

MHD Boundary Layer Flow of Casson Fluid Over a Stretching/Shrinking Sheet Through Porous Medium

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Abstract
MHD boundary layer flow of Casson fluid over a permeable stretching/shrinking sheet in a porous medium is investigated. Applying similarity transformations, the governing partial differential equation are converted into ordinary differential equation and solved it analytically. The effects of magnetic parameter, permeability parameter and Casson parameter on are presented graphically on velocity profiles while the local skin friction coefficient are presented through graphs. Our analysis reveals that the increase of Casson parameter the velocity is decrease and also reduce the boundary layer thickness.
Keywords: MHD, Casson parameter, permeability parameter, boundary layer, stretching/shrinking sheet.

INTRODUCTION

In the past few decades there has been an increasing interest in the flows of Newtonian and non-Newtonian fluids over stretching/ shrinking sheet because of their applications in processing industries such as polymer processing, glass fiber production and many others. The study of MHD flow is important because the study on the influence of magnetic field on electrically conducting fluid is applicable in many industrial process, such as in magnetic materials processing purification of crude oil and magneto hydrodynamic electrical power generation. Since the behavior of industrial fluids is very complex, it became necessary to use non-Newtonian fluid models to describe the physical phenomena substances such as Jelly, tomato Sause fruit juices etc. belong to the category of non-Newtonian fluids and they may be explained well through common model. Human blood is also be treated as a Casson fluid. Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear.

The pioneering work on steady boundary layer flow due to linear stretching sheet was done by Crane [1]. Exact analysis for the effects of heat transfer of MHD and radiation Marangoni boundary layer Nano fluid flow past a surface embedded in a porous medium is studied by Emad H Aly et al. [2]. Turkyilmazoglu [3] reported on flow of a micro polar fluid due to porous stretching sheet and heat transfer. Shalini Jain et al. [4] studied effects of MHD on boundary layer flow in porous medium due to exponentially shrinking sheet with slip MHD flow of a Casson fluid over an exponentially inclined permeable stretching surface with thermal radiation and chemical reaction was discussed by BalaAnkireddy[5] . Vishnu Ganesh et al. [6] discussed on Darcy forchheimer flow of hydro magnetic Nano fluid over a stretching/shrinking sheet in a thermally stratified porous medium with 2nd order slip, viscous and ohmic dissipations effects. MHD viscoelastic boundary layer flow and heat transfer past a convectively heated radiating stretching/shrinking sheet with temperature dependent heat source/sink embedded in saturated porous media was analyzed by Subhas Abel et al. [7].

Krishnendu Bhatacharyya et al. [8] considered dual solution in boundary flow of Maxwell fluid over a porous shirking sheet. Numerical solution for MHD stagnation point flow towards a stretching/ shrinking surface in a saturated porous medium is obtained by Susheela Chaudhary et al. [9] Alam et al. [10] analyzed effects of variable fluid properties and thermophoresis an unsteady forced convective boundary layer flow along a permeable stretching/shrinking wedge with variable Prandtl and Schimidt numbers. Santosh Chaudhary et al. [11] studied heat and mass transfer by MHD flow near the stagnation point over a stretching/shrinking sheet in a porous medium. Azhar Ali et al. [12] reported an analytic solution for fluid flow over an exponentially stretching porous sheet with surface heat flux in porous medium by means of homotopic analysis methods. Series solutions of non-similarity boundary layer flow in porous medium was reported by Nabeelakausar et al. [13]. Boundary layer stagnation point slip flow and heart transfer towards a stretching/shrinking cylinder over a permeable surface is investigated by nor AzaiahAihi Mat [14]. Analytical solution for MHD boundary layer flow of Casson fluid over a stretching/shrinking sheet with wall mass transfer is obtained by Krishnendu Bhattacharya et al. [15]. MHD flow of Newtonian fluid, is investigated by Chakrabarti and Gupta [16].

In this paper we analyze MHD boundary layer flow of Casson fluid over a stretching/shrinking sheet through a porous medium. The closed form analytical solution is obtained in both stretching/shrinking sheet cases.

FORMULATION OF THE PROBLEM

Consider the steady incompressible Casson fluid flow over a stretching/shrinking sheet through porous medium. The sheet is situated at y=0 and the flow is combined in y>0. A Uniform magnetic field of strength B_0 is applied perpendicular to the stretching sheet. The induced magnetic field is assumed to be negligible. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is given by Eldabe and Salwa [17]

$$\tau_{ij} = \begin{cases} (\mu_B + p_y/\sqrt{2\pi})2e_{ij}, & \pi > \pi_{c,} \\ (\mu_B + p_y/\sqrt{2\pi})2e_{ij}, & \pi < \pi_{c,} \end{cases} \quad (1)$$

Where μ_B is plastic dynamic viscosity of the non-Newtonian fluid, p_y is the yield stress of fluid,π is the product of the component of deformation rate with itself , namely$\pi = e_{ij}e_{ij}$,e_{ij} is the (i, j)th component of the deformation rate, and π_c is the critical value of this product based on the non-Newtonian model. Under the usual boundary layer approximation, the governing equations that describe the flow are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\left(1 + {}^{1}/_{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - \frac{\nu}{k}u \quad (3)$$

where u and v are the velocity components in x and y directions respectively, x is the distance along the sheet , y is the distance perpendicular to the sheet, ν is the kinematic fluid viscosity, ρ is the fluid density , $\beta = \mu_B\sqrt{2\pi_C}/p_y$ is the non-Newtonian(Casson) parameter. σ is the electrical conductivity of the fluid, ν kinematic viscosity, k permeability and B_0 is the strength of magnetic field applied in the y direction.

The boundary conditions are

$$u = U_w, v = -v_w \quad at \quad y = 0; \; u \to 0 \;\; as \;\; y \to \infty \quad (4)$$

where U_w is stretching /shrinking velocity of the sheet with $U_w = cx$ for stretching sheet case and $U_w = -cx$ for shrinking sheet case with c(>0) being the stretching/shrinking constant. Here v_w is the wall mass transfer velocity with $v_w > 0$ for mass suction and $v_w < 0$ for mass injection.

