Combined Influence of Thermal Radiation and Radiation Absorption on MHD Mixed Convective Heat and Mass Transfer Flow in Circular Annulus

G. Sreedevi[1] G. V. Ramana Reddy[1] D.R.V. Prasada Rao[2]
1.Department of Mathematics, K.L. University, Green Fields, Vaddeswaram, Guntur 522502, A.P, India
2.Department of Mathematics, S.K. University, Anantapur-515003, A.P, India

Abstract
The aim of the problem is to analyze theoretically the combined effects of thermal radiation and radiation absorption on hydromagnetic Brinkman-Forchhimer mixed convection flow of optically dense fluid flow under the influence of magnetic field and chemical reaction on through a vertical circular annulus filled with a saturated porous medium. The outer cylinder is maintained at constant heat flux while the inner cylinder is at a constant temperature. Based on the assumptions, the nonlinear model equations of momentum, energy and species concentration balance are obtained and tacked numerically using an efficient implicit Galerkin finite element method with quadratic polynomial approximation technique. The behavior of velocity, temperature and concentration is analyzed for different parametric values at different axial positions numerically. The local skin friction, local Nusselt number and local Sherwood number are illustrated to show interesting features of the solution.
Keywords: Heat & Mass Transfer; Thermal Radiation; Radiation absorption; Soret and Dufour effects; Concentric Annulus; Chemical reaction.

Nomenclature

C	Concentration of the fluid
C_o	Concentration of the fluid at outer cylinder
C_i	Concentration of the fluid at inner cylinder
C_p	Specific heat coefficient
D_1	Molecular diffusivity
D^{-1}	Inverse Darcy parameter
Du	Dufour parameter
F	Function depends on Reynolds number
g	Acceleration due to gravity
G	Grashof number
H_0	Constant applied magnetic field
k	Permeability of porous medium
k_c	Chemical reaction parameter
M	Hartmann number
N_t	Non dimensional temperature gradient
N_c	Non dimensional Concentration gradient
N_1	Thermal radiation parameter
Q_1	Radiation absorption parameter
q_R	radiative heat flux
Pr	Prandtl number
Sc	Schmidt number
So	Soret parameter

Greek Symbols

α	Heat source parameter
β	Thermal expansion coefficient
$\beta 0$ & $\beta 1$	Coefficient of thermal expansion of the fluid
β^*	coefficient of volume expansion
ρ	density of the fluid friction
Λ	Inertia parameter/Forchheimer number
θ	non-dimensional temperature
σ	electrical conductivity of the fluid
σ^*	Stefan–Boltzmann constant parameter
(x, y, z)	Cartesian coordinates
(u, v)	velocity components along x and y axes
W	axial velocity

Subscripts

i	inner cylinder temperature of the fluid
o	outer cylinder temperature of the fluid

1. Introduction

The Convective flow and heat transfer in porous media has been attracting the attention of large number of investigators due to its wide applications in engineering as geophysical thermal and insulation engineering, design of pebble-bed nuclear reactors, crude oil drilling, ceramic processing, geothermal energy conversion, use of fibrous material in the thermal insulation of buildings, catalytic reactors and compact heat exchangers, heat transfer from storage of agricultural products which generate heat as a result of metabolism, petroleum reservoirs, storage of nuclear wastes, etc. The derivation of the empirical equations which govern the flow and heat transfer in a porous medium has been discussed by Vafai [2000], Ingham and Pop [1998, 2002], Pop and Ingham [2001], and Nield and Bejan [2006]. Further, thermal radiation heat transfer effects on natural convection flow in porous media are very important in the context of space technology and processes involving high temperatures, and very little is known about the effects of radiation on the boundary layer flow of radiating fluid past a body. Recent developments in hypersonic flight, missile reentry, rocket combustion chambers, power plants for interplanetary flight, and gas cooled nuclear reactors have focused attention on thermal radiation as a mode of energy transfer and have emphasized the need for an improved understanding of radiative transfer in these processes. Abdus Sattar and Hamid [1996] studied unsteady Free-Convection Interaction with Thermal Radiation in a Boundary Layer Flow Past a Vertical Porous Plate. Yih[1999] studied the effect of radiation on the heat transfer characteristics in natural convection over an isothermal vertical cylinder embedded in porous medium. Nagaraju et al. [2001] examined the combined radiative and convective heat transfer in a medium with variable porosity. Chamkha et al. [2004] studied the free convection from a vertical cylinder embedded in a porous medium. Makinde [2005] studied Free Convection Flow with Thermal Radiation and Mass Transfer Past a Moving Vertical Plate. EL-Hakiem and Rashad [2007] investigated the effect of temperature-dependent viscosity on the non-Darcy natural convection flow over a vertical cylinder saturated porous medium. Cortell [2007] studied Viscoelastic fluid flow and heat transfer over a stretching sheet under the effects of a non-uniform heat source, viscous dissipation and thermal radiation. EL-Kabeir et al. [2008] have studied the effect of thermal radiation on the combined heat and mass transfer on MHD non-Darcy free convection about a horizontal cylinder embedded in a saturated porous medium. Rashad[2008] examined the effect of thermal radiation on heat and mass transfer by natural convection in fluid over a vertical flat plate embedded in porous medium. Makinde and Ogulu [2008] studied the effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field. Dulal pal [2009] studied heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. Chamkha et al. [2011] studied the effect of radiation on combined heat and mass transfer by non-Darcy natural convection about an impermeable horizontal cylinder embedded in porous medium. Sarkar et al [2015] studied Magnetohydrodynamic Peristaltic Flow of Nanofluids in a Convectively Heated Vertical Asymmetric Channel in Presence of Thermal Radiation.

