction. The aim of the present paper (which is an extension of Srinivas et al. [20]) is to study the MHD Free convection boundary layer flow of a nanofluid over a permeable shrinking sheet in the presence of thermal radiation and chemical reaction. In this paper, a similarity analysis is performed to reduce the governing equations to ordinary differential equations which are subsequently solved numerically using Method of line in Mathematica tool. The effects of governing parameters on fluid velocity, temperature and particle concentration have been discussed and shown graphically and tables as well.The present study is of immediate interest to all those which are highly affected with heat enhancement concept.

2. Mathematical Formulation

Consider a two dimensional unsteady magneto hydrodynamic boundary layer flow of a nanofluid which is electricity conducting (σ) and past a permeable shrinking sheet with the uniform velocity U in the presence of magnetic field of strength B0 is applied parallel to the y axis and chemical reaction with thermal radiation is considered in the flow region it is assumed that the induced magnetic field, the external electric field are negligible due to polarization of charges. Let us consider x axis to be focused along the shrinking sheet and y axis normal to the surface.

The governing equation of conservation of mass, momentum, energy and nano particle volume in the presence of magnetic field towards a permeable shrinking sheet can be written in Cartesian coordinates x & y.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+v\,\frac{\partial v}{\partial y} = v\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{k_p}u-\frac{\sigma_e B_0^2 u}{\rho}+(1-C_\infty)\rho f_\infty \beta g(T-T_\infty)-(\rho_f-\rho f_\infty)g(C-C_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t}+u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\propto\frac{\partial T}{\partial y}\right)+\tau\left\{D_B\left(\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}+\frac{Q_0}{(\rho c)_f}(T-T_\infty)-\frac{1}{(\rho c)_f}\left(\frac{\partial q_r}{\partial y}\right) \tag{3}$$

$$\frac{\partial C}{\partial t}+u\,\frac{\partial C}{\partial x}+v\,\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-K_r(C-C_\infty) \tag{4}$$

And the Boundary conditions are:

$$u=u_w(x,t)=-\frac{cx}{(1-\lambda t)},\ v=v_w(x,t),\ T=T_w(x,t),\ C=C_w(x,t)\ at\ y=0$$
$$u=0\ ,T\to T_\infty\ ,C\to C_\infty\ as\ y\to\infty \tag{5}$$

The wall mass transfer velocity then becomes

$$v_w(x,t)=-\sqrt{\frac{c}{(1-\lambda t)}}\,s \tag{6}$$

Where s is the constant wall mass transfer parameter with $s > 0$ for suction and $s < 0$ for injection, respectively.

Where u and v are velocity components along x and y directions, α is a thermal diffusivity, Q_0 is a heat generation coefficient, ρ is the density of nanofluid, ρ_p is the nanoparticle density, ρ_c is specific heat of nanofluid at constant pressure, τ is the ratio of nanoparticle heat capacity, σ_e is the electrical conductivity, C_p is the specific heat and constant pressure, β is volumetric thermal expansion coefficient, μ is the thermal viscosity, D_B is the Brownian diffusion coefficient , D_T is the thermophoresis diffusion coefficient and Kr is the rate of chemical reaction.

The Radiative heat flux term by using The Rosseland approximation is given by

$$q = \frac{4\sigma^*}{3k_1^*}\frac{\partial T^{*^2}}{\partial y^*} \tag{7}$$

Where σ^* and k_1^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. We assume that the temperature difference within the flow are sufficiently small such that may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor Series about and neglecting higher order terms. Thus,

$$T^{*4} \cong 4T_\infty^{*4} - 3T_\infty^{*4} \tag{8}$$

By using equation (6) and (7), into equation (3) is reduced to

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\propto \frac{\partial T}{\partial y}\right) + \tau\left\{D_B\left(\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} + \frac{Q_0}{(\rho c)_f}(T - T_\infty) - \frac{1}{(\rho c)_f}\left(-\frac{16\sigma^* T_\infty^{*^3}}{3k_1^*}\right)\frac{\partial^2 T^*}{\partial y^2} \tag{9}$$

The equations (2), (4) and (9) can be transformed into the ordinary differential equation by using the following similarity transformations.

$$\eta = y\sqrt{\frac{c}{v(1-\lambda t)}}, \qquad \psi = \sqrt{\frac{cv}{v(1-\lambda t)}}\; x\, f(\eta) \tag{10}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