Introducing the stream function , the velocity components u and v can be written as

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \quad (5)$$

For relations of (5), the mass conservation of equation (2) satisfied automatically, and the momentum equation (3) takes the following forms

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma B_0^2}{\rho}\frac{\partial \psi}{\partial y} - \frac{\nu}{k}\frac{\partial \psi}{\partial y} \quad (6)$$

and also the boundary conditions in (4) reduce to

$$\frac{\partial \psi}{\partial y} = U_w \;, -\frac{\partial \psi}{\partial x} = v_w \;\; at \; y = 0 \;\; \frac{\partial \psi}{\partial y} \to 0 \;\; as \; y \to \infty \quad (7)$$

We now introduce the following transformations:

$$\psi = \sqrt{c\nu}x\, f(\eta) \;\; \text{and } \eta = y\sqrt{\frac{c}{\nu}} \quad (8)$$

Using (8) and equation (6) finally takes the following forms

$$\left(1+\frac{1}{\beta}\right)f''' + ff'' - f'^2 - (M + \lambda_1)f' = 0 \quad (9)$$

Where $M = \frac{\sigma B_0^2}{\rho c}$ is the magnetic parameter $\lambda_1 = \frac{\nu}{ck}$ =permeability (or) porous parameter

The boundary conditions are described as follows.

For stretching sheet

$$f(\eta) = S\,, f'(\eta) = 1 \; at \; \eta = 0\,; \;\; f'(\eta) \to 0 \;\; as \; \eta \to \infty \quad (10)$$

For shrinking sheet

$$f(\eta) = S\,, f'(\eta) = -1 \; at \; \eta = 0\,; \;\; f'(\eta) \to 0 \;\; as \; \eta \to \infty \quad (11)$$

where $S = v_w/(c\nu)^{1/2}$ is wall mass transfer parameter with S>0 (i.e.$v_w > 0$) for wall mass suction and S<0 (i.e.$v_w < 0$) for wall mass injection.

SOLUTION OF THE PROBEM

a) Stretching sheet case

We assume the solution in the form

$$f(\eta) = a + be^{-\lambda\eta} \quad (12)$$

where a, b and λ are constants with $\lambda > 0$. Substituting the relation (12) into equation (9) and (10), we obtain.

$$b = -\frac{1}{\lambda} \;\; , \;\; a = s + \frac{1}{\lambda} \;\; \text{and } \lambda = \frac{s + \sqrt{s^2 + 4\left(1+\frac{1}{\beta}\right)(1 + (M + \lambda_1))}}{2\left(1+\frac{1}{\beta}\right)} \quad (13)$$

Now the exact analytical solution reduces to

$$f(\eta) = s + \frac{1}{\lambda} - \frac{1}{\lambda} e^{-\frac{s+\sqrt{s^2+4\left(1+\frac{1}{\beta}\right)(1+(M+\lambda_1))}}{2\left(1+\frac{1}{\beta}\right)}\eta} \quad (14)$$

hence

$$f'(\eta) = be^{-\lambda\eta}(-\lambda) = e^{-\lambda\eta}$$

$$f''(\eta) = e^{-\lambda\eta}(-\lambda) = -\lambda e^{-\lambda\eta} \quad (15)$$

$$f''(0) = -\lambda = -\frac{s+\sqrt{s^2+4\left(1+\frac{1}{\beta}\right)(1+(M+\lambda_1))}}{2\left(1+\frac{1}{\beta}\right)} \quad (16)$$

b) Shrinking sheet case

The shrinking sheet flow is more interesting than the stretching sheet flow. Putting the equation (12) into equation (9) and (11), we obtain

$$b = \frac{1}{\lambda} \;, \quad a = s - \frac{1}{\lambda} \quad \text{and} \quad \lambda = \frac{s \pm \sqrt{s^2-4\left(1+\frac{1}{\beta}\right)(1-(M+\lambda_1))}}{2\left(1+\frac{1}{\beta}\right)} \quad (17)$$

Now the exact analytical solution reduces to

$$f(\eta) = a + be^{-\lambda\eta}$$

$$f(\eta) = s - \frac{1}{\lambda} + \frac{1}{\lambda} e^{-\frac{s\mp\sqrt{s^2-4\left(1+\frac{1}{\beta}\right)(1-(M+\lambda_1))}}{2\left(1+\frac{1}{\beta}\right)}\eta} \quad (18)$$

hence

$$f'(\eta) = be^{-\lambda\eta}(-\lambda) = -e^{-\lambda\eta} = -e^{-\left(\frac{s\pm\sqrt{s^2-4\left(1+\frac{1}{\beta}\right)(1-(M+\lambda_1))}}{2\left(1+\frac{1}{\beta}\right)}\right)\eta}$$

$$f''(\eta) = be^{-\lambda\eta}(\lambda)$$

$$f''(0) = \lambda = \frac{s\mp\sqrt{s^2-4\left(1+\frac{1}{\beta}\right)(1-(M+\lambda_1))}}{2\left(1+\frac{1}{\beta}\right)}$$

For MHD flow of Casson fluid the existence and uniqueness of the similarity solution strongly depends on magnetic parameter M. Three cases arise to be considered which are $M+\lambda_1<1$, $M+\lambda_1=1$, $M+\lambda_1>1$.

Case 1: $M+\lambda_1<1$

The steady flow is possible when the following condition is satisfied.

$$s^2 \geq 4\left(1+\frac{1}{\beta}\right)(1-(M+\lambda_1)) \quad (19)$$

Also , the similarity solution is unique if $s^2 = 4\left(1+\frac{1}{\beta}\right)(1-(M+\lambda_1))$ and it is of dual nature if $s^2 < 4\left(1+\frac{1}{\beta}\right)(1-(M+\lambda_1)$ no similarity found.

Case 2: $M+\lambda_1=1$

The steady flow of Casson fluid occurs for mass suction only. i.e. for s>o and in this case the solution is always unique.

Case 3: $M+\lambda_1>1$

There is no restriction on the steady flow of Casson fluid over a shrinking sheet, i.e. the similarity solution exists for any values of the mass transfer parameter, Casson parameter, porous parameter and the solution is unique.

Local skin friction coefficient

The physical quantities of interest are the local skin friction coefficient c_f which are defined as

$$c_f = \frac{\mu\left(1+\frac{1}{\beta}\right)}{\frac{1}{2}\rho u^2}\left(\frac{\partial U}{\partial y}\right)_{y=0} \quad (20)$$

Which in the present case can be expressed in the following form

$$c_f = -(1+\frac{1}{\beta})f''(0) \quad (21)$$

RESULTS AND DISCUSSION

The objective of the present paper is to study the MHD boundary layer flow of Casson fluid over a stretching/shrinking sheet through a porous medium. The analytical solution is performed for several values of dimensionless parameter involved in the equations. viz. the Casson parameter β magnetic parameter M, porous parameter λ_1, suction/injection parameter s. To illustrate the computed results, some figures are plotted and physical explanation are given.