Natural convection heat transfer within an annulus has been a subject of interest for many decades. Natural convection in a finite space is important for many applications, including the design of electronic equipment cooling systems, nuclear reactor waste transport and storage, solar collectors and thermal storage systems, and thermal management of aviation. Laminar convection in both horizontal and vertical concentric annuli has gained much importance because of its wide spread applications, such as in heat exchangers, cooling systems in electrical devices, solar collectors, the cooling of turbine rotors and high speed gas bearings. Heat transfer in porous medium thermal insulation within vertical cylindrical annuli provide us insight into the mechanism of energy transport and enable engineers to use insulation more efficiently. Rokerya and Iqbal [1971] investigated the effect of viscous dissipation in the thermal energy balance for viscous flow in vertical concentric annuli. Caltagirone [1976] studied thermo convective instabilities in a porous medium bounded by two concentric horizontal cylinders. Free convection in a vertical or inclined annulus has also been investigated by many authors, including Keyhani et al. [1983, 1989]. Al- Nimir [1993] worked on analytical solutions for transient laminar fully developed free convection in vertical annuli. Kuznetsov [1996] conducted an analysis of non-thermal equilibrium fluid flow in concentric tube annulus filled with a porous medium. Solutions for fully developed natural convection in open-ended vertical concentric annuli under a radial magnetic field have been presented by Singh et al. [1997]. Barletta [1999, 1999, 2000] conducted an analytical method of perturbation to solve momentum balance and energy balance equations for investigating flow and heat transfer in vertical ducts. El-Shaarawi et al [1999] have studied the fully developed laminar natural convection in open ended vertical concentric annuli. Effects of viscous dissipation on mixed convection in an inclined channel studied by Barletta and Zanchini [2001] and in a vertical tube are also reported, using the perturbation method studied by Barletta and Rossi [2001]. Habib and Negm [2001] investigated non-uniform circumferential heating in horizontal concentric annuli. Yilbas [2001] analyzed entropy generation in cylindrical annuli due to conduction and viscous dissipation. Chen and Zhang [2002] simulated nonlinear thermal convection in a fluid-filled gap between two corotating and concentric cylindrical annuli. Categorization of the flow regimes, according to the number of eddies, are established on the Ra–Re plane for various numbers presented by Khanafer and Chamkha [2003]. Also, Al-Emin [2003] found that heat transfer and fluid flow over a non-isothermal horizontal cylinder in a porous medium can be controlled using electromagnetic fields. In the article of Mahmud and Fraser [2003], the first and second law (of thermodynamics) characteristics of fluid flow and heat transfer inside a cylindrical annulus have been investigated analytically. The study of magnetic field effects on an electrically conducting fluid has also been presented Kurt et al. [2004]. In the paper by Sankar et al. [2006], it is shown that the flow and heat transfer can be suppressed by imposing an external magnetic field. The effect of viscous dissipation in a porous medium in vertical annulus was investigated by Badruddin et al. [2007] and Barletta et al. [2007]. The study of flow and heat transfer characteristics in the vertical concentric cylinders in porous systems received attention in Kiwan and Alzahrani [2008]. Barletta and Magyari [2008] studied buoyant flow with viscous heating in a vertical circular duct filled with a porous medium. Zanchini [2006] conducted an analytical study of laminar mixed convection with a temperature-dependent viscosity in a vertical annular duct with uniform wall temperatures. Salman et al. [2008] studied Study of mixed convection in an annular vertical cylinder filled with saturated porous medium, using thermal non-equilibrium model. Morocco et al. [2012] conducted Experimental investigation of the turbulent heavy liquid metal heat transfer in the thermal entry region of a vertical annulus with constant heat flux on the inner surface. Malik et al. [2012] extended the experimental study of conjugate heat transfer within a bottom heated vertical concentric cylindrical enclosure. Recently, Forooghi et al. [2015] studied Buoyancy induced heat transfer deterioration in vertical concentric and eccentric annuli.

The heat and mass transfer simultaneously affect each other that create cross-diffusion. The heat transfer caused by concentration gradient is called the diffusion-thermo or Dufour effect. On the other hand, mass transfer caused by temperature gradients is called Soret or thermal diffusion effect. Thus Soret effect is referred to species differentiation developing in an initial homogeneous mixture submitted to a thermal gradient and the Dufour effect referred to the heat flux produced by a concentration gradient. Most of the studies were based on Soret and Dufour effects on free and mixed convection boundary layer flow in a porous medium Kafoussias and Williams [1995], Anghel et al. [2000], Postelnicu [2004] and Alam and Rahman [2006]. Chamkha and Ben-Nakhi [2008] analyzed the MHD mixed convection flow under radiation interaction along a vertical permeable surface immersed in a porous medium in the presence of Soret and Dufour's effects. Chamkha et al. [2011] studied unsteady double-diffusive natural convective MHD flow along a vertical cylinder in the presence of chemical reaction, thermal radiation and Soret and Dufour effects. Mallikarjuna et al. [2014] studied Soret and Dufour effects on double Diffusive convective flow through a Non-Darcy porous medium in a cylindrical annular region in the presence of heat sources.

Motivated by the above-mentioned researchers, the objective of this paper is to study the combined effect of thermal radiation and radiation absorption on mixed convection flow with Soret and Dufour effects through a porous medium in a circular duct where the outer cylinder is maintained at constant heat flux and the inner cylinder is at a constant temperature. The concentration on the cylinders is taken to be uniform. The Brinkman-Forchhimer extended Darcy equations which takes into account the boundary and inertia effects are used in the governing linear momentum equations. The effect of density variation is confined to the buoyancy term under Boussinesq approximation. The momentum, energy and diffusion equations are coupled equations. In order to obtain a better insight into this complex problem, we make use of Galerkin finite element analysis with quadratic polynomial approximations. The Galerkin finite element analysis has two important features. Firstly, the approximation solution is written directly as a linear combination of approximation functions with unknown nodal values as coefficients. Secondly, the approximation polynomials are chosen exclusively from the lower order piecewise polynomials restricted to contiguous elements. The behavior of velocity, temperature and concentration is analyzed at different axial positions. The rate of heat and mass transfer has been obtained for variations in the governing parameters.

2. Problem Formulation

We assumed the mixed convection flow in a vertical circular annulus through a porous medium whose walls are maintained at a constant temperature and concentration. We considered the temperature and concentration fully developed in the entire fluid flow. Both the fluid and porous region have constant physical properties and the flow is a mixed convection flow taking place under thermal and molecular buoyancies and uniform axial pressure gradient. The Boussinesq approximation is invoked so that the density variation is confined to the thermal and molecular buoyancy forces.

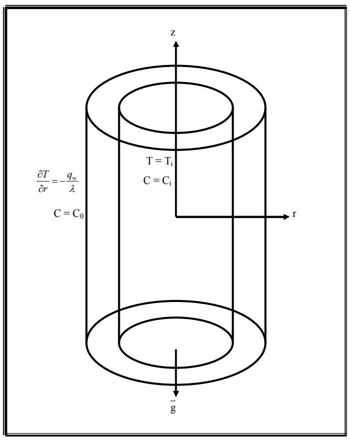

Figure 1. Schematic Diagram of the Problem

The Brinkman-Forchheimer-extended Darcy model which accounts for the inertia and boundary effects has been used for the momentum equation in the porous region. The momentum, energy and diffusion equations are coupled and non-linear. Also the flow is unidirectional along the axial direction of the cylindrical annulus.

