

# Influence of Porosity and Magnetic Field with Dissipative Heat Transfer Flow over a Stretching Surface through UCM Fluid

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## Abstract

The purpose of present analysis is to examine the effects of transverse magnetic field within a boundary layer of an upper-Convected Maxwell (UCM) fluid over a stretching surface through porous media. The requisite partial differential equations are converted into ordinary differential equations by using similarity transformations. Resultant equations are highly non-linear which cannot be solved analytically. Hence those equations are solved numerically by using efficient numerical shooting technique with fourth order Runge-Kutta method. The main aim of the present work is to analyze the effect of elastic parameter  $\beta$ , magnetic parameter  $Mn$  and thermal conductivity  $k_2$  on the temperature field above the sheet. The previous results are compared with our present results and are shown in tabulation and represented graphically

**Keywords:** Upper-Convected Maxwell fluid, Boundary layer, Stretching surface, Similarity transformation, Magnetic parameter, Porous media, Viscous dissipation.

## Nomenclature:

$u$	Velocity in x direction
$v$	Velocity in y direction
$B_0$	Strength of the magnetic field
$\nu$	Kinematic viscosity of the fluid
$\lambda$	Relaxation time parameter of the fluid
$T_w$	Wall Temperature
$T_\infty$	Temperature far away from the sheet.
$T_0$	Melt Temperature at the die exit
$T-T_s$	Melt solidification temperature
$L$	Distance between the die exit and the point which the melt solidifies
$k_2$	Thermal conductivity of the fluid
$b$	Constant whose value also depends on the fluid
$Mn$	Magnetic parameter
$\beta$	Elastic parameter
$Ec$	Eckert number
$Pr$	Prandtl number
$\rho$	Density
$\mu$	Dynamic viscosity
$C_p$	Specific constant pressure
$f$	Dimensionless stream function
$g$	Acceleration due to gravity
$c$	Stretching parameter

## Introduction

In recent years behaviors of non-Newtonian fluids have been studied due to the wide range of engineering and industrial applications. The dynamics of non-Newtonian fluids is a popular area of research owing to its ever increasing applications in chemical and process engineering. Hence several constitutive equations of non-Newtonian fluids have been presented over the past decades.

In view of these applications Hayat et al. [1] have studied about melting heat transfer in a boundary layer flow of a second grade fluid under Soret and Dufour effects. Pop et al. [2] have discussed MHD flow and heat transfer of a UCM fluid over stretching surface with variable thermo physical properties. Vimala and Loganathan [3] have analyzed the MHD flow of nano-fluids over an exponentially stretching sheet embedded in a stratified medium with suction and radiation effects. Shateyi and Marewo [4] have attained the numerical approach of MHD flow, heat and mass transfer for the UCM fluid over a stretching surface in the presence of thermal radiation. Rahman and Salahuddin [5] have experimented through hydro magnetic field, heat and mass transfer flow over an inclined heated surface with variable viscosity. Prasad et al. [6] have investigated the effect

of variable viscosity on MHD viscoelastic fluid flow and heat transfer over a stretching sheet. Rohni et al. [7] have investigated the flow and heat transfer over an unsteady shrinking sheet with suction in nano-fluids. Jaluria et al. [8] have discussed heat transfer in nanofluids. Bachok et al. [9] have elaborated an unsteady boundary layer flow and heat transfer of a nano-fluid over a permeable shrinking sheet. Singh et al. [10] have illustrated the influence of thermal radiation and magnetic field on unsteady stretching permeable sheet in presence of free stream velocity. Motsa [11] has studied on a new spectral local linearization method for non-linear boundary layer flow problems. Mahian et al. [12] have studied the applications of nano-fluids in solar energy system. Animasaun [13] has studied casson fluid flow of variable viscosity and thermal conductivity along exponentially stretching surface. Abbas et al. [14] have analyzed the MHD boundary layer flow of an UCM fluid through porous channel. Rahman and Eltayeb [15] have made a study on radiative heat transfer in a hydro magnetic flow nano-fluid past a non-linear stretching surface with convective boundary condition. Abel et al. [16] have analyzed MHD flow and heat transfer for the UCM fluid over a stretching sheet. Prasad et al. [17] have examined the influence of internal heat generation/ absorption, thermal radiation, magnetic field, variable fluid property and viscous dissipation on heat transfer characteristics of a Maxwell fluid over a stretching sheet. Mahmoud and Megahed [18] have studied the non-uniform heat generation effect on heat transfer of a non-Newtonian power-law fluid over a non-linearly stretching sheet. Bhattacharyya Krishnendu [19] have discussed in their experiment that the boundary layer flow and heat transfer over an exponentially shrinking sheet. Shateyi et al. [20] have made study on spectral relaxation method for entropy generation on a MHD flow and heat transfer of a Maxwell fluid.