Stretching sheet case

The variation in velocity $f'(\eta)$ for different values casson parameter β is shown in figure 1. We observed that the velocity is decreases with increasing Casson parameter β and consequently the thickness of the boundary layer decreases.

The variation in velocity $f'(\eta)$ for different values of casson parameter β with mass injection S. Shown in figure 2. We observe that the velocity is decreases with increasing β and consequently the thickness of the boundary layer decreases.

The variation in velocity $f'(\eta)$ for different value of magnetic parameter M with mass suction/injection are shown in figure 3 and figure 4. We observe that the velocity is decreases with increasing M. Reduction is caused by the Lorentz force, a mechanical force arising due to the interaction of magnetic and electric fields for the motion of an electrically conduction fluid. The Lorentz force increases when M increases and consequently boundary layer thickness in decreases.

The variation in velocity $f'(\eta)$ for different values of porous parameter λ_1 with mass suction/injection s are shown in figure 5. and figure 6. We observe that the velocity is decreases with increasing λ_1 . Consequently, the boundary layer thickness is decreases.

The local skin friction coefficient $f''(0)$ against s. for different values of casson parameter β as shown figure 7 we observe that the skin friction coefficient $f''(0)$ is decreases with increasing β .

The variation in local skin friction coefficient $f''(0)$ against s for different values of magnetic parameter M is shown in figure 8. We observe that the local skin friction coefficient $f''(0)$ is decreases with increasing M.

The variation in local skin friction coefficient $f''(0)$ against s for different values of porous parameter $\lambda_1$is shown in figure 9. We observe that the local skin friction coefficient $f''(0)$ is decreases with increasing λ_1.