Making use of the above assumptions the governing equations are:

$$-\frac{\partial p}{\partial z}+\frac{\mu}{\delta}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-(\frac{\mu}{k})w-(\frac{\sigma\mu_e^2H_o^2}{r^2})w+\frac{\rho\delta F}{\sqrt{k}}w^2+\rho g[\beta_0(T-T_0) + \rho g\beta^\bullet(C-C_0)=0 \tag{2.1}$$

$$\rho c_p w\frac{\partial T}{\partial z}=\lambda\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+Q(T_o-T)+k_{12}\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)+Q_1^{'}(C-C_o)-\frac{\partial}{\partial r}(\frac{1}{r}q_R) \tag{2.2}$$

$$w\frac{\partial C}{\partial z}=D_1\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)-k_c^{'}C+k_{11}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right), \tag{2.3}$$

where w is the axial velocity in the porous region, T, C are the temperature and concentration of the fluid,k is the permeability of porous medium, F is a function that depends on Reynolds number, the microstructure of the porous medium, D_1 is the molecular diffusivity, $\beta_0$andβ_1 are the coefficient of the thermal expansion, $\beta^\bullet$ is the coefficient of volume expansion, C_p is the specific heat, ρ is density, g is gravity and $k_c$is chemical reaction parameter. The relevant boundary conditions are

$$w=0,\ T=T_i,\ C=C_i,\quad \text{at}\quad r=a,$$

$$w=0,\ \frac{\partial T}{\partial r}=-\frac{q_w}{\lambda},\ C=C_0,\quad \text{at}\quad r=a+s. \tag{2.4}$$

Invoking Rosseland approximation Raptis and Perdikis[1999] and using Taylors expansion we find that

$$\frac{\partial}{\partial r}(\frac{1}{r}q_r)\cong\frac{16\sigma^*T^3}{3\beta R}(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}).$$

We now define the following non-dimensional variables

$$z^*=\frac{z}{a},\ r^*=\frac{r}{a},\ w^*=\frac{a}{\gamma}w,\ p^*=\frac{pa\delta}{\rho\gamma^2},\ \theta^*=\frac{\lambda(T-T_0)}{aq_w},$$

$$s^*=\frac{s}{a},\ C^*=\frac{C-C_i}{C_0-C_i},\ A=\frac{\partial T}{\partial z},\ B=\frac{\partial C}{\partial z},.$$

Introducing these non-dimensional variables, the governing equations in the non-dimensional form are (on removing the stars for clarity)

$$\frac{d^2 w}{dr^2}+\frac{1}{r}\frac{dw}{dr}=\pi+\delta(D^{-1}+\frac{M^2}{r^2})u+\delta^2\Lambda w^2-\delta G(\theta+N\,C), \tag{2.5}$$

$$(1+4N_1/3)\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}-\alpha\,\theta=P_rN_tw+Du(\frac{d^2C}{dr^2}+\frac{1}{r}\frac{dC}{dr}), \tag{2.6}$$

$$\frac{d^2C}{dr^2}+\frac{1}{r}\frac{dC}{dr}-k_c\,C=Sc\,N_cw+ScSr(\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}), \tag{2.7}$$

with

$$w=0;\ \theta=0;\ C=0,\quad \text{at}\quad r=1, \tag{2.8}$$

$$w=0;\ \frac{d\theta}{dr}=-1;\ C=1,\quad \text{at}\quad r=1+s, \tag{2.9}$$

where $\Lambda=FD^{-1}$ (Inertia parameter or Forchheimer number), $G=\frac{g\beta q_w a^4}{\nu^2\lambda}$ (Grashof number), $D^{-1}=\frac{a^2}{k}$ (Inverse Darcy parameter), $N_t=\frac{A\lambda}{q_w}$ (Non-dimensional temperature gradient), $N_c=\frac{Ba}{C_1-C_0}$ (Non-dimensional concentration gradient), $P_r=\frac{\rho\upsilon C_p}{\lambda}$ (Prandtl number), $Sc=\frac{\nu}{D_1}$ (Schmidt number), $S_o=\frac{\lambda k_{11}\Delta C}{\nu a q_w}$ (Soret parameter), $Du = \frac{k_{12} a q_w}{\lambda k_f \Delta C}$ (Dufour parameter), $k_c = \frac{k_c^{'} a^2}{D_1}$ (Chemical reaction parameter), $\alpha = \frac{Q a^2}{k_f C_p}$ (Heat Source parameter), $N_1 = \frac{4\sigma^{\bullet} T_e^3}{\beta_R k_f}$ (Radiation parameter), and $\pi = \frac{\partial P}{\partial z}$ (Pressure gradient parameter). Other important parameters of interest in this present study are the local skin friction C_f), local Nusselt number Nu and the local Sherwood number Sh defined by

$$C_f = \left.\frac{dw}{dr}\right|_{r=1,1+s}, Nu = -\left(1+\frac{4N_1}{3}\right)\left.\frac{d\theta}{dr}\right|_{r=1}, Sh = -\left.\frac{dC}{dr}\right|_{r=1,1+s} \tag{2.10}$$

3. Numerical procedure

The Finite Element Method (FEM) is numerical and computer-based technique of solving a variety of practical engineering problems that arise in different fields. It has been applied to a number of physical problems, where the governing differential equations are available. The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution. The steps involved in the finite element analysis are as follows:

- Discretization of the domain into set of finite elements.
- Weighted integral formulation of the differential equation.
- Defining an approximate solution over the element.
- Substitution of the approximate solution and the generation of the element equations.
- Asslembly of the Stiffness matrices for each element.
- Imposition of the boundary conditions.
- Solution of assembled equations.

The entire flow domain is divided into 10000 quadratic elements of equal size. Each element is three-noded and therefore the whole domain contain 20001 nodes.A system of equations has been obtained which is solved by the Gausse limination method.The code of the algorithm has been executed in MATLAB running on a PC. Excellent convergence was achieved for all results.

4. Results and Discussion

In this analysis, we discuss the combined influence of non-linear density relation and thermal radiation effects on convective heat and mass transfer flow of a viscous electrically conducting fluid through a porous medium confined in an annular region between the cylinder $r = a$ and $r = b$ in the presence of heat generating sources. The governing equations of flow, heat and mass transfer are solved by employing Galerkin finite element analysis. Also we consider the chemical reaction effect on flow phenomenon. It should be mentioned that the results obtained herein are compared with the results of the thermal radiation N_1=0 the results obtained herein are compared with Mathew [2009] as shown in Table 1. In the comparison, the results are found to be in good agreement.

Table 1. Comparison of Nu and Sh at r=1 and r=2 with Mathew [2009] with thermal radiation N_1= 0

α	Mathew [2009]			Present results		
	Nu	Sh		Nu	Sh	
	r=1	r=1	r=2	r=1	r=1	r=2
2	-4.796	2.8935	-0.51765	-4.7987	2.8938	-0.51768
4	-8.8858	4.17348	-1.40724	-8.8860	4.17350	-1.40723
6	-12.9413	5.437	-2.2884	-12.9415	5.4372	-2.2885

5.1 Velocity profiles

The axial velocity (w) is shown in figures 2-8 for different values of ʌ, Q_1, S_0, Du, k_c. The actual axial velocity w is in vertically upward direction and hence w<0 represents a reversal flow. Fig. 2 represents the variation of axial velocity w with ʌ. It is found that lesser the Forchheimer number, axial velocity decreases in the flow region. Fig. 3 depicts the axial velocity w with radiation absorption parameter Q_1. It is found that the axial velocity reduces with increase in $Q_1 \le 1.5$ and enhances with higher values of $Q_1 \ge 2.5$. From Fig. 4, we observed the variation of axial velocity w with Soret parameter $S_0$and Dufour parameter Du. It is found that the axial velocity w depreciates with increase in S_0 (or decrease in Du). Fig. 5 is exhibits the variation of axial velocity w with chemical reaction parameter k_c. It is noticed that the axial velocity w enhances in the degenerating chemical reaction case and depreciates in the generating chemical reaction case.