### Mathematical Formulation:

The governing equations of continuity, momentum and energy for the magneto hydro dynamic flow of an incompressible Upper Convected Maxwell fluid over the stretching surface through porous media are presented as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \lambda \left[ u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right] - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) + \mu \left( \frac{\partial u}{\partial y} \right)^2 \quad (3)$$

Here adopted two kinds of heating boundary conditions namely PST & PHF

- (i) **Prescribed Power-Law Surface Temperature (PST):** In this case the respective boundary conditions are as follows.

$$u = Bx; \quad v = 0; \quad T = T_w(x) = T_0 - T_s \left( \frac{x}{L} \right)^2 \quad \text{at} \quad y = 0$$

$$u \rightarrow 0; \quad T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty;$$

- (ii) **Prescribed Power-Law Heat Flux (PHF):** In this case the boundary conditions are

$$u = Bx; \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_w = b \left( \frac{x}{L} \right)^2 \quad \text{at} \quad y = 0$$

$$u \rightarrow 0; \quad T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \quad (4)$$

### Method of Solution

Introducing the following dimensionless similarity variables

$$u = Bx f'(\eta), \quad v = \sqrt{\nu B} f(\eta), \quad \eta = \sqrt{\frac{B}{\nu}} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad g(\eta) = \frac{T - T_\infty}{b \left( \frac{x}{L} \right)^2 \frac{1}{k} \sqrt{\frac{\nu}{b}}} \quad (5)$$

Governing equations (1)-(3) can be transformed exactly into a set of ordinary differential equations as

$$f'''' - Mnf' - (f')^2 + ff'' + \beta(2ff'f'' - f^2 f''') - k_2 f' = 0 \quad (6)$$

$$\theta'' = Pr[2f'\theta - f\theta' - \beta\theta - Ec(f'')^2] \quad \text{in PST case} \quad (7)$$

$$g'' = Pr[2f'g - fg' - \beta g - Ec(f'')^2] \quad \text{in PHF case} \quad (8)$$

And their associated boundary conditions are

$$f = 0; f' = 1; \quad \theta = 1; \quad g' = -1, \quad \text{at} \quad \eta = 0 \quad (9)$$

$$f' = 0; \quad \theta = 0; \quad g = 0, \quad \text{as} \quad \eta \rightarrow \infty \quad (10)$$

Where  $Mn = \frac{\sigma B_0^2}{\rho B}$  is a Magnetic parameter and  $\beta = 2B$  is the elastic parameter,

$k_2 = \frac{\nu}{k}$  is the porous parameter. The non-linear differential equations (6), (7) and (8) of with appropriate boundary conditions given in (9) and (10) are first decomposed into a system of first order differential equations. The resulting initial value problem (IVP) then can be solved numerically by the shooting technique. The convergence criterion largely depends on fairly good guesses of the initial conditions in the shooting technique. Once the convergence is achieved then integrating the resultant ordinary differential equations using standard Runge-Kutta method with the given set of parameters to obtain the required solution.

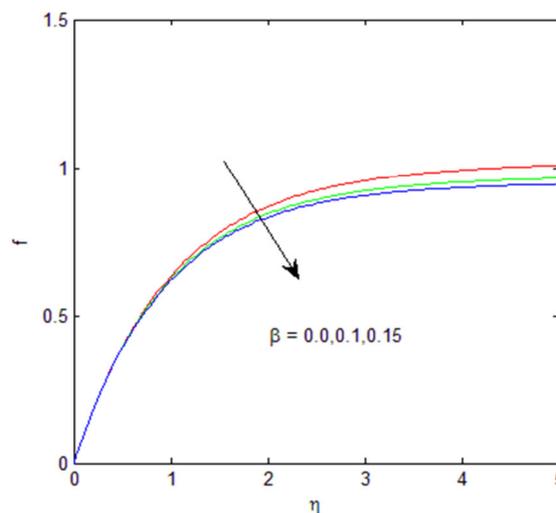


Fig.1, Transverse Velocity Profiles for Different Values of Elastic Parameter  $\beta$  and  $k_2=0.2$