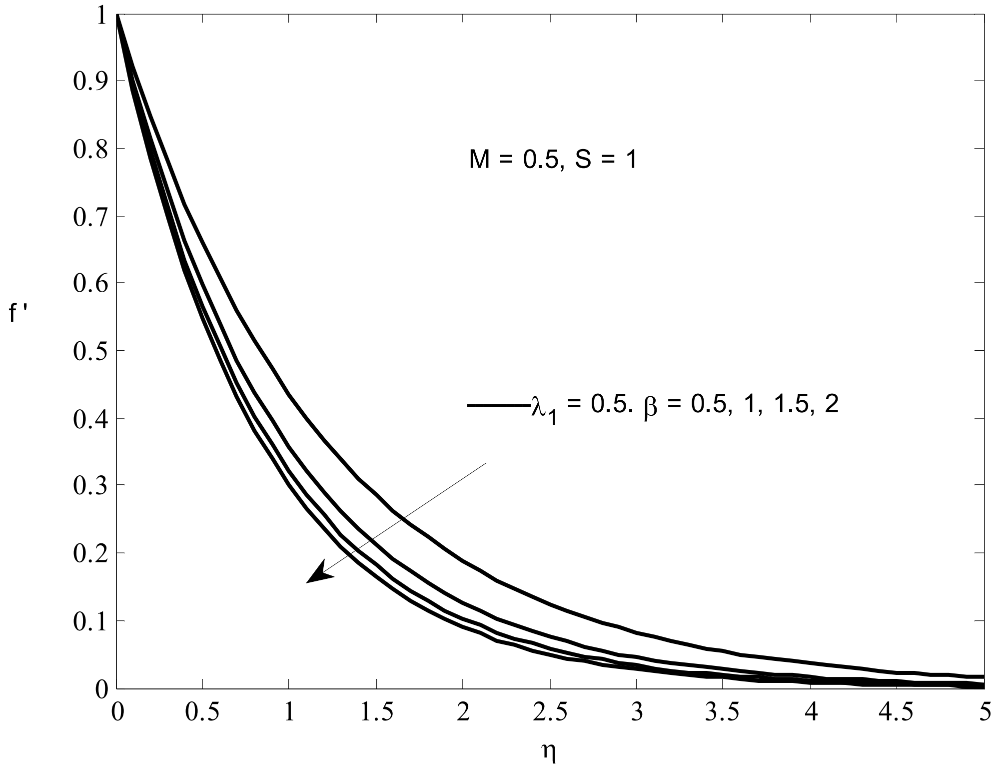

Fig 1.Velocity profiles for several values of β with mass suction

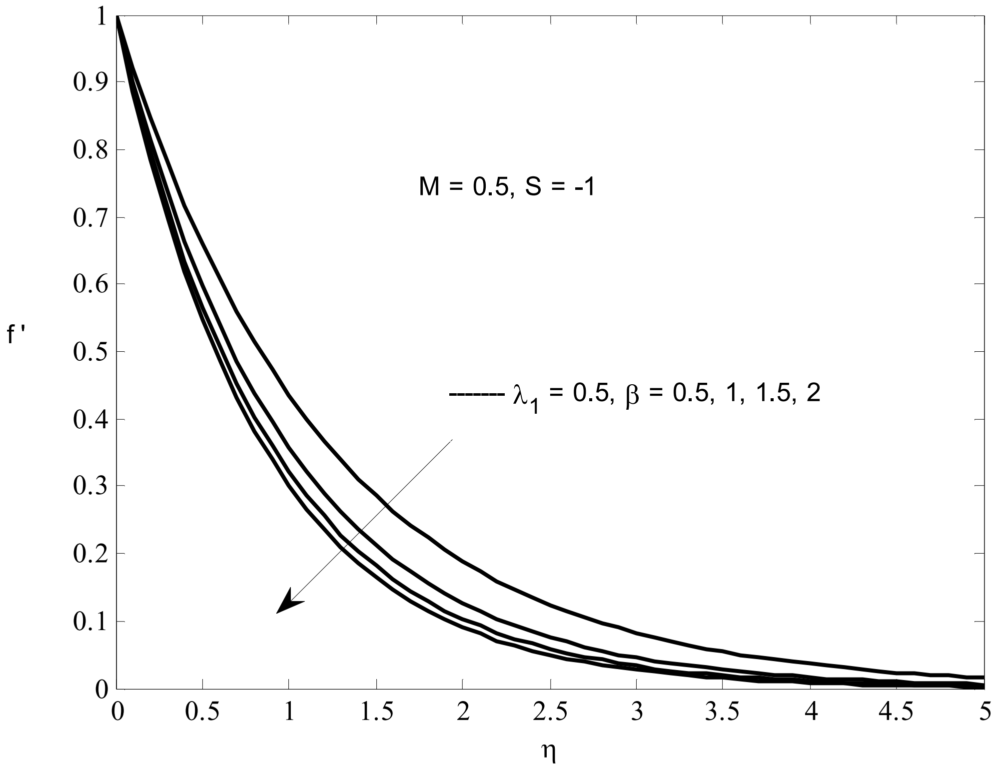

Fig 2.Velocity profiles for several values of β with mass injection

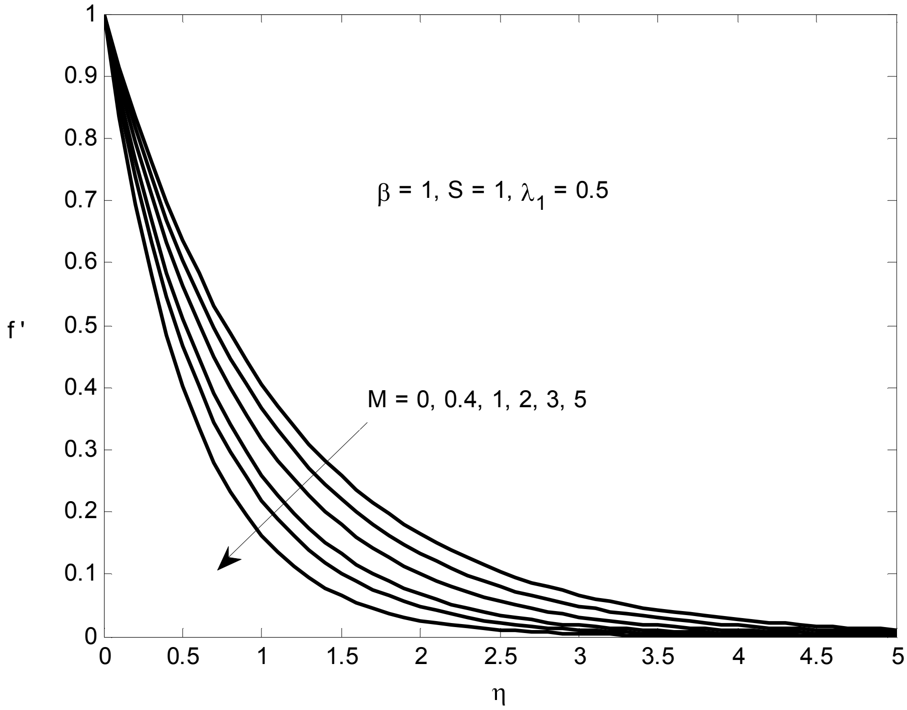

Fig 3.Velocity profiles for several values of M with mass suction

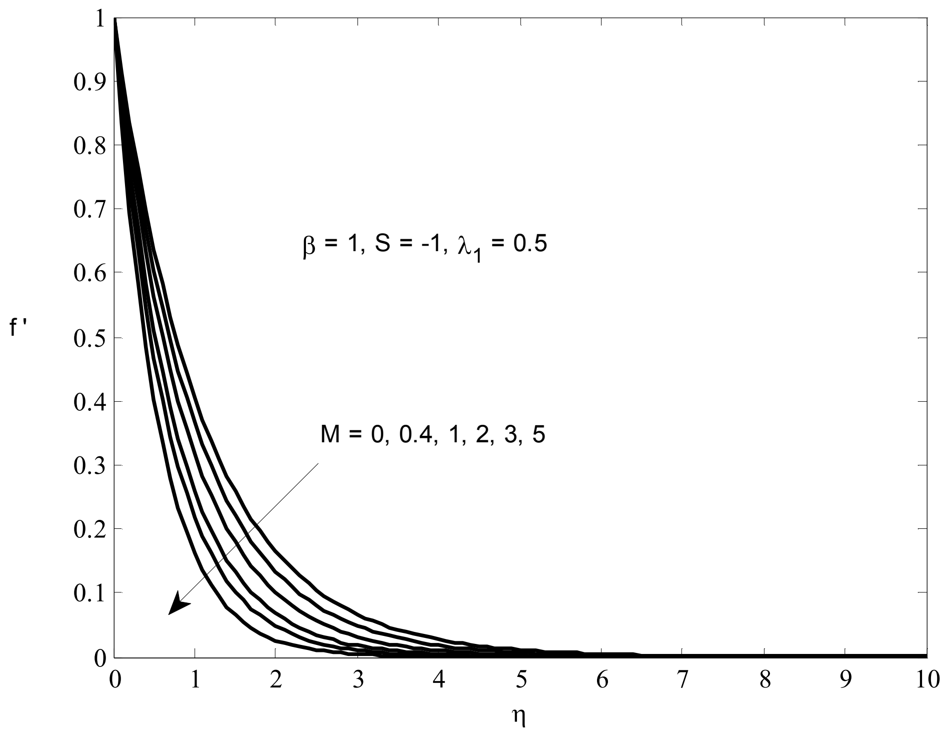