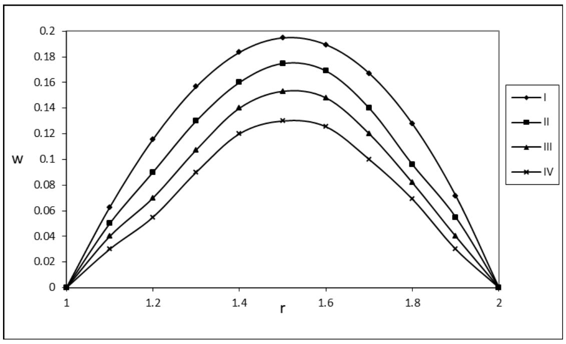

Figure 2. Effect of Forchheimer Number ʌon axial velocity profile w; G=10^3, M=2, Sc=1.3, N=1, K_c=0.5, γ=0.01, So=2, Du=0.03, Q_1=0.5, N_1=0.5, α=2; for different values of ʌ (0.5, 1, 1.5, 2)

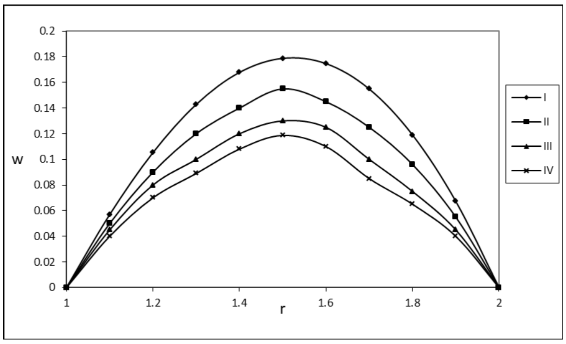

Figure 3. Effect of Radiation Absorption Parameter Q_1 on axial velocity profile w; G=10^3, M=2, Sc=1.3, N=1,K_c=0.5,γ=0.01,So=2,Du=0.03, N_1=0.5, D^{-1}=0.5,α=2; for different values of Q_1 (0.5, 1, 1.5, 2)

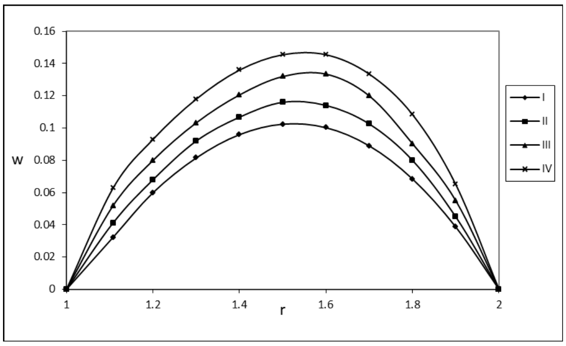

Figure 4. Effect of Soret So, Dufour Du Parameters on axial velocity profile w; G=10^3, M=2, Q_1=0.5, Sc=1.3, K_c=0.5, γ=0.01, N=1, N_1=0.5, D^{-1}=0.5, α=2; for different values of So (0.6, 1, 1.5, 2), Du (1, 0.6, 0.4, 0.3)

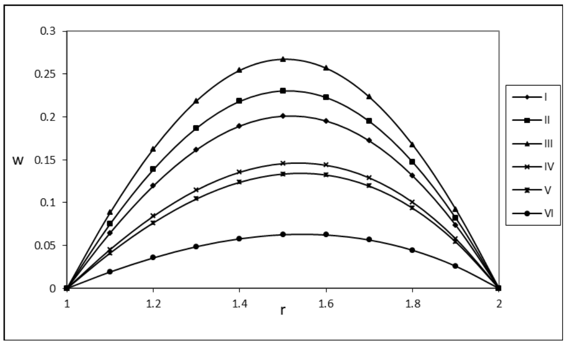

Figure 5. Effect of Chemical Reaction Parameter K_c on axial velocity profile w; G=10^3, M=2, Q_1=0.5, Sc=1.3,So=2, Du=0.03,γ=0.01, N=1, N_1=0.5, D^{-1}=0.5,α=2; for different values of K_c (0.5, 1.5, 2.5, -0.5, -1.5, -2.5)

5.2 Temperature Profiles

The non-dimensional temperature (θ) is shown in Figs. 6-9 for different parametric values of ʌ, Q_1, So, Du, k_c. We follow the convention that the non-dimensional temperature is positive or negative according as the actual temperature (T) is greater/lesser than the ambient temperature. The variation of temperature θ with Forchheimer number ʌ is exhibited in Fig. 6. It is observed from the Figure that lesser the Forchheimer number larger the actual temperature. Higher the radiation heat flux larger the temperature in the flow region. Fig.7 shows the variation of temperature profile θ with radiation absorption parameter Q_1. It is observed from the Figure that higher the radiation absorption larger the actual temperature in the flow region. This is due to the fact that the thermal boundary layer thickness increases with radiation absorption parameter Q_1.

The variation θ with Soret parameter S_0 (or decrease in Dufour parameter Du) is shown in Fig. 8. We observed form the graphical notation that increase in the Soret parameter (or decrease in the Dufour parameter Du) results an enhancement in the actual temperature in the flow region. The variation temperature profile θ with chemical reaction parameter k_c is shown in Fig. 9. It is found that the actual temperature reduces with enhances with generating chemical reaction parameter k_c>0, while it reduces in the generating chemical reaction case. It is due to the fact that the thermal boundary layer thickness decreases with increase of chemical reaction parameter and enhances with decrease in chemical reaction parameter.

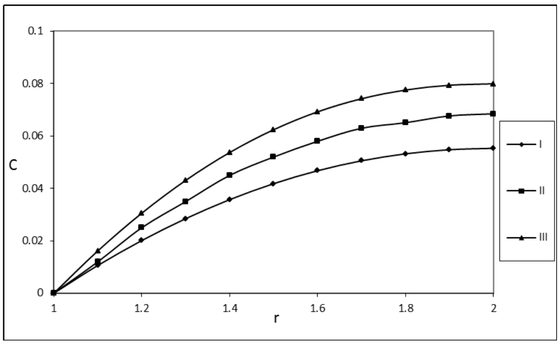

Figure 6. Effect of Inertia Parameter Λ on Temperature profile θ; $G=10^3$, M=2,Sc=1.3, N=1, $K_c=0.5$,γ=0.01, So=2, Du=0.03, $Q_1=0.5$, $N_1=0.5$, α=2; for different values of Λ (0.5, 1, 1.5, 2)