And the stream function $\psi(x, y)$ is defined such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{11}$$

From the above transformations the non-dimensional, non-linear, coupled differential equations are obtained as: (12)

$$f''' + ff'' - f'^2 - A\left(f' + \frac{\eta}{2}f''\right) - Mf' - \delta f + Ra_x(\theta - Nr\varphi) = 0$$

$$\frac{1}{Pr_{eff}}\theta'' + f\,\theta' - A\frac{\eta}{2}\theta' + Nb\varphi'\theta' + Nt\,\theta'^2 = 0 \tag{13}$$

$$\varphi'' + Le\left(f' - \frac{\eta}{2}f''\right)\varphi' + \left(\frac{Nt}{Nb}\right)\theta'' + \gamma\varphi = 0 \tag{14}$$

Where

$$Pr_{eff} = \frac{Pr}{(1 + \frac{4R}{3})}\,(Prandtl\ number)$$

$$A = \frac{\lambda}{c}\ (Heat\ Source\ Parameter)$$

$$\lambda = \frac{Q_0 x^{1/2}}{(\rho c)_f \sqrt{(1-C_\infty) g\beta(T_w - T_\infty)}} \text{(Heat Source Parameter)}$$

$$M = \frac{\sigma_e B_0^2}{(\rho C)_f} \text{ (Magnetic Parameter)}$$

$$\delta = \frac{\mu}{k_\rho} u \text{ (Permeable Parameter)}$$

$$v = \frac{\mu}{\rho_f} \text{ (Kinematic viscocity)}$$

$$Le = \frac{\alpha}{D_B} \text{(Lewis number)}$$

$$R = \frac{4\sigma^* {T_\infty}^3}{k\, \alpha_m \rho_f\, c} \text{(Radiation parameter)}$$

$$Ra_x = \frac{(1 - C_\infty)\beta g f_\infty (T_w - T_\infty)}{c^2 x} \text{(Local Rayleigh Number)}$$

$$Nr = \frac{\left(\rho_p - \rho_{f_\infty}\right)\beta g f_\infty (C_w - C_\infty)}{(1 - C_\infty) f_\infty \beta (T_w - T_\infty)} \text{(Buoyancy ratio parameter)}$$

$$Nb = \frac{\tau D_B (C_w - C_\infty)}{v} \text{ (Brownian motion parameter)}$$

$$Nt = \frac{\tau D_T (T_w - T_\infty)}{v T_\infty} \text{ (Thermo phoresis parameter)}$$

$$Re_x = \frac{a x^2}{v_\infty} \text{ (Local Reynolds Number)}$$

$$\gamma = \frac{ku(C_w - C_\infty)}{v} \text{ (Chemical reaction Parameter)}$$

The corresponding Boundary conditions are

$$f = s, f' = -1, \theta = 1, \varphi = 1 \text{ at } \eta = 0$$
$$f = 0, \theta = 0, \varphi = 0 \text{ as } \eta \to \infty \quad (15)$$

The physical quantities of Skin friction C_f, the local Nusselt number Nu, and the local Sherwood number Sh are calculated by the following equations:

$$\left.\begin{aligned} C_f (Re_x)^{-1/2} &= f''(0) \\ N_u (Re_x)^{-1/2} &= -\theta'(0) \text{ and} \\ S_h (Re_x)^{-1/2} &= -\varphi'(0) \end{aligned}\right\} \quad (16)$$

3. Numerical Analysis

The set of non-dimensional, non-linear couple boundary layer equations (12) – (14) subject to boundary conditions (15) are non-linear and possess no analytical solution and must be solved numerically. The governing equations are solved by using Method of line .The code of the algorithm has been executed in MATHEMATICA running on PC.In this study, we get The validity of the present computations has been confirmed via benchmarking with several earlier studies. Excellent convergence was achieved for all the results.

4. Results and Discussions

The heat and mass transfer problem associated with MHD flow of the nanofluids over a permeable shrinking sheet in the presence of thermal radiation and chemical reaction has been studied. Table 1 indicates the values of skin friction, Nusselt Number and Sherwood Number for different values of the physical parameters. The present results are compared with that of Rohni et al.[18] and Samir et al.[19] (reduced cases) and found that there is an excellent agreement (Table 1)

The variations in velocity field, temperature distribution and concentration profiles with the effect of magnetic field parameter M is monitored in Fig.1-3 respectively. It is seen in Fig.1 that the effect of magnetic field M increases, the dimensionless velocity profiles increases and Fig.2 the reverse phenomenon observed in the temperature distribution. From Fig.3, it can be observed that increase the magnetic field M effect is to decreases the concentration profiles.