Fig 4.Velocity profiles for several values of M with mass injection

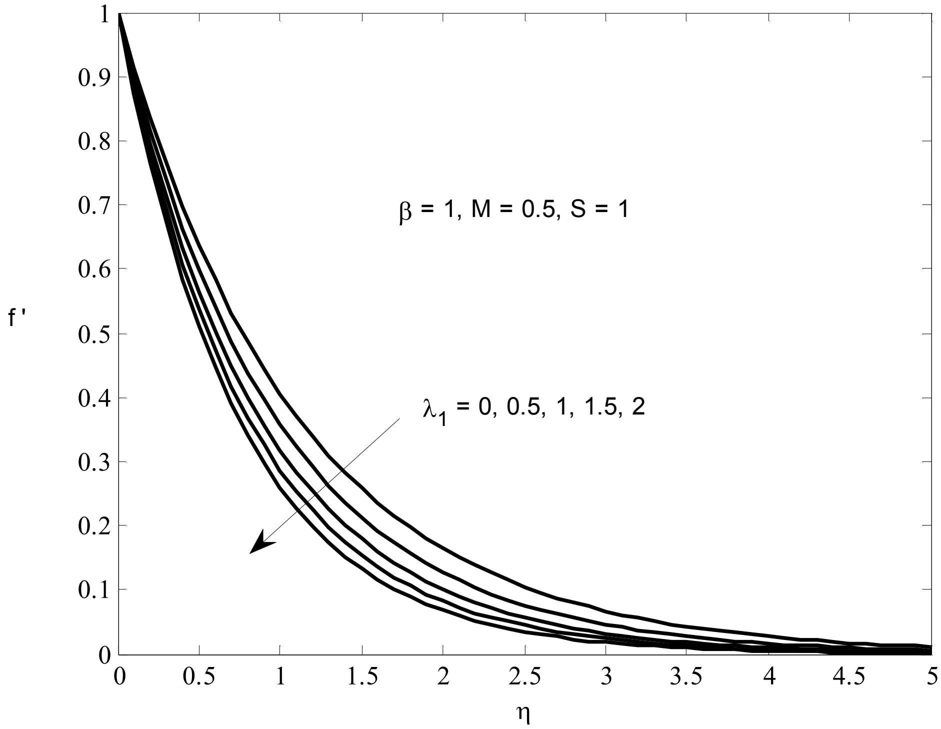

Fig 5.Velocity profiles for different values of λ_1 with mass suction

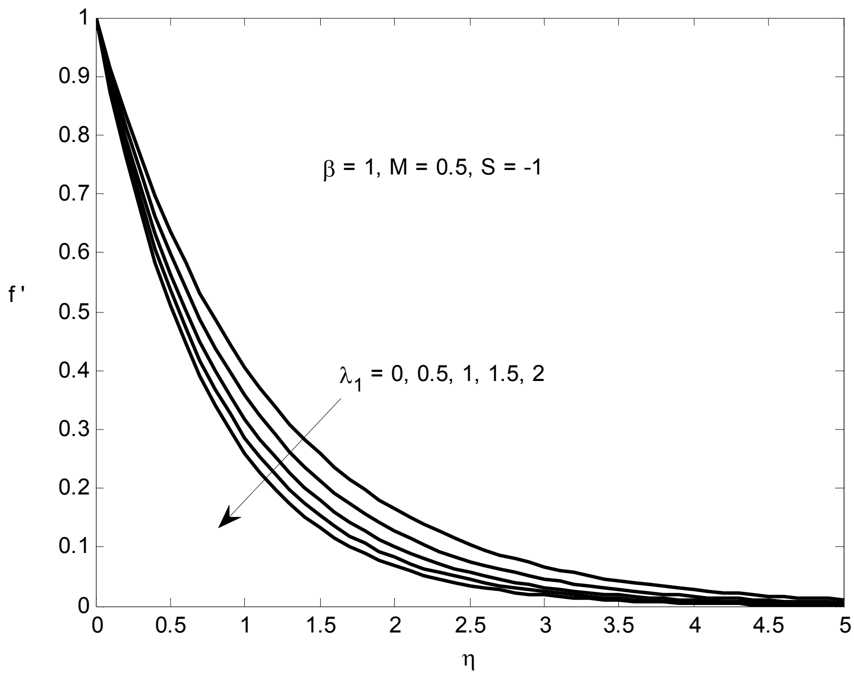

Fig 6.Velocity profiles for different values of λ_1 with mass injection

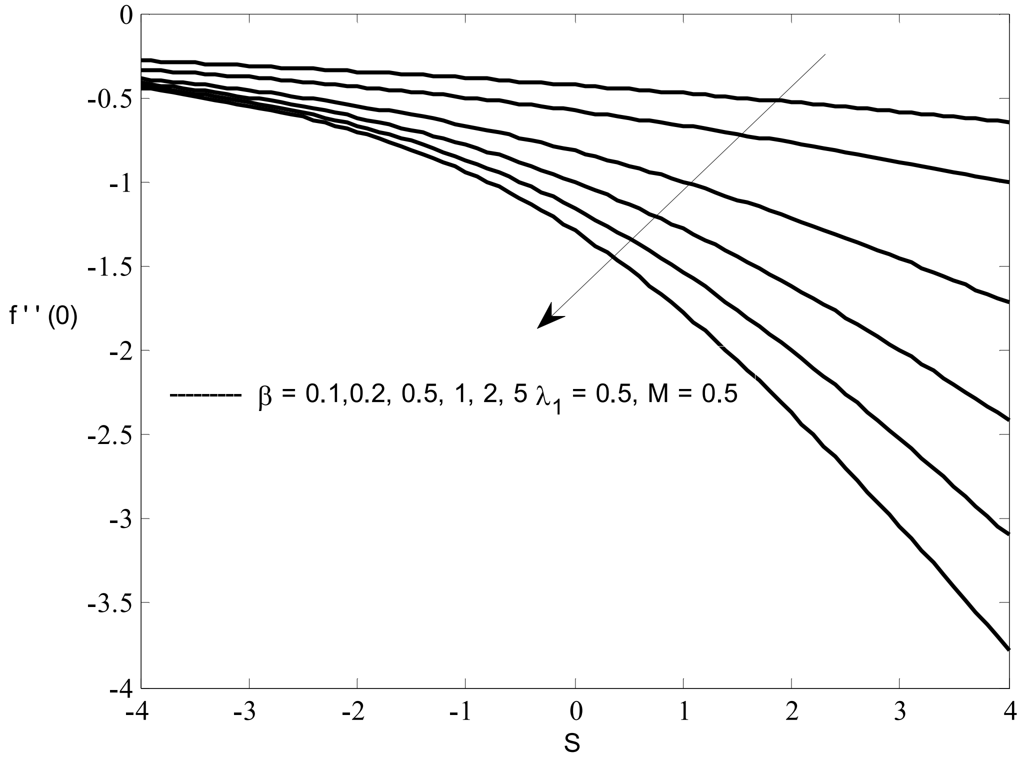

Fig 7.Skin friction coefficient versus S for several values of β .