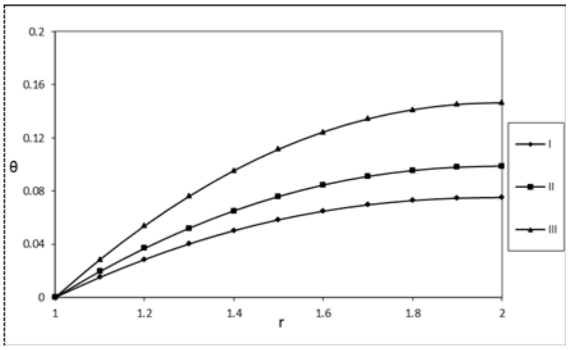

Figure 7. Effect of Radiation Absorption Parameter Q_1 on Temperature profile θ; $G=10^3$, M=2,Sc=1.3, N=1, $K_c=0.5$, γ=0.01, So=2, Du=0.03, $N_1=0.5$, $D^{-1}=0.5$,α=2; for different values of Q_1 (0.5, 1, 1.5, 2)

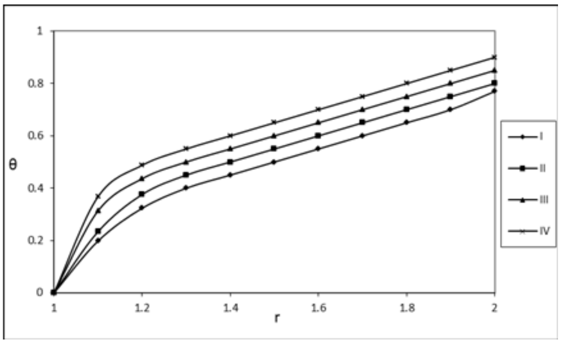

Figure 8. Effect of Soret So, Dufour Du Parameters on Temperature profile θ; $G=10^3$, M=2, $Q_1=0.5$, Sc=1.3, $K_c=0.5$, γ=0.01, N=1, $N_1=0.5$, $D^{-1}=0.5$, α=2; for different values of So (0.6, 1, 1.5, 2), Du (1, 0.6, 0.4, 0.3)

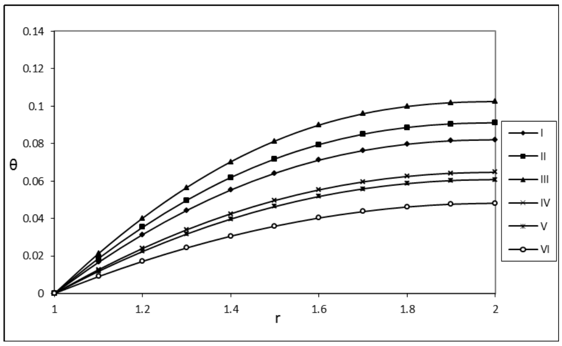

Figure 9 Effect of Chemical Reaction Parameter K_c on Temperature profile θ; $G=10^3$, M=2,$Q_1=0.5$, Sc=1.3, So=2,Du=0.03,γ=0.01, N=1, $N_1=0.5$, $D^{-1}=0.1$,α=2; for different values of K_c (0.5, 1.5, 2.5, -0.5, -1.5, -2.5)

5.3 Concentration Profiles

The non-dimensional concentration profile C is shown in Figs. 10-13 for different parametric values of Λ, Q_1, So, Du, k_c. We follow the convention that the non-dimensional concentration positive / negative according as actual concentration is greater/lesser than the non-dimensional concentration.

The variation of concentration profile C with Forchheimer number $\Lambda$is shown in Fig. 10. From the figure we identified that smaller the Forchheimer number, the temperature in the flow region is reduces. The variation of the concentration profile C with radiation absorption parameter Q_1 is exhibited in Fig. 11. It is found from the graphical notation that the actual concentration enhances with increase in radiation absorption parameter Q_1. Fig.12 represents the variation of concentration profiles C with Soret parameter So (or decrease in Dufour parameter Du). It is observed from the figure that the actual concentration reduces with increase in So (or decrease in Dufour parameter Du) in the entire flow region. It can be seen from the profiles that the increase in the Soret parameter So (or decrease in Dufour parameter Du) results in a depreciation in the concentration in the flow region. The effect of chemical reaction parameter k_c with concentration profiles C is represented in Fig. 13. From the figure we identified that the actual concentration increases with increase in the degenerating chemical reaction case and the concentration reduces in the generating chemical reaction case in the entire flow region.

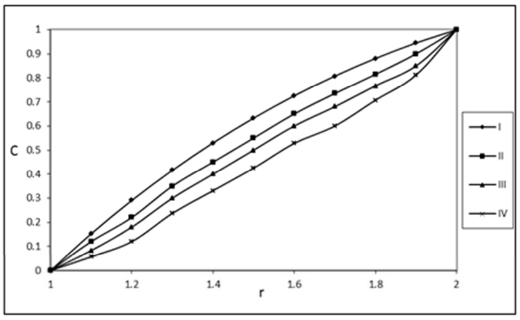

Figure 10. Effect of Forchheimer Number on Concentration profile C; $G=10^3$, M=2, Sc=1.3, N=1, K_c=0.5, γ=0.01, So=2, Du=0.03, Q_1=0.5, N_1=0.5, α=2; for different values of Δ (0.5, 1, 1.5, 2)

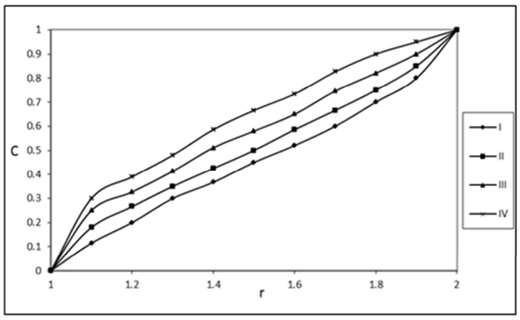

Figure 11. Effect of Radiation Absorption Parameter Q_1 on Concentration profile C; $G=10^3$, M=2, Sc=1.3, N=1, K_c=0.5, γ=0.01, So=2, Du=0.03, N_1=0.5, D^{-1}=0.5, α=2; for different values of Q_1 (0.5, 1, 1.5, 2)

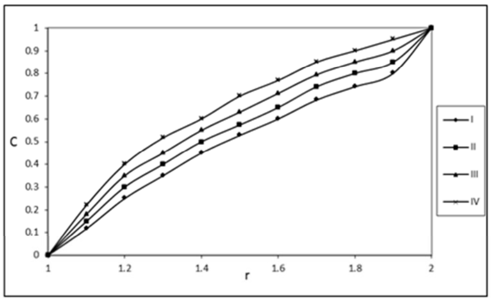

Figure 12. Effect of Soret So, Dufour Du Parameters on Concentration profile C; $G=10^3$, M=2, Q_1=0.5, Sc=1.3, K_c=0.5, γ=0.01, N=1, N_1=0.5, D^{-1}=0.5, α=2; for different values of So (0.6, 1, 1.5, 2), Du (1, 0.6, 0.4, 0.3)