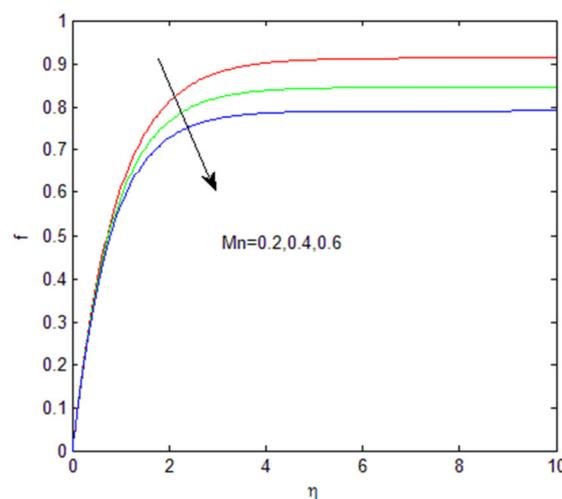


Fig.2, Transverse Velocity Profiles for Fixed Values of  $\beta = 0.05$  and Different Values of  $Mn$

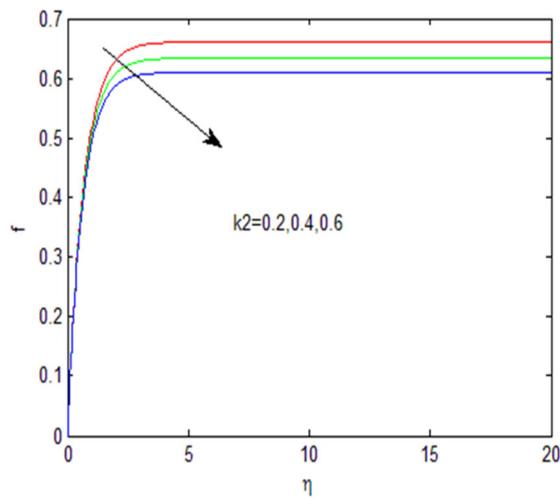


Fig.3, Transverse Velocity Profiles for Fixed Value of  $Mn=0.2$ ,  $\beta = 0.05$  & Different Values of  $k_2$

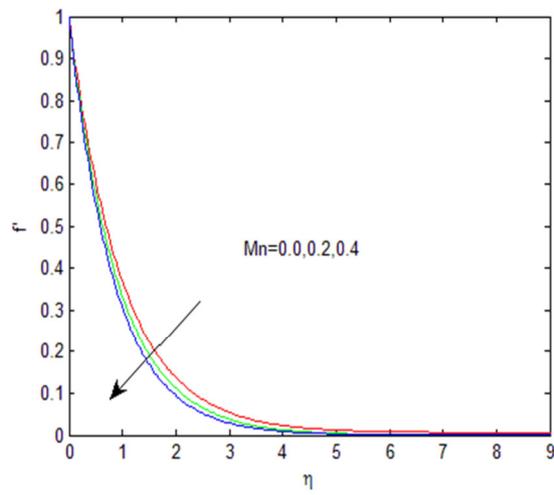


Fig.4 Longitudinal Velocity Profiles for Fixed Value of  $\beta = 0.05$ ,  $k_2=0.2$  & Different Values of  $Mn$

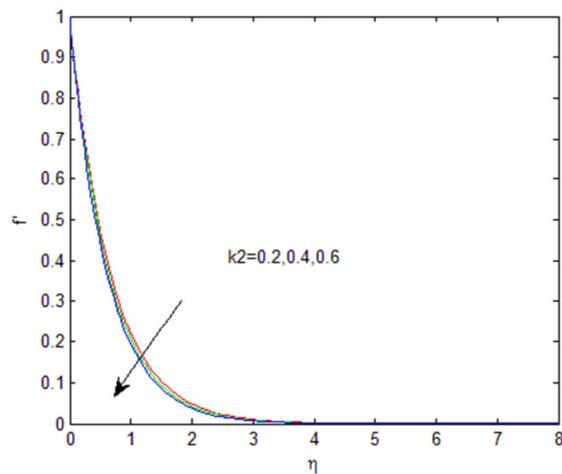


Fig.5, Longitudinal Velocity Profiles for Fixed Values of  $Mn = 0.1$  and Different Values of  $k_2$