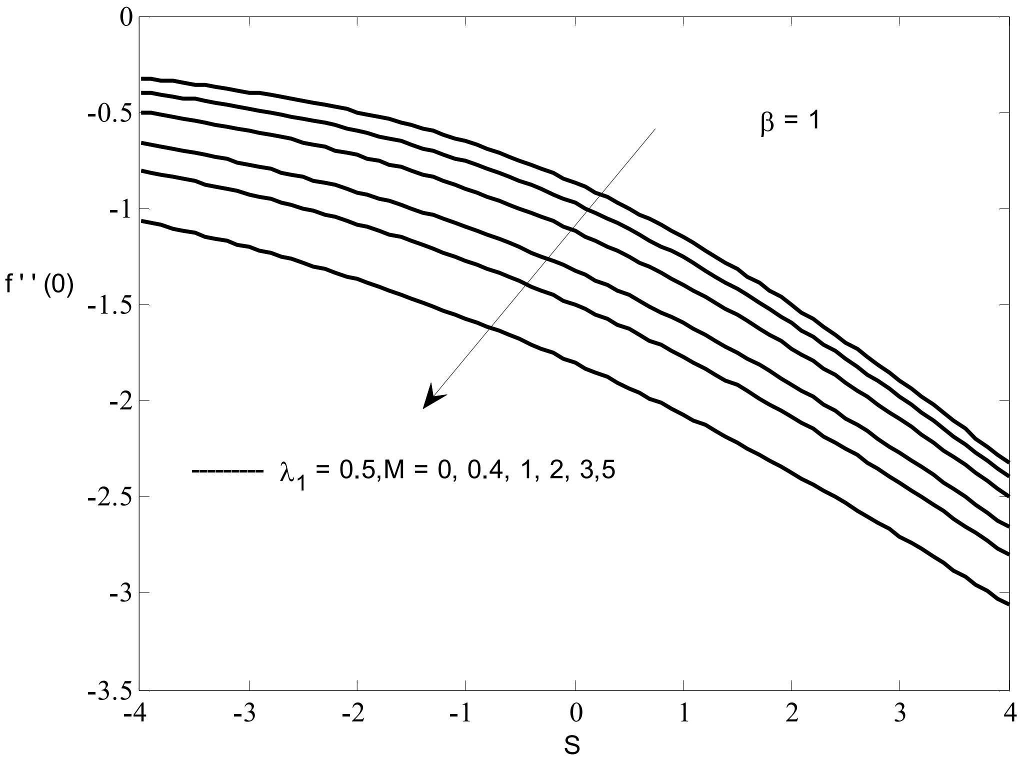

Fig 8.Skin friction coefficient versus S for several values of M

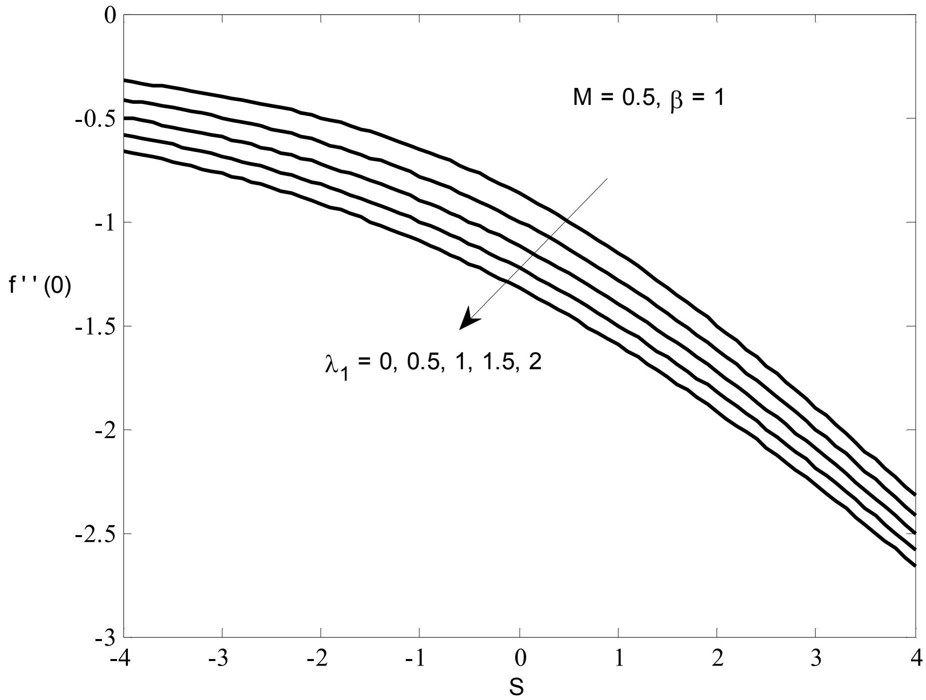

Fig 9.Skin friction coefficient versus S for several values of λ_1.

Shrinking sheet case

The variation in velocity $f'(\eta)$ for different values Magneticparameter M is shown in figure [10]. We observed that the velocity is increases with increasing Magnetic parameter M.

The variation in velocity $f'(\eta)$ for different values Casson parameter β is shown in figure [11]. We observed that the velocity is increases with increasing Casson parameter β.

The variation in dual velocity $f'(\eta)$ profiles are evalueate for different values Magnetic parameter M. We observed that the the first solution of the velocity is increases with increasing Magnetic parameter M and opposite behaviour of the second solution in velocity profile are shown in figure [12].

The variation in local skin friction coefficient $f''(0)$ against s for different values of Casson parameter $\beta$is shown in figure [13]. We observe that the local skin friction coefficient $f''(0)$ is increases with increasing β.

The variation in local skin friction coefficient $f''(0)$ against s for different values of porous parameter λ_1 is shown in figure [14]. We observe that the local skin friction coefficient $f''(0)$ is increases with increasing λ_1.

The variation in local skin friction coefficient $f''(0)$ against s for different values of Magnetic parameter Mis shown in figure [15]. We observe that the local skin friction coefficient $f''(0)$ is increases with increasing M.