Fig.4-6 have been plotted to demonstrate the effects of Suction parameter on different profiles. In Fig.4 the velocity inside the boundary layer increases with an increase in Suction parameter for the shrinking case and it reveals the opposite phenomena for temperature and concentration profile. As may be expected, it is because of the fact that suction cause the reduction of momentum boundary layer thickness and consequently enhances the flow near the solid surface.

Fig.7-9 shows the variation of different profiles for several values of the unsteadiness parameter. In Fig.7 It is analyzed that the velocity of the nanofluid increases when the influence of unsteadiness parameter increases. From Fig.8-9 reveals that the temperature and concentration at a point decreases as the magnitude of the unsteadiness parameter increases. This is due to the fact that the heat transfer rate increases with the increase in unsteadiness parameter which in turn reduces the temperature of the fluid.

In order to understand the influence of Prandtl number on velocity, temperature and concentration profiles are plotted in Fig.10-12.Fig.10 demonstrates whereas the velocity profile increases with the increase in Prandtl number. Fig.11-12 depicts that the temperature and concentration profile decreases when the values of Prandtl number increases. This is due to fact that a higher Prandtl number fluid has relatively low thermal conductivity, which reduces conduction and thereby the thermal conductivity which reduces conduction and thereby the thermal boundary layer thickness and as a result temperature and concentration decreases.

Fig13-18 shows the influence of the changes in Brownian motion and thermophoresis parameter on different profiles. It is noticed that as thermophoresis parameter increases and the temperature and concentration gradient at the surface decreases as both Brownian motion and thermophoresis parameter. We noticed that thermophoresis parameter, Brownian motion increases and the velocity gradient decreases at the surface.

Table 1. Comparison of the critical values of unsteadiness parameter A for several values of suction parameters with *M, δ,* $Ra_x = 0$.

S	Rohni et al. [18]	Samir et al. [19]	Present
2.10	-1.6550	-1.654850	-1.6548483
2.15	-3.8605	-3.860346	-3.8603249
2.20	-8.3408	-8.349103	-8.340097