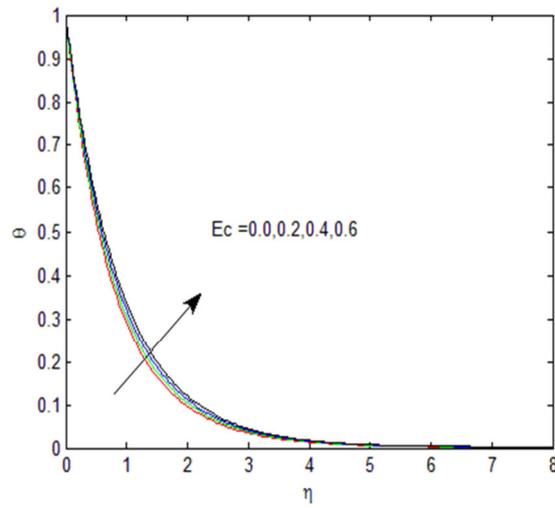


Fig.6. Temperature Profiles for Fixed Value of  $Pr=1$  and Different Values of  $Ec$

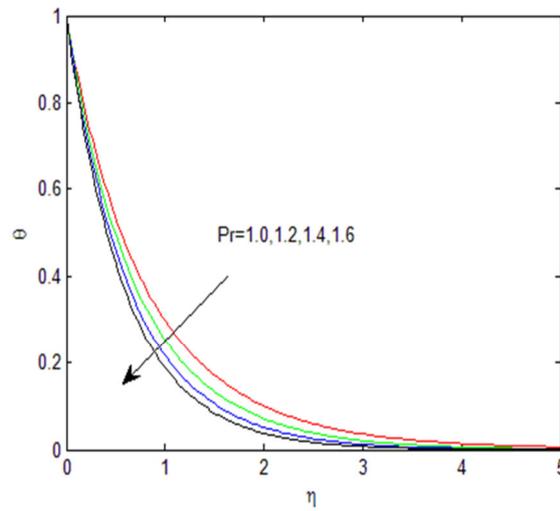


Fig.7, Temperature Profiles for Fixed Value of  $Ec=0.2$  and Different Values of  $Pr$

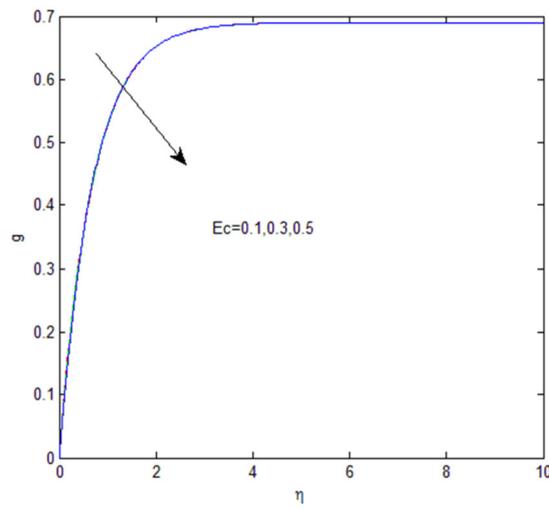


Fig.8. Temperature Profiles for Fixed Value of  $Pr=3$  and for Different Values of  $Ec$

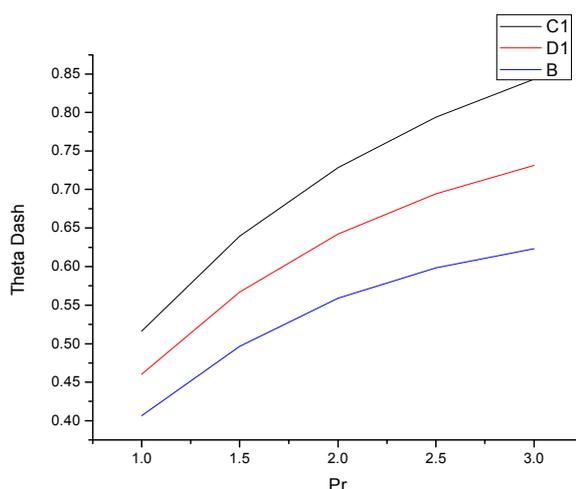


Fig.9, Temperature Gradient in the PST case for  $Ec=1$ ,  $\beta=0.1$ ,  $Mn=1$  and for Different Values of Pr