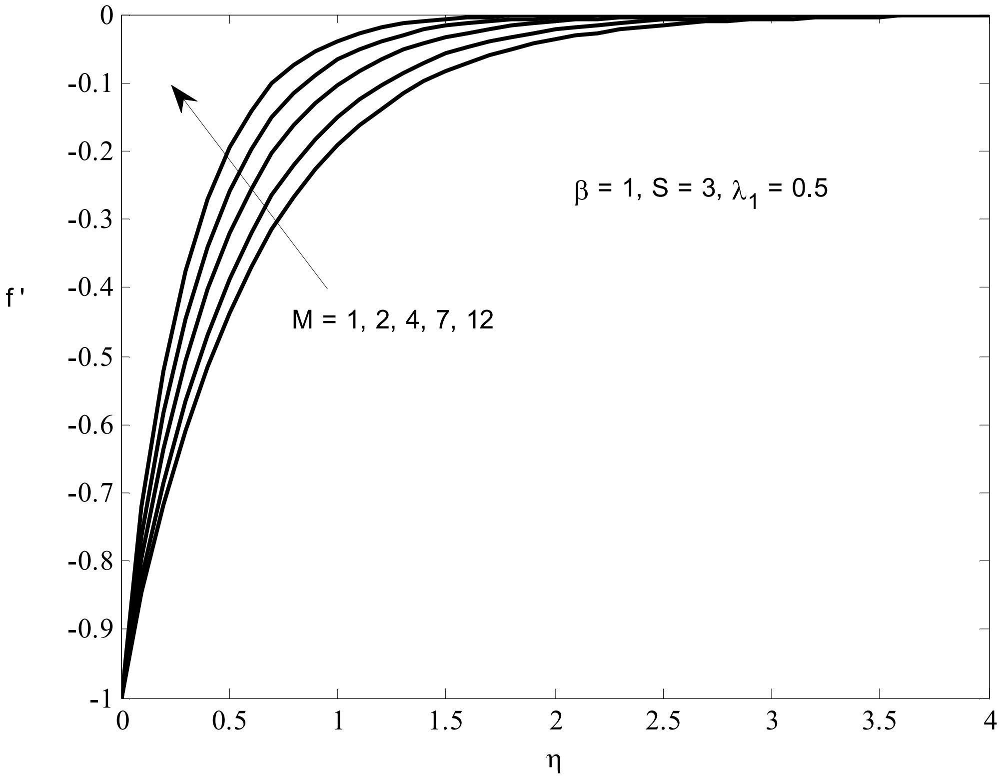

Figure 10. Velocity profiles for several values of M.

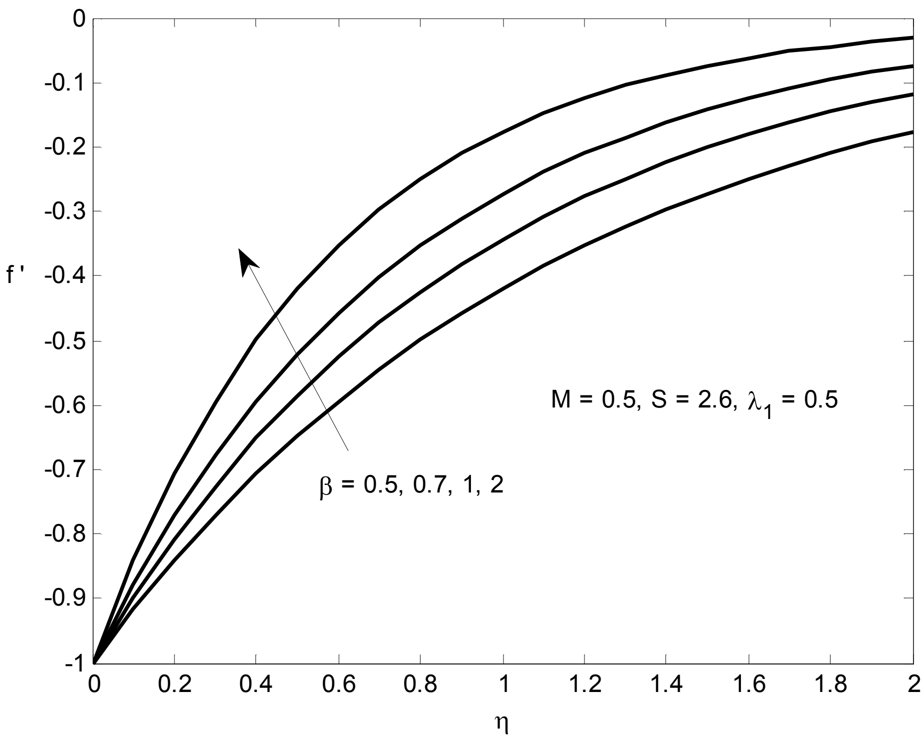

Figure 11.Velocity profile for several values of beta.

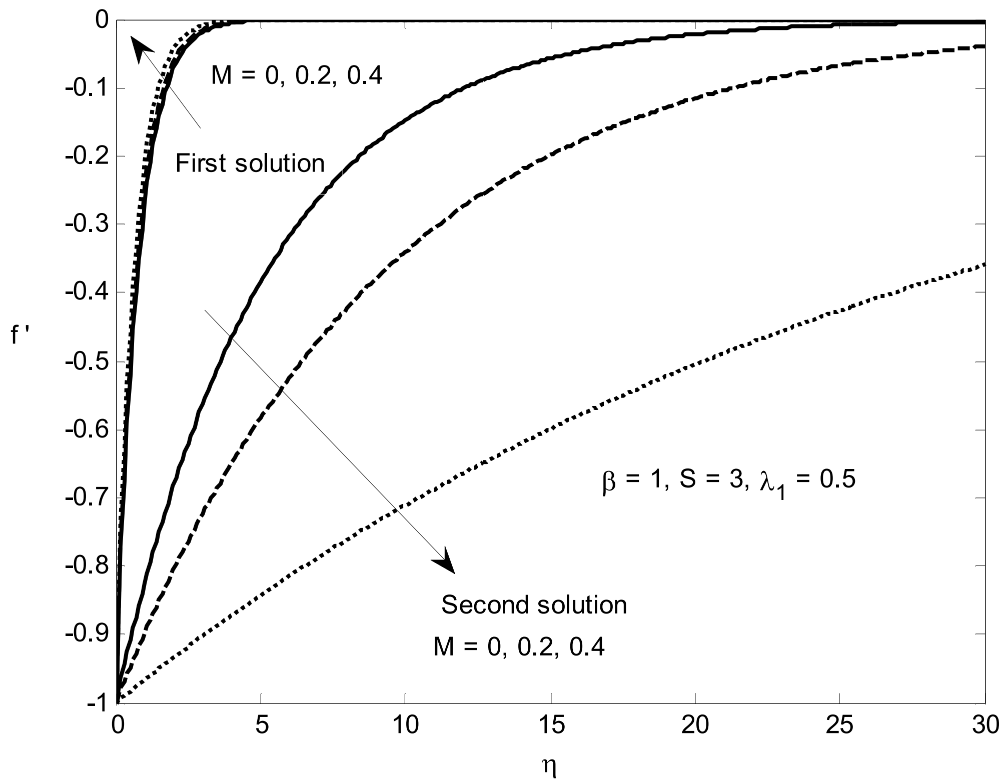

Figure 12.Dual Velocity profile for several values of M.