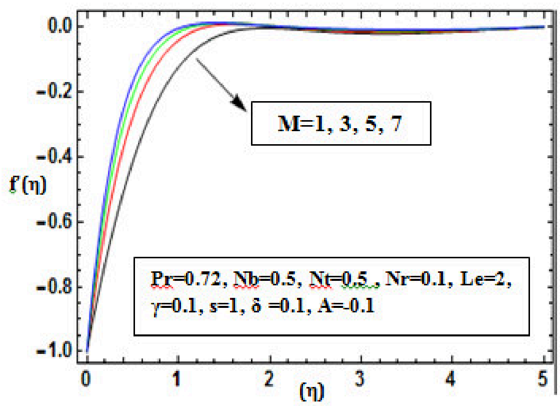

Fig.1: Effects of Magnetic parameter (*M*) on velocity Profiles

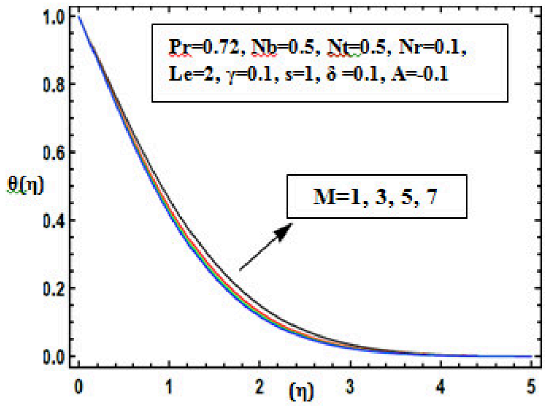

Fig.2: Effects of Magnetic parameter (*M*) on temperature Profiles

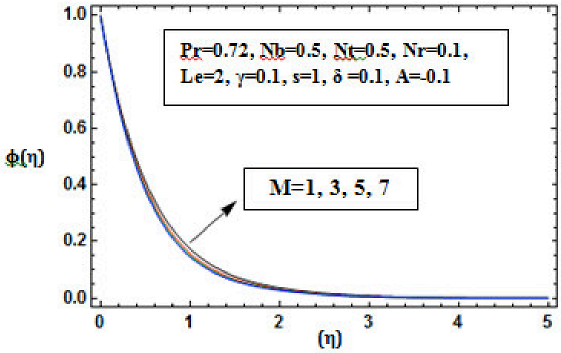

Fig.3: Effects of Magnetic parameter (*M*) on concentration Profiles

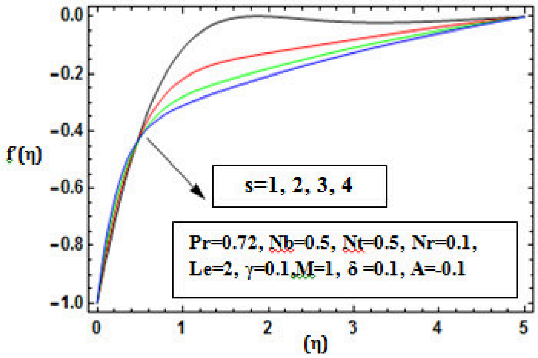

Fig.4: Effects of suction parameter (*s*) on velocity Profiles

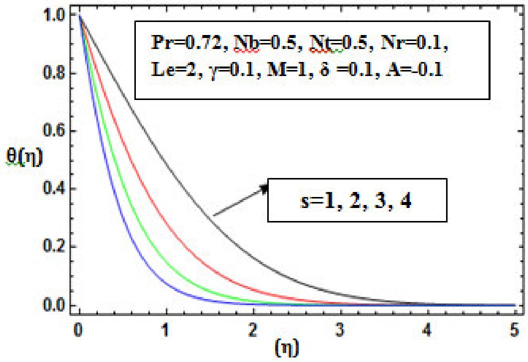

Fig.5: Effects of suction parameter (*s*) on temperature Profiles

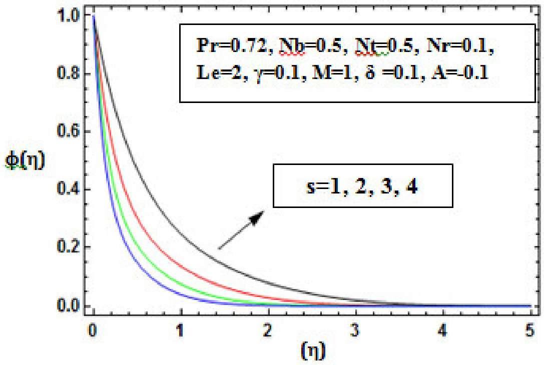

Fig.6: Effects of suction parameter (*s*) on concentration Profiles

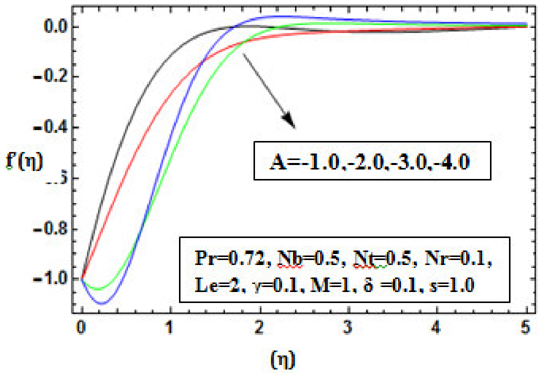

Fig.7: Effects of Unsteadiness parameter (*A*) on velocity Profiles

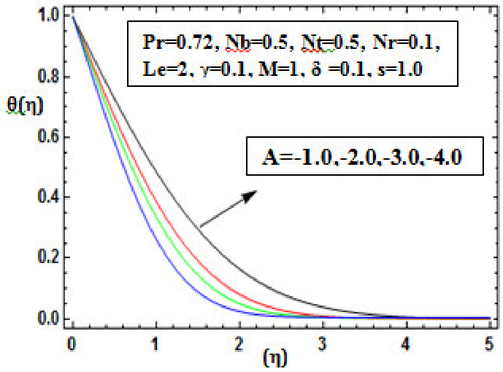

Fig.8: Effects of Unsteadiness parameter (*A*) on temperature Profiles

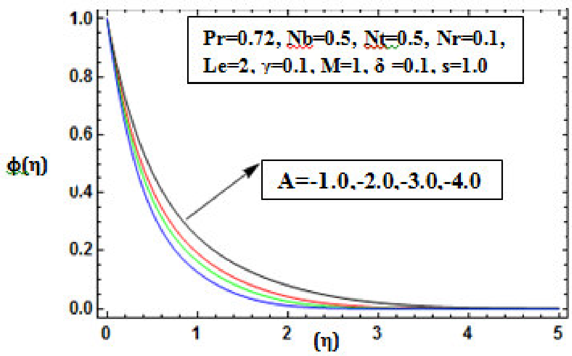

Fig.9: Effects of Unsteadiness parameter (*A*) on concentration Profiles

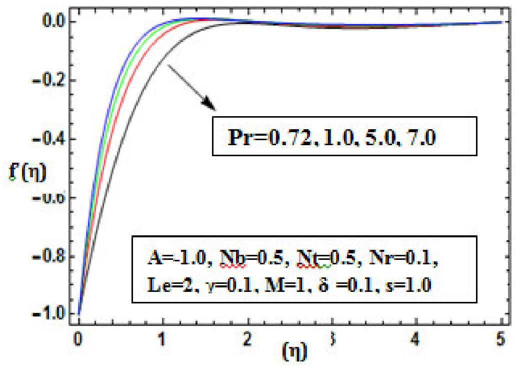

Fig.10: Effects of Prandtl Number (*Pr*) on velocity Profiles

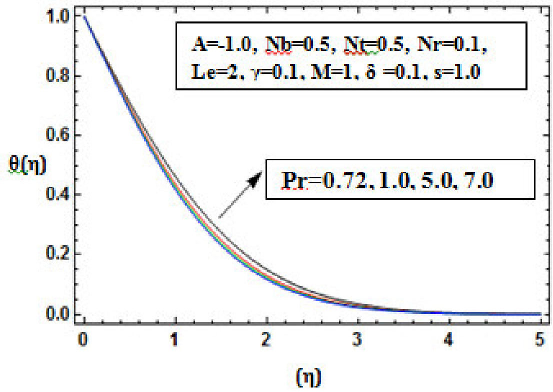

Fig.11: Effects of Prandtl Number (*Pr*) on temperature Profiles