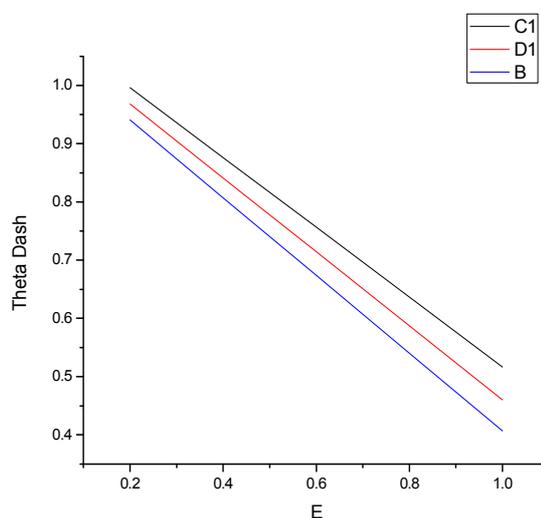


Fig. 10. Distribution of Wall Temp. in PHF Case for  $Mn=1$ ,  $\beta=0.1$ ,  $Pr=1$  and Different Values of Ec

### Results and Discussions

The study of ordinary differential equations (6)- (8) subject to the boundary conditions are solved numerically by Runge-Kutta fourth-fifth order method. Higher order non-linear differential equations (6) – (8) are converted into simultaneous linear differential equations of first order and they are transformed into initial value problem by applying shooting technique. The numerical calculation for the distribution of velocity, temperature and concentration across the boundary layer for different values of parameters are carried out.

The effect of elastic parameter  $\beta$  on the velocity profiles is shown in fig.1. It is observed from the graph that, the velocity decreases with increasing values of  $\beta$ . Effect of magnetic parameter Mn on the velocity profile is shown in fig. 2 with constant values elastic parameter  $\beta$ . It is noticed from the graph that, velocity decreases with increasing values of Mn. Fig.3 shows the variation of velocity for different values of permeability parameter  $k_2$ . The velocity boundary layer thickness decreases with increasing values of  $k_2$ . Longitudinal velocity profile is shown for different values of magnetic parameter Mn in fig.4. Variation of magnetic parameter Mn and other values keeping as constant, thickness of the boundary layer decreases. Fig.5 depicts longitudinal velocity profile for different values of  $k_2$ . As we increase the values of permeability parameter  $k_2$ , velocity profile decreases. Fig.6 illustrates the effect of Eckert number Ec on the temperature field. With increasing the values of Eckert number Ec, the boundary layer thickness increases. The effect of Prandtl number Pr on the temperature profile is shown in fig.7. It is observed from the graph that, thickness of the boundary layer decreases with increasing values of Pr. Temperature profile is shown for different values of Eckert number Ec in fig.8. It is noticed from the graph that, the boundary layer thickness decreases with increasing the values of

Eckert number  $Ec$ . The graph of skin friction for different values of Prandtl number  $Pr$  and Eckert number  $Ec$  are shown in fig.9-fig.10. It is observed from two graphs that the skin friction increases and decreases with increasing the values of  $Pr$  and  $Ec$ .

### Conclusions

In the present study of the flow, the effects of transverse magnetic field within a boundary layer on an upper-convected maxwell (UCM) fluid over a stretching surface through porous media is analysed. The requisite partial differential equations are converted into ordinary differential equations by using similarity transformations. These equations are highly non-linear which cannot be solved analytically. Therefore resulting ordinary differential equations are then solved numerically by using efficient numerical shooting technique with fourth order Runge-Kutta method. The effects of various parameters on velocity, temperature profiles are discussed and presented graphically. The conclusions are as follows:

- The magnetic field parameter has a tendency to reduce the skin friction coefficient.
- An increase in viscous dissipation parameter enhances the thermal boundary layers.
- An increase in prandtl number decreases the temperature profile.

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