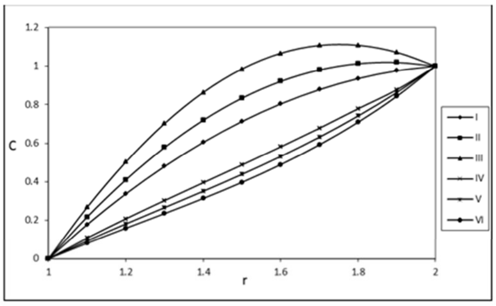

Figure 13. Effect of Chemical Reaction Parameter K_c on Concentration profile C; $G=10^3$, M=2, Q_1=0.5, Sc=1.3, So=2, Du=0.03, γ=0.01, N=1, N_1=0.5, D^{-1}=0.5, α=2; for different values of K_c (0.5, 1.5, 2.5, -0.5, -1.5, -2.5)

The skin friction, the rate of heat and mass transfer are exhibited in Figures (14-17) for different variations of Q_1. The rate of heat transfer enhances on r=1 and r=2 with increase in Hartmann number M. The rate of mass transfer decreases on r=1 and r=2 with increase in Hartmann number in the case of radiation absorption parameter Q_1.

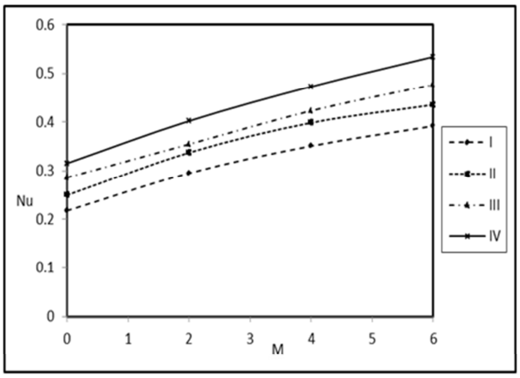

Figure14. Effect of Radiation absorption on the rate of Heat transfer Nu at r=1

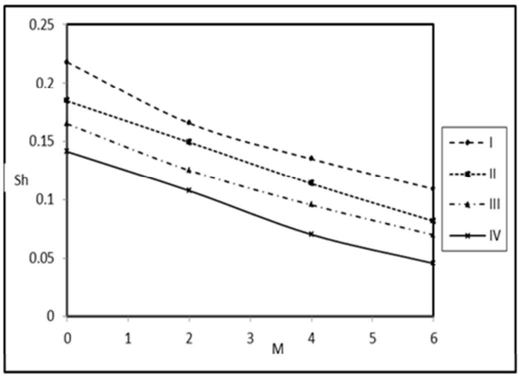

Figure 15. Effect of Radiation absorption on the rate of Mass transfer Sh at r=1

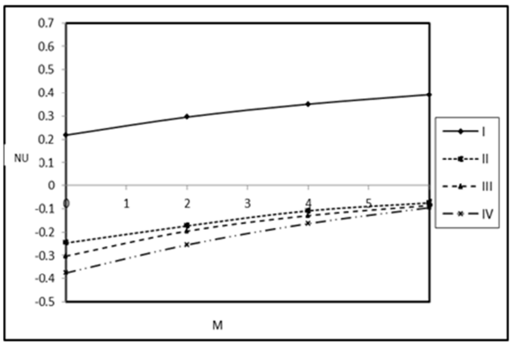

Figure 16. Effect of Radiation absorption on the rate of Heat transfer Nu at r=2

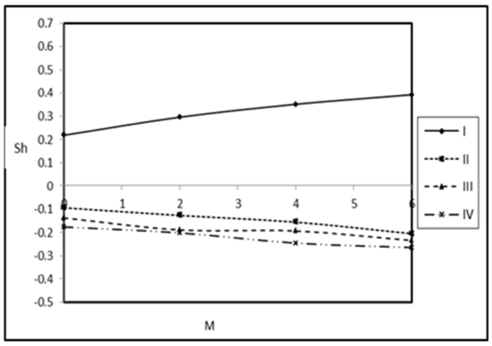

Figure 17. Effect of Radiation absorption on the rate of Mass transfer Sh at r=2

The rate of heat transfer (Nusselt number) at the inner cylinder r = 1 is shown in Table 3 for different parametric values of N_1, So & Du, $K_c$and Λ. It is observed form the Table that the skin friction (τ) enhances with increase in M or N_1. The magnitude of τ enhances on r=1 and r=2 in both the degenerating and generating chemical reaction cases. Increasing Soret parameter So (or decreasing Dufour parameter Du) leads to a depreciation in $|\tau|$ on r=1 and r=2.

The rate of heat transfer enhances on r=1 and reduces r=2 with increase in Hartmann number (M). The rate of heat transfer reduces on r=1 and r=2 in the degenerating chemical reaction case, while in generating chemical reaction case |Nu| enhances at r=1 and reduces at r=2. Higher the radiative heat flux, larger |Nu| on r=1 and smaller on r=2. Increasing Soret parameter So (or decreasing Dufour parameter Du) leads to an increase in |Nu|on r=1 and r=2. An increase in the Forchheimer (Λ) enhances |Nu| on both the cylinders.

The rate of mass transfer (Sherwood number Sh) enhances with increase in M on r=1 and r=2. The rate of mass transfer increases in both degenerating and generating chemical reaction case on r=1 and r=2. Higher the radiative heat flux larger the rate of mass transfer. Increasing Soret parameter So (or decrease in Dufour parameter Du) results in a depreciation in |Sh| on both the cylinders. An increase in Forchheimer (Λ) reduces the rate of mass transfer on both the cylinders. The rate of heat transfer enhances on r=1 and r=2 with increase in Hartmann number.

Table 3. Variation of Nusselt number and Sherwood number on different parametric values

Parameter	Parameter Values	Nu1	Nu2	Sh1	Sh2
(So,Du)	(0.6, 1.0)	13.1026	9.61497	13.0888	14.4877
	(1, 0.06)	13.1055	9.64311	13.1323	14.5265
	(1.5, 0.04)	13.1834	9.65395	13.1857	14.63472
	(2,0.03)	13.2925	9.65969	13.24403	14.96326
Λ	0.1	0.115003	-0.09485	2.06931	0.092955
	0.3	0.315624	-0.17684	1.57606	0.522915
	0.5	0.748408	-0.19061	1.51467	0.579067
k_c	0.5	0.115003	-0.09485	2.06931	0.092955
	1.5	0.640303	-0.17032	1.83508	0.182446
	2.5	0.740992	-0.19082	2.23636	0.27837
	-0.5	0.760992	-0.21082	2.26367	0.28379
	-1.5	0.8263985	-0.23976	2.284597	0.314675
	-2.5	0.887649	-0.25673	2.321575	0.325763
N_1	0.5	1.58887	-0.25836	1.45106	0.634429
	1.5	1.5997	-0.28367	1.47206	0.64529
	5	1.61887	-0.32836	1.51006	0.653429

6. Conclusions

The problem relating to the combined effect of thermal radiation, radiation absorption on mixed convective flow and heat and mass transfer in a circular annulus in the presence of heat generation/absorption has been analyzed. The numerical results were obtained and compared with previously reported cases available in the literature and they were found to be in good agreement. Graphical results for various parametric conditions were presented and discussed for different values. The main findings are summarized below

- Smaller the Forchheimer number Λ larger the axial velocity, temperature, concentration and the rate of heat and mass transfers.