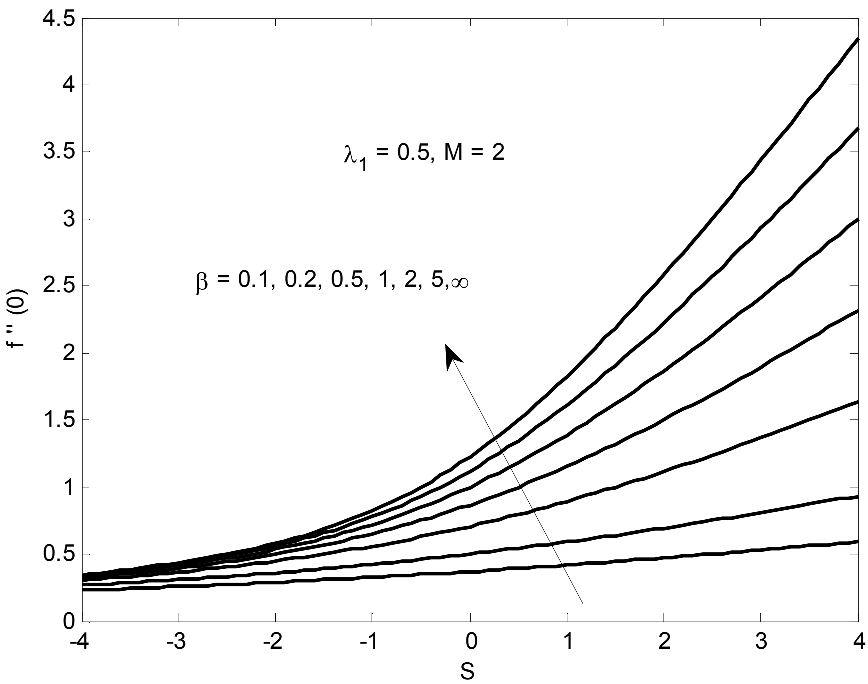

Figure 13. Skin friction coefficient versus S for several values of beta

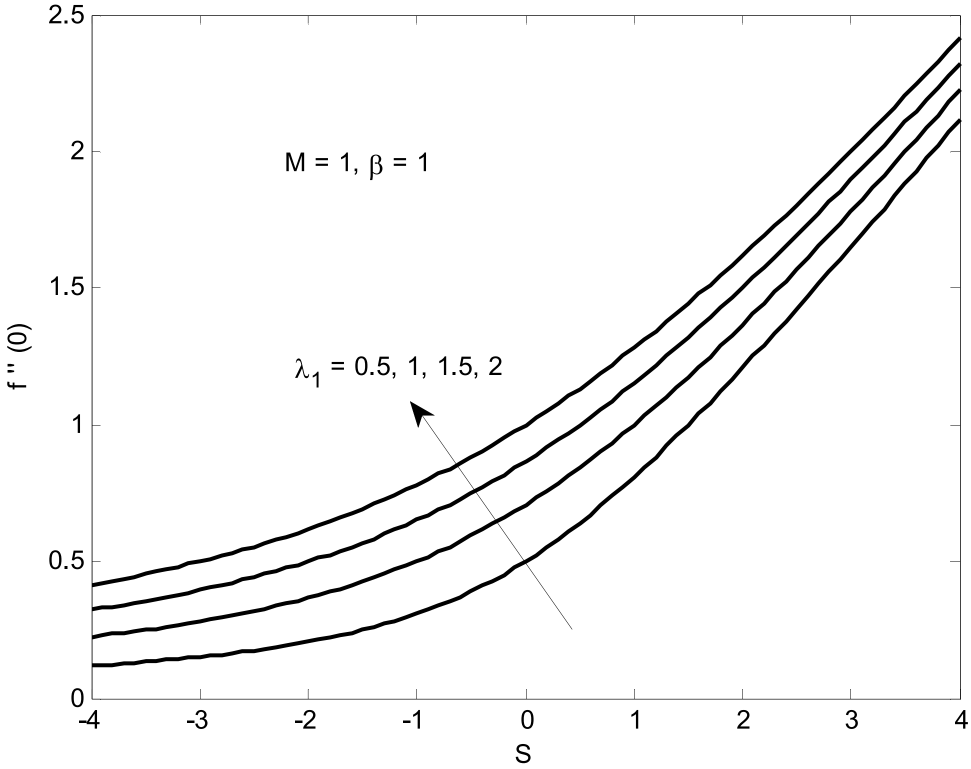

Figure 14.Skin friction coefficient versus S for several values of lambda

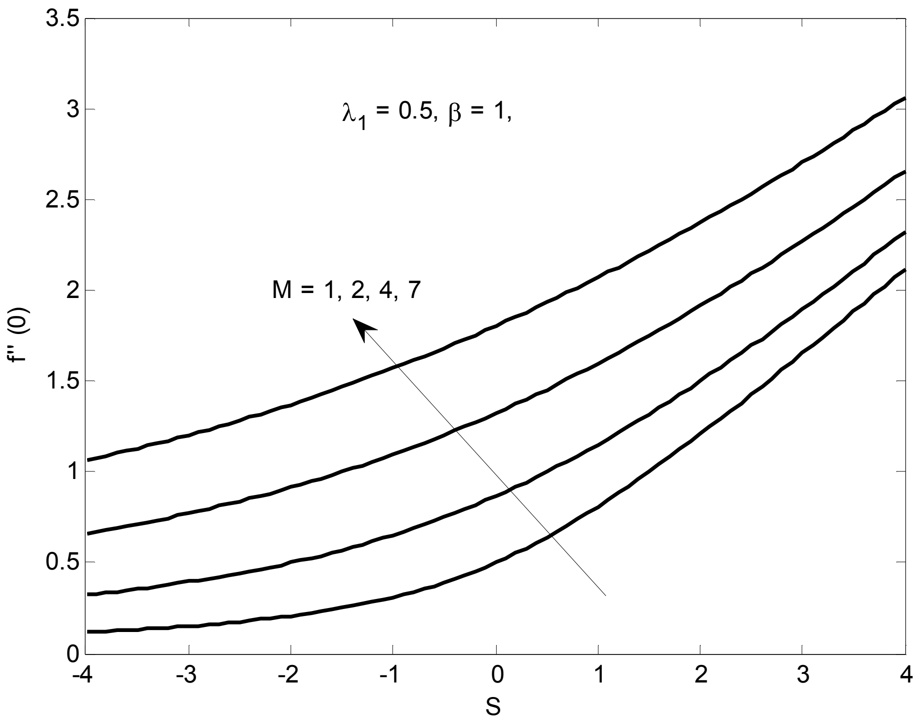

Figure 15. Skin friction coefficient versus S for several values of M

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