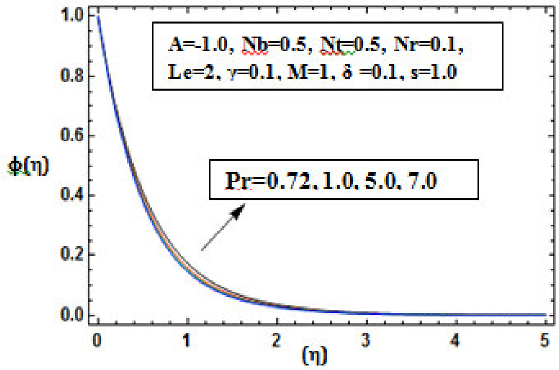

Fig.12: Effects of Prandtl Number (*Pr*) on concentration Profiles

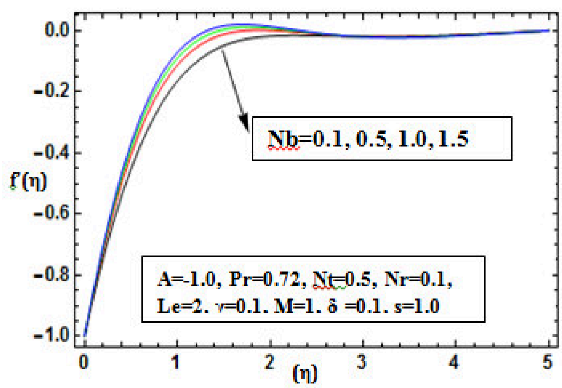

Fig.13: Effects of Brownian Motion parameter (*Nb*) on velocity Profiles

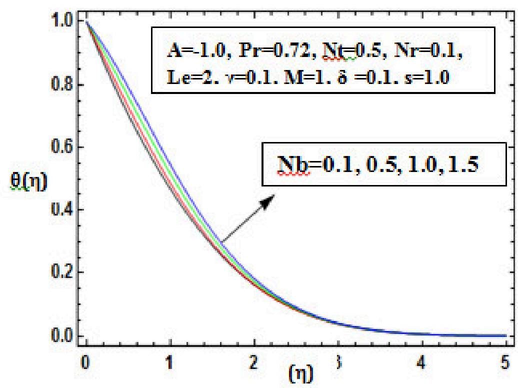

Fig.14: Effects of Brownian Motion parameter (*Nb*) on temperature Profiles

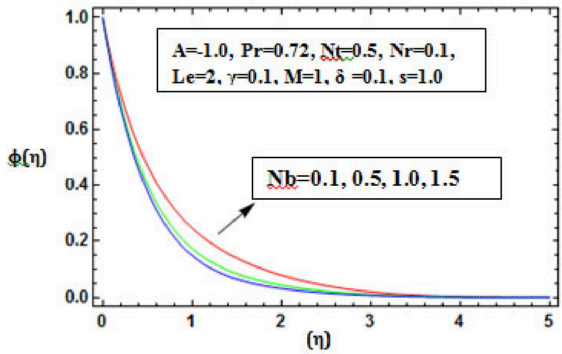

Fig.15: Effects of Brownian Motion parameter (*Nb*) on concentration Profiles

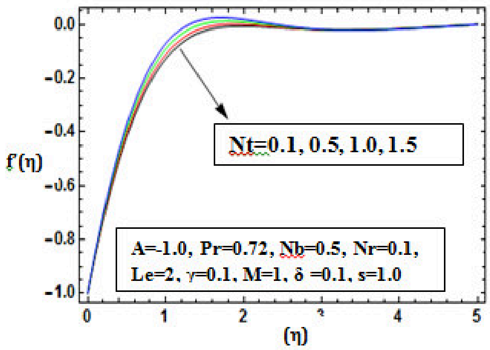

Fig.16: Effects of Thermophoresis parameter (*Nt*) on velocity Profiles

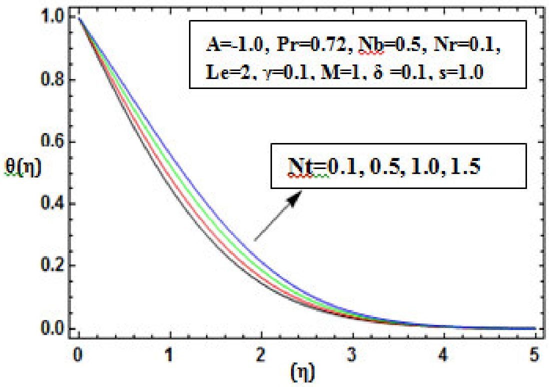

Fig.17: Effects of Thermophoresis parameter (*Nt*) on Temperature Profiles

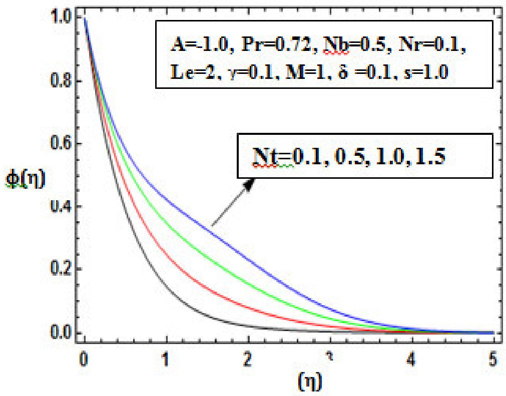

Fig.18: Effects of Thermophoresis parameter (*Nt*) on concentration Profiles

5.Conclusion

In this study, a mathematical model for a two dimensional unsteady free convection MHD flow and heat transfer of nanofluid over a permeable shrinking sheet in the presence of thermal radiation and chemical reaction. The validity of the present computations has been confirmed via benchmarking based on several earlier studies. The present results are compared with the existing literature and found a good agreement. The entire computation procedure is implemented using a program written in Mathematica software.

- The increase in magnetic parameter is to increase in velocity and decrease in concentration and temperature profiles.
- The increasing values of suction parameter accelerate the velocity and decrease in concentration and temperature profiles.
- As Prandtl number Pr decreases the thickness of thermal boundary layer becomes thickness of the velocity boundary layer. So the thickness of thermal boundary layer increases as Prandtl number Pr decreases and hence temperature profiles decreases with the increase of Prandtl number Pr.
- The increase of Unsteadiness parameter is to increases in velocity. the heat transfer rate increases with the increase of unsteadiness parameter A which in turn reduces the temperature of fluid.
- The thermophoresis parameter Nt phenomenon describes the fact that small micron size particular suspended in non-isothermal fluid will acquire a velocity in the direction of decreases temperature.
- Brownian motion parameters increase both the velocity and decrease in the concentration profile temperature profiles.

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