- The enhancement of radiation absorption parameter Q_1, axial velocity reduces for some region and enhances in remaining flow region. For larger the radiation absorption parameter Q_1, the actual temperature and concentration enhances in the fluid region. The rate of heat transfer enhances with increase in radiation absorption parameter. Whereas reversal effect observed in the rate of mass transfer.
- Increasing of Soret parameter So (decrease in the Dufour parameter Du), the axial velocity and concentration depreciates and an enhancement occurs in the actual temperature. The rate of heat and mass transfer enhances when an enhancement in the Soret parameter So (depreciation in the Dufour parameter Du).
- Increase in degenerating chemical reaction parameter kc>0, the axial velocity and concentration enhances whereas temperature reduces. In the case of increase in generating chemical reaction parameter kc<0 reduces the axial velocity, actual temperature and concentration in the entire flow region. The rate of heat and mass transfer enhances in the case of degenerating chemical reaction case and reduces in the case of generating chemical reaction.

References

Vafai, K (Ed.): Handbook of Porous Media, Marcel Dekker, New York, (2000).
Ingham, D. B and Pop, I (Eds.): Transport Phenomena in Porous Media, Pergamon, Oxford, (1998).
Ingham, D. B and Pop, I (Eds.): Transport Phenomena in Porous Media, Pergamon, Oxford, (2002).
Pop, I and Ingham, D. B: Convective Heat Transfer: Mathematical and Computational Modelling of Viscous Fluids and Porous Media, Pergamon, Oxford, (2001)
Nield, D. A. and Bejan, A: Convection in Porous Media, third ed., Springer, New York, (2006).
Abdus-Sattar, M.D and Hamid Kalim, M. D: Unsteady free convection interaction with thermal radiation in a boundary-layer flow past a vertical porous plate, Journal of Mathematical and Physical Sciences, 30, pp. 25–37 (1996).
Yih, K. A: Radiation effect on natural convection over a vertical cylinder embedded in porous media, Int. Commun. Heat Mass Transfer, vol. 26, pp. 259–267 (1999).
Nagaraju, P, Chamkha, A. J, Takhar, H. S and Chandrasekhara, B. C: Simultaneous radiative and convective heat transfer in a variable porosity medium, Heat Mass Transfer, vol. 37, pp. 243–250 (2001).
Chamkha, A. J, Jaradat, M and Pop, I: Thermophoresis free convection from a vertical cylinder embedded in a porous medium, Int. J. Appl. Mech. Eng., vol. 9, pp. 471–481 (2004).
Makinde, O. D: Free Convection Flow with Thermal Radiation and Mass Transfer Past a Moving Vertical Plate. International Communications in Heat and Mass Transfer;32(10):1411-9 (2005).
EL-Hakiem, M. A and Rashad, A. M: Effect of radiation on non-Darcy free convection from a vertical cylinder embedded in a fluid-saturated porous medium with a temperature dependent viscosity, J. Porous Media, vol. 10, no. 2, pp. 209–218 (2007).
Cortell, R: Viscoelastic fluid flow and heat transfer over a stretching sheet under the effects of a non-uniform heat source, viscous dissipation and thermal radiation. Int. J. Heat Mass Transfer;50(15–16):3152-62 (2007).
EL-Kabeir, S. M. M, EL-Hakiem, M. A and Rashad, A. M: Group method analysis of combined heat and mass transfer by MHD non-Darcy non-Newtonian natural convection adjacent to horizontal cylinder in a saturated porous medium, Appl. Math. Model., vol. 32, pp. 2378–2395 (2008).
Rashad, A. M: Influence of radiation on MHD free convection from a vertical flat plate embedded in porous media with ther-mophoretic deposition of particles, Commun. Nonlinear Sci. Numer. Simul., vol. 13, pp. 2213–2222 (2008).
Makinde, O. D and Ogulu, A: The effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field, Chemical Engineering Communications, Volume 195, Issue 12 (2008).
Pal, D: Heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. Meccanica; 44: pp. 145-58 (2009).
Chamkha, A. J., EL-Kabeir, S. M. M and Rashad, A. M: Heat and mass transfer by non-Darcy free convection from a vertical cylinder embedded in porous media with a temperature dependent viscosity, Int. J. Numer. Methods Heat Fluid Flow, vol. 21, no. 7, pp. 847–863 (2011).
Sarkar, B. C, Das, S, Jana, R. N and Makinde, O. D: Magnetohydrodynamic Peristaltic Flow of Nanofluids in a Convectively Heated Vertical Asymmetric Channel in Presence of Thermal Radiation; Journal of Nanofluids, pp. 461-473 (2015).
Rokerya, M.S and Iqbal, M: Effects of viscous dissipation on combined free and forced convection through vertical concentric annuli, International Journal of Heat and Mass Transfer, 14, pp. 491–495 (1971).
Caltagirone, J.P: Thermo convective instabilities in a porous medium bounded by two concentric horizontal cylinders, J. fluid Mech, v. 76, p. 337-362 (1976).
Keyhani, M, Kulacki, F. A and Christensen, R. N: Free Convection in a Vertical Annulus with Constant Heat

Flux on the Inner Wall, Trans. ASME, vol. 105, pp. 454–459 (1983).

Hamad, F. A. W: Experimental Study of Natural Convection Heat Transfer in Inclined Cylindrical Annulus, Solar & Wind Technol., vol. 6, pp. 573–579 (1989).

Al-Nimir, M. A: Analytical solutions for transient laminar fully developed free convection in vertical annuli, Int. J. Heat and mass transfer, v. 36, pp. 2388–2395 (1993).

Kuznetsov, A. V: Analysis of non-thermal equilibrium fluid flow in concentric tube annulus filled with a porous medium, Int. Commun. Heat Mass Transfer, vol. 23, pp. 929–938 (1996).

Singh, S. K, Jha, B. K. and Singh, A. K: Natural convection in vertical concentric annuli under a radial magnetic field, Heat and Mass Transfer, 32, pp. 399–401 (1997).

Barletta, A: Combined forced and free convection with viscous dissipation in a vertical circular duct, International Journal of Heat and Mass Transfer, 42, pp. 2243–2253 (1999).

Barletta, A: Laminar convection in a vertical channel with viscous dissipation and buoyancy effects, International Communications in Heat and Mass Transfer, 26, pp. 153–164 (1999).

Barletta, A: Combined forced and free flow of a power-law fluid in a vertical annular duct, International Journal of Heat and Mass Transfer, 43, pp. 3673–3686 (2000).

El-Shaarawi, M.A. I and AL-Nimir, M. A: Fully developed laminar natural convection in open ended vertical concentric annuli, Int. J. Heat and mass transfer pp. 1873–1884 (1999).

Barletta, A and Zanchini E: Mixed convection with viscous dissipation in an inclined channel with prescribed wall temperatures, International Journal of Heat and Mass Transfer, 44, pp. 4267–4275 (2001).

Barletta, A and Rossi, E: Effect of viscous dissipation on mixed convection heat transfer in a vertical tube with uniform wall heat flux, Heat and Mass Transfer, 38, pp. 129–140 (2001).

Habib, M.A and Negm, A.A.A: Laminar mixed convection in horizontal concentric annuli with non- uniform circumferential heating, Heat and Mass Transfer, 37, pp. 427–435 (2001).

Yilbas, B.S: Entropy analysis of concentric annuli with rotating outer cylinder, Entropy an International Journal, 1, pp. 60–66 (2001).

Chen, C.X and Zhang, K: Nonlinear convection in a rotating annulus with a finite gap, Geophysical & Astrophysical Fluid Dynamics, 96, pp. 499–518 (2002).

Khanafer, K. H and Camkha, A. J: Mixed convection within a porous heat generating horizontal annulus, International Journal of Heat and Mass Transfer, 46, pp. 1725–1735 (2003).

El-Amin, M. F: Combined effect of viscous dissipation and joule heating on MHD forced convection over a non-isothermal horizontal cylinder embedded in a fluid saturated porous medium, Journal of Magnetism and Magnetic Materials, 263, pp. 337–343 (2003).

Mahmud, S and Fraser, R.A: Analysis of entropy generation inside concentric cylindrical annuli with relative rotation, International Journal of Thermal Sciences, 42, pp. 513–521 (2003).

Kurt, E, Busse, F.H and Pesch, W: Hydromagnetic convection in a rotating annulus with an azimuthal magnetic field, Theoretical and Computational Fluid Dynamics, 18, pp. 251–263 (2004).

Sankar, M, Venkatachalappa, M and Shivakumara, I.S: Effect of magnetic field on natural convection in a vertical cylindrical annulus, International Journal of Engineering Science, 44, pp. 1556–1570 (2006).

Badruddin, I.A, Zainal, Z.A, Zahid, A, Khan, Z and Mallick, E: Effect of viscous dissipation and radiation on natural convection in a porous medium embedded within vertical annulus'', International Journal of Thermal Sciences, 46, pp. 221–227 (2007).

Barletta, A, Magyari, E, Pop, I and Storesletten, L: Mixed convection with viscous dissipation in a vertical channel filled with a porous medium, Acta Mechanica, 194, pp. 123–140 (2007).

Kiwan, S and Alzahrani, M.S: Effect of using porous inserts on natural convection heat transfer between two concentric vertical cylinders, Numerical Heat Transfer, 53, pp. 870–889 (2008).

Barletta, A, Magyari, E, Pop, I and Storesletten, L: Buoyant flow with viscous heating in a vertical circular duct filled with a porous medium, Transport in Porous Media, 74, pp. 133–151 (2008).

Zanchini, E: Mixed convection with variable viscosity in a vertical annulus with uniform wall temperatures, International Journal of Heat and Mass Transfer, 5, pp. 30–40 (2006).

Venkata Reddy, P, and Narasimham, G.S.V.L: Natural convection in a vertical annulus driven by acentral heat generating rod, International Journal of Heat and Mass Transfer, Volume 51, Issues 19–20, September, pp. 5024-5032 (2008).

Salman Ahmed, N.J, Anjum Badruddin, I, JeevanKanesan, Z.A, Zainal, and NazimAhamed, K.S: Study of mixed convection in an annular vertical cylinder filled with saturated porous medium, using thermal non-equilibrium model, International Journal of Heat and Mass Transfer, Volume 54, 17–18, pp. 3822-3825 (2011).

Luca Marocco, André Loges, Thomas Wetzel and Robert Stieglitz: Experimental investigation of the turbulent heavy liquid metal heat transfer in the thermal entry region of a vertical annulus with constant heat flux on the inner surface; International Journal of Heat and Mass Transfer, Volume 55, 23–24, pp. 6435-6445

(2012).

Asif Hussain Malik, Alvi, M.S.I, Shahab Khushnood, Mahfouz, F.M, Ghauri, M.K.K and Ajmal Shah, Experimental study of conjugate heat transfer within a bottom heated vertical concentric cylindrical enclosure, International Journal of Heat and Mass Transfer, Volume 55(4), 31, pp. 1154-1163 (2012).

Pourya Forooghi, Iman AshtianiAbdi, Mahidzal Dahari and Kamel Hooman: Buoyancy induced heat transfer deterioration in vertical concentric and eccentric annuli, International Journal of Heat and Mass Transfer, Volume 81, pp. 222-233 (2015).

Kafoussias, N. G and Williams, E. M: Thermal-diffusion and Diffusion-thermo effects on free convective and mass transfer boundary layer flow with temperature dependent viscosity, Int. J. Eng. Science 33, pp. 1369-1376 (1995).

Anghel, M, Takhar, H. S and Pop, I: Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium, J. Heat and Mass Transfer, 43, pp.1265- 1274 (2000).

Postelnicu, A: Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Int. J. Heat Mass Transfer, 47, pp. 1467-1472 (2004).

Alam, M. S and Rahman, M. M: Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction, 11, pp. 3-12 (2006).

Chamkha, A.J and Ben-Nakhi, A: MHD Mixed Convection-Radiation Interaction along A Permeable Surface Immersed in A Porous Medium in The Presence of Soret and Dufour’s Effect, Heat Mass Transfer, vol. 44, pp. 845–856 (2008).

Chamkha, A. J, El-Amin M. F and Aly, A. M: Unsteady double-diffusive natural convective MHD flow along a vertical cylinder in the presence of chemical reaction, thermal radiation and Soret and Dufour effects, Journal of Naval Architecture and Marine Engineering 8, pp. 25-36 (2011).

Mallikarjuna, B, Chamkha, A. J and Bhuvana Vijaya, R: Soret and dufour effects on double diffusive convective flowthrough a non-darcy porous medium in a cylindrical annular region in the presence of heat sources, Journal of Porous Meedia, Begell House Publications, Volume 17 (7), (2014)

Raptis, A and Perdikis, C: Radiation and free convection flow past a moving plate, Int. J. Applied Mech. and Engg, 4(4), pp. 817-821 (1999).

Sulochana C and Tayappa H: Non-Darcy Convective Heat and Mass Transfer Flow in a Circular Annulus with Soret, Dufour Effects and Constant Heat Flux, International Journal of Scientific and Innovative Mathematical Research, Volume 2(8), 684-697 (2014).

Mathew, S: Hydro magnetic mixed convective heat and mass transfer through a porous medium in a vertical channel with thermo-diffusion effect, Ph.D thesis, S.K. University, (2009).